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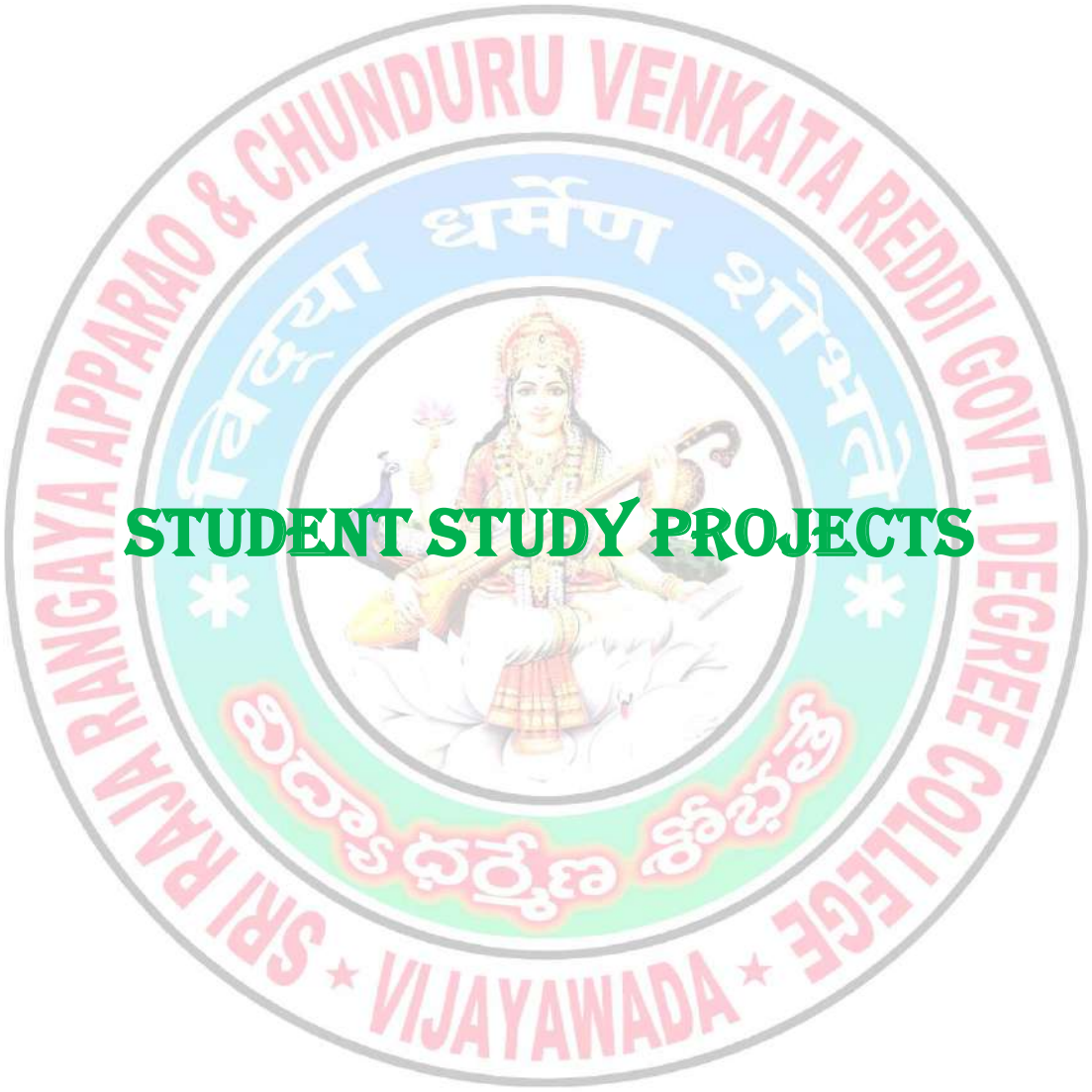

STUDENT STUDY PROJECTS

SRR & CVR Govt. Degree College (A), Vijayawada

DEPARTMENT OF FOOD TECHNOLOGY

Final Report of Community Service Project

"Study on NUTRITIONAL STATUS OF ADOLESCENT GIRL'S Machavaram down vijayawada,NTR dt, Andhra Pradesh"

SUBMITTED

BY

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UNDER SUPERVISION

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SRR & CVR Govt. Degree College (A), Vijayawada

DECLARATION

I KILARI.NAGA SRIYA A Student of I B.Sc (MBF), Studying in SRR & CVR Govt. Degree College (A), Vijayawada with Register No: 21315203 hereby declare that the Community Service Project report on NUTRITIONAL STATUS OF ADOLESCENT GIRL'S Machavaram down,NTR dt, Andhra Pradesh" has been carried out and submitted by me. The work contained in this report is original and I declare that it has not been submitted.

Vijayawada

Dt. 22-9-2022

K. Naga Sriya

(KILARI.NAGA SRIYA)

SIGNATURE OF THE MENTOR

SRR & CVR Govt. Degree College (A), Vijayawada

CERTIFICATE

This is to Certify that **Community Service Project** entitled" **"Study on NUTRITIONAL STATUS OF ADOLESCENT GIRL'S Machavaram down,NTR dt, and Andhra Pradesh"** to **SRR & CVR Govt. Degree College (A),Vijayawada** in partial fulfillment to requirements for the **DEGREE OF BACHELOR OF SCIENCE** is done BY **KILARI. NAGA SRIYA**B. Sc (MBF), with Register No: 21315203 during the academic year 2021-22 from **SRR & CVR Govt. Degree College (A), Vijayawada, NTR District, Andhra Pradesh**.

SIGNATURE OF THE MENTOR

(Dr. Anu Radha)

SIGNATURE OF THE CSP COORDINATOR

(Mrs. A Rosalind)

SIGNATURE OF THE PRINCIPAL

(DR. KODALI BHAGYA LAKSHMI)

II

Preface

The present study focus on the nutritional status of adolescent girls.By this community service project we come to learn about the metabolic activities. growth. development and other major aspects of adolescent girl child .By this project we are focusing on the diet that should be followed by adolescent girls.we had known the under diet ,over diet and normal diet minimum requirement of the diet by the candidates and their health conditions and their daily active participation in everyday life

We also noted that the diet followed by the candidates are playing major roles in their daily life status and active participation in everyday activities.Diet play a major role in human activity.The child with well nourished vary with the child with malnourished.our theme of project help in such a way to guide the adolescent girls about the amount of diet they should maintain and the nutrients required by their body particularly at this particular period.As the fashion is becoming passion now a days many adolescent girls are running behind so called zero size etc.. which are non sence things playing vital role these days such activities must be avoided. By this project we are providing awareness about the importance of nutrition and how it is important at this particular growth period. This helps to improve the knowledge on nutrition and the diet must be followed at this growing age

Hence the present investigation was taken up to address about the awareness about the nutritional status must be maintained by the adolescent girls.

K. Naga Sriya

(KILARI .NAGA SRIYA)

ACKNOWLEDGEMENT

I am indebted to **Department of Food Technology, SRR & CVR Govt. Degree College (A), Vijayawada.** Where I am studying I B.Sc (MBF) encouraging me to take up Community Service Project in the field of Nutritional status of adolescent girl's.

It is pleasure to thank the many people who extended their valuable co-operation towards to make this Community Service Project Report Possible.

I expressed my deep sense of gratitude to **Dr Anu Radha**, Mentor, Lecturer in Food Technology, **Department of Food Technology, SRR & CVR Govt. Degree College (A), Vijayawada, NTR District, Andhra Pradesh** for her constant encouragement, support, motivation and assistance for successful completion of CSP has come to shape in time.

I express my sincere thanks to **Mrs. A. Rosalind, Community Service Project Coordinator** for providing guidance and support.

I deem it a great privilege and pleasure to express my deep sense of gratitude to **Dr. Kodali Bhagya Lakshmi, Principal,SRR & CVR Govt. Degree College (A), Vijayawada, NTR District, Andhra Pradesh** for her valuable suggestions and encouragement.

I Express my thank fullness to my classmates and friends.

K. Naga Sriya
(KILARI.NAGA SRIYA)

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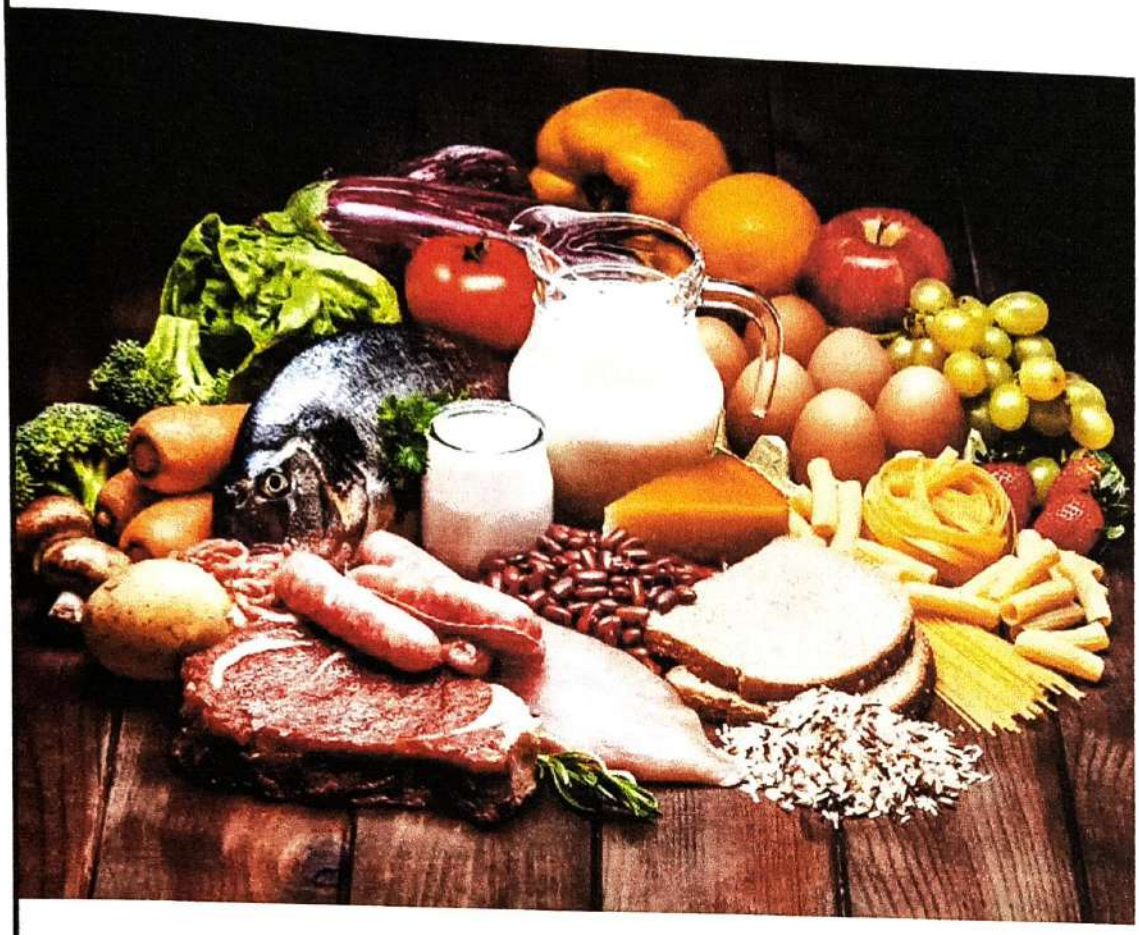

Introduction

Introduction

Adolescence is the transition period between childhood and adulthood where rapid physical, mental, emotional and social development takes place. Adolescent girls are at greatest risk for nutrient deficiency and it badly affects their overall development. With this background this study was conducted to study some aspects of the socio-demographic profile and to assess the nutritional status of adolescent girls.

This is about the same period puberty sets in, typically between the ages of 10 and 13 years in girls. Adolescence is characterised by the growth spurt, a period in which growth is very fast. During this time, physical changes affect the body's nutritional needs, while changes in one's lifestyle may affect eating habits and food choices. Adolescent nutrition is therefore important for supporting the physical growth of the body and for preventing future health problems. All parents should therefore pay particular attention to the nutritional needs of their teenagers.Any nutritional deficiency experienced during this critical period of life can have an effect on the future health of the individual and their offspring. For example, failure to consume an adequate diet at this time can result in delayed sexual maturation and delayed or retarded physical growth. The rapid physical changes of adolescence have a direct influence on a person's nutritional needs. The growth spurt that occurs in adolescence, second only to that in the first year of life, creates increased demands for energy and nutrients. Nutritional status and physical growth are dependent on one another such that optimal nutrition is a requisite for achieving full growth potential.

Any deficiency or inadequate dietary pattern can lead to poor nutrition which can further influence both growth and development throughout from infancy to adolescence. Since adolescents represent the next

generation of parents, it is important to monitor their nutritional status at this crucial stage. Thus, this study aimed to explore the factors associated with nutritional status among adolescent girls belonging to these tea gardens.

Objectives:

1. To find out the prevalence of underweight, over weight and stunning weight in correlation with dietery pattern, socio-demographic profile and prevalance of clinical anemia.
2. To ensure that adolescent girls receive enough food based on basic five food groups.
3. The Opportunities and challenges that Adolescence presents in the life cycle from a nutrition stand point.

Review of literature

Review of Literature

Smitha Malenahalli Chandrashekarappa et al., (2021) conducted a cross-sectional study among adolescent girls aged 16–19 years. Data regarding sociodemographic variables were collected by administration of a prestructured, pretested questionnaire. Height and weight were measured by standardized techniques in a sample of 700 adolescents. BMI was calculated. Results found that 36.2% of adolescent girls were malnourished, among whom 33.7% were obese and 66.3% were undernourished. Age and education of the mother and father were found to be significantly associated with malnutrition.

Gebrehiwot Hadush et al., (2021) conducted a cross-sectional study design among 736 adolescent girls from February15 to March 05, 2017 in Afar, Northeastern Ethiopia. Multi-stage sampling technique was used to select study participants. A pretested and structured interviewer-administered questionnaire and anthropometric measurements was used to collect the data. Body Mass Index for age (thinness) and height for age (stunting) was used to assess undernutrition of adolescent girls by using the new 2007 WHO Growth Reference. Data were analyzed using bivariate and multivariable logistic regression. The degree of association between dependent and independent variables were assessed using odds ratio with 95% confidence interval, and variables with p value < 0.05 were considered significant. The study revealed that the prevalence of thinness and stunting were 15.8% (95% CI 13.3–18.5%) and 26.6% (95% CI 23.5–29.9%), respectively. Being at an early adolescent age (AOR = 2.89, 95% CI 1.23–6.81) for thinness and being at an early adolescent age (AOR = 1.96, 95% CI 1.02–3.74), household food insecure (AOR = 2.88, 95% CI 1.15– 7.21), menstruation status (AOR = 2.42, 95% CI 1.03–5.71), and availability of home latrine (AOR = 3.26, 95% CI 1.15–4.42) for stunting were the independent predictors among the adolescent girls. The prevalence of thinness and stunting is above the public health importance threshold level. Thus, Multi-sector-centered nutrition interventions to improve nutritional status of disadvantaged adolescent girls through providing comprehensive nutritional assessment and counseling services at community, school, and health facilitiand creating household's income-generating activities are recommended before they reach conception to break the intergenerational cycle effect of malnutrition

Sharja Phuljhele et al (2021) studied whether the prevalence of malnutrition among adolescent girls is high in India. Malnutrition has long term effects on the physical and mental health of adolescents. The main objective is to find out the prevalence of underweight, overweight and stunting in correlation with dietary pattern, socio-demographic profile and prevalence of clinical anemia and signs of vitamin and mineral deficiency. This is a community-based cross-sectional observational study conducted for 1 year (Jan 2019 – Dec 2019), including a total of 480 adolescent girls who were attending government high school. 480 girls were interviewed. As per WHO nutritional measurement criteria, adolescent girls were classified as per their anthropometric criteria, 50.63% of girls were moderately stunted and 7.29% had severe stunting. 7.29% of girls were in the category of severe thinness, 36.04% had thinness and 9.37% were overweight. 28.12% had conjunctival pallor (anaemia), 4.80% had dental caries, 1.46% had vitamin B complex deficiency and 1.04% had signs of vitamin A deficiency. More than half of the (57.92%) late adolescent school girls were stunted, 43.33% were having some form of thinness and 28.12% had clinical anaemia. Effective implementation of government programs such as the RKSK and Sabla Yojana program will be a paradigm shift from the existing clinic-based services to promotion and prevention and reaching adolescents in their environment, such as in schools, families and communities. Skills-based nutrition education for the family and effective infection control and routine health assessment of school-going girls should be done.

Pompy Konwar et al., (2019) conducted community-based cross-sectional study to assess the nutritional status of adolescent girls belonging to the tea garden community and the association of the sociodemographic factors with it. Anthropometric measurement was taken among adolescent girls in the tea estates of Nazira subdivision of Sivasagar district, Assam. The pattern of dietary intake among adolescents was also studied. The prevalence of thinness and stunting across 265 adolescent girls was 49.4% and 50.6%, respectively. Calorie and

protein deficits were found to be 76.60% and 65%, respectively. Majority of the respondents, i.e., 66.80% of the participants, had a poor intake of essential food constituents. Moreover, 76.21% of the respondents were anemic. The association of different sociodemographic factors with thinness, inadequate protein intake, and anemia were found during the study. Thinness and stunting along with protein–energy malnutrition and inadequate intake of important food groups were prevalent in adolescent tea community girls. Overall, the public health burden of malnutrition is still a persisting health problem in the tea gardens of Assam.

Sheloj Joshi et al., (2014) studied to determine the nutritional status of adolescent girls in rural area of Bhopal district and to study the socio-demographic factors affecting the nutritional status of adolescent girls in rural area of Bhopal district. Material & method: A community based cross sectional study was carried out amongst adolescent girls in the age group of 10-19 years. Two Government schools of Ratua village of Bhopal district were selected for study purpose. Study period was from June to August2013.Total 200 adolescent girls were included. Variables included were age, socioeconomical status, weight, Height and dietary intake pattern. Statistical analysis: Data was analysed using SPSS version 20 statistical software and Odds ratio and Chi square test was applied to test the strength of association between the variables. Result:

Amongst all adolescent girls 69% suffered from under nutrition. Only 31% girls had normal nutritional status. Majority i.e. 87.20% of adolescent girls belonging to the age group of 10-14 years were undernourished Conclusion: Majority of rural adolescent girls were under nourished.

The World Health Organization (WHO) defines adolescence as 10–19 years .It is a key decade in the life course with implications on adult health, socio-economic well-being of a country and even the health of the future children. Adolescents comprise 16% of the total world population .Asia has more than half of the world psychological's adolescents while according to the Census 2011, 20% of India's population are adolescents .

Adolescence is a period of rapid growth and development, second only to infancy, with dramatic biological, psychological changes often shaped by socio-cultural factors. It is usually divided into two phases: early adolescence (10–14 years) and late adolescence (15–19 years) [5]. Physiologically, the early years are dominated by pubertal changes and the later stages by sexual maturation and development of adult roles and responsibilities. The nutritional issues in this age group have commonalities with children and adults with some added dimensions of puberty, psychological changes, and growth spurt which are crucial for current, future and intergeneration health [6]. Poor nutrition, on the one hand, can lead to delay or failure in achieving maturation with a stunted linear growth perpetuating the cycle of poverty and intergenerational undernutrition. On the other hand, there is an increased risk of non-communicable diseases (NCDs) [7]. The nutritional transition that is occurring in some low-middle income countries is resulting in a double burden of overweight and obesity in some population groups, along with the existing high proportion of undernutrition in others .

The National Family Health Surveys (NFHS)are periodic health and demographic surveys that includeincludee adult men and women in the age group of 15–54 years and 15–49 respectively and the adolescents (15–19 years) are included in them [11,12].The NFHS reports have so far used the adult cutoffs for nutritional assessment for this age group; i.e., those with a BMI <18.5 kg/m2 are considered thin, those between 25–29.9 kg/m2 are overweight, and BMI >30kg/m2 are considered obese [11–13]. The proportions thus classified as thin were 52.1% and 46.8% in NFHS-3,and 45% and 42% in NFHS-4 in boys and girls respectively [11,12]. However, their nutritional assessment should be done using the WHO 2007 Growth reference curves [14].

They re-analyze the nutritional status of adolescents in the age group of 15–19 years(referred to as adolescents hereafter) included in NFHS-3 and 4 using the age and sex-specific WHO 2007 Growth reference to obtain revised estimates of thinness, overweight and stunting in boys and girls and compare them with currently reported estimates. They also compare these estimates across urban and rural populations, wealth quintiles and explore sub-national heterogeneity and inequalities based on socio-economic class.

Methods : Data source—National Family Health Survey

The NFHS is conducted at regular intervals in India since 1992–93 under the stewardship of the Ministry of Health and Family Welfare and coordinated by the International Institute of Population Sciences (IIPS), Mumbai. The NFHS-3 was conducted in 2005–6 and NFHS-4 in 2015–16. Data of both these surveys were extracted in Microsoft Excel from the Demographic and Health Surveys (DHS) database [15].

Study population and sample size

The NFHS-3 covered a representative sample of 109,041 households from the Census of 2001and included adult men and women in the age group of 15–54 years and15-49 years respectively. The number of boys and girls in the age group of 15–19 years was 26,086 and 26,238 respectively, making upto a total of 52,324 participants.

TheNFHS-4 covered a representative sample of 601,509 households from the Census of 2011 with adult men and women in the same age categories as NFHS-3. A total of 277,059 adolescents in the age group of 15–19 years: 142,162 boys and 134,897 girls were included from these households.

Anthropometry

Trained field staff visited the households and any adult member capable of providing information served as the respondent for the 'Household Questionnaire'. Other members were consulted only if necessary and this was followed by a 'Biomarker or Measurement Questionnaire'. This included among other things, the measurement of weight and height using standard techniques for the available members in the under-five and 15-49-year-old adults. In the NFHS-3, weight was measured using a solar-powered electronic SECA scale with a digital screen (manufactured under the guidance of UNICEF) and measuring board designed by Shorr Productions for use in survey settings [11]. In the NFHS-4, digital weighing machine (SECA 874) to the nearest of one gram and stadiometer (SECA 213) to the nearest of 0.1cm were used. This was done by trained field staff and monitored by field supervisors. The percentage of missing anthropometric information was 6% in the under-five participants, 12.2% in men (15–54) and 5.7% in women (15–49) in NFHS-4 [12], but corresponding information for participants in the NFHS-3 is not available.

They used the WHO 2007 Growth Reference to assess the nutritional status of adolescents (14). Age, sex, weight, and height recorded in the NFHS-3 and NFHS-4 surveys, were accessed from DHS database, extracted in Microsoft Excel, and imported as a TXT file into the 'nutritional survey tool' of the WHO AnthroPlus software (v1.04) [16]. This is specifically used for children and adolescents in the age group of 6–19 years. Height-for-age z-scores (HAZ) and BMI-for-age z-scores (BAZ) were used to identify stunting and thinness (and overweight/obesity) respectively. The nutritional survey tool of the software generates 'standard reports' and gives the prevalence of nutritional abnormalities as percentages and confidence intervals (CI), stratified by sex along with mean z-scores and standard deviations (SD). The software uses default lower and upper SD boundary as flag limits to identify any extreme or potentially incorrect z-scores. These are -6SD and +6SD for HAZ scores and -5SD and +5SD for BAZ scores and any values beyond these limits get automatically removed.Interpretation of anthropometric data

All the height-for-age z-scores <-2 SD compared to the WHO median were considered as stunting; while BAZ scores < -2SD and <-3SD defined thinness and severe thinness respectively [14]. The BAZ score at +1SD for 19 years coincides with the adult BMI of 25 kg/m2; which is the cut-off for overweight in adults [13]. As a result, BAZ >+1SD was used to classify overweight and >+2SD for obesity .

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Machavaram, Vijayawada, Krishna District, AP-520 004

Questionnaire

1. Name:
2. Age:
3. Place:
4. Address:
5. Urban/Rural:
6. Height:
7. Weight:
8. BMI:
9. Do you have any health issues:

   Anemia

   PCOD

   Any others
10. Are you vegetarian or non-vegetarian:
11. Number of meals consumed in a day:
12. How is your appetite: Good / Fair / Poor
13. Do you know about dietary recommendations:
14. Do you have knowledge on Nutrients and their sources:
15. Do you skip meals:
16. Do you eat breakfast regularly:
17. What do you generally consume in between meals:

18. Do you eat outside: Yes/No if yes what type of food

19. How many days in a week do you have sweets and soft drinks:

20. Do you exercise regularly:

21. Dietary intake from basic Five Food groups

Food Group	Amount to be consumed/Serving	No. of servings	Amounts consumed by the individual	Weather met or not
Cereals	25gm	9 – 16		
Pulses Dairy Meat products	25gm 150ml 30gm	3 – 5		
Yellow, orange, red, green colour fruits & vegetables	50-75gms	3 -5		
Other fruits and vegetables	50-75gms			
Sugars/oils	5gms/5ml	5		

Suggestions:

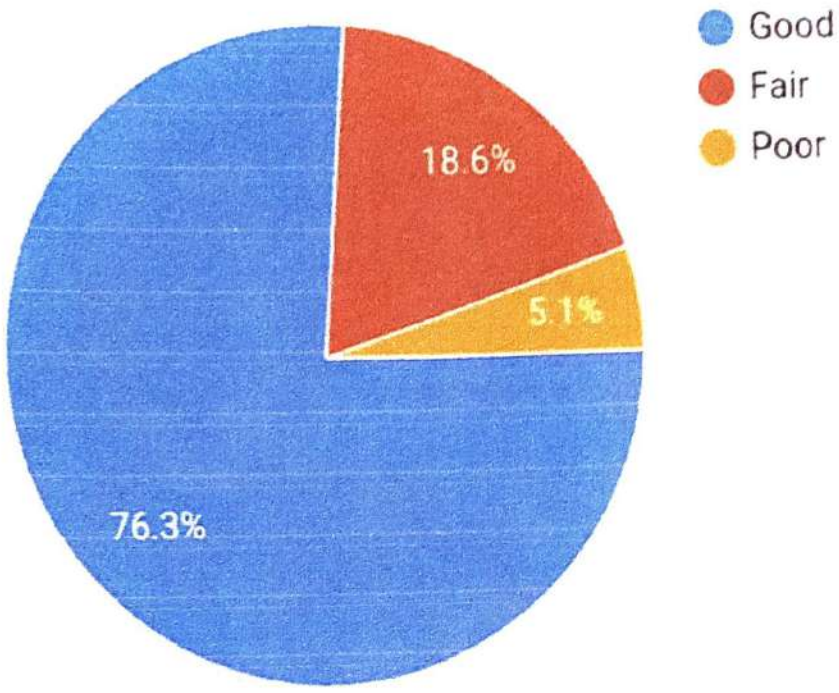
RESULTS

Results:

Name	Height	Weight	BMI	Total servings taken per day	Total servings should be taken	Meeting of RDA	Well nourished	Under nourished	Over nourished
Ch. Likhitha	4.11	52kgs	23.1	03	03	Yes	Yes	-	-
P. Hema	5.2	50kgs	20.1	03	03	Yes	Yes	-	-
P. Chandini	5	46 kgs	19.8	03	03	Yes	Yes	-	-
P. Yasmeen	5	52 kgs	22.3	04	03	Yes	Yes	-	-
T. Sravanthi	5.2	47 kgs	18.9	03	03	No	-	Yes	-
R. Sai vardhani	4.10	42 kgs	19.2	03	03	Yes	Yes	-	-
Sk. Sajida	5.5	56 kgs	20.5	03	03	Yes	Yes	-	-
Ch. Jyothi	5.3	52 kgs	20.3	03	03	Yes	Yes	-	-
K. Naga hema	5.5	60 kgs	22.0	03	03	Yes	Yes	-	-
M. Pujitha	5.1	40 kgs	16.5	03	03	No	-	Yes	-
Ch. Deepika	5.4	50 kgs	18.9	03	03	No	-	Yes	-
K. Nandini	4.11	36 kgs	16	03	03	No	-	Yes	-
P. Prathyusha	5.2	40 kgs	16.1	03	03	No	-	Yes	-
K. Geetha	5.6	60 kgs	21.3	03	03	Yes	Yes	-	-
T. Vineetha	5.3	60 kgs	23.4	03	03	Yes	Yes	-	-
G. Bhavani	4.11	45kgs	17.5	03	03	No	-	Yes	-
M. Manjula	4.10	42kgs	19.3	03	03	Yes	Yes	-	-
K. Mounika	5.2	48kgs	19.2	03	03	Yes	Yes	-	-
P. Silvi Angeleena	5.1	55 kgs	22.9	03	03	Yes	Yes	-	-
P. Priya	5.2	50 kgs	20.2	03	03	Yes	Yes	-	-

How is your appetite

59 responses



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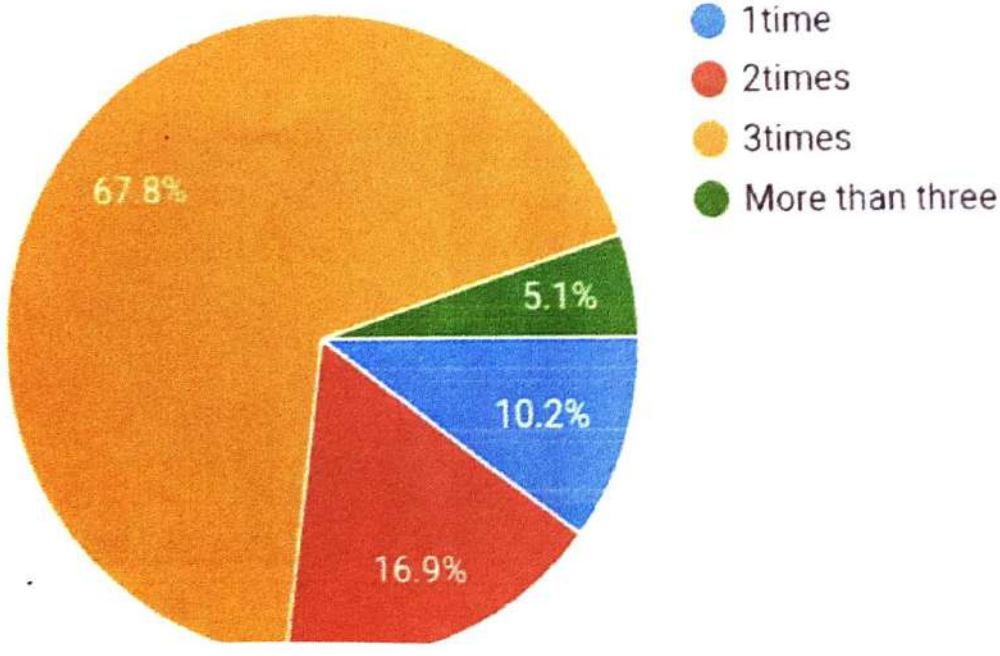

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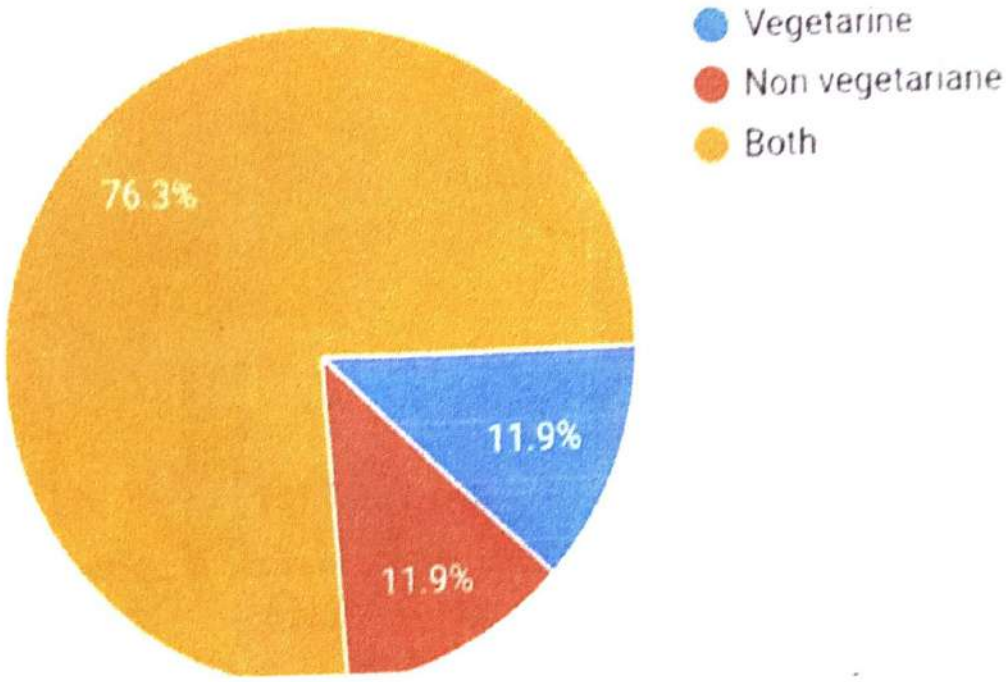

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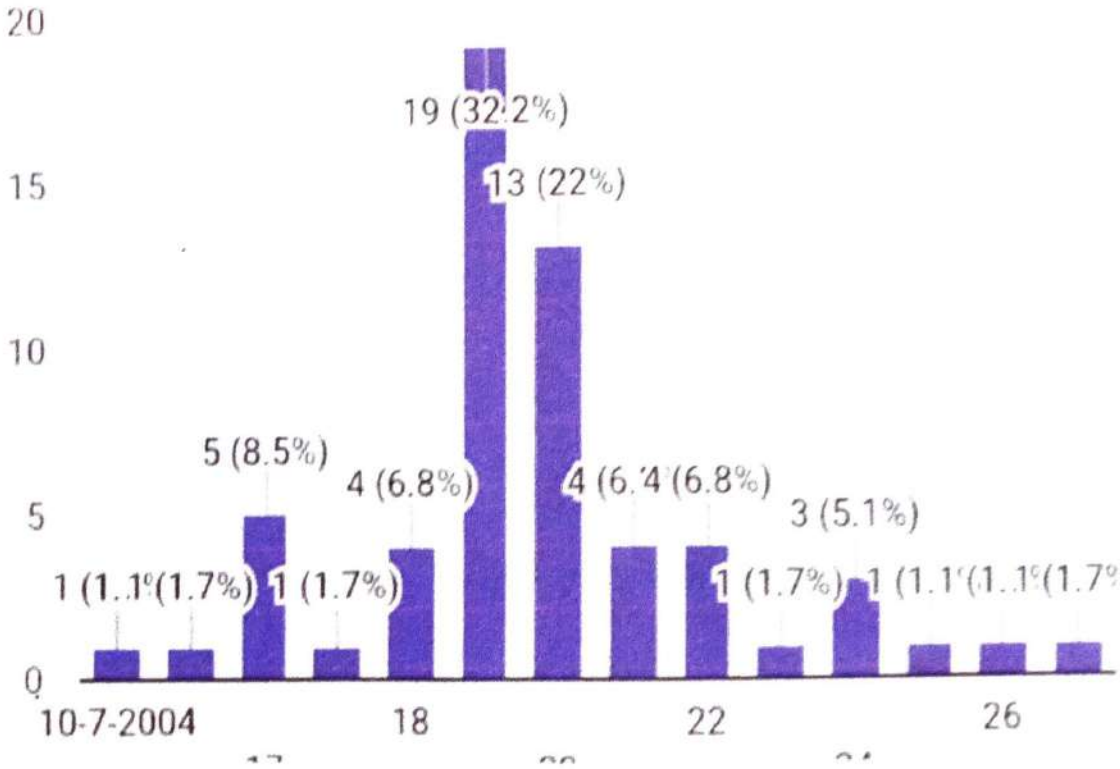

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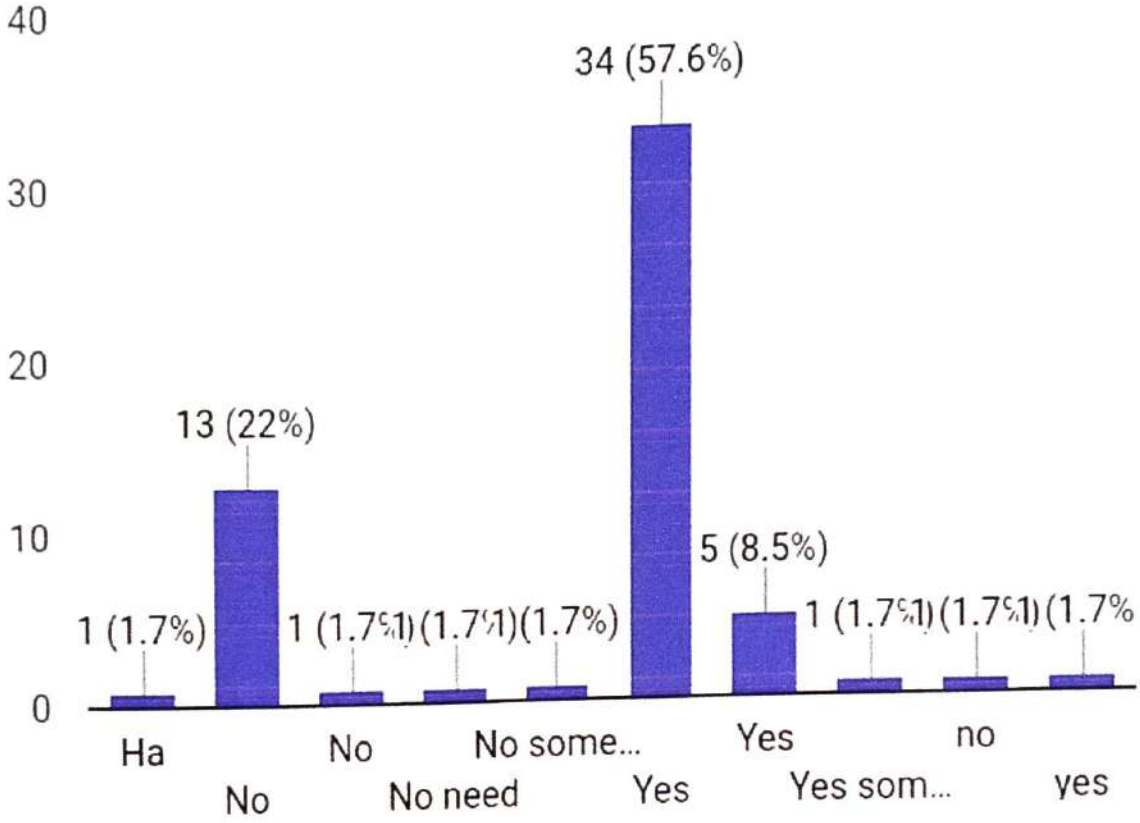

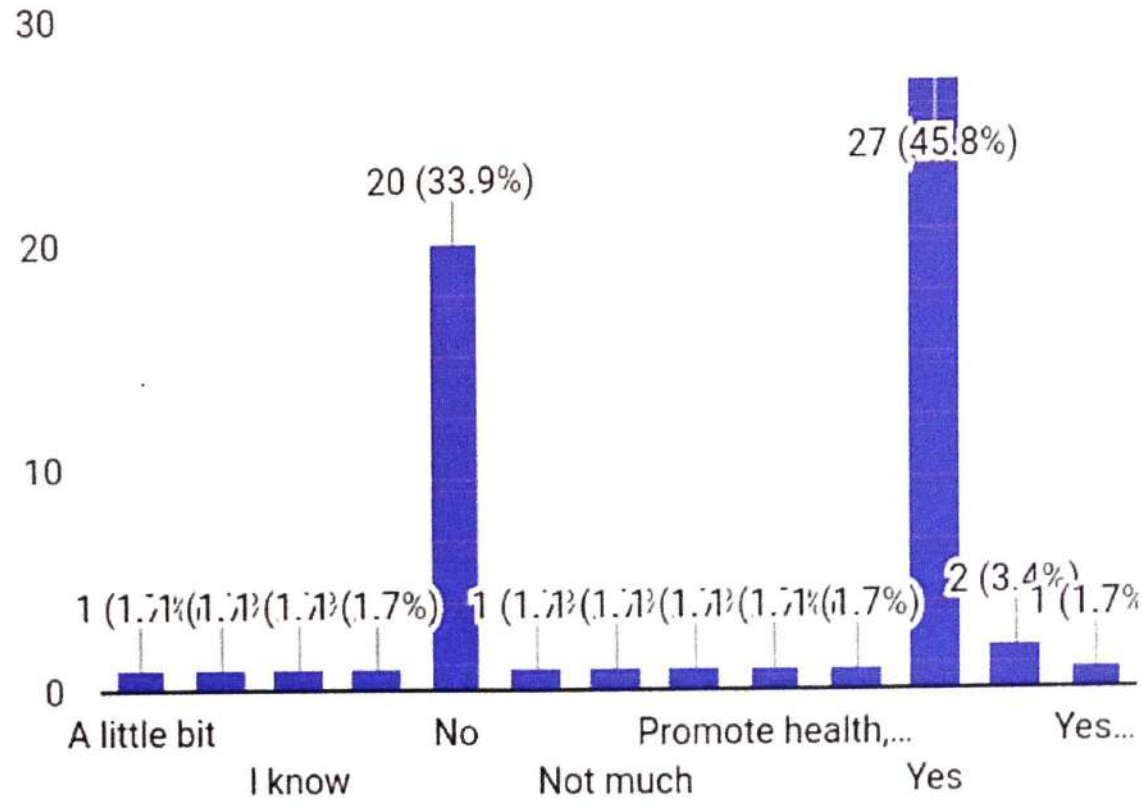

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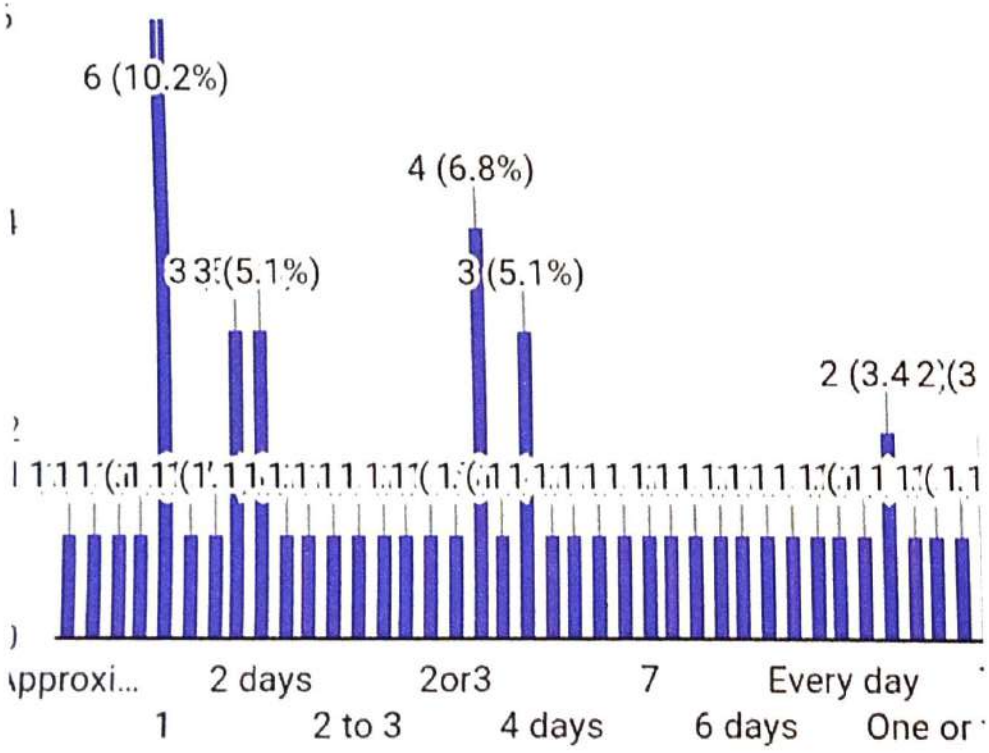

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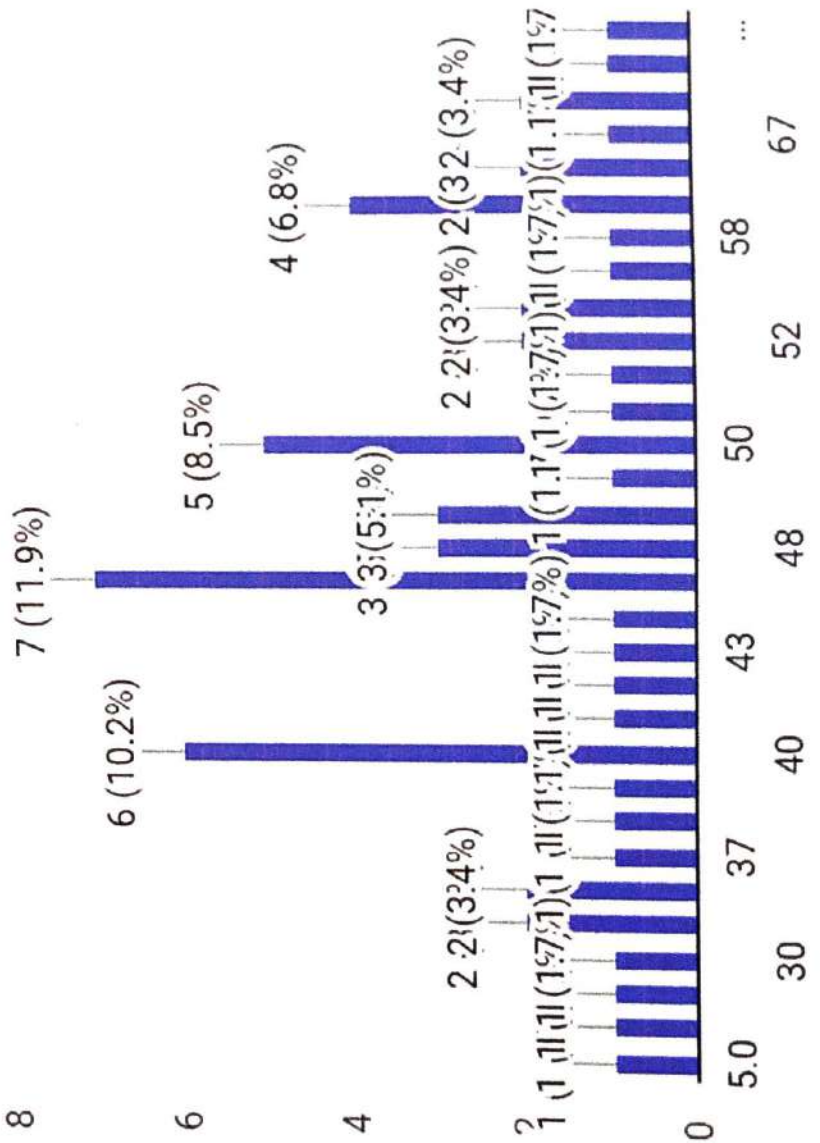

issues

59 responses

Discussion

The nutritional status of adolescent girls, that contributes significantly to the nutritional status of the community. The nutritional requirement increases during adolescent period. In present study 40% of adolescent girls are under nourished.

Total 85% adolescent girls consume calories <1400kcal/day and 40% of which are under nourished. This shows that their average nutrient intake is much below the recommended dietary allowences . Thus most likely reason for being underweight is not having enough calories . In present day study there is no significant association between calorie intake and BMI of adolescent girls. The average BMI of an adolescent girl must be 18.5 to 24.9 . If BMI is less than 18.5 it is termed as Underweight (malnutrition) and the BMI is more than 24.9 then it is over weight (obesity).

Conclusion

By the above report i conclude that the adolescent girls in the taken area majority are well nourished. This is a good sign. Also some girls are under nourished. This will effect their health nutritional status during adolescent period

The conclusion was taken on the basis of significant association between age of adolescent girls and Body Mass Index. The girls with under nourishment may face future problems like anemia PCOD which will effect their entire life. So, they must take certain steps to be nourished. For anemia intake of fruits and green leafy vegetables are very necessary in their diet. The adolescent period plays a vital role in every girl child life so, as it is in our hands we must nourished ourselves. Intake of junk food's must be avoided.

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View at: Publisher Site | Google Scholar

Commissionerate of Collegiate Education, Government of Andhra Pradesh

SRR & CVR Govt. Degree College (A), Vijayawada

_Community Service Project (CSP) - Student Daily Progress Report

Name of the Student : K.NAGA SRIYA

Year : 2021-2024

Regd No: 21315203

Programme Studying with Group: B.Sc (MBFT)

Name of the Mentor: DR.K.ANURADHA

Title of the CSP: NUTRITIONAL STATUS OF ADOLESCENT GIRLS

Place of CSP Execution: MACHAVARAM

Student Daily Progress Report

S No	D ate	Work Done	No of Hours Done	Signature of the CSP Mentor
1.	01-06-2022	I visited K.MOUNIKA house D. No 13-1-120 Ganesh appartments; Machavaram ,Vijayawada.I collected information regarding their nutrional status.	2	
2.	02-06-2022	I visited P.CHANDINI house D.No-13-1-120;Ganesh appartments; Machavaram ,Vijayawada. I collected information regarding their nutrional status.	2	

3.	03-06-2022	I visited P.YASMEEN house D.No-13-1-120;Ganesh appartments Machavaram ,Vijayawada. I collected information regarding their nutrional status.	2	
4.	04-06-2022	I visited T.SRAVANTHI house D.NO 4 -55,Arundel peta , Sanjeevaiah colony, Vijayawada. I collected information regarding their nutrional status.	2	
5.	05-06-2022	I visited R.SAI VARDHNAI house D.NO 4 -44,KONDAPALLI. I collected information regarding their nutrional status.	3	
6.	06-06-2022	I visited SK.SAJIDA house D.NO 21-A Machavaram down. Vijayawada. I collected information regarding their nutrional status.	3	
7.	07-06-2022	I visited CH.JYOTHI house D.NO 5-55A ,Vyjayanthi appartments,Vijayawada. I collected information regarding their nutrional status.	3	
8.	08-06-2022	I visited K.NAGA HEMA house D.NO 21-A, Machavaram down. Vijayawada. I collected information regarding their nutrional status.	3	
9.	09-06-2022	I visited P.PRIYA house D.NO 13-1-120.Ganesh appartments,Vijayawada. I collected information regarding their nutrional status.	3	
10.	10-06-2022	I visited M.PUJITHA house D.NO 1-12C,Raja rajeswari peta.Vijayawada. I collected information regarding their nutrional status.	3	

11.	11-06-2022	I visited CH.DEEPIKA house D.NO 1-23B,YSR Colony, Vijayawada. I collected information regarding their nutrional status.	3	
12.	12-06-2022	I visited K.NANDINI house D.NO 3-33,Chittinagar, Vijayawada. I collected information regarding their nutrional status.	3	
13.	13-06-2022	I visited P.PRATHYUSHA house D.NO 4-44, Krishnalanka,Vijayawada. I collected information regarding their nutrional status.	3	
14.	14-06-2022	I visited K.GEETHA house D.No-2-75;Near Modern Mart,Vijayawada. I collected information regarding their nutrional status.	3	
15.	15-06-2022	I visited T.VINEETHA house, D NO:2-22;Ramavarpadu,Vijayawada. I collected information regarding their nutrional status.	3	
16.	16-06-2022	I visited G.BHAVANI house, D NO:1-132,Machavaram down,Vijayawada. I collected information regarding their nutrional status.	1	
17.	17-06-2022	I visited M.MANJULA house, D NO:41-1/14-222,Krishanalanka,Vijayawada. I collected information regarding their nutrional status.	2	
18.	18-06-2022	I visited P.SILVI ANGELEENA, house, D NO:4-1-30C ,Raj appartments,Vijayawada. I collected information regarding their nutrional status.	3	

19.	19-06-2022	I visited P.PRIYA house, D NO:4-34A Lakshmi appartments, Vijayawada.	2	
20.	20-06-2022	I visited CH.LIKITHA house DNO:5-55,K.Tadepalli, Vijayawada. I collected information regarding their nutrional status.	2	

SRR & CVR Government Degree College

Machavaram, Vijayawada, Krishna District, AP-520 004

Questionnaire

1. Name: CH. Likitha
2. Age: 22 years
3. Place: K. Tadepalli
4. Address: 5-55, K. Tadepalli Vijayawada (Rural) - 520012
5. ~~Urban~~/Rural: Rural
6. Height: 4'11"
7. Weight: 52 Kgs
8. BMI: 23.1
9. Do you have any health issues:

   Anemia —

   PCOD – Yes

   Any others No
10. Are you vegetarian or non-vegetarian: Non Vegetarian
11. Number of meals consumed in a day: 3
12. How is your appetite: Good / Fair / Poor
13. Do you know about dietary recommendations: Yes
14. Do you have knowledge on Nutrients and their sources: Yes
15. Do you skip meals: Yes
16. Do you eat breakfast regularly: No
17. What do you generally consume in between meals: coffee

18. Do you eat outside: Yes/No if yes what type of food Junk-food ie., Noodles etc.

19. How many days in a week do you have sweets and soft drinks: 3

20. Do you exercise regularly: No

21. Dietary intake from basic Five Food groups

Food Group	Amount to be consumed/Serving	No. of servings	Amounts consumed by the individual	Weather met or not
Cereals	25gm	9 – 16	250gm	Met
Pulses	25gm	3 – 5	50 gms	NO
Dairy	150ml		100 ml	NO
Meat products	30gm		90gm	Yes
Yellow, orange, red, green colour fruits & vegetables	50-75gms	3 -5	120gms	NO
Other fruits and vegetables	50-75gms		150gms	Yes
Sugars/oils	5gms/5ml	5	25gms	Yes

Suggestions: Eat lots of fruits & Vegetables.

SRR & CVR Government Degree College

Machavaram, Vijayawada, Krishna District, AP-520 004

Questionnaire

1. Name: P. Hema
2. Age: 19 yrs
3. Place: Vijayawada
4. Address: 13-1-120 ; Ganesh appartments ; Vijayawada
5. Urban/~~Rural~~: Urban
6. Height: 5' 2"
7. Weight: 50 kgs
8. BMI: 20.1
9. Do you have any health issues:

   Anemia - No

   PCOD - No

   Any others - No
10. Are you vegetarian or non-vegetarian: Vegetarian
11. Number of meals consumed in a day: 3
12. How is your appetite: Good / Fair / Poor ✓
13. Do you know about dietary recommendations: Yes
14. Do you have knowledge on Nutrients and their sources: Yes
15. Do you skip meals: No
16. Do you eat breakfast regularly: Yes
17. What do you generally consume in between meals: Fruits

18. Do you eat outside: Yes/No if yes what type of food Fast Foods

19. How many days in a week do you have sweets and soft drinks: 7

20. Do you exercise regularly: Yes

21. Dietary intake from basic Five Food groups

Food Group	Amount to be consumed/Serving	No. of servings	Amounts consumed by the individual	Weather met or not
Cereals	25gm	9 – 16	500gms	Yes
Pulses	25gm	3 – 5	200gms	Yes
Dairy	150ml		150ml	No
Meat products	30gm		120gms	Yes
Yellow, orange, red, green colour fruits & vegetables	50-75gms	3 -5	150 gms	Yes
Other fruits and vegetables	50-75gms		150gms	Yes
Sugars/oils	5gms/5ml	5	25gms	Yes

Suggestions: Don't skip Breakfast

SRR & CVR Government Degree College

Machavaram, Vijayawada, Krishna District, AP-520 004

Questionnaire

1. Name: P. Chandini
2. Age: 18 yrs
3. Place: Vijayawada
4. Address: 13-1-120 ; Ganesh appartments ; Vijayawada
5. Urban/~~Rural~~: Urban
6. Height: 5
7. Weight: 46
8. BMI: 19.8
9. Do you have any health issues:

   Anemia No

   PCOD No

   Any others No
10. Are you vegetarian or non-vegetarian: Non-Vegetarian
11. Number of meals consumed in a day: 3
12. How is your appetite: Good / Fair / Poor
13. Do you know about dietary recommendations: Yes
14. Do you have knowledge on Nutrients and their sources: Yes
15. Do you skip meals: Yes
16. Do you eat breakfast regularly: No
17. What do you generally consume in between meals: Snacks & fruits

18. Do you eat outside: Yes/No if yes what type of food ✓ Fast foods

19. How many days in a week do you have sweets and soft drinks: 2

20. Do you exercise regularly: No

21. Dietary intake from basic Five Food groups

Food Group	Amount to be consumed/Serving	No. of servings	Amounts consumed by the individual	Weather met or not
Cereals	25gm	9 – 16	400gms	Yes
Pulses	25gm	3 – 5	200gms	Yes
Dairy	150ml		500ml	Yes
Meat products	30gm		120gms	Yes
Yellow, orange, red, green colour fruits & vegetables	50-75gms	3 -5	200gms	Yes
Other fruits and vegetables	50-75gms		150gms	Yes
Sugars/oils	5gms/5ml	5	25gms	Yes

Suggestions: Eat more fish, including a portion of oily fish.

SRR & CVR Government Degree College

Machavaram, Vijayawada, Krishna District, AP-520 004

Questionnaire

1. Name: P. Yasmeen
2. Age: 17 Years
3. Place: Vijayawada
4. Address: 13/1/120, Ganesh Appartments, Vijayawada
5. Urban/~~Rural~~: Urban
6. Height: 5' Feet
7. Weight: 52 kgs
8. BMI: ~~18.5~~ 22.3
9. Do you have any health issues:

   Anemia No

   PCOD Yes

   Any others No
10. Are you vegetarian or non-vegetarian: Non vegetarian
11. Number of meals consumed in a day: 4
12. How is your appetite: Good / Fair / Poor
13. Do you know about dietary recommendations: Yes
14. Do you have knowledge on Nutrients and their sources: Yes
15. Do you skip meals: Yes
16. Do you eat breakfast regularly: No
17. What do you generally consume in between meals: Snacks

18. Do you eat outside: Yes/No if yes what type of food ✓ Fast foods

19. How many days in a week do you have sweets and soft drinks: 2

20. Do you exercise regularly: Yes

21. Dietary intake from basic Five Food groups

Food Group	Amount to be consumed/Serving	No. of servings	Amounts consumed by the individual	Weather met or not
Cereals	25gm	9 – 16	250gms	Yes
Pulses	25gm	3 – 5	90 gms	Yes
Dairy	150ml		400ml	
Meat products	30gm		120gms	
Yellow, orange, red, green colour fruits & vegetables	50-75gms	3 -5	150gms	Yes
Other fruits and vegetables	50-75gms		180 gms	Yes
Sugars/oils	5gms/5ml	5	15gms	No

Suggestions: Base your meals on high fiber and starch, carbohydrates.

SRR & CVR Government Degree College

Machavaram, Vijayawada, Krishna District, AP-520 004

Questionnaire

1. Name: T. Sravanthi
2. Age: 20 Yrs
3. Place: Vijaywada
4. Address: 4-55, Arundel peta, Sanjeevaiah colony, Vijaywada
5. Urban/~~Rural~~: Urban
6. Height: 5' 2"
7. Weight: 47 Kgs
8. BMI: 18.9
9. Do you have any health issues:

   Anemia —

   PCOD —

   Any others No
10. Are you vegetarian or non-vegetarian: Non Vegeterian
11. Number of meals consumed in a day: 3
12. How is your appetite: Good / Fair / Poor
13. Do you know about dietary recommendations: Yes
14. Do you have knowledge on Nutrients and their sources: Yes
15. Do you skip meals: Yes
16. Do you eat breakfast regularly: Yes
17. What do you generally consume in between meals: Fruits

18. Do you eat outside: Yes/No if yes what type of food ✓ Junkfood

19. How many days in a week do you have sweets and soft drinks: No

20. Do you exercise regularly: No

21. Dietary intake from basic Five Food groups

Food Group	Amount to be consumed/Serving	No. of servings	Amounts consumed by the individual	Weather met or not
Cereals	25gm	9 – 16	250gms	Yes
Pulses	25gm	3 – 5	75gms	Yes
Dairy	150ml		300ml	Yes
Meat products	30gm		250gms	Yes
Yellow, orange, red, green colour fruits & vegetables	50-75gms	3 -5	200gms	Yes
Other fruits and vegetables	50-75gms		150gms	Yes
Sugars/oils	5gms/5ml	5	15gms	No

Suggestions: Follow healthy diet.

SRR & CVR Government Degree College

Machavaram, Vijayawada, Krishna District, AP-520 004

Questionnaire

1. Name: R. Sai Vardhani
2. Age: 19 yrs
3. Place: Vijayawada
4. Address: 4-44, Kondapalli
5. Urban/~~Rural~~: Urban
6. Height: 4' 10"
7. Weight: 42
8. BMI: 19.2
9. Do you have any health issues:

   Anemia – No

   PCOD – No

   Any others – No
10. Are you vegetarian or non-vegetarian: Non-Vegetarian
11. Number of meals consumed in a day: 3
12. How is your appetite: Good / Fair / Poor
13. Do you know about dietary recommendations: Yes
14. Do you have knowledge on Nutrients and their sources: Yes
15. Do you skip meals: Yes
16. Do you eat breakfast regularly: No
17. What do you generally consume in between meals: Snacks

18. Do you eat outside: Yes/No if yes what type of food Fastfoods

19. How many days in a week do you have sweets and soft drinks: 3

20. Do you exercise regularly: Yes

21. Dietary intake from basic Five Food groups

Food Group	Amount to be consumed/Serving	No. of servings	Amounts consumed by the individual	Weather met or not
Cereals	25gm	9 – 16	250gms	Yes
Pulses	25gm	3 – 5	75gm	Yes
Dairy	150ml		150ml	No
Meat products	30gm		150gms	Yes
Yellow, orange, red, green colour fruits & vegetables	50-75gms	3 -5	200gms	Yes
Other fruits and vegetables	50-75gms		150gms	Yes
Sugars/oils	5gms/5ml	5	25gm	Yes.

Suggestions: Get Active and be a healthy weight.

SRR & CVR Government Degree College

Machavaram, Vijayawada, Krishna District, AP-520 004

Questionnaire

1. Name: SK. Sajida
2. Age: 19yrs
3. Place: Vijayawada
4. Address: 21-A, Machavaram Down, Vijayawada
5. Urban/~~Rural~~: Urban
6. Height: 5'5"
7. Weight: 56
8. BMI: 20.5
9. Do you have any health issues:

   Anemia - No

   PCOD - No

   Any others - No
10. Are you vegetarian or non-vegetarian: Non-Vegetarian
11. Number of meals consumed in a day: 3
12. How is your appetite: Good / Fair / Poor
13. Do you know about dietary recommendations: Yes
14. Do you have knowledge on Nutrients and their sources: Yes
15. Do you skip meals: No
16. Do you eat breakfast regularly: Yes
17. What do you generally consume in between meals: Snacks

18. Do you eat outside: Yes/No if yes what type of food ✓ Fast-foods

19. How many days in a week do you have sweets and soft drinks: 4

20. Do you exercise regularly: No

21. Dietary intake from basic Five Food groups

Food Group	Amount to be consumed/Serving	No. of servings	Amounts consumed by the individual	Weather met or not
Cereals	25gm	9 – 16	450gms	Yes
Pulses	25gm	3 – 5	120gms	Yes
Dairy	150ml		300ml	Yes
Meat products	30gm		200gms	Yes
Yellow, orange, red, green colour fruits & vegetables	50-75gms	3 -5	150gms	Yes
Other fruits and vegetables	50-75gms		150gms	Yes
Sugars/oils	5gms/5ml	5	15gms	NO

Suggestions: Cut down on saturated fat & sugar

SRR & CVR Government Degree College

Machavaram, Vijayawada, Krishna District, AP-520 004

Questionnaire

1. Name: CH. Jyothi
2. Age: 19 Yrs
3. Place: Vijayawada
4. Address: 5-55A ; Vyjayanthi Appartments ; Vijayawada
5. Urban/~~Rural~~: Urban
6. Height: 5' 3"
7. Weight: 52 kgs
8. BMI: 20.3
9. Do you have any health issues:

   Anemia – No

   PCOD – Yes

   Any others – No
10. Are you vegetarian or non-vegetarian: Vegetarian
11. Number of meals consumed in a day: 3
12. How is your appetite: Good / Fair / Poor
13. Do you know about dietary recommendations: Yes
14. Do you have knowledge on Nutrients and their sources: Yes
15. Do you skip meals: No
16. Do you eat breakfast regularly: Yes
17. What do you generally consume in between meals: Fruits

18. Do you eat outside: Yes/No if yes what type of food ✓ Fast foods

19. How many days in a week do you have sweets and soft drinks: 2

20. Do you exercise regularly: Yes

21. Dietary intake from basic Five Food groups

Food Group	Amount to be consumed/Serving	No. of servings	Amounts consumed by the individual	Weather met or not
Cereals	25gm	9 – 16	550gms	Yes
Pulses	25gm	3 – 5	125 gms	Yes
Dairy	150ml		450 ml	Yes
Meat products	30gm		30gms	No
Yellow, orange, red, green colour fruits & vegetables	50-75gms	3 -5	450gms	Yes
Other fruits and vegetables	50-75gms		200gms	Yes
Sugars/oils	5gms/5ml	5	15gms	Yes

Suggestions: Eat good food & Do exercise regularly.

SRR & CVR Government Degree College

Machavaram, Vijayawada, Krishna District, AP-520 004

Questionnaire

1. Name: K. Naga Hema
2. Age: 21 yrs
3. Place: Vijayawada
4. Address: A-21, Machavaram down, Vijayawada.
5. Urban/~~Rural~~: Urban
6. Height: 5' 5"
7. Weight: 60 kgs
8. BMI: 22.0
9. Do you have any health issues:

   Anemia – No

   PCOD – No

   Any others – No
10. Are you vegetarian or non-vegetarian: Non-Vegetarian
11. Number of meals consumed in a day: 3
12. How is your appetite: Good / Fair / Poor
13. Do you know about dietary recommendations: Yes
14. Do you have knowledge on Nutrients and their sources: Yes
15. Do you skip meals: No
16. Do you eat breakfast regularly: Yes
17. What do you generally consume in between meals: Fruits & Snacks

18. Do you eat outside: Yes No if yes what type of food Fast foods

19. How many days in a week do you have sweets and soft drinks: 4

20. Do you exercise regularly: No

21. Dietary intake from basic Five Food groups

Food Group	Amount to be consumed Serving	No. of servings	Amounts consumed by the individual	Weather met or not
Cereals	25gm	9 – 16	500gms	Yes
Pulses	25gm	3 – 5	75gms	Yes
Dairy	150ml		450ml	Yes
Meat products	30gm		120gms	Yes
Yellow, orange, red, green colour fruits & vegetables	50-75gms	3 -5	200gms	Yes
Other fruits and vegetables	50-75gms		150gms	Yes
Sugars/oils	5gms/5ml	5	25gms	Yes

Suggestions:

SRR & CVR Government Degree College

Machavaram, Vijayawada, Krishna District, AP-520 004

Questionnaire

1. Name: M. Pujitha
2. Age: 20 yrs
3. Place: Vijayawada (Machavaram)
4. Address: 1-12 C ; Rajarajeswari peta , Vijayawada
5. Urban/~~Rural~~: Urban
6. Height: 5'1"
7. Weight: 40kgs
8. BMI: 16.5
9. Do you have any health issues:

   Anemia - No

   PCOD - No

   Any others - No
10. Are you vegetarian or non-vegetarian: Non-vegetarian
11. Number of meals consumed in a day: 3
12. How is your appetite: Good / Fair / Poor
13. Do you know about dietary recommendations: Yes
14. Do you have knowledge on Nutrients and their sources: Yes
15. Do you skip meals: No
16. Do you eat breakfast regularly: Yes
17. What do you generally consume in between meals: Dry fruits

18. Do you eat outside: Yes/No if yes what type of food ✓ Fast Foods

19. How many days in a week do you have sweets and soft drinks: 7

20. Do you exercise regularly: No

21. Dietary intake from basic Five Food groups

Food Group	Amount to be consumed/Serving	No. of servings	Amounts consumed by the individual	Weather met or not
Cereals	25gm	9 – 16	300gms	Yes
Pulses	25gm	3 – 5	75 gms	Yes
Dairy	150ml		300 ml	
Meat products	30gm		120gms	
Yellow, orange, red, green colour fruits & vegetables	50-75gms	3 -5	150gms	Yes
Other fruits and vegetables	50-75gms		150 gms	Yes
Sugars/oils	5gms/5ml	5	25gms	Yes

Suggestions: Eat fresh fruits & Vegetables.

SRR & CVR Government Degree College

Machavaram, Vijayawada, Krishna District, AP-520 004

Questionnaire

1. Name: CH. Deepika
2. Age: 19yrs
3. Place: Vijayawada (Machavam)
4. Address: 1-93 B, YSR colony, Vijayawada
5. Urban/~~Rural~~: Urban
6. Height: 5'4"
7. Weight: 50 Kgs
8. BMI: 18.9
9. Do you have any health issues:

   Anemia - No

   PCOD - No

   Any others - No
10. Are you vegetarian or non-vegetarian: Non-vegetarian
11. Number of meals consumed in a day: 3
12. How is your appetite: Good / Fair / Poor
13. Do you know about dietary recommendations: Yes
14. Do you have knowledge on Nutrients and their sources: Yes
15. Do you skip meals: No
16. Do you eat breakfast regularly: Yes
17. What do you generally consume in between meals: Snack, fruits

18. Do you eat outside: Yes/No if yes what type of food ✓ Fast foods

19. How many days in a week do you have sweets and soft drinks: 7

20. Do you exercise regularly: Yes

21. Dietary intake from basic Five Food groups

Food Group	Amount to be consumed/Serving	No. of servings	Amounts consumed by the individual	Weather met or not
Cereals	25gm	9 – 16	400gms	Yes
Pulses	25gm	3 – 5	90 gms	Yes
Dairy	150ml		300ml	
Meat products	30gm		120gms	
Yellow, orange, red, green colour fruits & vegetables	50-75gms	3 -5	150 gms	Yes
Other fruits and vegetables	50-75gms		150 gms	Yes
Sugars/oils	5gms/5ml	5	25 gms	Yes

Suggestions: Take Healthy food & do exercise regularly.

SRR & CVR Government Degree College

Machavaram, Vijayawada, Krishna District, AP-520 004

Questionnaire

1. Name: K. Nandini
2. Age: 19 yrs
3. Place: Vijayawada
4. Address: 3-33, chittinagar, Vijayawada
5. Urban/~~Rural~~: Urban
6. Height: 4' 11"
7. Weight: 36kgs
8. BMI: 16.
9. Do you have any health issues:

   Anemia – No

   PCOD – No

   Any others – No.
10. Are you vegetarian or non-vegetarian: Non-Vegetarian
11. Number of meals consumed in a day: 3
12. How is your appetite: Good / Fair / Poor
13. Do you know about dietary recommendations: Yes
14. Do you have knowledge on Nutrients and their sources: Yes
15. Do you skip meals: No
16. Do you eat breakfast regularly: Yes
17. What do you generally consume in between meals: No

18. Do you eat outside: Yes/No if yes what type of food ✓ Junk Foods

19. How many days in a week do you have sweets and soft drinks: 7

20. Do you exercise regularly: No

21. Dietary intake from basic Five Food groups

Food Group	Amount to be consumed/Serving	No. of servings	Amounts consumed by the individual	Weather met or not
Cereals	25gm	9 – 16	200gms	No
Pulses	25gm	3 – 5	50gms	No
Dairy	150ml		450 ml	Yes
Meat products	30gm		150gms	Yes
Yellow, orange, red, green colour fruits & vegetables	50-75gms	3 -5	200gms	Yes
Other fruits and vegetables	50-75gms		150gms	Yes
Sugars/oils	5gms/5ml	5	10gms	No

Suggestions: Take fresh fruits & vegetables in diet.

SRR & CVR Government Degree College

Machavaram, Vijayawada, Krishna District, AP-520 004

Questionnaire

1. Name: P. Prathyusha
2. Age: 20 yrs
3. Place: Vijayawada
4. Address: 4-44, Krishnalanka, Vijayawada
5. Urban/~~Rural:~~ Urban
6. Height: 5' 2"
7. Weight: 40 kgs
8. BMI: 16.1
9. Do you have any health issues:

   Anemia –

   PCOD –

   Any others No
10. Are you vegetarian or non-vegetarian: Non-Vegetarian
11. Number of meals consumed in a day: 2 ✓
12. How is your appetite: Good / Fair / Poor
13. Do you know about dietary recommendations: Yes
14. Do you have knowledge on Nutrients and their sources: Yes
15. Do you skip meals: Yes
16. Do you eat breakfast regularly: Yes
17. What do you generally consume in between meals: Fruits

18. Do you eat outside: ✓Yes No If yes what type of food Fast foods

19. How many days in a week do you have sweets and soft drinks: 4

20. Do you exercise regularly: Yes

21. Dietary intake from basic Five Food groups

Food Group	Amount to be consumed/ Serving	No. of servings	Amounts consumed by the individual	Weather met or not
Cereals	25gm	9 – 14	200gms	No
Pulses	25gm		50 gms	No
Dairy	150ml	3 – 5	200 ml	No
Meat products	30gm		250gms	Yes
Yellow, orange, red, green colour fruits & vegetables	50-75gms	3 -5	200 gms	Yes
Other fruits and vegetables	50-75gms		200gms	Yes
Sugars/oils	5gms/5ml	5	25gms	Yes

Suggestions: Take Healthy diet.

SRR & CVR Government Degree College

Machavaram, Vijayawada, Krishna District, AP-520 004

Questionnaire

1. Name: K. Geetha
2. Age: 19
3. Place: Vijayawada
4. Address: 2-75 ; near modern mart , Vijayawada
5. Urban/~~Rural~~: Urban
6. Height: 5'8"
7. Weight: 60 kgs
8. BMI: 21.3
9. Do you have any health issues:

   Anemia –

   PCOD –

   Any others –
10. Are you vegetarian or non-vegetarian: Non-Vegetarian
11. Number of meals consumed in a day: 3
12. How is your appetite: Good / Fair / Poor
13. Do you know about dietary recommendations: Yes
14. Do you have knowledge on Nutrients and their sources: Yes
15. Do you skip meals: No
16. Do you eat breakfast regularly: No
17. What do you generally consume in between meals: Snacks

18. Do you eat outside: Yes/No if yes what type of food Fast-foods

19. How many days in a week do you have sweets and soft drinks: 7

20. Do you exercise regularly: No

21. Dietary intake from basic Five Food groups

Food Group	Amount to be consumed/Serving	No. of servings	Amounts consumed by the individual	Weather met or not
Cereals	25gm	9 – 16	250gms	Yes
Pulses	25gm	3 – 5	100gms	Yes
Dairy	150ml		200ml	No
Meat products	30gm		240gms	Yes
Yellow, orange, red, green colour fruits & vegetables	50-75gms	3 -5	150gms	Yes
Other fruits and vegetables	50-75gms		150gms	Yes
Sugars/oils	5gms/5ml	5	25gms	Yes

Suggestions: Follow healthy diet.

SRR & CVR Government Degree College

Machavaram, Vijayawada, Krishna District, AP-520 004

Questionnaire

1. Name: T. Vineetha
2. Age: 20yrs
3. Place: Vijayawada
4. Address: 8-20; Ramavarpadu, Vijayawada
5. Urban ~~Rural~~: Urban
6. Height: 5'3"
7. Weight: 60 kgs
8. BMI: 23.4
9. Do you have any health issues:

   Anemia – No

   PCOD – NO

   Any others – NO
10. Are you vegetarian or non-vegetarian: Non Vegetarian
11. Number of meals consumed in a day: 3
12. How is your appetite: Good / Fair / Poor
13. Do you know about dietary recommendations: Yes
14. Do you have knowledge on Nutrients and their sources: Yes
15. Do you skip meals: No
16. Do you eat breakfast regularly: Yes
17. What do you generally consume in between meals: Snacks

18. Do you eat outside: Yes/No if yes what type of food Fast Foods

19. How many days in a week do you have sweets and soft drinks: 4

20. Do you exercise regularly: No

21. Dietary intake from basic Five Food groups

Food Group	Amount to be consumed/Serving	No. of servings	Amounts consumed by the individual	Weather met or not
Cereals	25gm	9 – 16	250gms	Yes
Pulses	25gm	3 – 5	75gms	Yes
Dairy	150ml		300ml	Yes
Meat products	30gm		120gms	Yes
Yellow, orange, red, green colour fruits & vegetables	50-75gms	3 -5	150gms	Yes
Other fruits and vegetables	50-75gms		150gms	Yes
Sugars/oils	5gms/5ml	5	25gms	Yes

Suggestions: Take healthy diet regularly.

SRR & CVR Government Degree College

Machavaram, Vijayawada, Krishna District, AP-520 004

Questionnaire

1. Name: G. Bhavani
2. Age: 18 yrs
3. Place: Vijayawada
4. Address: 1-132 ; Machavaram town Vijayawada
5. Urban/~~Rural~~: Urban
6. Height: 4' 11"
7. Weight: 45
8. BMI: 17.5
9. Do you have any health issues:

   Anemia – No

   PCOD – No

   Any others – No
10. Are you vegetarian or non-vegetarian: Non-Vegetarian
11. Number of meals consumed in a day: 3
12. How is your appetite: Good / Fair / Poor
13. Do you know about dietary recommendations: Yes
14. Do you have knowledge on Nutrients and their sources: Yes
15. Do you skip meals: No
16. Do you eat breakfast regularly: Yes
17. What do you generally consume in between meals: Fruits

18. Do you eat outside: Yes/No if yes what type of food ✓ Fast foods

19. How many days in a week do you have sweets and soft drinks: 3

20. Do you exercise regularly: Yes

21. Dietary intake from basic Five Food groups

Food Group	Amount to be consumed/Serving	No. of servings	Amounts consumed by the individual	Weather met or not
Cereals	25gm	9 – 16	250gms	Yes
Pulses	25gm	3 – 5	75gms	Yes
Dairy	150ml		300ml	No
Meat products	30gm		120gms	Yes
Yellow, orange, red, green colour fruits & vegetables	50-75gms	3 -5	150gms	Yes
Other fruits and vegetables	50-75gms		100gms	NO
Sugars/oils	5gms/5ml	5	25gms	Yes

Suggestions: Do exercise Regularly.

SRR & CVR Government Degree College

Machavaram, Vijayawada, Krishna District, AP-520 004

Questionnaire

1. Name: M. Manjula
2. Age: 19
3. Place: Vijayawada
4. Address: 41-1/14-222, Krishnalanka, Vijayawada
5. Urban/~~Rural~~: Urban
6. Height: 4'10"
7. Weight: 45 kgs.
8. BMI: 19.3
9. Do you have any health issues;

   Anemia –

   PCOD –

   Any others – No
10. Are you vegetarian or non-vegetarian: Non-Vegetarian
11. Number of meals consumed in a day: 3
12. How is your appetite: Good / Fair / Poor
13. Do you know about dietary recommendations: Yes
14. Do you have knowledge on Nutrients and their sources: Yes
15. Do you skip meals: Yes
16. Do you eat breakfast regularly: No
17. What do you generally consume in between meals: No

18. Do you eat outside: Yes/No if yes what type of food - Noodles, etc- -

19. How many days in a week do you have sweets and soft drinks: 1

20. Do you exercise regularly: No

21. Dietary intake from basic Five Food groups

Food Group	Amount to be consumed/Serving	No. of servings	Amounts consumed by the individual	Weather met or not
Cereals	25gm	9 – 16	250gms	Yes
Pulses	25gm	3 – 5	50gms	No
Dairy	150ml		200ml	Yes
Meat products	30gm		250gms	Yes
Yellow, orange, red, green colour fruits & vegetables	50-75gms	3 -5	150gms	Yes
Other fruits and vegetables	50-75gms		150gms	Yes
Sugars/oils	5gms/5ml	5	10gms	No

Suggestions: Take Citrus fruits frequently.

SRR & CVR Government Degree College

Machavaram, Vijayawada, Krishna District, AP-520 004

Questionnaire

1. Name: K. Mounika
2. Age: 19yrs
3. Place: Vijayawada (Machavaram)
4. Address: 13-1-120 ; Ganesh appartments ; Vijayawada
5. Urban/Rural:
6. Height: 5'2"
7. Weight: 48Kgs
8. BMI: 19.2
9. Do you have any health issues:

   Anemia – No

   PCOD – No

   Any others – No
10. Are you vegetarian or non-vegetarian: Vegetarian
11. Number of meals consumed in a day: 3
12. How is your appetite: Good / Fair / Poor
13. Do you know about dietary recommendations: Yes
14. Do you have knowledge on Nutrients and their sources: Yes
15. Do you skip meals: Yes
16. Do you eat breakfast regularly: Yes
17. What do you generally consume in between meals: No

18. Do you eat outside: Yes/No if yes what type of food ✓ Fast Foods

19. How many days in a week do you have sweets and soft drinks: No

20. Do you exercise regularly: NO

21. Dietary intake from basic Five Food groups

Food Group	Amount to be consumed/Serving	No. of servings	Amounts consumed by the individual	Weather met or not
Cereals	25gm	9 – 16	400gms	Yes
Pulses Dairy Meat products	25gm 150ml 30gm	3 – 5	75gms 300ml No	No
Yellow, orange, red, green colour fruits & vegetables	50-75gms	3 -5	150gms	Yes
Other fruits and vegetables	50-75gms		200gms	Yes
Sugars/oils	5gms/5ml	5	25gms	Yes

Suggestions: Eat a lot of fruits & Vegetables.

SRR & CVR Government Degree College

Machavaram, Vijayawada, Krishna District, AP 520 004

Questionnaire

1. Name: P. Silvi Angeleena
2. Age: 17 yrs
3. Place: Vijayawada
4. Address: 4-1-30 C ; Raj Appartments ; Vijayawada
5. Urban/~~Rural~~: Urban
6. Height: 5' 1"
7. Weight: 55 kgs
8. BMI: 22.9
9. Do you have any health issues:

   Anemia - No

   PCOD - Yes

   Any others - No
10. Are you vegetarian or non-vegetarian: Non-Vegetarian
11. Number of meals consumed in a day: 3
12. How is your appetite: Good / Fair / Poor ✓
13. Do you know about dietary recommendations: Yes
14. Do you have knowledge on Nutrients and their sources: Yes
15. Do you skip meals: Yes
16. Do you eat breakfast regularly: Yes
17. What do you generally consume in between meals: Snacks, drinks or juices

18. Do you eat outside: Yes/No if yes what type of food ✓ Fast Foods

19. How many days in a week do you have sweets and soft drinks: 5

20. Do you exercise regularly: Yes

21. Dietary intake from basic Five Food groups

Food Group	Amount to be consumed/Serving	No. of servings	Amounts consumed by the individual	Weather met or not
Cereals	25gm	9 – 16	350gms	Yes
Pulses	25gm	3 – 5	75gms	Yes
Dairy	150ml		350ml	NO
Meat products	30gm		300gms	Yes
Yellow, orange, red, green colour fruits & vegetables	50-75gms	3 -5	200gms	Yes
Other fruits and vegetables	50-75gms		150gms	Yes
Sugars/oils	5gms/5ml	5	25gms	Yes

Suggestions: Eat Fresh fruits & Vegetables and drink plenty of water.

SRR & CVR Government Degree College

Machavaram, Vijayawada, Krishna District, AP-520 004

Questionnaire

1. Name: P. Priya
2. Age: 20 yrs
3. Place: Vijayawada
4. Address: 4-34 A ; Lakshmi Appartments ; Vijayawada
5. Urban/~~Rural~~: Urban
6. Height: 5'2"
7. Weight: 50 Kgs
8. BMI: 20.2
9. Do you have any health issues:

   Anemia - No

   PCOD - No

   Any others - No
10. Are you vegetarian or non-vegetarian: Non-Vegetarian
11. Number of meals consumed in a day: 3
12. How is your appetite: Good / Fair / Poor
13. Do you know about dietary recommendations: Yes
14. Do you have knowledge on Nutrients and their sources: Yes
15. Do you skip meals: No
16. Do you eat breakfast regularly: Yes
17. What do you generally consume in between meals: Snacks & Fruits

18. Do you eat outside: Yes/No if yes what type of food ✓ Fast-foods

19. How many days in a week do you have sweets and soft drinks: 4

20. Do you exercise regularly: Yes

21. Dietary intake from basic Five Food groups

Food Group	Amount to be consumed/Serving	No. of servings	Amounts consumed by the individual	Weather met or not
Cereals	25gm	9 – 16	250gms	Yes
Pulses	25gm	3 – 5	50gms	No
Dairy	150ml		100ml	No
Meat products	30gm		350gms	Yes
Yellow, orange, red, green colour fruits & vegetables	50-75gms	3 -5	100gms	No
Other fruits and vegetables	50-75gms		150gms	Yes
Sugars/oils	5gms/5ml	5	25gms	Yes

Suggestions: Eat proteinecous food and be healthy.

A STUDY PROJECT ON

"BIOINFOMATICS"

Study of a protein using different databases

BSc (Biochemistry) Project

Department of Biochemistry

In partial fulfilment of the requirement for the
Award of Degree in
BACHELOR OF SCIENCES

Submitted by
KesarapalliPavani, GuruvelliNeeraja
Redg.No-18313204, 18313218

UNDER THE GUIDENCE OF

Sd. V. Tahaseen
LECTURER IN biochemistry
S.R.R & C.V.R Government Degree College (A)
VIJAYAWADA – 520004

DEPARTMENT OF BIOCHEMISTRY

S.R.R & C.V.R Government Degree College (A)

Vijayawada-52004

CERTIFICATE

This is to certify that project work entitled **"BIOINFOMATICS-Study of a protein using different databases"** is a bonafide report carried out by**KesarapalliPavani, Guruvelli Neeraja-18313204, 18313218**of **III BSc MBC**Bioinformatics Cluster students submitted to the **Department of Biochemistry, S.R.R & C.V.R Government Degree College (A), Vijayawada** during**August2021.**

Sd. V. Tahaseen

Under the guidance of Head of the departmentSd. V. Tahaseen MSc, B.Ed, M.Phill
LECTURER IN biochemistry

Sd. V. Tahaseen

Sd. V. Tahaseen MSc, B.Ed, M.Phil
LECTURER IN biochemistry

A. Pallavi
29/8/2021
External Examiner.

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INTRODUCTION:-

A family of peptides with broad-spectrum antimicrobial activity has been isolated from the skin of the African clawed frog **Xenopus laevis.** It consists of two closely related peptides that are each 23 amino acids and differ by two substitutions. These peptides are water soluble, nonhemolytic at their effective antimicrobial concentrations, and potentially amphiphilic. At low concentrations they inhibit growth of numerous species of bacteria and fungi and induce osmotic lysis of protozoa. The sequence of a partial cDNA of the precursor reveals that both peptides derive from a common larger protein. These peptides appear to represent a previously unrecognized class of vertebrate antimicrobial activities.

Function:-

The **magainins** are a class of antimicrobial peptides found in the African clawed frog (*Xenopus laevis*).The peptides are cationic, generally lack a stable conformation in water but form amphipathic α-helix in membranes; their mechanism against micro-organisms is unclear but they disrupt the cell membranes of a broad spectrum of bacteria, protozoa, and fungi.

Xenopus laevis :-

The **African clawed frog** (*Xenopus laevis*, also known as the **xenopus**, **African clawed toad**, **African claw-toed frog** or the ***platanna***) is a species of African aquatic frog of the family Pipidae. Its name is derived from the three short claws on each hind foot, which it uses to tear apart its food. The word *Xenopus* means "strange foot" and *laevis* means "smooth".

The species is found throughout much of Sub-Saharan Africa (Nigeria and Sudan to South Africa), and in isolated, introduced populations in North America, South America, Europe, and Asia.All species of the family Pipidae are tongueless, toothless and completely aquatic. They use their hands to shove food in their mouths and down their throats and a hyobranchial pump to draw or suck things in their mouth. Pipidae have powerful legs for swimming and lunging after food. They also use the claws on their feet to tear pieces of large food. They have no external eardrums, but instead subcutaneous cartilaginous disks that serve the same function.[3] They use their sensitive fingers and sense of smell to find food. Pipidae are scavengers and will eat almost anything living, dying, or dead and any type of organic waste.

1. Retrieval of a Genbank Entry using an accession number

Aim:

To retrieve a nucleotide sequence of interest from Genbank entry with Specific accession number

Procedure:

1. Open the web browser and type the database address http://www.ncbi.nlm.nih.gov/genbank/ in the address bar.
2. Type the query (i.e., the strain name for which the nucleotide sequence has to be retrieved) in the search bar.
3. Select the particular sequence which is required.
4. Select FASTA format in the display settings and click apply.
5. Copy the sequence data and paste it in the notepad.
6. Save the notepad file in FASTA format.
7. Report the result.

Result:

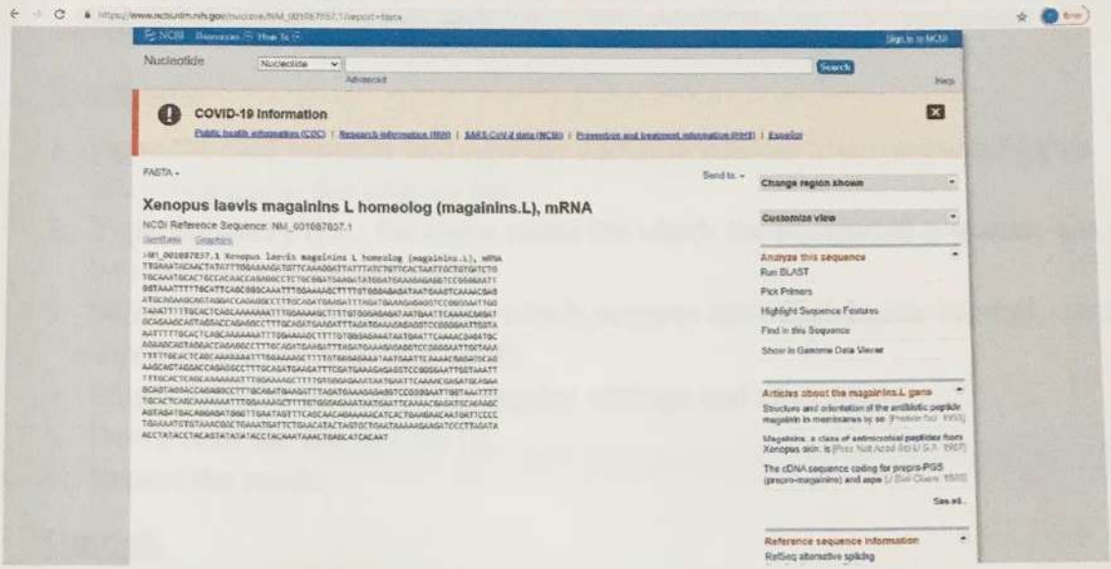
Thus, the nucleotide sequence (**Accession Number:** NM_001087837.1) was retrieved from GenBank Database.

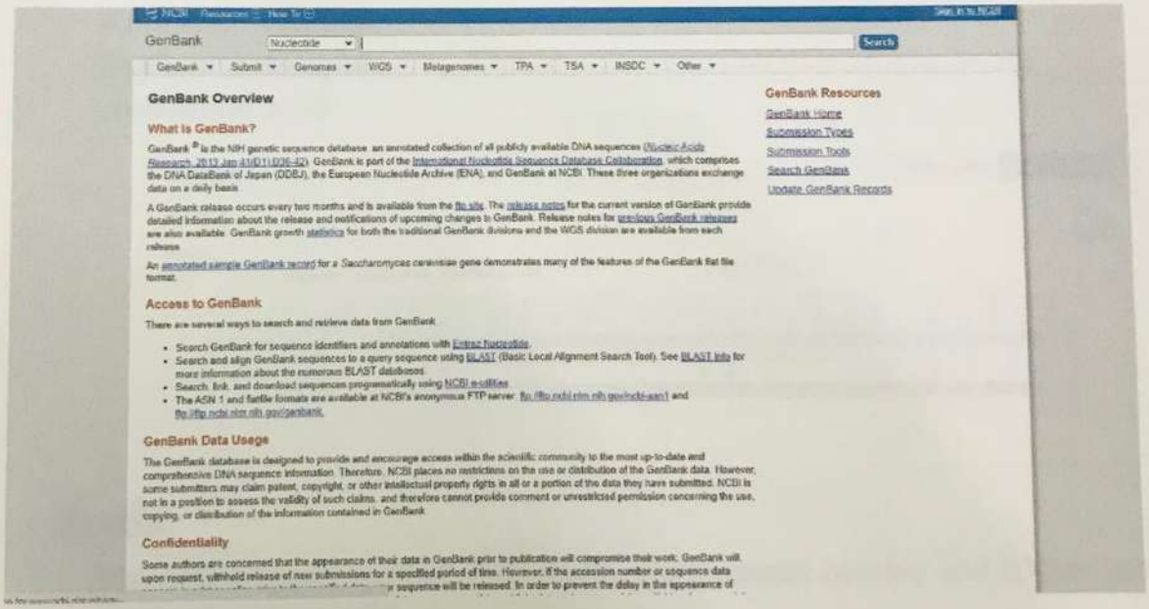
Xenopus laevis magainins L homeolog

DNA sequence in FASTA format (Nucleotides: 1098bp)

>NM_001087837.1 Xenopus laevis magainins L homeolog (magainins.L), mRNA

TTGAAATACAACTATATTTGGAAAAGATGTTCAAAGGATTATTTATCTGTTCACTAATTGCTGTG
ATCTGTGCAAATGCACTGCCACAACCAGAGGCCTCTGCGGATGAAGATATGGATGAAAGAGAG
GTCCGGGGAATTGGTAAATTTTTGCATTCAGCGGGCAAATTTGGAAAAGCTTTTGTGGGAGAGA
TAATGAAGTCAAAACGAGATGCAGAAGCAGTAGGACCAGAGGCCTTTGCAGATGAAGATTTAG
ATGAAAGAGAGGTCCGGGGAATTGGTAAATTTTTGCACTCAGCAAAAAAATTTGGAAAAGCTTT
TGTGGGAGAGATAATGAATTCAAAACGAGATGCAGAAGCAGTAGGACCAGAGGCCTTTGCAGA
TGAAGATTTAGATGAAAGAGAGGTCCGGGGAATTGGTAAATTTTTGCACTCAGCAAAAAAATTT
GGAAAAGCTTTTGTGGGAGAAATAATGAATTCAAAACGAGATGCAGAAGCAGTAGGACCAGAG
GCCTTTGCAGATGAAGATTTAGATGAAAGAGAGGTCCGGGGAATTGGTAAATTTTTGCACTCAG
CAAAAAAATTTGGAAAAGCTTTTGTGGGAGAAATAATGAATTCAAAACGAGATGCAGAAGCAG
TAGGACCAGAGGCCTTTGCAGATGAAGATTTCGATGAAAGAGAGGTCCGGGGAATTGGTAAATT
TTTGCACTCAGCAAAAAAATTTGGAAAAGCTTTTGTGGGAGAAATAATGAATTCAAAACGAGAT
GCAGAAGCAGTAGGACCAGAGGCCTTTGCAGATGAAGATTTAGATGAAAGAGAGGTCCGGGGA
ATTGGTAAATTTTTGCACTCAGCAAAAAAATTTGGAAAAGCTTTTGTGGGAGAAATAATGAATT
CAAAACGAGATGCAGAAGCAGTAGATGACAGGAGATGGGTTGAATAGTTTCAGCAACAGAAAA
ACATCACTGAAGAACAATGATTCCCCTGAAAATGTGTAAACGGCTGAAATGATTCTGAACATAC
TAGTGCTGAATAAAAAGAAGATCCCTTAGATAACCTATACCTACAGTATATATACCTACAAATA
AACTGAGCATCACAAT


Nucleotide
COVID-19 Information
FASTA
Xenopus laevis magainins L homeolog (magainins.L), mRNA
NCBI Reference Sequence: NM_001087837.1
Change region shown
Customize view
Analyze this sequence
Run BLAST
Pick Primers
Highlight Sequence Features
Find in this Sequence
Show in Genome Data Viewer
Articles about the magainins.L gene
Reference sequence information


GenBank
Nucleotide
Search
GenBank
Submit
Genomes
WGS
Metagenomes
TPA
TSA
INSDC
Other
GenBank Overview
What is GenBank?
GenBank Resources
GenBank Home
Submission Types
Submission Tools
Search GenBank
Update GenBank Records
Access to GenBank
GenBank Data Usage
Confidentiality

2. FINDING THE OFFICIAL SYMBOL, ALIAS NAME, CHROMOSOME NUMBER AND ID FOR GENE USING NCBI

Aim:

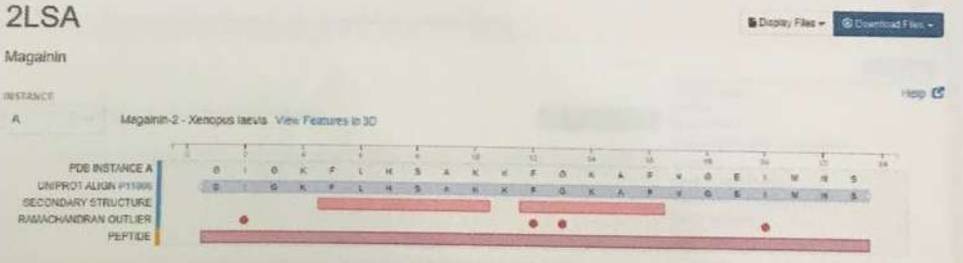
To analyze the NCBI web site and find the official gene symbol, its alias name, chromosome number and its ID.

Procedure:

1. Open the web browser and type the database address http://www.ncbi.nlm.nih.gov/gene/in the address bar.
2. Type the query (i.e., the strain name for which the nucleotide sequence has to be retrieved) in the search bar.
3. Select the particular sequence which requires the official gene symbol, name, chromosome number and its ID.
4. Select FASTA format in the display settings and click apply.
5. Down load the file in FASTA.
6. Report the result.

Output:

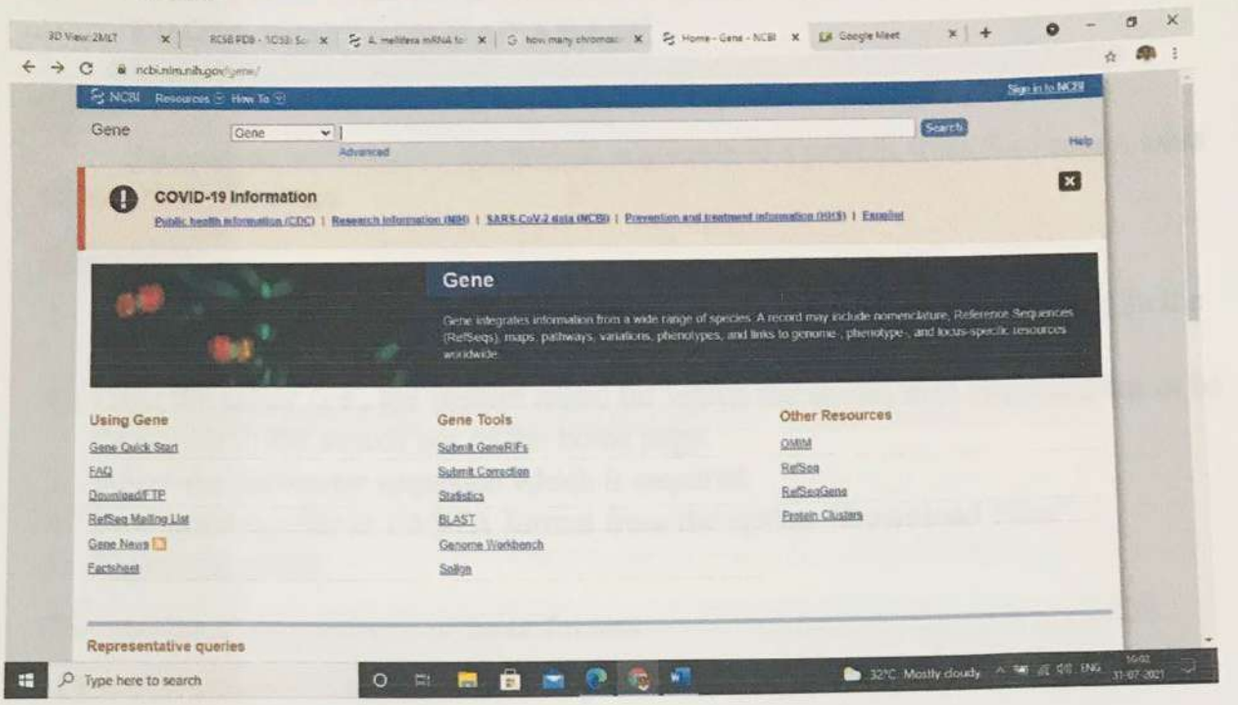
Kingdom: Animalia
Phylum: Arthropoda
chromosome: 36

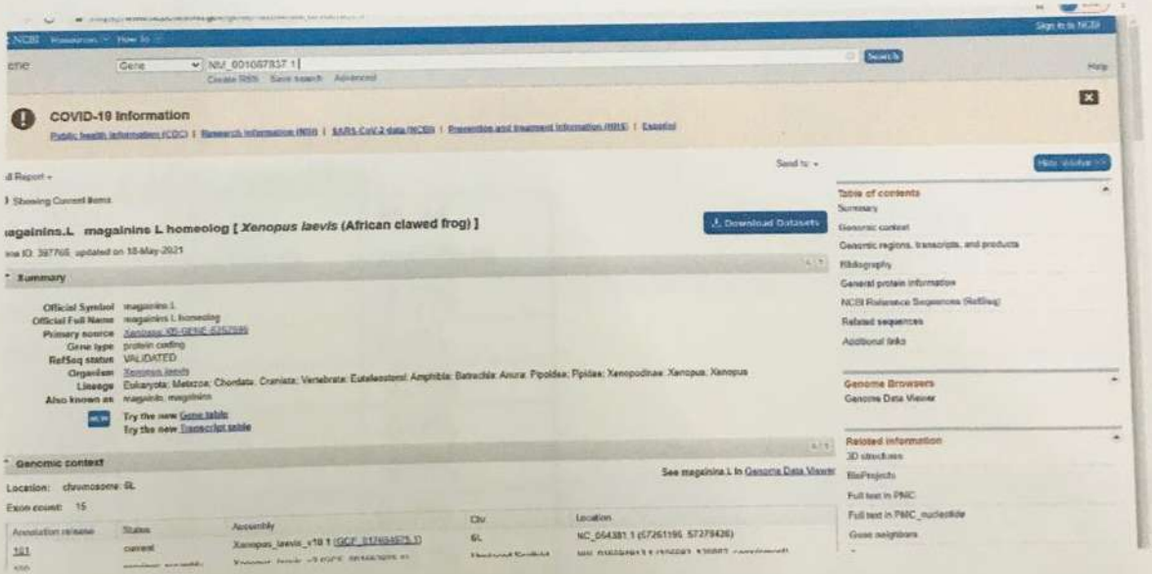


Result:

The official gene symbol, alias name, chromosome number and ID has been retrieved from the NCBI

observation :





3. RETRIEVAL AND ANALYSIS OF PROTEIN SEQUENCE FROM PROTEIN DATABASE

Aim:

To analyse and retrieve the protein sequence of a protein from the Protein Data Bank (PDB) database.

Procedure:

1. Open the web browser and type the database address https://www.rcsb.org/ in the address bar.
2. Type the query (i.e., the protein name for which the amino acid sequence has to be retrieved) in the search bar of the home page.
3. Select the particular sequence which is required.
4. Download the file in FASTA format from the option "Download Files".
5. Report the result.

The retrieved sequence is in fasta format

>2LSA_1|Chain A|Magainin-2|Xenopus laevis (8355)

GIGKFLHSAKKFGKAFVGEIMNS

Kingdom: Animalia
Phylum: Arthropoda
chromosome: 36
Accession No: **2lsa**
NUMBER OF AMONO ACIDS:-23

Amino acid composition:

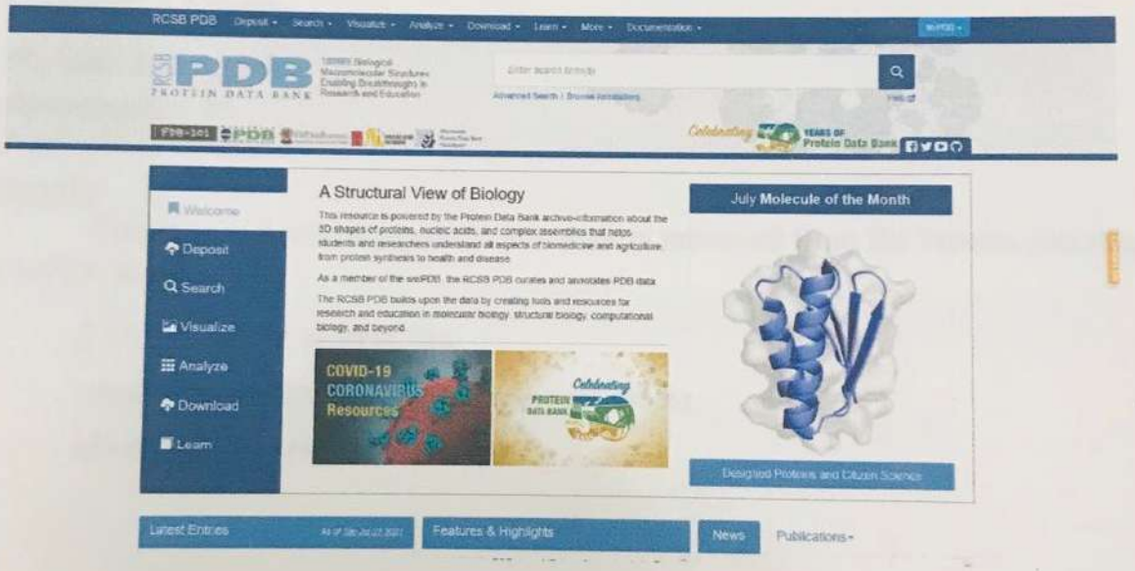
Amino acid	No of residues	% of residues
Ala(A)	2	8.7%
Arg(R)	0	0.0%
Asn(N)	1	4.3%
Asp(D)	0	0.0%
Cys(C)	0	0.0%
Gln(Q)	0	0.0%
Glu(E)	1	4.3 %
Gly(G)	4	17.4%
His(H)	1	4.3%
Ile(I)	2	8.7%
Leu(L)	1	4.3%
Lys(K)	4	17.4%
Met(M)	1	4.3%

Phe(F)	3	13.0%
Pro(P)	0	0.0%
Ser(S)	2	8.7%
Thr(T)	0	0.0%
Trp(W)	0	0.0%
Tyr(Y)	0	0.0%
Val(V)	1	4.3%
Pyl(O)	0	0.0%
Sec (U)	0	0.0%

(B) 0 0.0%
(Z) 0 0.0%
(X) 0 0.0%

Result:

Thus, the required protein sequence is retrieved from the PDB database.



4. PRIMARY STRUCTURE ANALYSIS OF A PROTEIN

(i). Retrieval of Primary Protein Structure

Aim:

To retrieve the structure of a protein from Protein Data Bank (PDB) database.

Procedure:

1. Open the web browser and type the database address www.pdb.org in the address bar.
2. Type the query (the protein name for which structure has to be retrieved) in the search bar of the home page.
3. Select the protein of interest.
4. Download the file in FASTA format from the option "Download Files".
5. Report the result.

Kingdom: Animalia

Phylum: Arthropoda
chromosome: 36
Accession No: **2lsa**

Result:

The required primary protein structure is retrieved from the Protein Database in FASTA format

FASTA format protein file

>2LSA_1|Chain A|Magainin-2|Xenopus laevis (8355)

GIGKFLHSAKKFGKAFVGEIMNS

(ii). Analysis of primary protein structure

Aim:

To compute the physical and chemical parameters of the given protein.

Procedure:

1. Open the web browser and type the database address www.expasy.org/tools in the address bar.
2. Select ProtParam from the list of displayed tools.
3. Paste the query sequence in the box given.
4. Click the option "Compute Parameters".
5. Report the result.

The computed parameters are,

Number of amino acids	**23**
Molecular Weight	2466.93
Theoretical pI	10
Total number of negatively charged residues (Asp + Glu)	1
Total number of positively charged residues (Arg + Lys)	4
Number of atoms	354

Result:

Thus, protein parameters were computed using ProtParam tool.

5. SECONDARY STRUCTURE ANALYSIS OF A PROTEIN

Aim:

To retrieve the secondary structure of a protein from Protein Data Bank (PDB) database

Procedure:

1. Open the web browser and type the database address https://www.rcsb.org/ in the address bar.
2. Type the query (i.e., the protein name for which structure has to be retrieved) in the search bar of the home page.
3. Select the protein of interest.
4. Click on the required part of the protein
5. Report the result

Output:

Kingdom: Animalia
Phylum: Arthropoda
chromosome: 36
Accession No: **2lsa**

Result:

The required secondary protein structure is retrieved from the Protein Database in pdb format

6. TERTIARY PROTEIN STRUCTURE ANALYSIS USING RASMOL

Aim:

To view the tertiary structure and analyze the protein using Rasmol.

Procedure:

1. Open the working page of offline tool.
2. Upload the pdb file of desired protein.
3. Structure will be displayed.
4. View the different forms of structure using display option.
5. Change the format into GIF in export option.

Result:

Thus, the tertiary structure of protein is viewed in Rasmol.(Magainin

7. PAIR-WISE SEQUENCE ALIGNMENT AND MULTIPLE SEQUENCE ALIGNMENT USING CLUSTALW

Aim:

To perform pair wise and multiple sequence alignment using clustalw for given sequences.

Procedure:

1. Open the web browser and type https://www.genome.jp/tools-bin/clustalw
2. Upload the sequences from the Notepad or paste the sequences in FASTA format.
3. Upload two sequences for pair-wise alignment or more than two sequences for multiple sequence alignment. After uploading, choose the "Execute Multiple Alignment" option in the alignment icon.
4. Sequence alignment results will be appeared within few seconds after execution.
5. Report the result.

Result:

Thus the given sequences are aligned using ClustalW.

Multiple Sequence Alignment by CLUSTALW

ETE3 | MAFFT | CLUSTALW | PRRN

Help

General Setting Parameters:

Output Format: CLUSTAL

Pairwise Alignment: FAST/APPROXIMATE SLOW/ACCURATE

Enter your sequences (with labels) below (copy & paste): PROTEIN DNA

Support Formats: FASTA (Pearson), NBRF/PIR, EMBL/Swiss Prot, GDE, CLUSTAL, and GCG/MSF

```
>2LSA_1|Chain A|Magainin-2|Xenopus laevis (8355)
GIGKFLHSAKKFGKAFVGEIMNS
>2MAG_1|Chain A|MAGAININ 2|Xenopus laevis (8355)
GIGKFLHSAKKFGKAFVGEIMNSX
```

Or give the file name containing your query

Choose File | No file chosen

Execute Multiple Alignment | Reset

More Detail Parameters...

Pairwise Alignment Parameters:

For FAST/APPROXIMATE:

K-tuple(word) size: 1, Window size: 5, Gap Penalty: 3

Number of Top Diagonals: 5, Scoring Method: PERCENT

For SLOW/ACCURATE:

Gap Open Penalty: 10.0, Gap Extension Penalty: 0.1

Select Weight Matrix: BLOSUM (for PROTEIN)

(Note that only parameters for the algorithm specified by the above "Pairwise Alignment" are

rcsb.pdb.2MAG.fasta ^ 2lsa.pdb ^

CLUSTALW Result

[clustalw.aln][clustalw.dnd][readme]

Select tree menu | Exec

```
 CLUSTAL 2.1 Multiple Sequence Alignments

Sequence type explicitly set to Protein
Sequence format is Pearson
Sequence 1: 2LSA_1|Chain     23 aa
Sequence 2: 2MAG_1|Chain     24 aa
Start of Pairwise alignments
Aligning...

Sequences (1:2) Aligned. Score: 100
Guide tree file created:   [clustalw.dnd]

There are 1 groups
Start of Multiple Alignment

Aligning...
Group 1: Sequences:   2      Score:375
Alignment Score 156

CLUSTAL-Alignment file created  [clustalw.aln]
```

clustalw.aln

```
CLUSTAL 2.1 multiple sequence alignment

2LSA_1|Chain      GIGKFLHSAKKFGKAFVGEIMNS-
2MAG_1|Chain      GIGKFLHSAKKFGKAFVGEIMNSX
                  ***********************
```

clustalw.dnd

```
(2LSA_1|Chain:0,2MAG_1|Chain:0);
```

Select tree menu | Exec

8. PAIR WISE AND MULTIPLE SEQUENCE ALIGNMENT USING BLAST

Aim:

To perform pair wise and multiple sequence alignment using BLAST tool.

BLAST

BLAST is an acronym for Basic Local Alignment Search Tool. BLAST is a sophisticated software package that has become the single most important piece of software in the field of Bioinformatics. The following tools are different type of blast programs used with respect to the aim of the user.

Blast programs:

BLASTp – Compares an amino acid query sequence against a protein sequence database.

BLASTn – Compares a nucleotide query sequence against a nucleotide sequence database.

BLASTx – Compares a nucleotide query sequence translated in all reading frames against a protein sequence database.

tBLASTn – Compares a protein query sequence against a nucleotide sequence database dynamically translated in all reading frames.

tBLASTx – Compares the six frame translation of a nucleotide query sequence against the 6 frame translation of a nucleotide sequence database.

Megablast – For highly similar sequences.

Procedure:

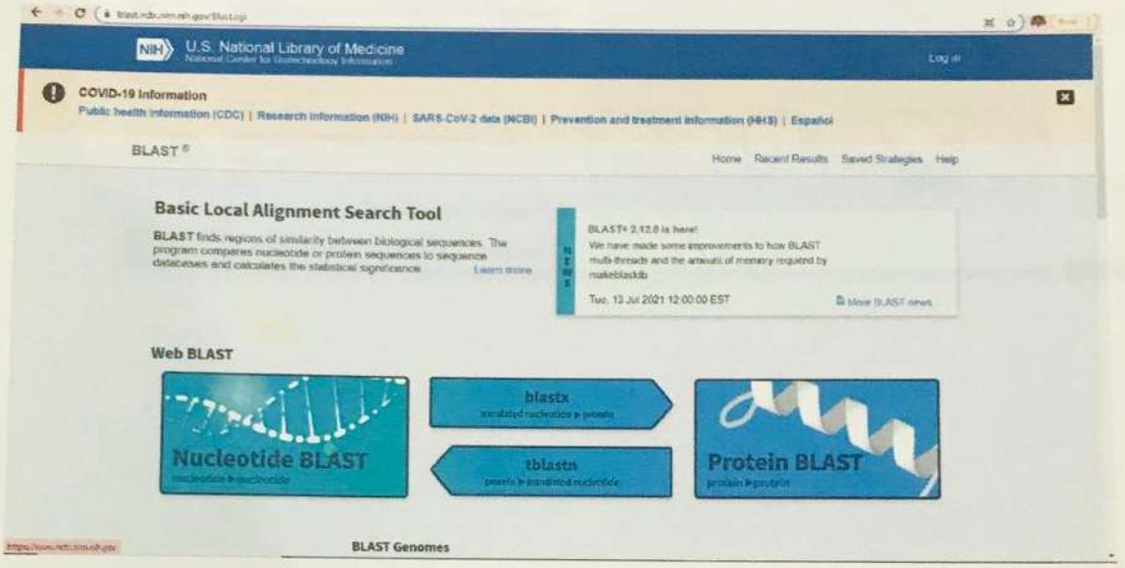
1. Open the web browser and type https://blast.ncbi.nlm.nih.gov/Blast.cgi
2. Click either nucleotide blast or protein blast icon according to the requirement
3. Select "Align two or more sequences" check box for opting multiple sequence alignment or deselect for pair wise alignment
4. Upload or paste a query sequence (in FASTA format) in the query box and execute BLAST for pair wise alignment. This will be identifying most similar sequences from the databank.
5. Upload or paste a query sequence (in FASTA format) in the query box and upload more than one sequences (in FASTA format) in the subject box and then

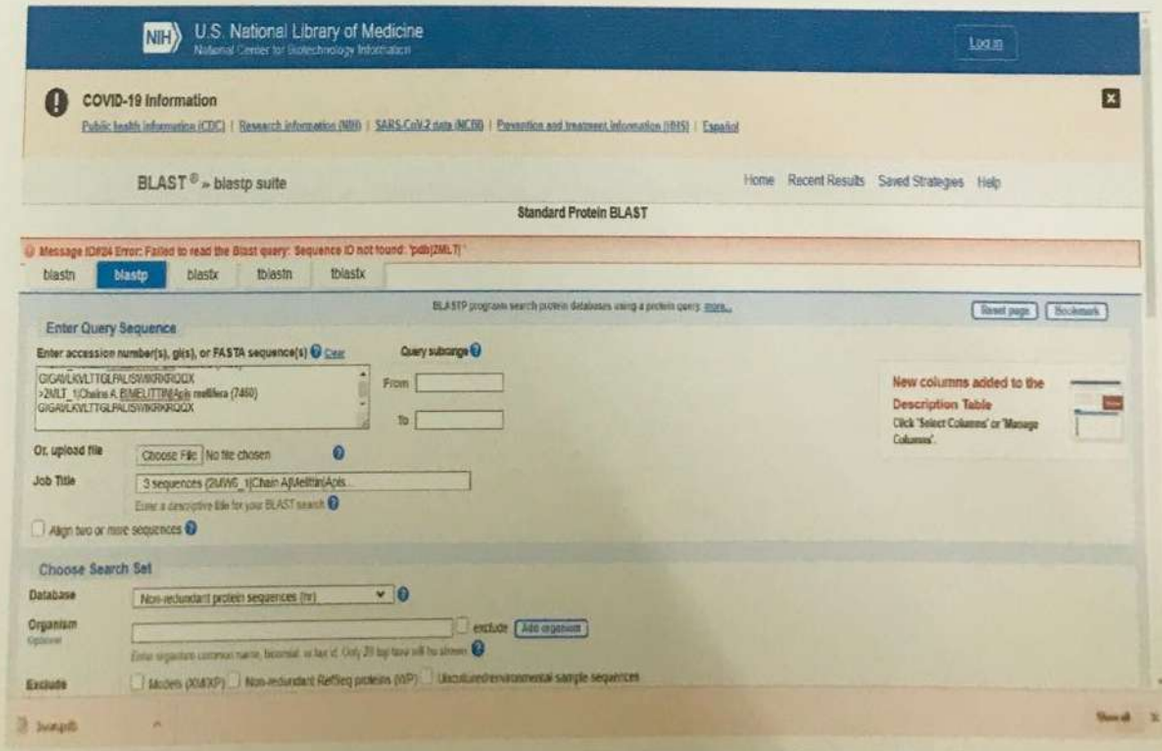
execute BLAST for multiple sequence alignment. This will be identifying the similarity /dissimilarity among the sequences.

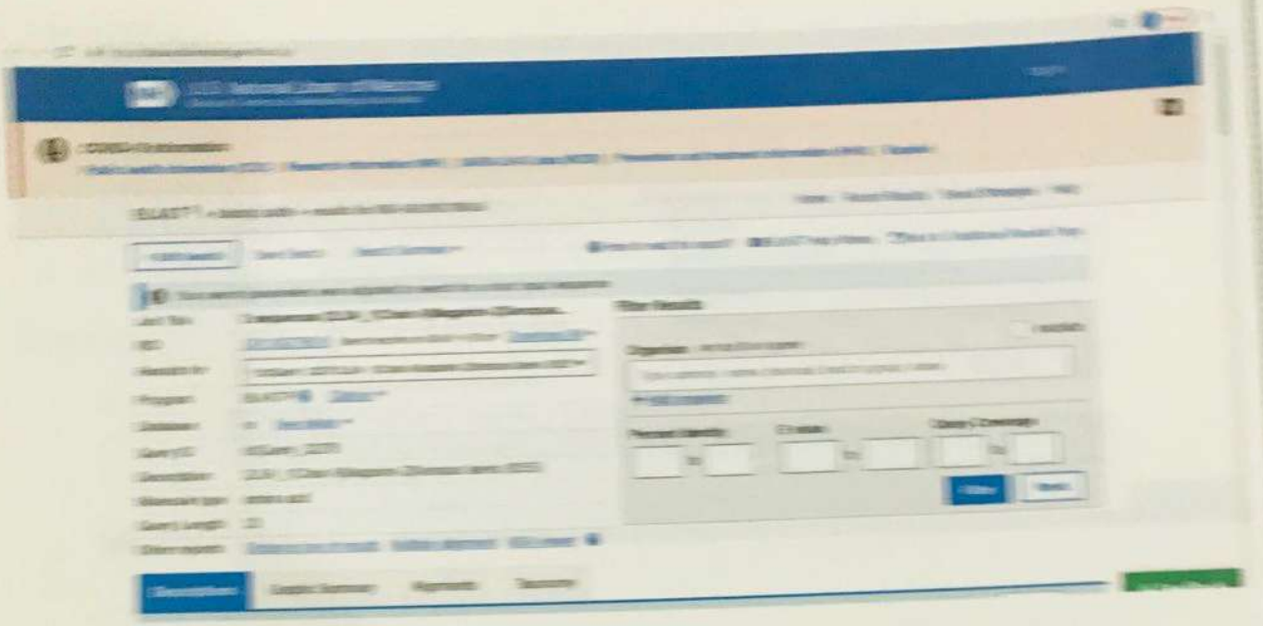
6. Report the result

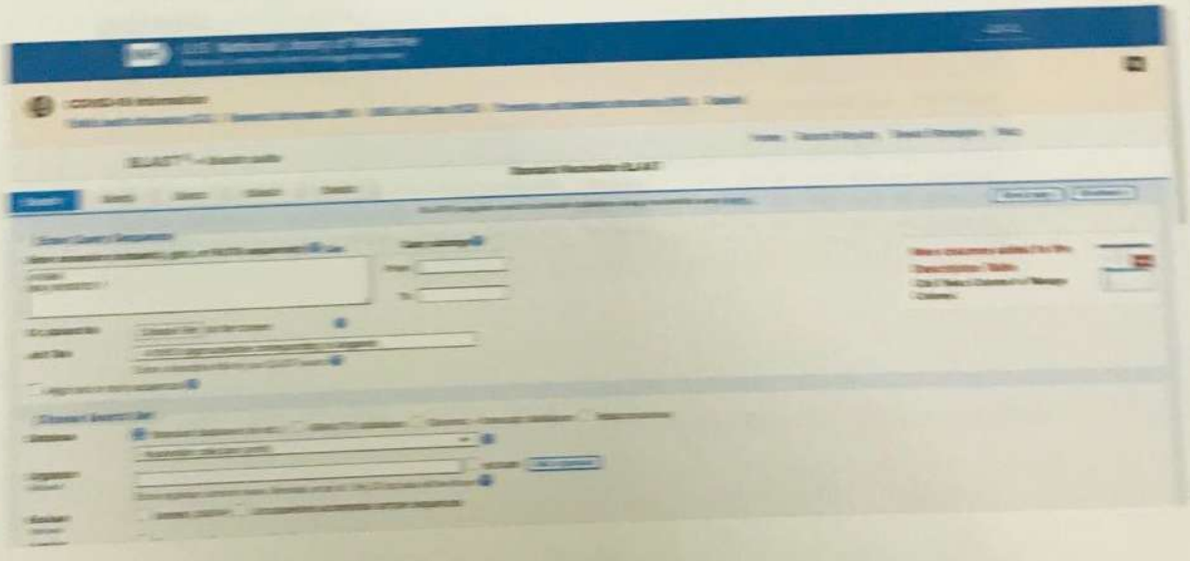
Result:

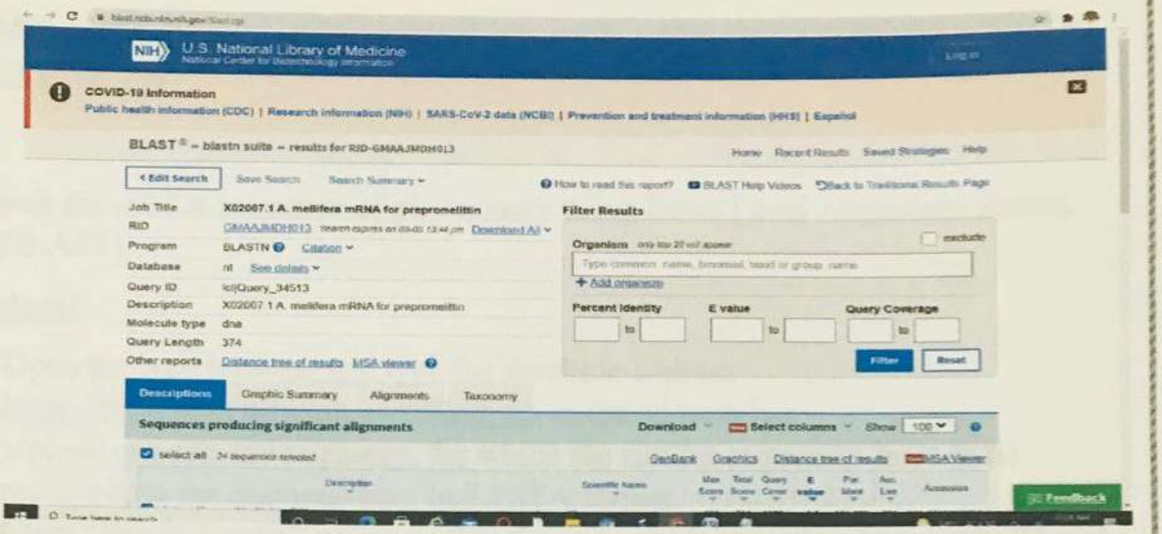
Thus the given sequences are aligned using BLAST

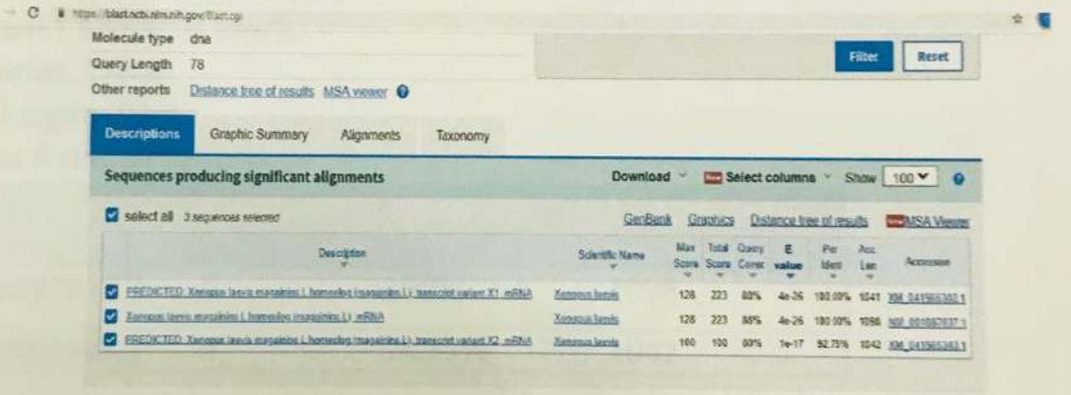









U.S. National Library of Medicine
COVID-19 Information
BLAST® – blastn suite – results for RID-GMAAJMDH013
Job Title X02067.1 A. mellifera mRNA for prepromelittin
RID GMAAJMDH013
Program BLASTN
Database nt
Query ID lcl|Query_34513
Description X02067.1 A. mellifera mRNA for prepromelittin
Molecule type dna
Query Length 374
Other reports Distance tree of results MSA viewer
Filter Results
Organism
Percent Identity
E value
Query Coverage
Filter
Reset
Descriptions
Graphic Summary
Alignments
Taxonomy
Sequences producing significant alignments
Download
Select columns
Show 100
select all
GenBank
Graphics
Distance tree of results
MSA Viewer
Feedback


Molecule type dna
Query Length 78
Other reports Distance tree of results MSA viewer
Filter
Reset
Descriptions
Graphic Summary
Alignments
Taxonomy
Sequences producing significant alignments
Download
Select columns
Show 100
select all 3 sequences selected
GenBank
Graphics
Distance tree of results
MSA Viewer
Description
Scientific Name
Max Score
Total Score
Query Cover
E value
Per Ident
Acc. Len
Accession
PREDICTED: Xenopus laevis myosin-I heavy chain-like (LOC108697837), transcript variant X1, mRNA
Xenopus laevis
126
223
80%
4e-26
100.00%
1041
XM_041965360.1
Xenopus laevis myosin-I heavy chain-like (LOC108697837), mRNA
Xenopus laevis
126
223
80%
4e-26
100.00%
1096
NM_001087837.1
PREDICTED: Xenopus laevis myosin-I heavy chain-like (LOC108697837), transcript variant X2, mRNA
Xenopus laevis
100
100
80%
1e-17
92.75%
1042
XM_041965363.1

9. SIMILARITY SEARCH USING BLAST AND INTERPRETATION OF RESULTS

Aim:

To search the similar sequence of given query using Basic Local Alignment Search Tool (BLAST).

Procedure:

1. Open the web browser and type the database address https://blast.ncbi.nlm.nih.gov/Blast.cgi in the address bar.
1. Upload or paste the sequence, for which the similar sequences have to be retrieved, in the dialogue box, in FASTA format.
2. Click "Begin Search" option so that the page containing the organism's ID and other details are displayed.
3. Click "View Reports" and the sequences similar to the given sequence of interest are displayed.
4. Report the sequences which have maximum identity and similarity.

Query ID: lcl|Query_A15950.1
Organism : **Xenopus laevis**
Nucleotide: DNA
Query length: 78bp
Retrieved similar sequences

ID NO.	E-value	%identity	Length of the sequence
NM001087837.1	0	100%	1098
XM041565303.1	0	98.85%	1042

Result:

The similar sequences were retrieved and the %identity and the expected value for the sequences were known

10. CONVERSION OF GENE SEQUENCE INTO ITS CORRESPONDING AMINO ACID SEQUENCE

Aim:

To convert the given gene sequence into its corresponding amino acid sequence.

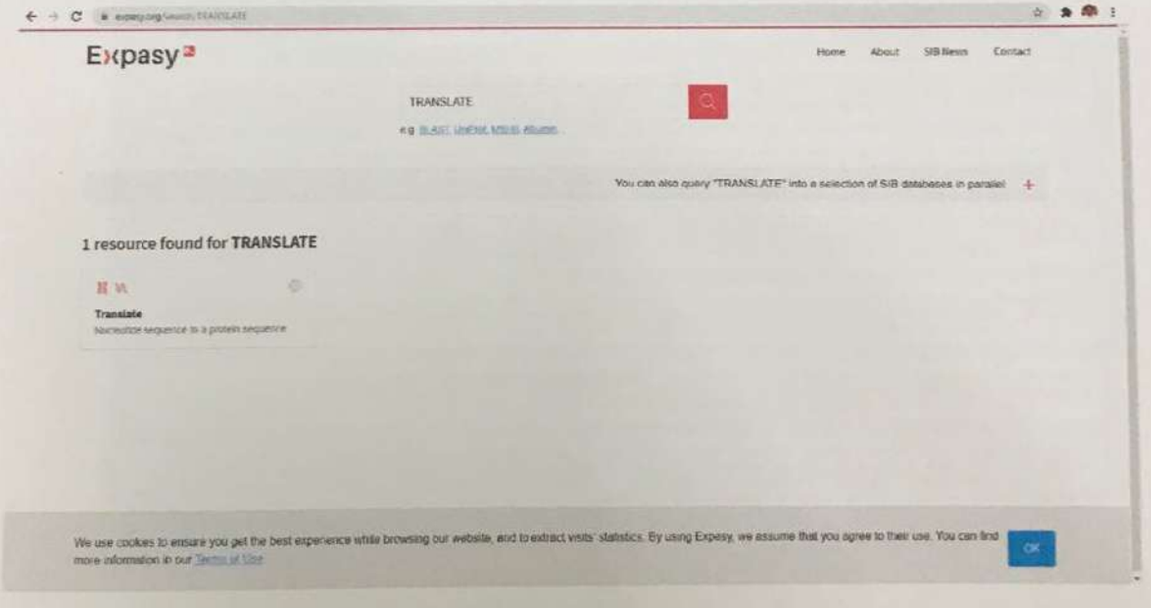
Procedure:

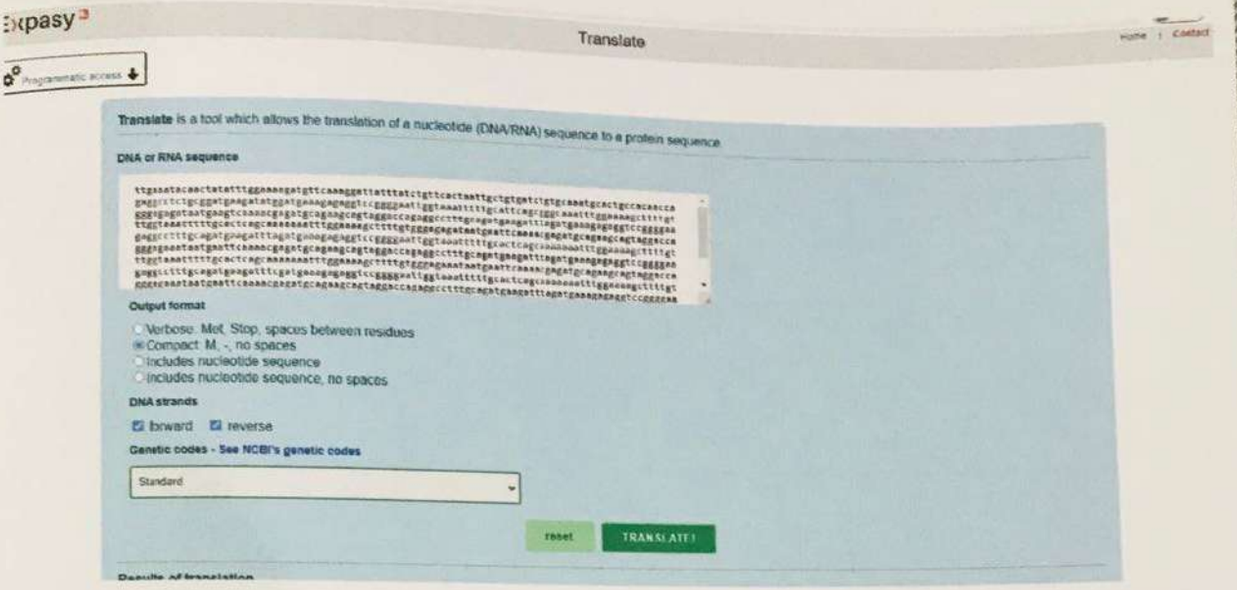
1. Open the web browser and type the database address www.expasy.org/tools in the address bar.
2. Select TRANSLATE from the list of displayed tools.
3. Paste the query sequence in the given box.
4. Click TRANSLATE option.
5. Report the result.

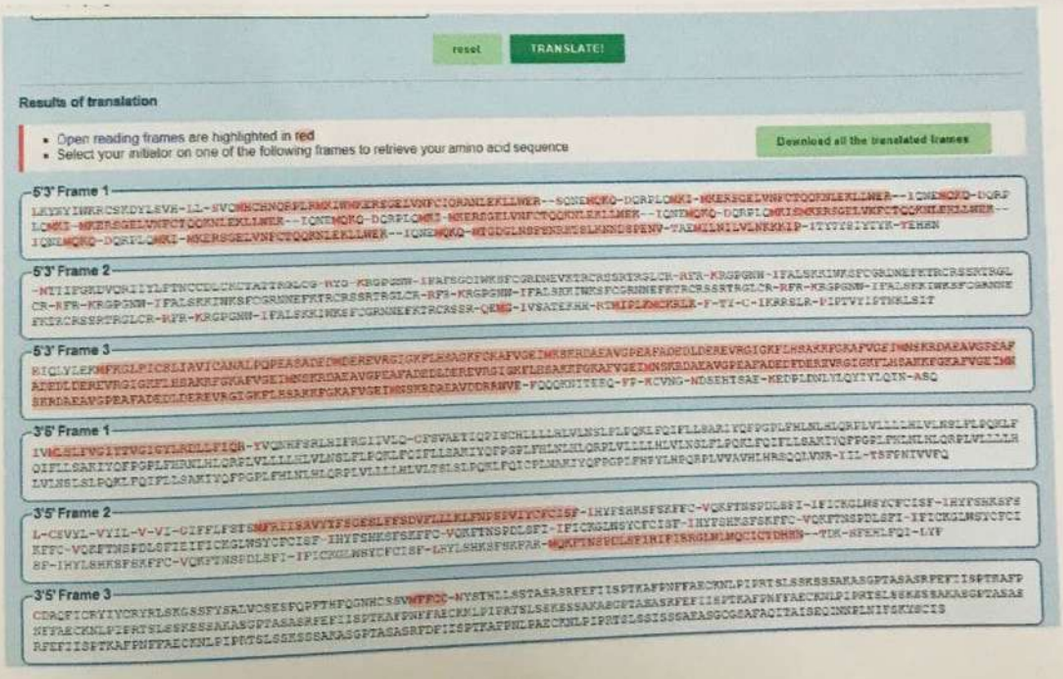
Organism: **Xenopus laevis**.
Protein: Magainins

Result:

Thus the given gene sequence is converted into its corresponding Protein sequence.




Expasy
Translate
Home | Contact
Programmatic access
Translate is a tool which allows the translation of a nucleotide (DNA/RNA) sequence to a protein sequence
DNA or RNA sequence
Output format
Verbose: Met, Stop, spaces between residues
Compact: M, -, no spaces
Includes nucleotide sequence
Includes nucleotide sequence, no spaces
DNA strands
forward
reverse
Genetic codes - See NCBI's genetic codes
Standard
reset
TRANSLATE!


reset
TRANSLATE!
Results of translation
Open reading frames are highlighted in red
Select your initiator on one of the following frames to retrieve your amino acid sequence
Download all the translated frames
5'3' Frame 1
5'3' Frame 2
5'3' Frame 3
3'5' Frame 1
3'5' Frame 2
3'5' Frame 3

A STUDY PROJECT ON

"BIOINFOMATICS"

Study of a protein using different databases

BSc (Biochemistry) Project
Department of Biochemistry

In partial fulfilment of the requirement for the
Award of Degree in
BACHELOR OF SCIENCES

Submitted by
Miriyala Vyshnavi
Redg.No-18313201

UNDER THE GUIDENCE OF
Sd. V. Tahaseen MSc, B.Ed, M.Phil
LECTURER IN biochemistry
S.R.R & C.V.R Government Degree College (A)
VIJAYAWADA – 520004

DEPARTMENT OF BIOCHEMISTRY

S.R.R & C.V.R Government Degree College (A)

Vijayawada-52004

CERTIFICATE

This is to certify that project work entitled **"BIOINFOMATICS- Study of a protein using different databases"** is a bonafide report carried out by **Miriyala Vyshnavi -18313201** of **III BSc MBC** Bioinformatics Cluster students submitted to the **Department of Biochemistry, S.R.R & C.V.R Government Degree College (A), Vijayawada** during **August2021.**

Sd.V.Tahaseen

Under the guidance of
Sd. V. Tahaseen MSc, B.Ed, M.Phill
LECTURER IN biochemistry

Sd.V.Tahaseen

Head of the department
Sd. V. Tahaseen MSc, B.Ed, M.Phil
LECTURER IN biochemistry

External Examiner.
A. Pallavi
22/8/2021

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*		Luciferase Introduction.		
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3.		Retrieval and analysis of protein sequence from protein database.		
4.		Primary structure analysis of protein		
		(i) Retrieval of Primary protein structure		
		(ii) Analysis of primary protein structure		
5.		Secondary structure analysis of protein		
6.		Tertiary structure analysis using RASMOL		
7.		Pairwise sequence alignment using CLUSTALW.		
8.		Pairwise sequence alignment using Blast (protein, DNA)		
9.		Similarity searching using Blast		
10.		Conversion of nucleotide sequence into protein sequence.		

LUCIFERASE

Luciferase is a generic term for the class of oxidative enzymes that produce bioluminescence, and is usually distinguished from a photoprotein. The name was first used by Raphaël Dubois who invented the words luciferin and luciferase, for the substrate and enzyme, respectively.[1] Both words are derived from the Latin word *lucifer*, meaning "lightbearer", which in turn is derived from the Latin words for "light" (*lux*) and "to bring or carry" (*ferre*).[2]

Luciferases are widely used in biotechnology, for microscopy and as reporter genes, for many of the same applications as fluorescent proteins. However, unlike fluorescent proteins, luciferases do not require an external light source, but do require addition of luciferin, the consumable substrate.

Luciferases are enzymes that use a substrate called luciferin, along with oxygen and ATP, in an energetic process that produces light—like the yellow glow of fireflies.

Classification: protein

Organism: Photinus pyralis (Common eastern firefly)

1.RETRIEVAL OF A GENBANK ENTRY USING AN ACCESION NUMBER

Aim: To retrieve a nucleotide sequence of interest from Genbank entry with Specific accession number.

Procedure:

1. Open the web browser and type the database address http://www.ncbi.nlm. nih.gov/genbank/ in the address bar.

2. Type the query (i.e., the strain name for which the nucleotide sequence has to be retrieved) in the search bar.

3. Select the particular sequence which is required.

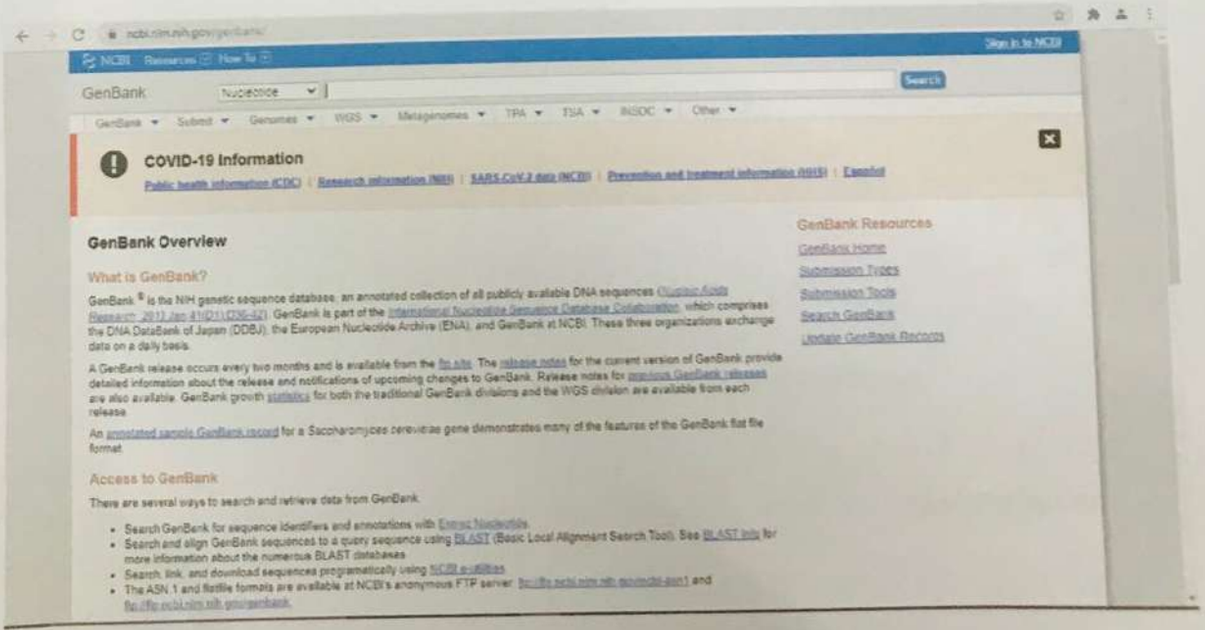
4. Select FASTA format in the display settings and click apply.

5. Copy the sequence data and paste it in the notepad.

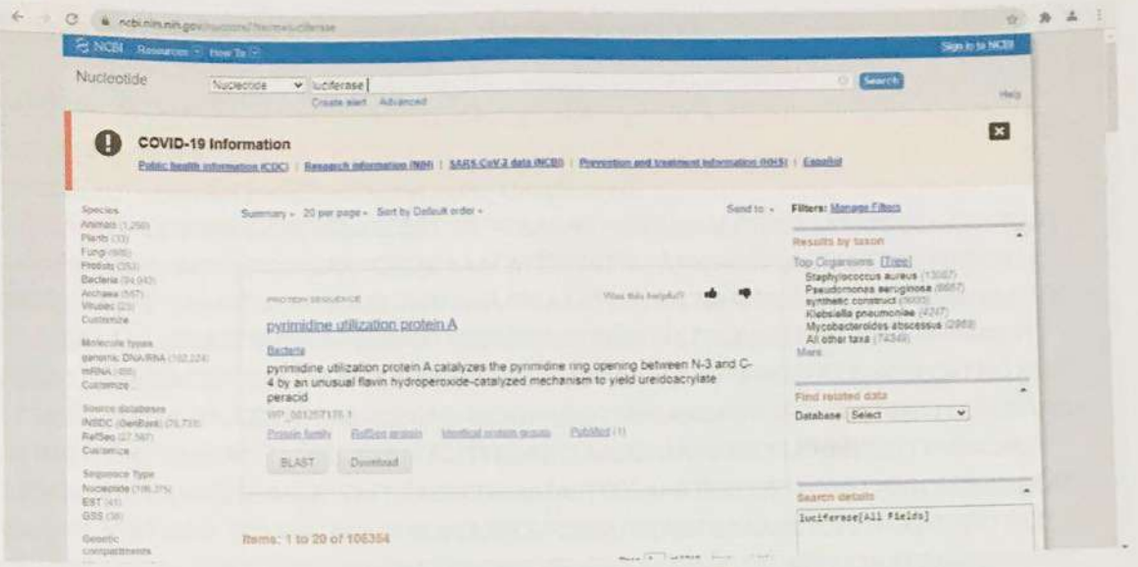
6. Save the notepad file in FASTA format.

7. Report the result.

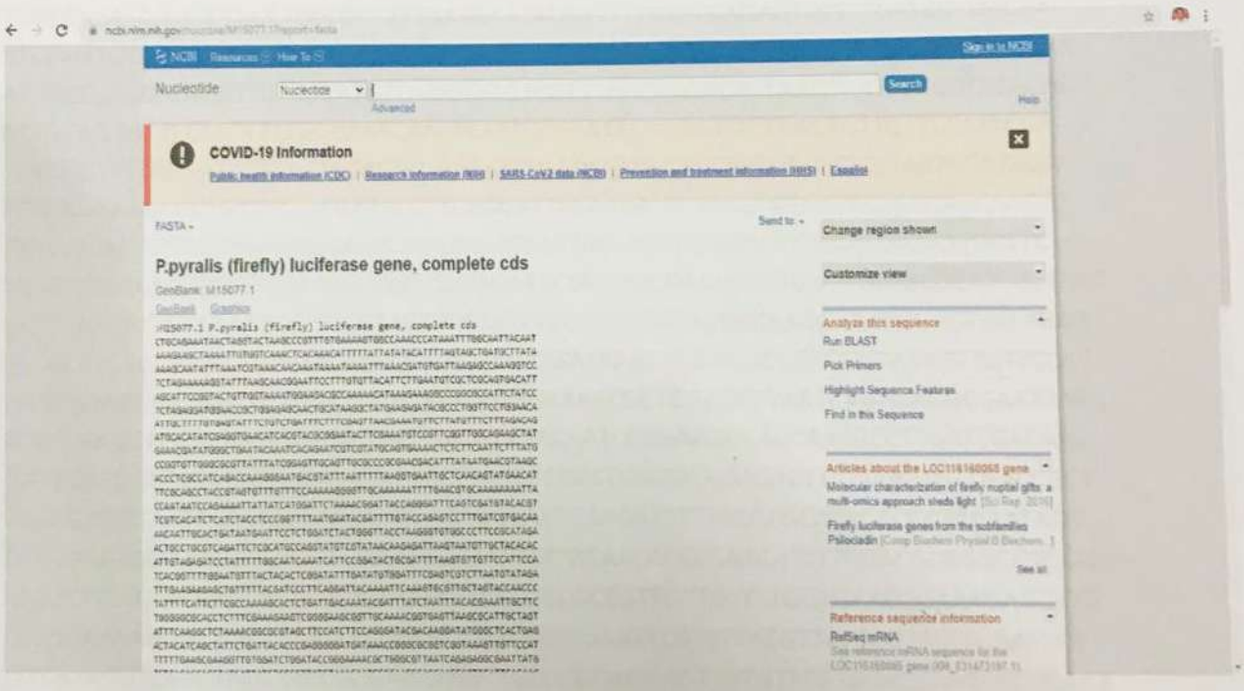
Observation:



Home page of Genbank Data base



Search Output from Gen bank database of NCBI



Retrieved Nucleotide Sequence in FASTA Format

Result:

Thus the nucleotide sequence (Accession Number: M15077.1) was retrieved from GenBank Database.

P.pyralis (firefly) luciferase gene, complete cds

DNA in FAST5A format

GenBank: M15077.1

GenBank Graphics

>M15077.1 P.pyralis (firefly) luciferase gene, complete cds

CTGCAGAAATAACTAGGTACTAAGCCCGTTTGTGAAAAGTGGCCAAACCCATAAATTTGGCAATTACAAT
AAAGAAGCTAAAATTGTGGTCAAACTCACAAACATTTTTATTATATACATTTTAGTAGCTGATGCTTATA
AAAGCAATATTTAAATCGTAAACAACAAATAAAATAAAATTTAAACGATGTGATTAAGAGCCAAAGGTCC
TCTAGAAAAAGGTATTTAAGCAACGGAATTCCTTTGTGTTACATTCTTGAATGTCGCTCGCAGTGACATT
AGCATTCCGGTACTGTTGGTAAAATGGAAGACGCCAAAAACATAAAGAAAGGCCCGGCGCCATTCTATCC
TCTAGAGGATGGAACCGCTGGAGAGCAACTGCATAAGGCTATGAAGAGATACGCCCTGGTTCCTGGAACA
ATTGCTTTTGTGAGTATTTCTGTCTGATTTCTTTCGAGTTAACGAAATGTTCTTATGTTTCTTTAGACAG
ATGCACATATCGAGGTGAACATCACGTACGCGGAATACTTCGAAATGTCCGTTCGGTTGGCAGAAGCTAT
GAAACGATATGGGCTGAATACAAATCACAGAATCGTCGTATGCAGTGAAAACTCTCTTCAATTCTTTATG
CCGGTGTTGGGCGCGTTATTTATCGGAGTTGCAGTTGCGCCCGCGAACGACATTTATAATGAACGTAAGC
ACCCTCGCCATCAGACCAAAGGGAATGACGTATTTAATTTTTAAGGTGAATTGCTCAACAGTATGAACAT
TTCGCAGCCTACCGTAGTGTTTGTTTCCAAAAAGGGGTTGCAAAAAATTTTGAACGTGCAAAAAAAATTA
CCAATAATCCAGAAAATTATTATCATGGATTCTAAAACGGATTACCAGGGATTTCAGTCGATGTACACGT
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ACTGCCTGCGTCAGATTCTCGCATGCCAGGTATGTCGTATAACAAGAGATTAAGTAATGTTGCTACACAC
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TCACGGTTTTGGAATGTTTACTACACTCGGATATTTGATATGTGGATTTCGAGTCGTCTTAATGTATAGA
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TGAAGTCTTTAATTAAATACAAAGGATATCAGGTAATGAAGATTTTTACATGCACACACGCTACAATACC
TGTAGGTGGCCCCCGCTGAATTGGAATCGATATTGTTACAACACCCCAACATCTTCGACGCGGGCGTGGC
AGGTCTTCCCGACGATGACGCCGGTGAACTTCCCGCCGCCGTTGTTGTTTTGGAGCACGGAAAGACGATG
ACGGAAAAAGAGATCGTGGATTACGTCGCCAGTAAATGAATTCGTTTTACGTTACTCGTACTACAATTCT
TTTCATAGGTCAAGTAACAACCGCGAAAAAGTTGCGCGGAGGAGTTGTGTTTGTGGACGAAGTACCGAAA
GGTCTTACCGGAAAACTCGACGCAAGAAAAATCAGAGAGATCCTCATAAAGGCCAAGAAGGGCGGAAAGT
CCAAATTGTAAAATGTAACTGTATTCAGCGATGACGAAATTCTTAGCTATTGTAATATTATATGCAAATT
GATGAATGGTAATTTTGTAATTGTGGGTCACTGTACTATTTTAACGAATAATAAAATCAGGTATAGGTAA
CTAAAAA

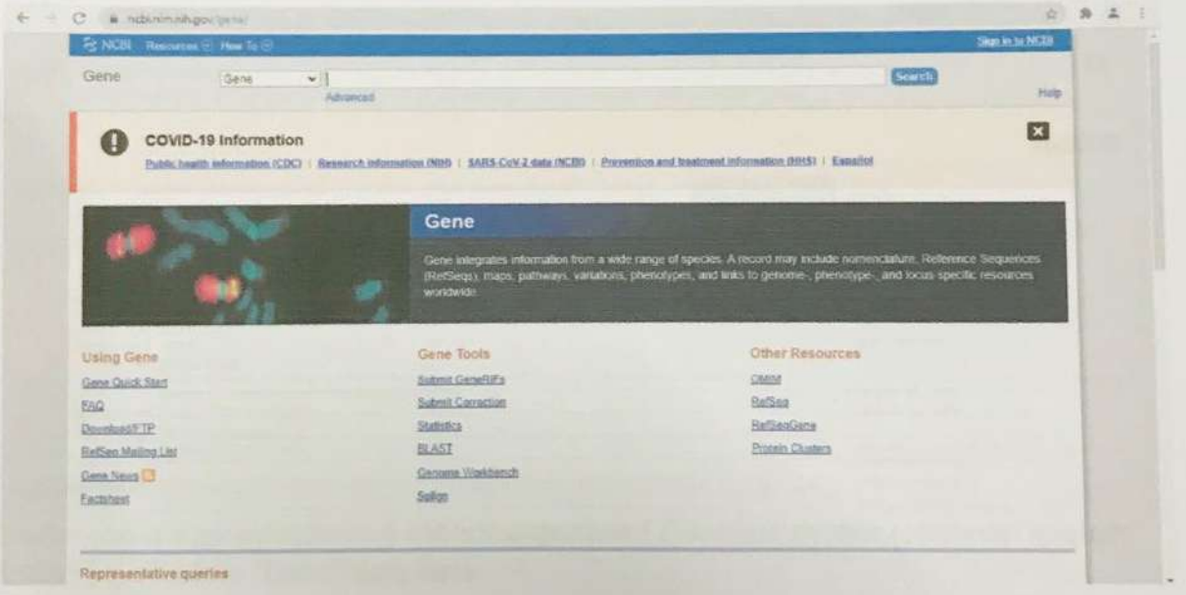
(Nucleotides = 2387)

2. FINDING THE OFFICIAL SYMBOL, ALIAS NAME, CHROMOSOME NUMBER AND ID FOR GENE USING NCBI

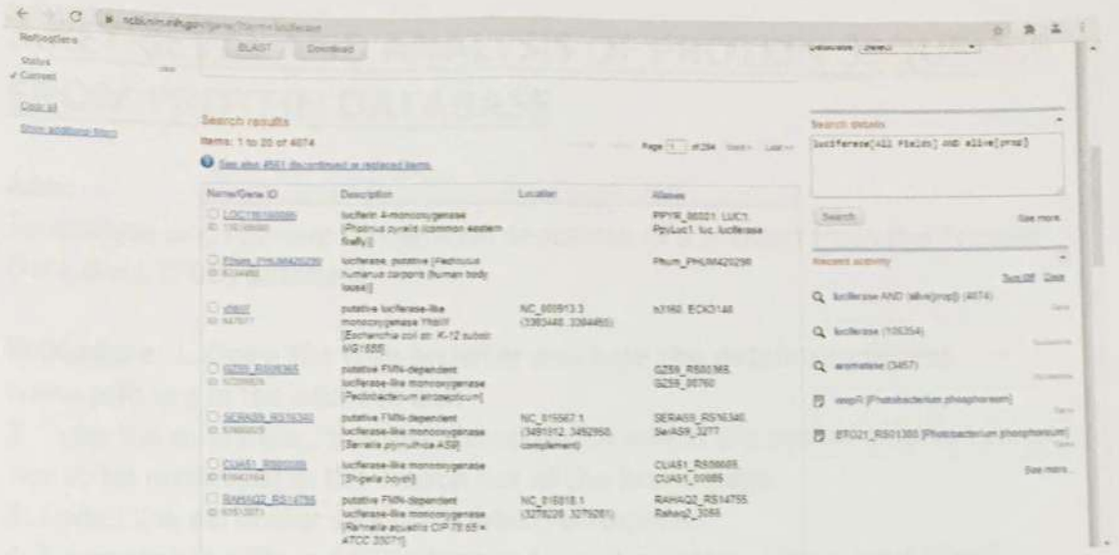
Aim: To analyze the NCBI web site and find the official gene symbol, its alias name, chromosome number and its ID.

Procedure:

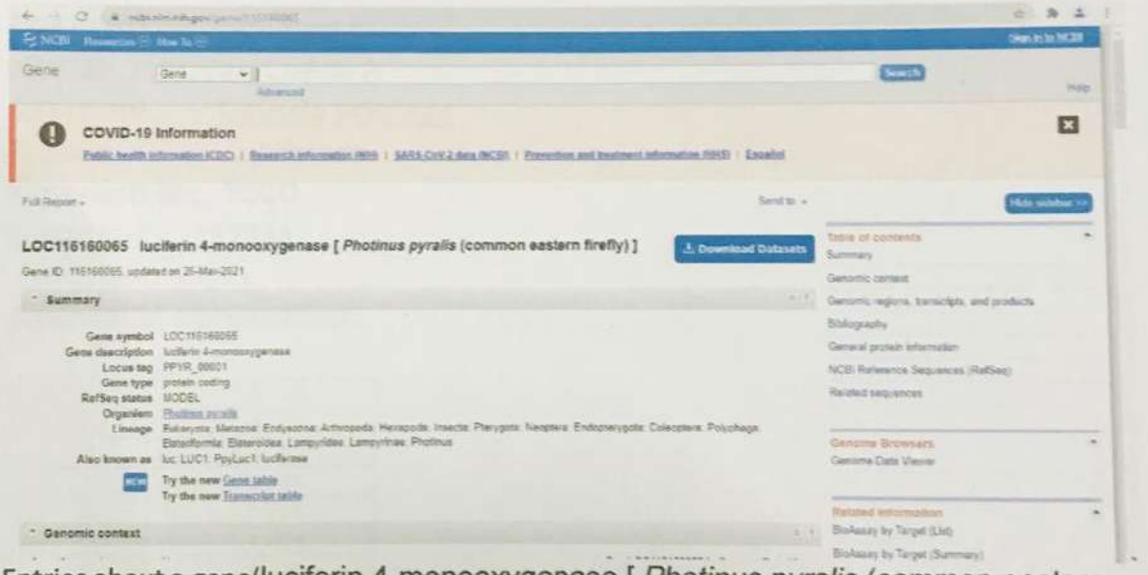
1. Open the web browser and type the database address http://www.ncbi.nlm. nih.gov/gene/ in the address bar.
2. Type the query (i.e., the strain name for which the nucleotide sequence has to be retrieved) in the search bar.
3. Select the particular sequence which requires the official gene symbol, name, chromosome number and its ID.
4. Select FASTA format in the display settings and click apply.
5. Down load the file in FASTA format.
6. Report the result.



Home page of the Data Bank "Gene"



Search output from the "Gene" data bank



Entries about a gene(luciferin 4-monooxygenase [*Photinus pyralis* (common eastern firefly)] available in "Gene" data bank

Output:

Kingdom: Eukaryota

Organism: Photinus pyralis (Common eastern firefly)

Chromosome location: chromosome: Un

Chromosomes: 20

Result: The official gene symbol, alias name, chromosome number and ID has been retrieved from the NCBI.

3.RETRIEVAL AND ANALYSIS OF PROTEIN SEQUENCE FROM PROTEIN DATABASE

Aim:
To analyse and retrieve the protein sequence of a protein from the Protein Data Bank (PDB) database.

Procedure: 1. Open the web browser and type the database address www.pdb.org in the address bar.
2. Type the query (i.e., the protein name for which the amino acid sequence has to be retrieved) in the search bar of the home page.
3. Select the particular sequence which is required.
4. Download the file in FASTA format from the option "Download Files".
5. Report the result.

The retrieved sequence is
>7D2O_1|PDBID|CHAIN A
Organism : Gaussia princeps
Protein : Luciferase
Accession No: 7D2O
Length :174aa

Aminoacid composition:

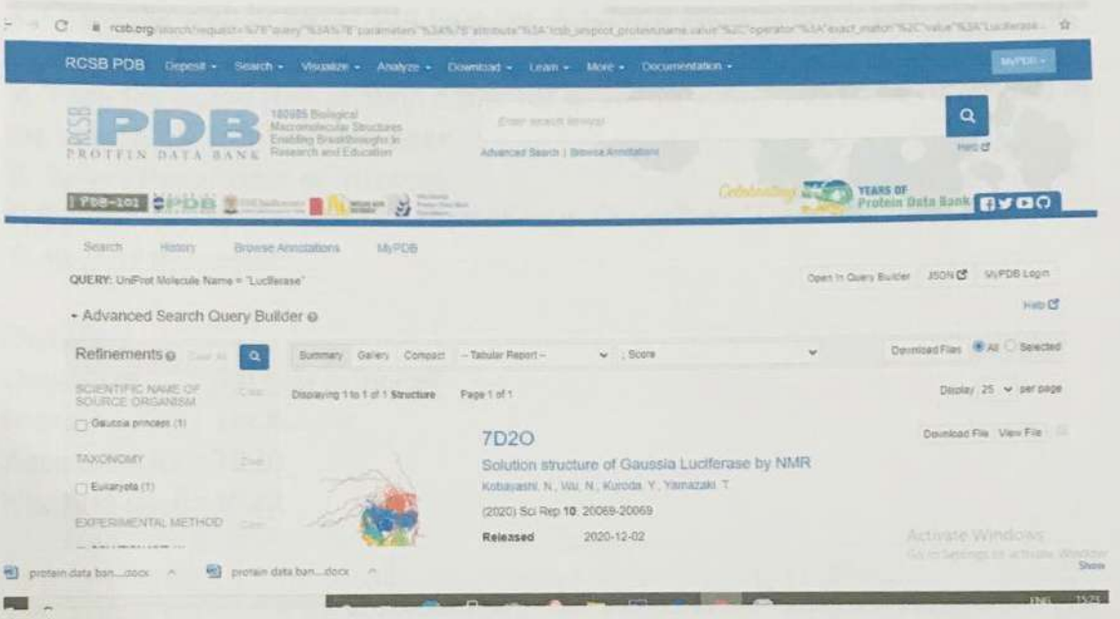
Ala (A)	15	8.6%
Arg (R)	7	4.0%
Asn (N)	6	3.4%
Asp (D)	12	6.9%
Cys (C)	10	5.7%
Gln (Q)	7	4.0%
Glu (E)	11	6.3%
Gly (G)	17	9.8%
His (H)	2	1.1%
Ile (I)	12	6.9%
Leu (L)	14	8.0%
Lys (K)	18	10.3%
Met (M)	3	1.7%
Phe (F)	6	3.4%
Pro (P)	9	5.2%
Ser (S)	5	2.9%
Thr (T)	10	5.7%
Trp (W)	1	0.6%
Tyr (Y)	1	0.6%
Val (V)	8	4.6%
Pyl (O)	0	0.0%
Sec (U)	0	0.0%

```
(B)    0          0.0%
(Z)    0          0.0%
(X)    0          0.0%
```

Result:

Thus the required protein sequence is retrieved from the PDB database.

Observation :



Information on an luciferase protein in PDB Database

4. PRIMARY STRUCTURE ANALYSIS OF A PROTEIN

(i) Retrieval of Primary Protein Structure

Aim:

To retrieve the structure of a protein from Protein Data Bank (PDB) database.

Procedure:

1. Open the web browser and type the database address www.pdb.org in the address bar.
2. Type the query (the protein name for which structure has to be retrieved) in the search bar of the home page.
3. Select the protein of interest.
4. Download the file in FASTA format from the option "Download Files".
5. Report the result.

Output :

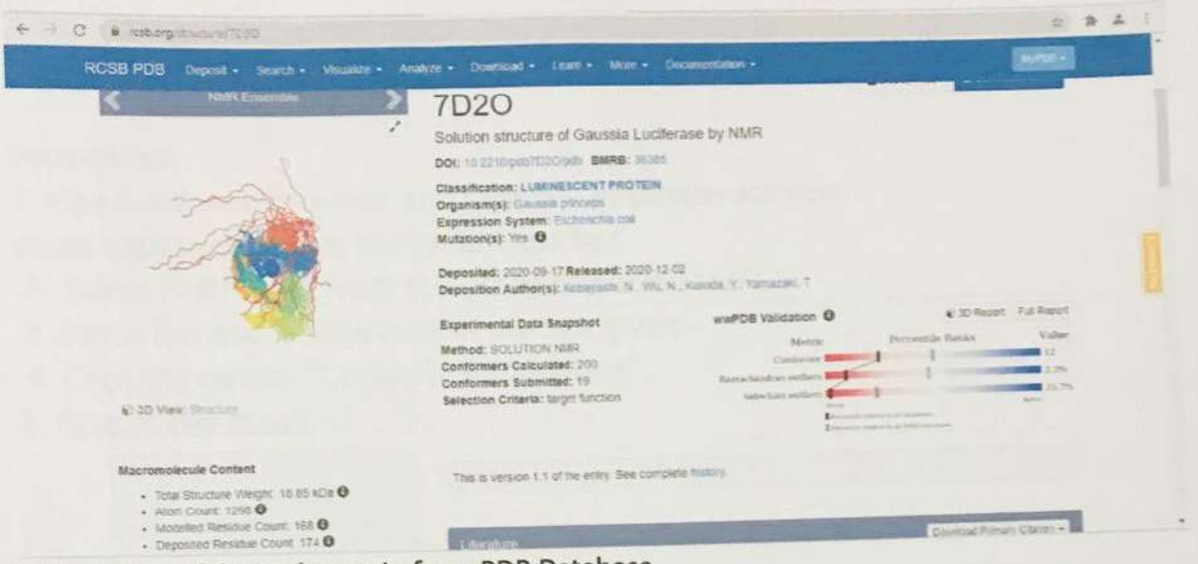
Organism : Gaussia princeps
Protein : Luciferase
Accession ID : 7D20
Method used : NMR

>7D2O_1|Chain A|Luciferase|Gaussia princeps (T. Scott, 1894) (148582)
TGKPTENNEDFNIVAVASNFATTDLDADRGKLPGKKLPLEVLKEMEANARKAGCTRGCLICLSHIKCTPKM
KKFIPGRCHTYEGDKESAQGGIGEAIVDIPAIPRFKDLEPMEQFIAQVDLCVDCTTGCLKGLANVQCSDLL
KKWLPQRCATFASKIQGQVDKIKGAGGDIEGR

Result :

The required primary protein structure is retrieved from the Protein Database in FASTA format.

Observation:



Information of desired protein from PDB Database

(ii). Analysis of primary protein structure

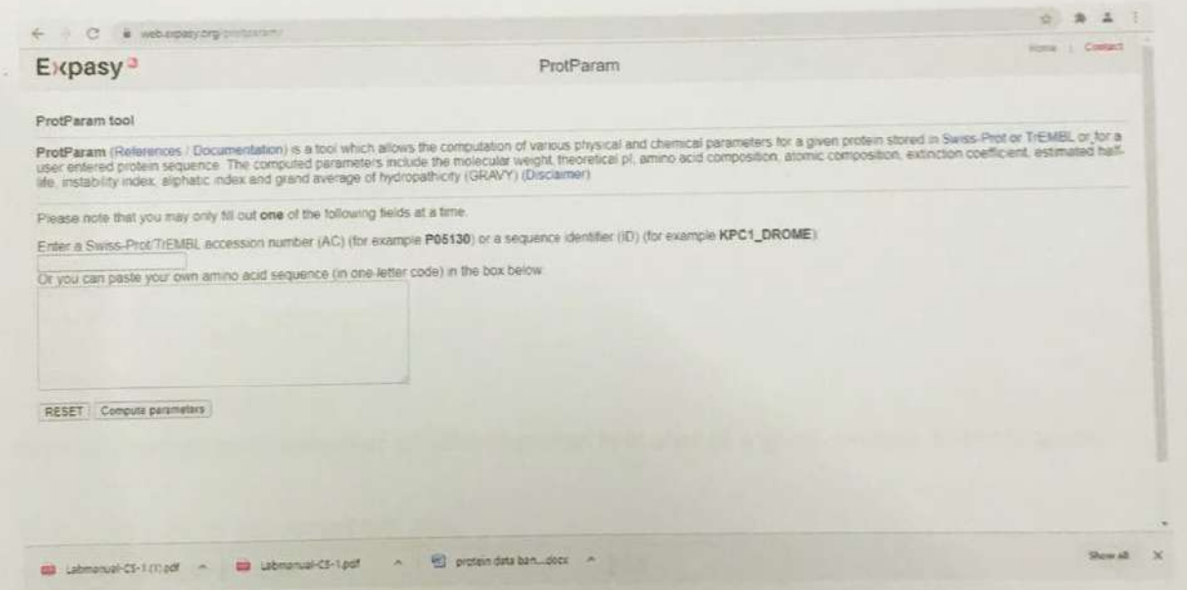
Aim:

To compute the physical and chemical parameters of the given protein.

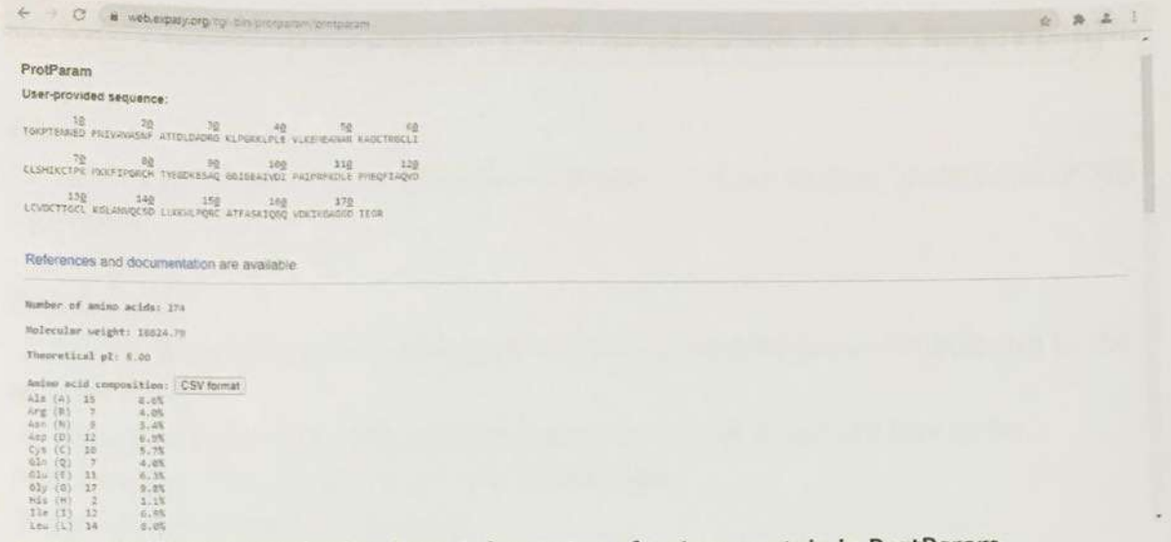
Procedure:

1. Open the web browser and type the database address www.expasy.org/tools in the address bar.
2. Select ProtParam from the list of displayed tools.
3. Paste the query sequence in the box given.
4. Click the option "Compute Parameters".
5. Report the result.

Observstion:



ProtParam Home page



ProtParam

User-provided sequence:

```
        10         20         30         40         50         60
TGKPTENNED PRIVVHASNP ATTDLDADRG KLPGRKLPLE VLKEMEARAH KADCTRGCLI

        70         80         90        100        110        120
CLSMIKCTPE FKKFIPGRHH TYEGDKESAQ GGIGEAIVDI PEIPRFKDLE PMEQFIAQVD

       130        140        150        160        170
LCVDCTTGCL KDLANVQCSD LLKKWLPQRC ATFASKIQGQ VDKIKGAGGD IEGR
```

References and documentation are available

Number of amino acids: 174

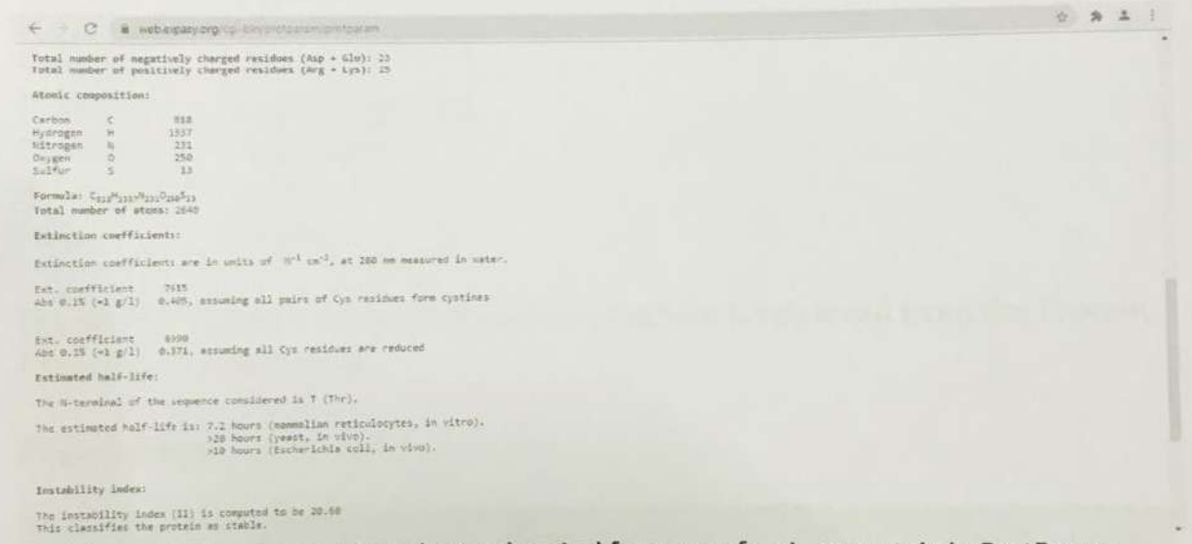
Molecular weight: 18824.79

Theoretical pI: 8.00

Amino acid composition: CSV format

```
Ala (A)  15       8.6%
Arg (R)   7       4.0%
Asn (N)   9       5.4%
Asp (D)  12       6.9%
Cys (C)  10       5.7%
Gln (Q)   7       4.0%
Glu (E)  11       6.3%
Gly (G)  17       9.8%
His (H)   2       1.1%
Ile (I)  12       6.9%
Leu (L)  14       8.0%
```

Amino acid content and other basic information of a given protein in ProtParam



Total number of negatively charged residues (Asp + Glu): 23
Total number of positively charged residues (Arg + Lys): 25

Atomic composition:

```
Carbon      C          918
Hydrogen    H         1357
Nitrogen    N          231
Oxygen      O          250
Sulfur      S           13
```

Formula: $C_{918}H_{1357}N_{231}O_{250}S_{13}$
Total number of atoms: 2649

Extinction coefficients:

Extinction coefficients are in units of M^{-1} cm^{-1}, at 280 nm measured in water.

```
Ext. coefficient    7615
Abs 0.1% (=1 g/l)   0.405, assuming all pairs of Cys residues form cystines

Ext. coefficient    8990
Abs 0.1% (=1 g/l)   0.371, assuming all Cys residues are reduced
```

Estimated half-life:

The N-terminal of the sequence considered is T (Thr).

The estimated half-life is: 7.2 hours (mammalian reticulocytes, in vitro).
>20 hours (yeast, in vivo).
>10 hours (Escherichia coli, in vivo).

Instability index:

The instability index (II) is computed to be 20.60
This classifies the protein as stable.

Atomic content and few other physio-chemical features of a given protein in ProtParam

The computed parameters are,

Number of amino acids	174
Molecular Weight	18824.79
Theoretical pI	8.00
Total number of negatively charged residues (Asp + Glu)	23
Total number of positively charged residues (Arg + Lys)	25
Total number of atoms	2649
Extinction Coefficient	7615

Result: Thus, protein parameters were computed using ProtParam tool.

5. SECONDARY STRUCTURE ANALYSIS OF A PROTEIN

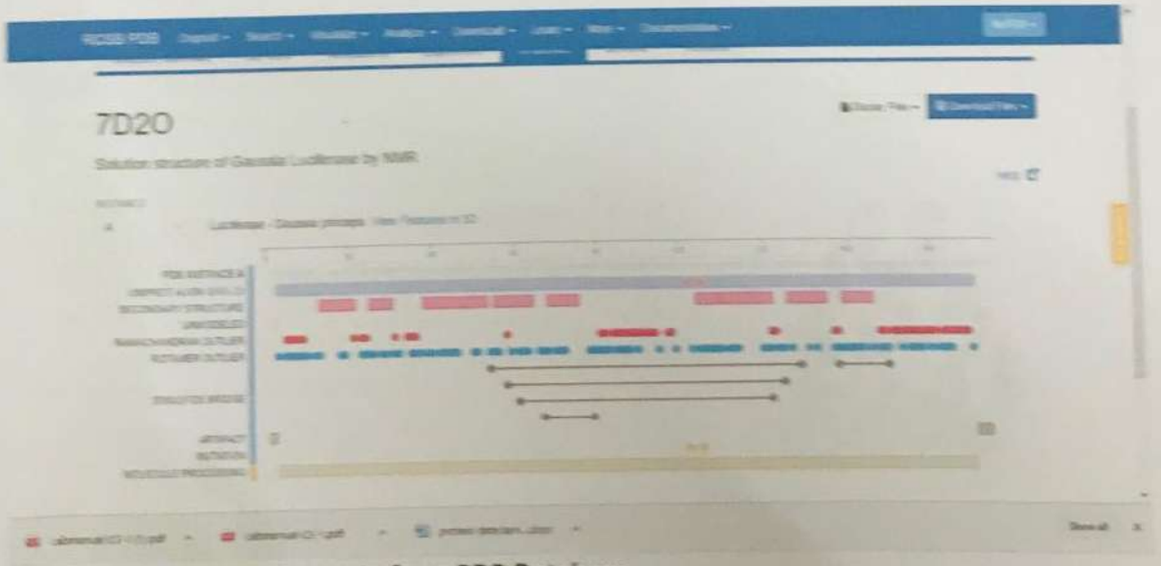
Aim:
To retrieve the secondary structure of a protein from Protein Data Bank (PDB) database.

Procedure:
1. Open the web browser and type the database address www.pdb.org in the address bar.
2. Type the query (i.e., the protein name for which structure has to be retrieved) in the search bar of the home page.
3. Select the protein of interest.
4. Click on the required part of the protein
5. Report the result.

Output:
Organism : Gaussia princeps
Protein : Luciferase
Accession ID : 7D20
Length : 174aa

Result: The required secondary protein structure is retrieved from the Protein Database in pdb format.

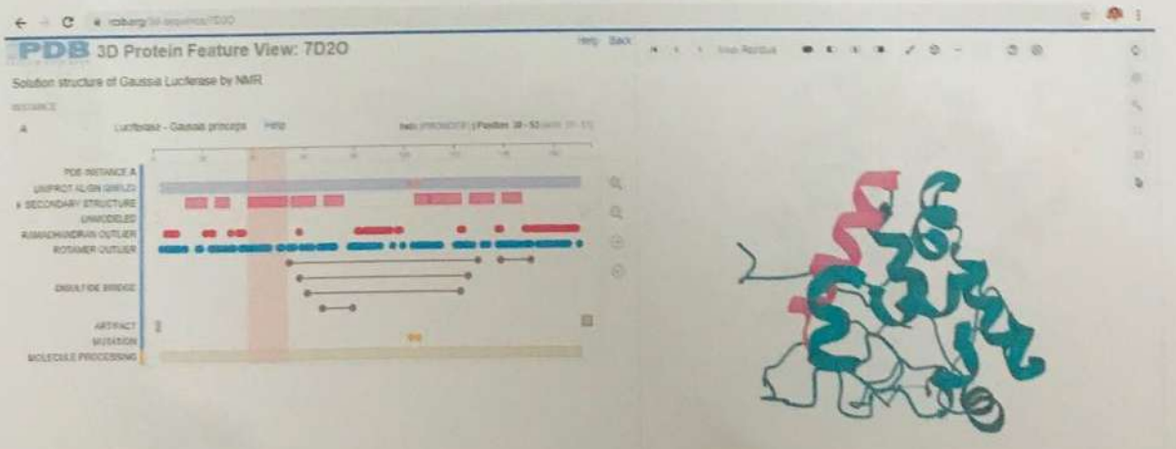
Observation:



Information of desired protein from PDB Database

PDB 3D Protein Feature View: 7D2O
Solution structure of Gaussia Luciferase by NMR
Luciferase - Gaussia princeps
PDB INSTANCE A
UNIPROT ALIGN (GRCL2)
SECONDARY STRUCTURE
UNMODELED
RAMACHANDRAN OUTLIER
ROTAMER OUTLIER
DISULFIDE BRIDGE
ARTIFACT
MUTATION
MOLECULE PROCESSING

PDB 3D Protein Feature View: 7D2O
Solution structure of Gaussia Luciferase by NMR
Luciferase - Gaussia princeps
PDB INSTANCE A
UNIPROT ALIGN (GRCL2)
SECONDARY STRUCTURE
UNMODELED
RAMACHANDRAN OUTLIER
ROTAMER OUTLIER
DISULFIDE BRIDGE
ARTIFACT
MUTATION
MOLECULE PROCESSING


PDB 3D Protein Feature View: 7D2O
Solution structure of Gaussia Luciferase by NMR
Luciferase - Gaussia princeps
PDB INSTANCE A
UNIPROT ALIGN (GRCL2)
SECONDARY STRUCTURE
UNMODELED
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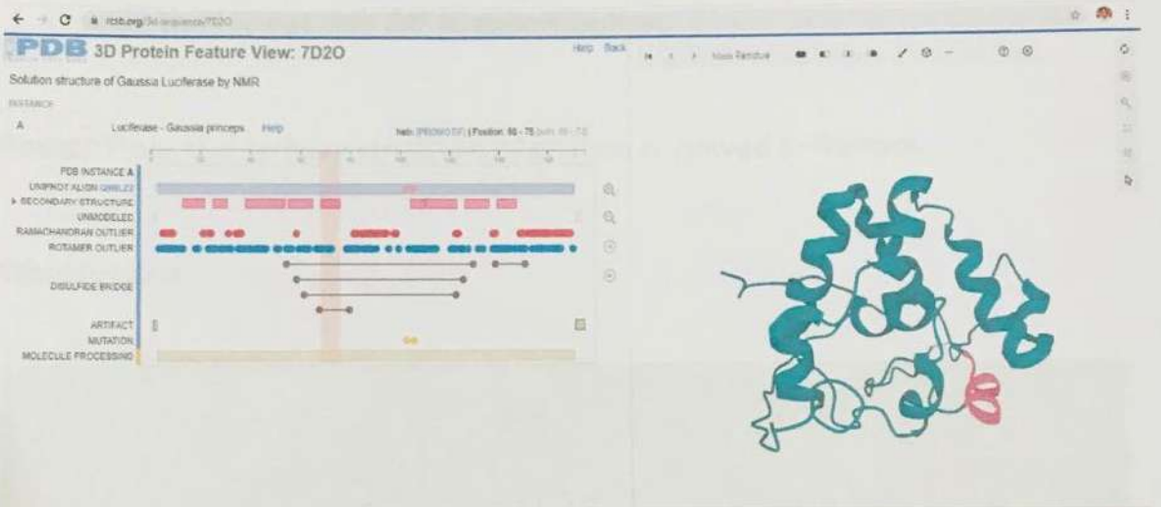
MUTATION
MOLECULE PROCESSING

PDB 3D Protein Feature View: 7D2O
Solution structure of Gaussia Luciferase by NMR
Luciferase - Gaussia princeps
PDB INSTANCE A
SECONDARY STRUCTURE
UNMODELED
RAMACHANDRAN OUTLIER
ROTAMER OUTLIER
DISULFIDE BRIDGE
ARTIFACT
MUTATION
MOLECULE PROCESSING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Luciferase - Gaussia princeps
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RAMACHANDRAN OUTLIER
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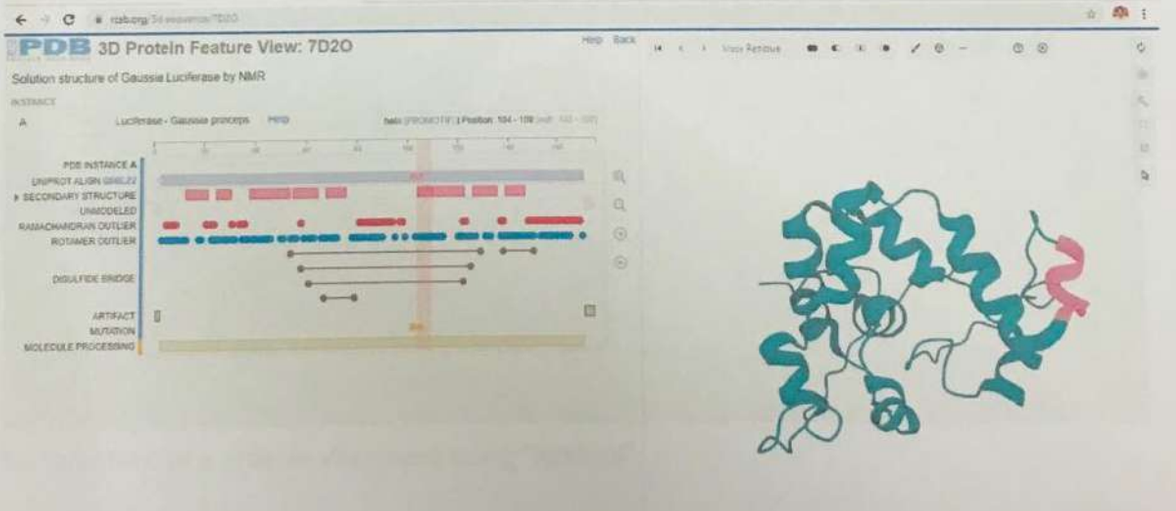
MOLECULE PROCESSING


PDB 3D Protein Feature View: 7D2O
Solution structure of Gaussia Luciferase by NMR
Luciferase - Gaussia princeps
PDB INSTANCE A
SECONDARY STRUCTURE
UNMODELED
RAMACHANDRAN OUTLIER
ROTAMER OUTLIER
DISULFIDE BRIDGE
ARTIFACT
MUTATION
MOLECULE PROCESSING

6. TERTIARY PROTEIN STRUCTURE ANALYSIS USING RASMOL

Aim:

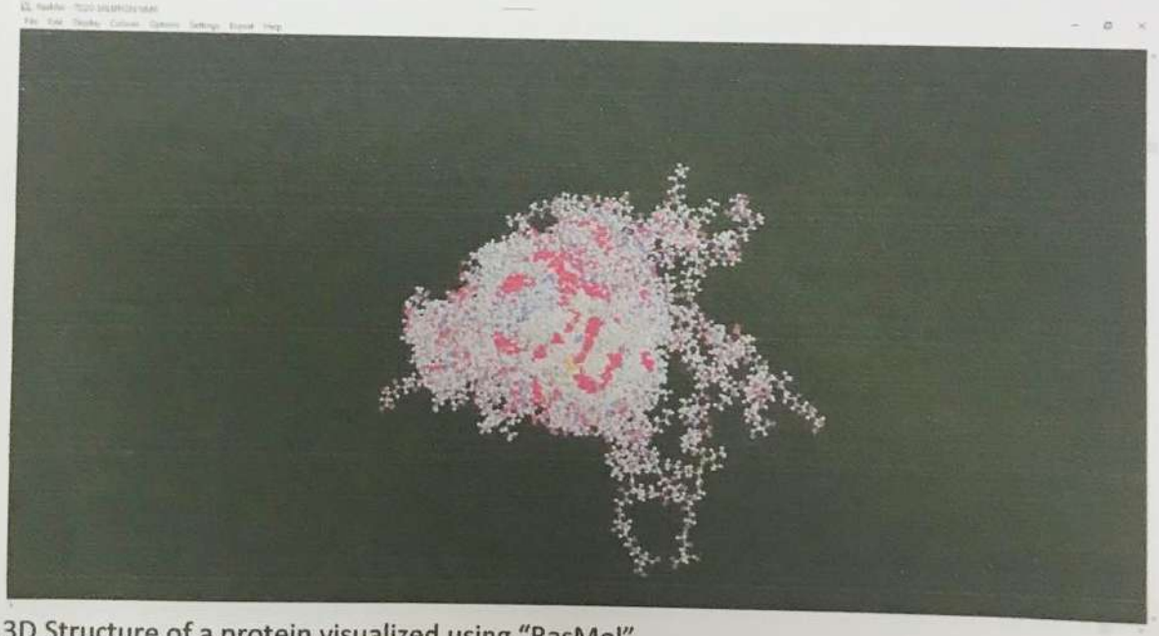
To view the tertiary structure and analyze the protein using Rasmol.

Procedure:

1. Open the working page of offline tool.
2. Upload the pdb file of desired protein.
3. Structure will be displayed.
4. View the different forms of structure using display option.
5. Change the format into GIF in export option.

Result: Thus, the tertiary structure of protein is viewed in Rasmol.

Observation:



3D Structure of a protein visualized using "RasMol"

7. PAIR-WISE SEQUENCE ALIGNMENT AND MULTIPLE SEQUENCE ALIGNMENT USING CLUSTALW

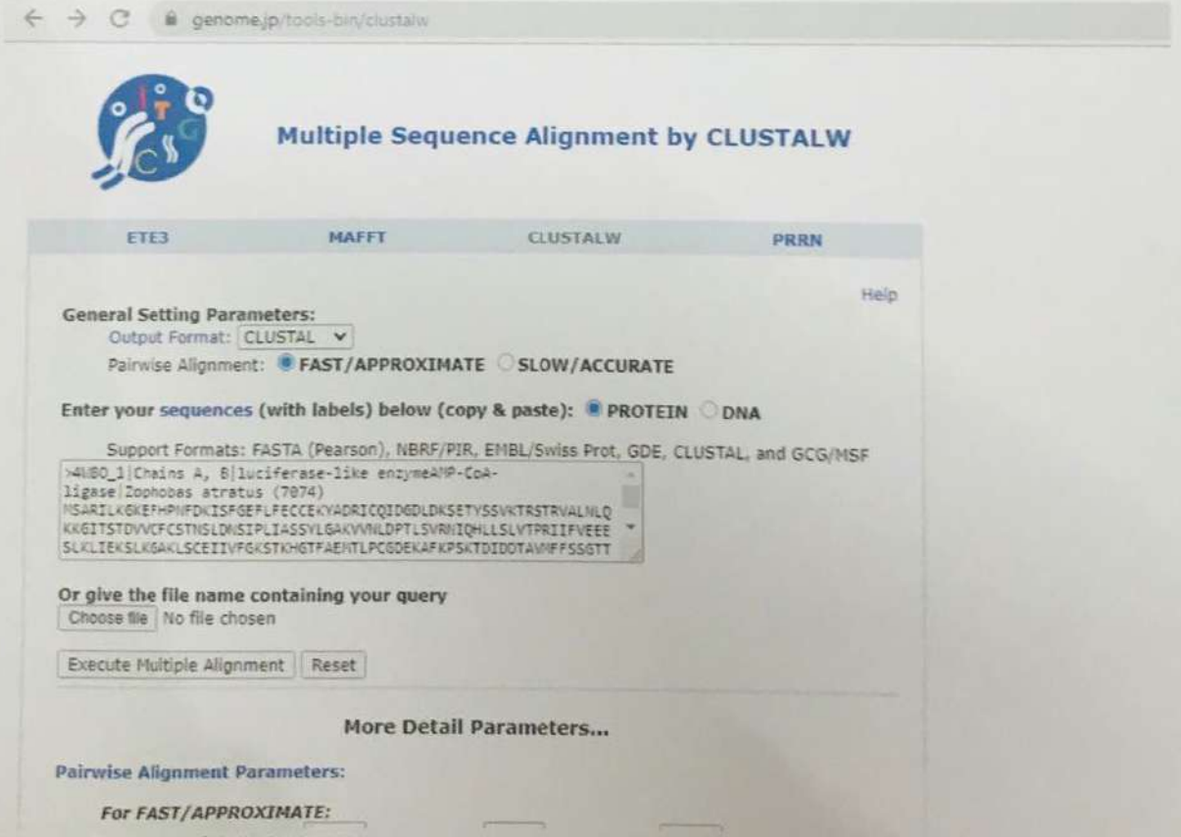
Aim:

To perform pair wise and multiple sequence alignment using clustalw for given sequences.

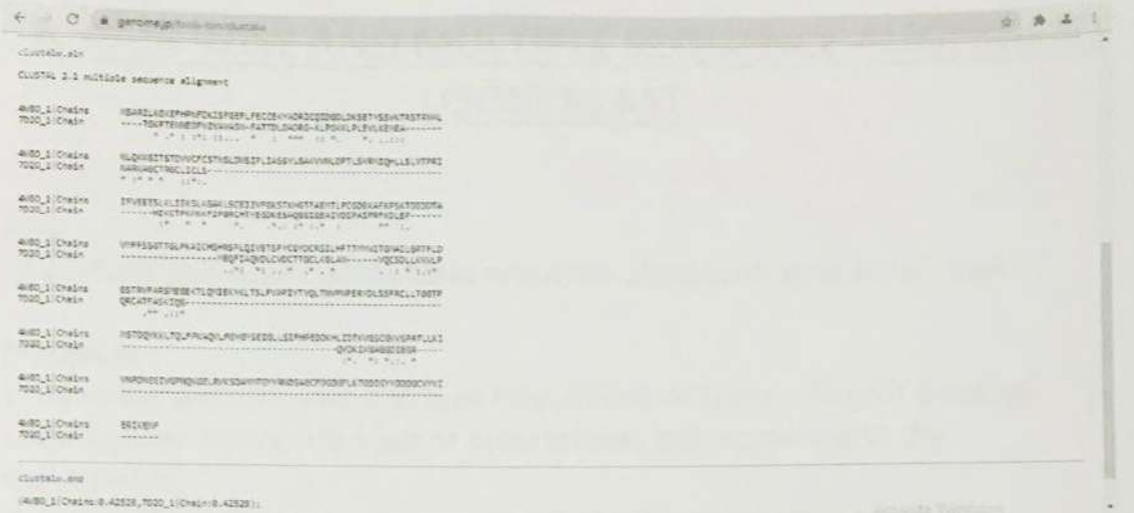
Procedure:

1. Open the web browser and type www.ebi.ac.uk/Tools/msa/ clustalw.
2. Upload the sequences from the Notepad or paste the sequences in FASTA format.
3. Upload two sequences for pair-wise alignment or more than two sequences for multiple sequence alignment After uploading, choose the "Execute Multiple Alignment" option in the alignment icon.
4. Sequence alignment results will be appeared within few seconds after execution.
5. Report the result.

Observation:



Query window of CLUSTALW



Alignment of retrieved sequences using CLUSTALW tool.

Result: *Thus the given sequences are aligned using ClustalW.*

8. PAIR WISE AND MULTIPLE SEQUENCE ALIGNMENT USING BLAST

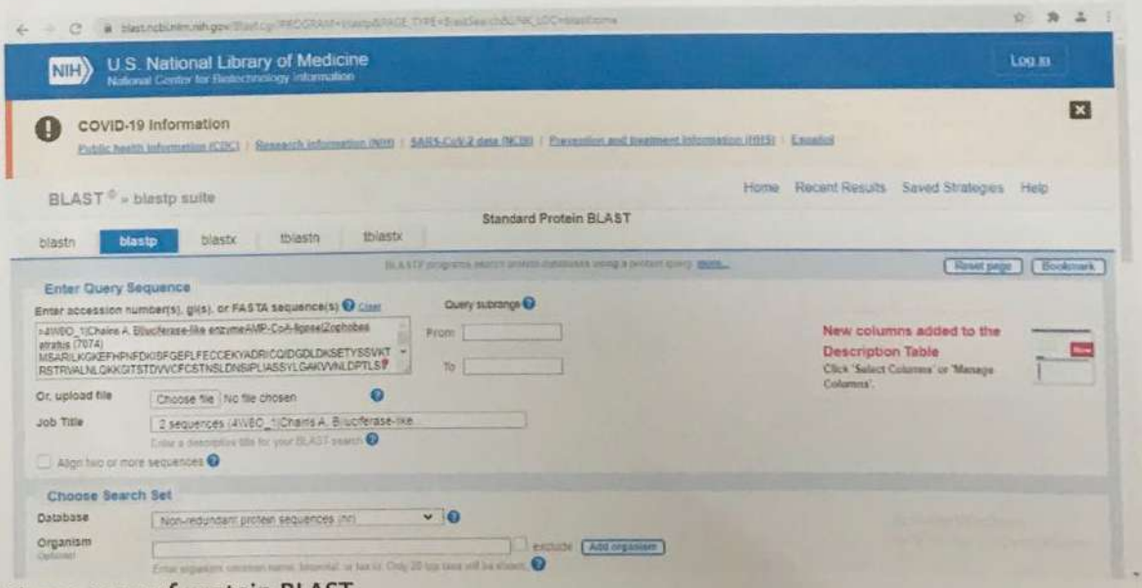
Aim:

To perform pair wise and multiple sequence alignment using BLAST tool.

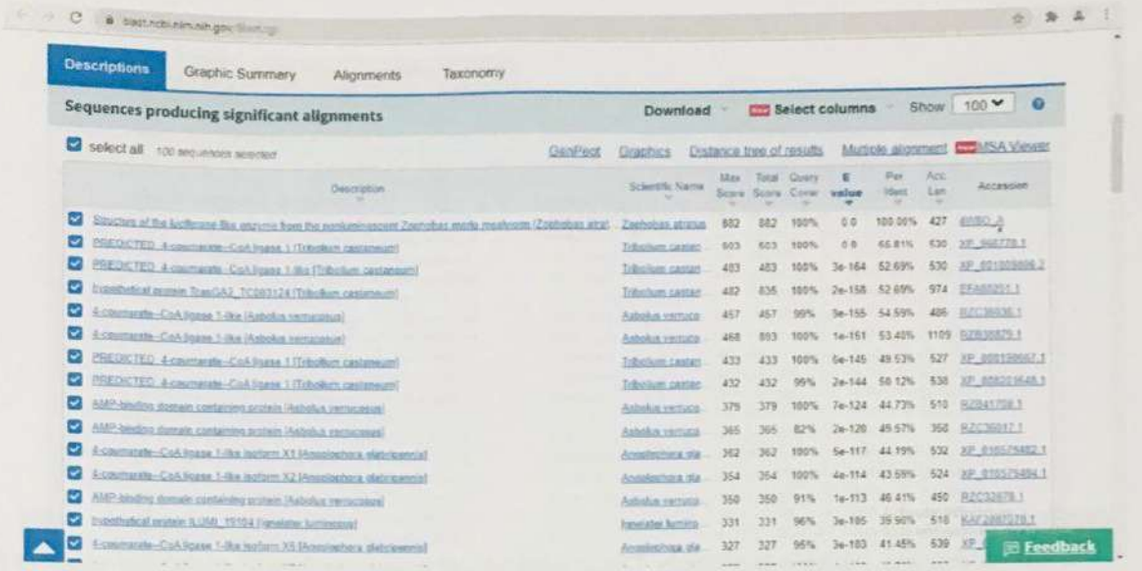
Procedure:

1. Open the web browser and type http://blast.ncbi.nlm.nih.gov/ Blast.cgi
2. Click either nucleotide blast or protein blast icon according to the requirement
3. Select "Align two or more sequences" check box for opting multiple sequence alignment or deselect for pair wise alignment
4. Upload or paste a query sequence (in FASTA format) in the query box and execute BLAST for pair wise alignment. This will be identifying most similar sequences from the databank.
5. Upload or paste a query sequence (in FASTA format) in the query box and upload more than one sequences (in FASTA format) in the subject box and then execute BLAST for multiple sequence alignment. This will be identifying the similarity /dissimilarity among the sequences.
6. Report the result.

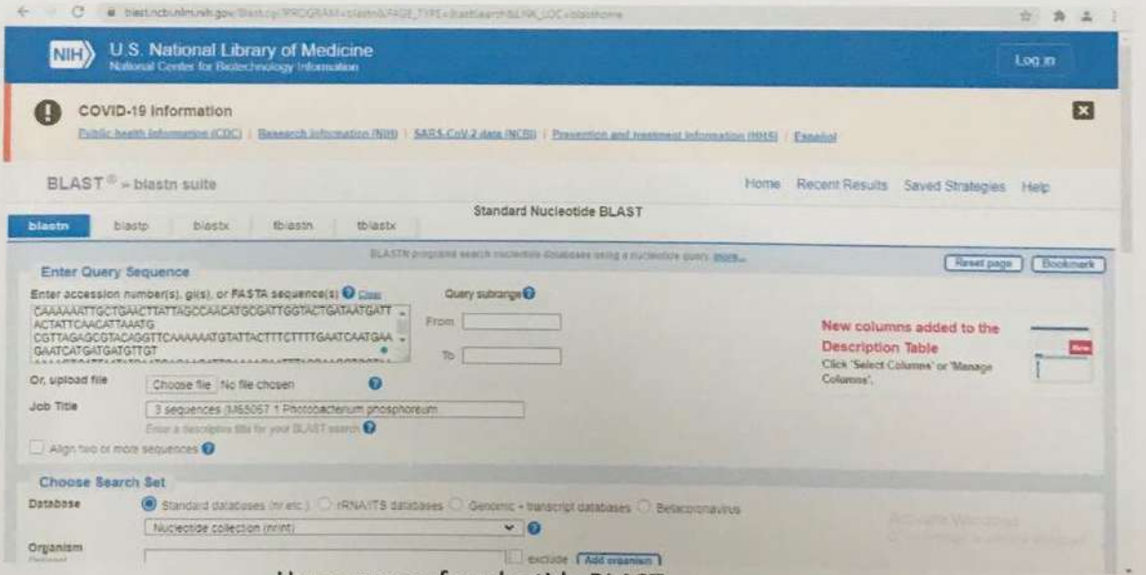
Observation:



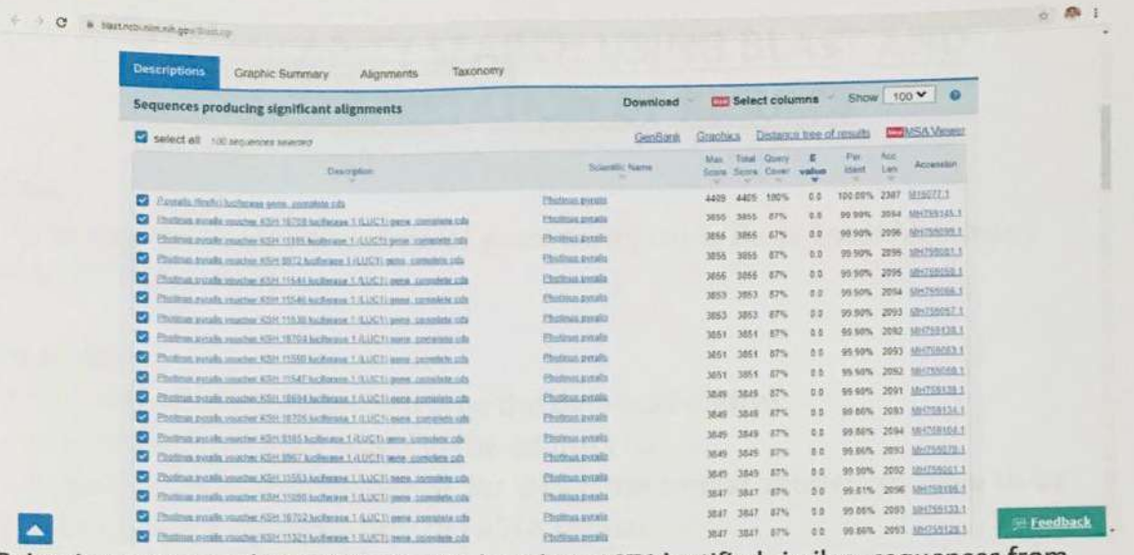
Home page of protein BLAST



Pair-wise sequence alignment using protein BLAST identified similar sequences from Databanks.



Home page of nucleotide BLAST



Pair-wise sequence alignment using Nucleotide BLAST identified similar sequences from Databanks.

Result: Thus the given sequences are aligned using BLAST.

9. SIMILARITY SEARCH USING BLAST AND INTERPRETATION OF RESULTS

Aim:

To search the similar sequence of given query using Basic Local Alignment Search Tool (BLAST).

Procedure:

1. Open the web browser and type the database address www.ncbi.nlm.nih.gov/ blast/ in the address bar.
2. Upload or paste the sequence, for which the similar sequences have to be retrieved, in the dialogue box, in FASTA format.
3. Click "Begin Search" option so that the page containing the organism's ID and other details are displayed.
4. Click "View Reports" and the sequences similar to the given sequence of interest are displayed.
5. Report the sequences which have maximum identity and similarity.

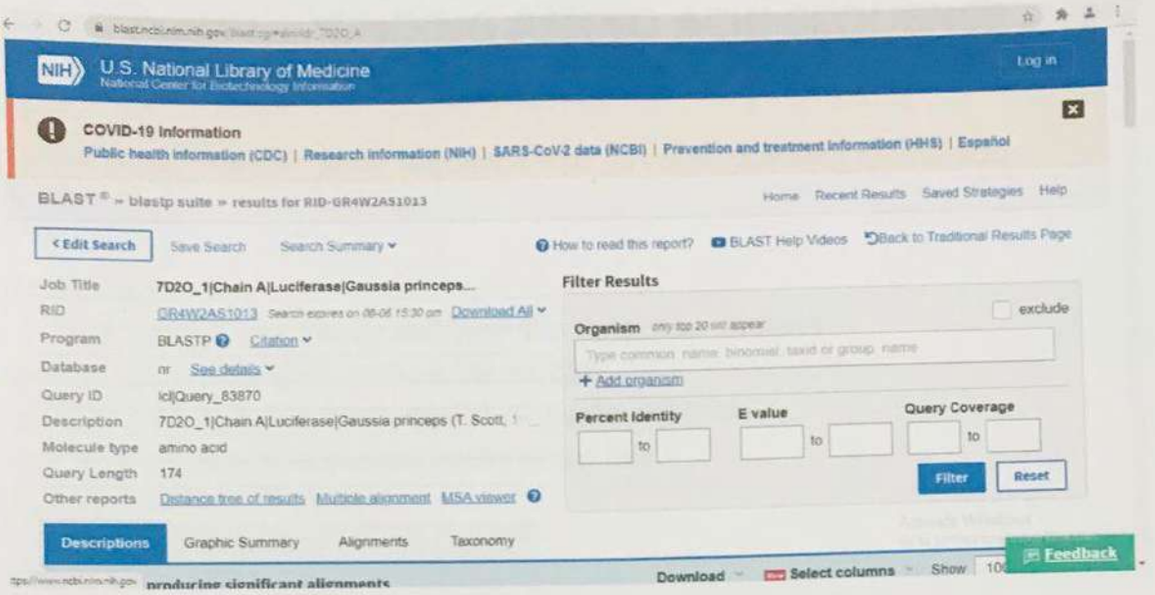
Query ID : lcl|Query_83870
Organism : Gaussia princeps
Query length : 174bp

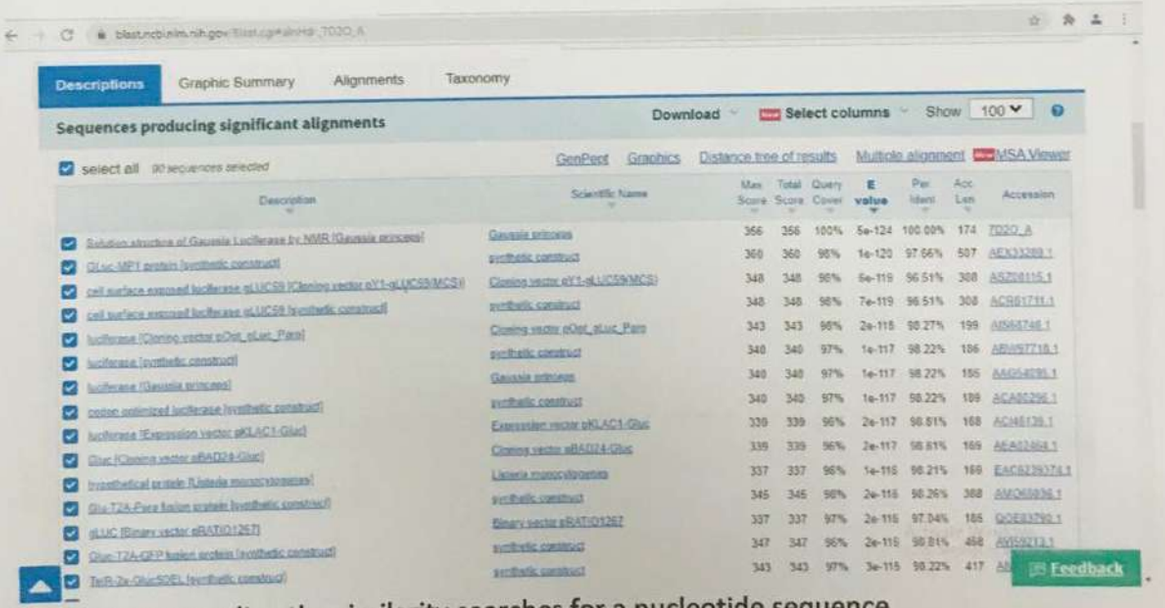
Retrieved similar sequences

ID NO.	E-value	%identity	Length of the sequence
7D20_A	5e-124	100.00%	174
AEX33289.1	1e-120	97.66%	507

Result: The similar sequences were retrieved and the %identity and the expected value for the sequences were known.

Observation:





Information regarding the similarity searches for a nucleotide sequence

10. CONVERSION OF GENE SEQUENCE INTO ITS CORRESPONDING AMINO ACID SEQUENCE

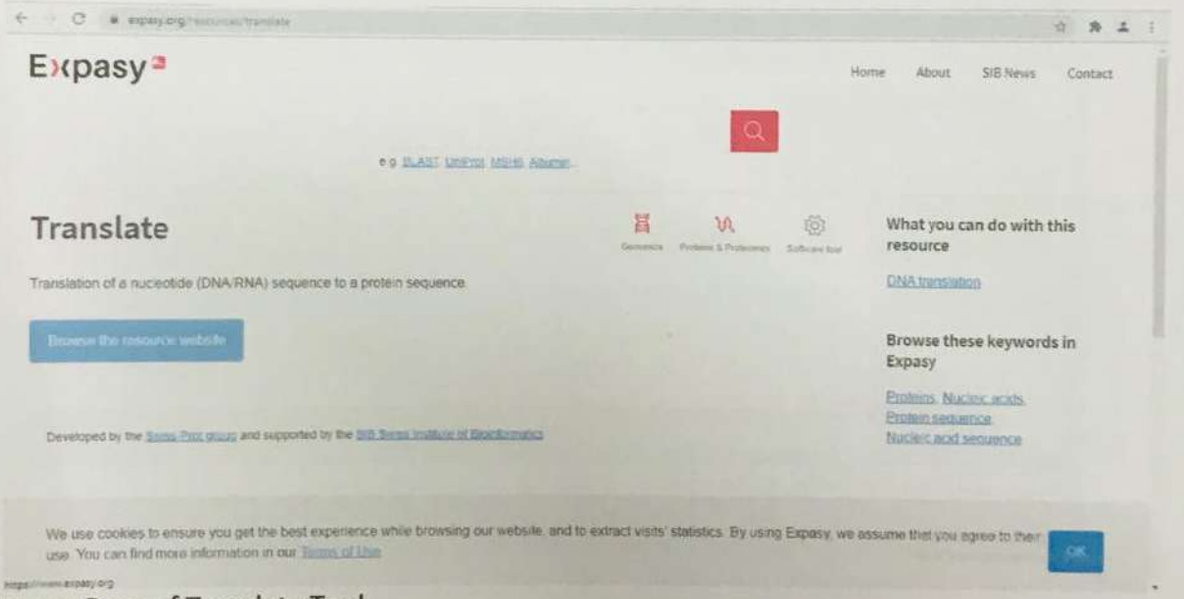
Aim:

To convert the given gene sequence into its corresponding amino acid sequence.

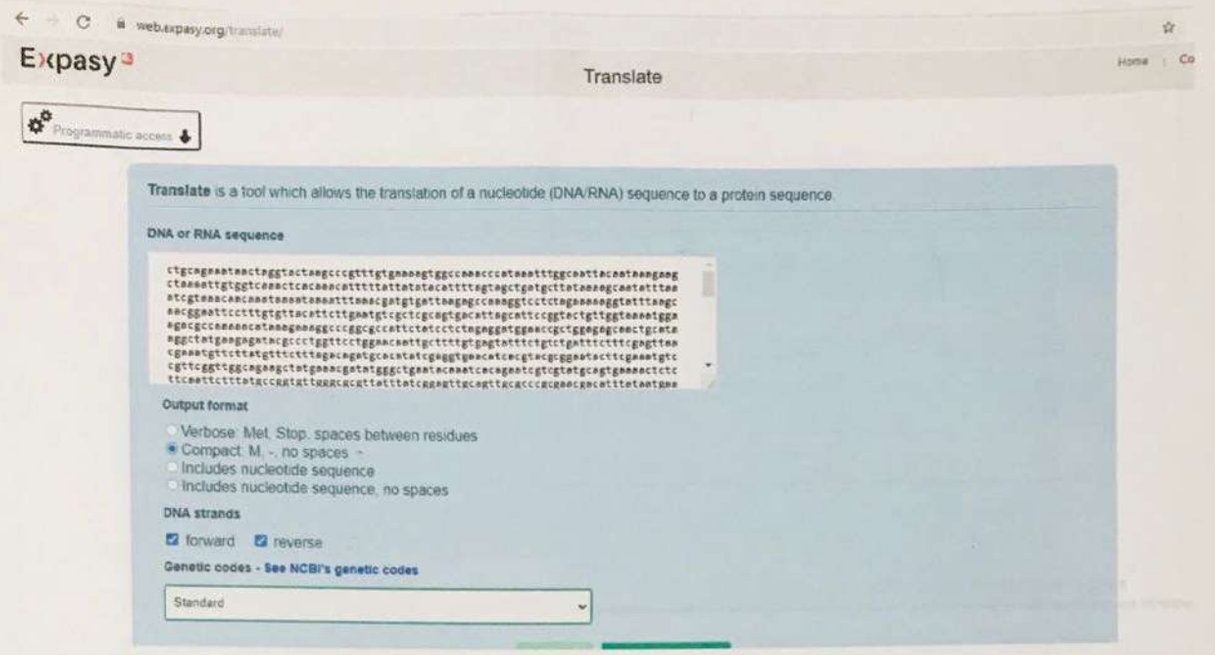
Procedure:

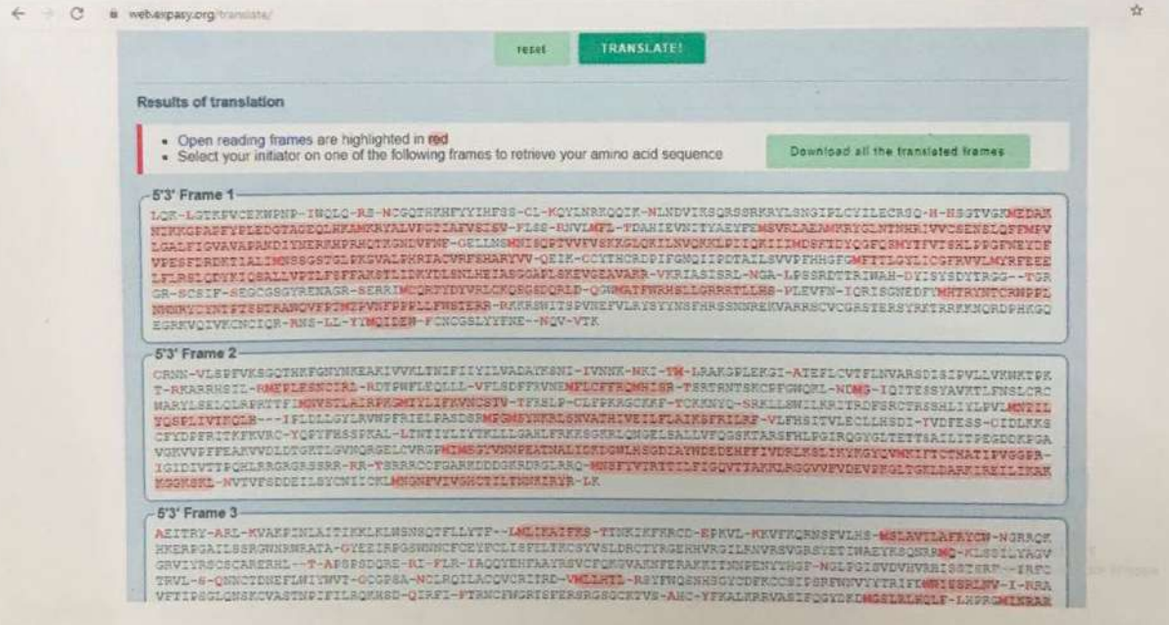
1. Open the web browser and type the database address www.expasy.org/tools in the address bar.
2. Select TRANSLATE from the list of displayed tools.
3. Paste the query sequence in the given box.
4. Click TRANSLATE option.
5. Report the result.

Observation:



Home Page of Translate Tool





Translated amino acid sequence

Organism : P.pyralis(firefly)

Accession number : M15077.1

Result : Thus the given gene sequence is converted into its corresponding protein sequence.

A STUDY PROJECT ON

"BIOINFOMATICS"

Study of a protein using different databases

BSc (Biochemistry) Project

Department of Biochemistry

In partial fulfilment of the requirement for the
Award of Degree in
BACHELOR OF SCIENCES

Submitted by

Bolisetti Teja Naga Sai, Gummapu Rohit Nanda

Redg.No-18313222, 18313219

UNDER THE GUIDENCE OF

Sd. V. Tahaseen MSc, B.Ed, M.Phil

LECTURER IN biochemistry

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VIJAYAWADA – 520004

DEPARTMENT OF BIOCHEMISTRY

S.R.R & C.V.R Government Degree College (A)

Vijayawada-52004

CERTIFICATE

This is to certify that project work entitled **"BIOINFOMATICS- Study of a protein using different databases"** is a bonafide report carried out by **Bolisetti Teja Naga Sai, Gummapu Rohit Nanda -18313222, 18313219** of **III BSc MBC** Bioinformatics Cluster students submitted to the **Department of Biochemistry, S.R.R & C.V.R Government Degree College (A), Vijayawada** during **August2021.**

Sd V Tahaseen

Under the guidance of
Sd. V. Tahaseen MSc, B.Ed, M.Phill
LECTURER IN biochemistry

S.d. V. Tahaseen

Head of the department
Sd. V. Tahaseen MSc, B.Ed, M.Phil
LECTURER IN biochemistry

A. Radhika 22/8/21
External Examiner

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Melittin

Introduction

- **Melittin** is the main component (40–60% of the dry weight) and the major pain producing substance of honeybee **(Apismellifera) venom**. Melittin is a basic peptide consisting of **26 amino acids.**

Classification:	Protein
organism	Apismellifera
Molecular weight:	2958.82 da

Function

- The principal function of **melittin** as a component of bee venom is to cause pain and destruction of tissue of intruders that threaten a beehive.
- However, in honey bees, melittin is not only expressed in the venom gland, but also in other tissues when infected with pathogens. The two venom molecules, melittin and secapin, that are over-expressed in honey bees infected with various pathogens, possibly indicate a role for melittin in the immuneresponse of bees to infectiousdiseases.

Apismellifera

The **western honey bee** or **European honey bee** (*Apismellifera*) is the most common of the 7–12 species of honey bees worldwide. The genus name *Apis* is Latin for "bee", and *mellifera* is the Latin for "honey-bearing", referring to the species' production of honey.
Like all honey bee species, the western honey bee is eusocial, creating colonies with a single fertile female (or "queen"), many normally non-reproductive females or "workers", and a small proportion of fertile males or "drones". Individual colonies can house tens of thousands of bees. Colony activities are organized by complex communication between individuals, through both pheromones and the dance language.
The western honey bee was one of the first domesticated insects, and it is the

1. Retrieval of a Genbank Entry using an accession number

Aim:

To retrieve a nucleotide sequence of interest from Genbank entry with Specific accession number

Procedure:

1. Open the web browser and type the database address http://www.ncbi.nlm. nih.gov/genbank/ in the address bar.
2. Type the query (i.e., the strain name for which the nucleotide sequence has to be retrieved) in the search bar.
3. Select the particular sequence which is required.
4. Select FASTA format in the display settings and click apply.
5. Copy the sequence data and paste it in the notepad.
6. Save the notepad file in FASTA format.
7. Report the result.

Result:

Thus, the nucleotide sequence (**Accession Number: X02007.1**) was retrieved from GenBank Database.

A. mellifera mRNA for prepromelittin

DNA sequence in FASTA format (Nucleotides: 374bp)

AGCGAATTAACAGAATTAACAGGAAGGAAGGAAGGAAGCGATCGG
AGAAATCATGAAATTCTTAGTCAACGTTGCCCTTGTTTTTATGGTCGT
GTACATTTCTTACATCTATGCGGCCCCTGAACCGGAACCGGCACCAG
AGCCAGAGGCGGAGGCAGACGCGGAGGCAGATCCGGAAGCGGGAA
TTGGAGCAGTTCTGAAGGTATTAACCACAGGATTGCCCGCCCTCATA
AGTTGGATTAAACGTAAGAGGCAACAGGGTTAGTCGGATCCATCGA
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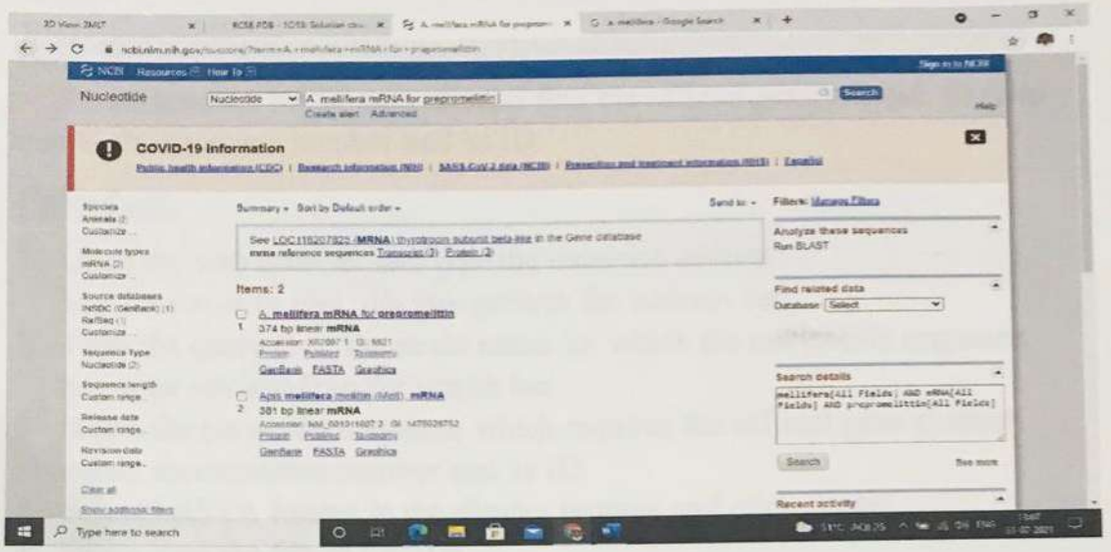
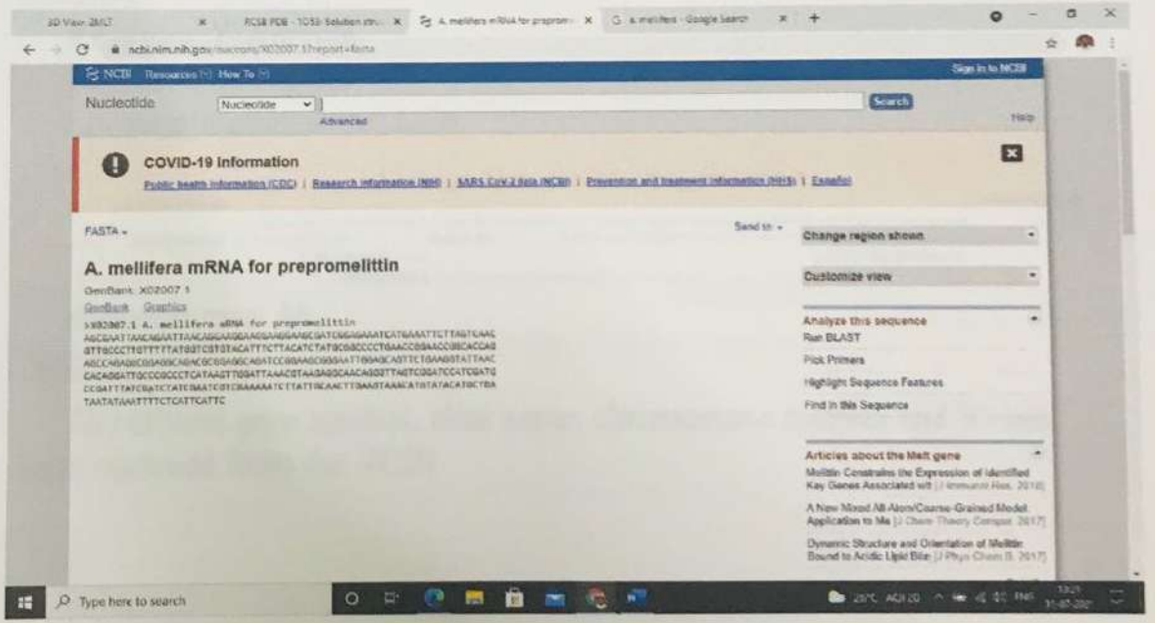
C


Nucleotide
A. mellifera mRNA for prepromelittin
COVID-19 Information
Items: 2
A. mellifera mRNA for prepromelittin
374 bp linear mRNA
Apis mellifera melittin (Mel) mRNA
381 bp linear mRNA
Search details
mellifera[All Fields] AND mRNA[All Fields] AND prepromelittin[All Fields]
Recent activity
Type here to search


Nucleotide
COVID-19 Information
FASTA
A. mellifera mRNA for prepromelittin
GenBank: X02007.1
Change region shown
Customize view
Analyze this sequence
Run BLAST
Pick Primers
Highlight Sequence Features
Find in this Sequence
Articles about the Melt gene
Type here to search

2. FINDING THE OFFICIAL SYMBOL, ALIAS NAME, CHROMOSOME NUMBER AND ID FOR GENE USING NCBI

Aim:

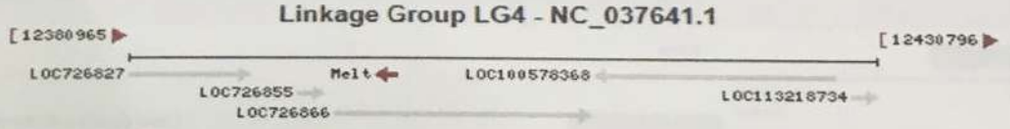
To analyze the NCBI web site and find the official gene symbol, its alias name, chromosome number and its ID.

Procedure:

1. Open the web browser and type the database address http://www.ncbi.nlm. nih.gov/gene/in the address bar.
2. Type the query (i.e., the strain name for which the nucleotide sequence has to be retrieved) in the search bar.
3. Select the particular sequence which requires the official gene symbol, name, chromosome number and its ID.
4. Select FASTA format in the display settings and click apply.
5. Down load the file in FASTA.
6. Report the result.

Output:

Kingdom: Eukaryota
Phylum: Arthropoda
Location:chromosome: LG4

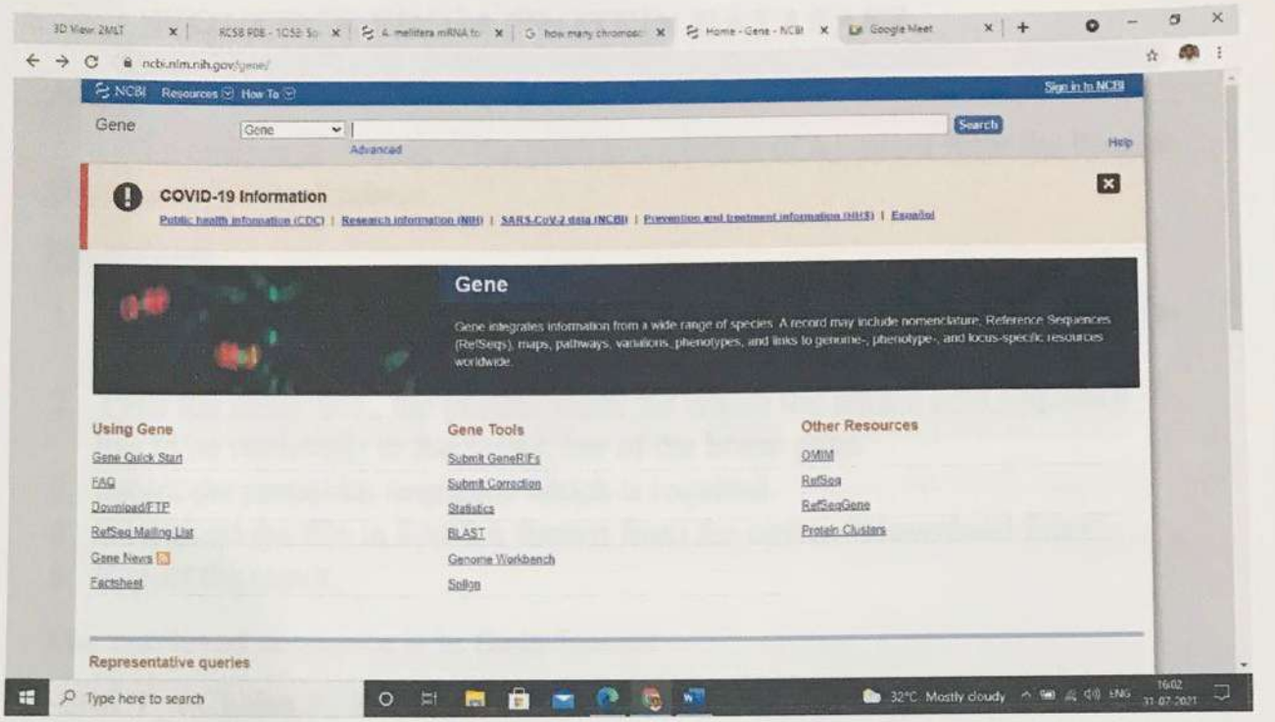


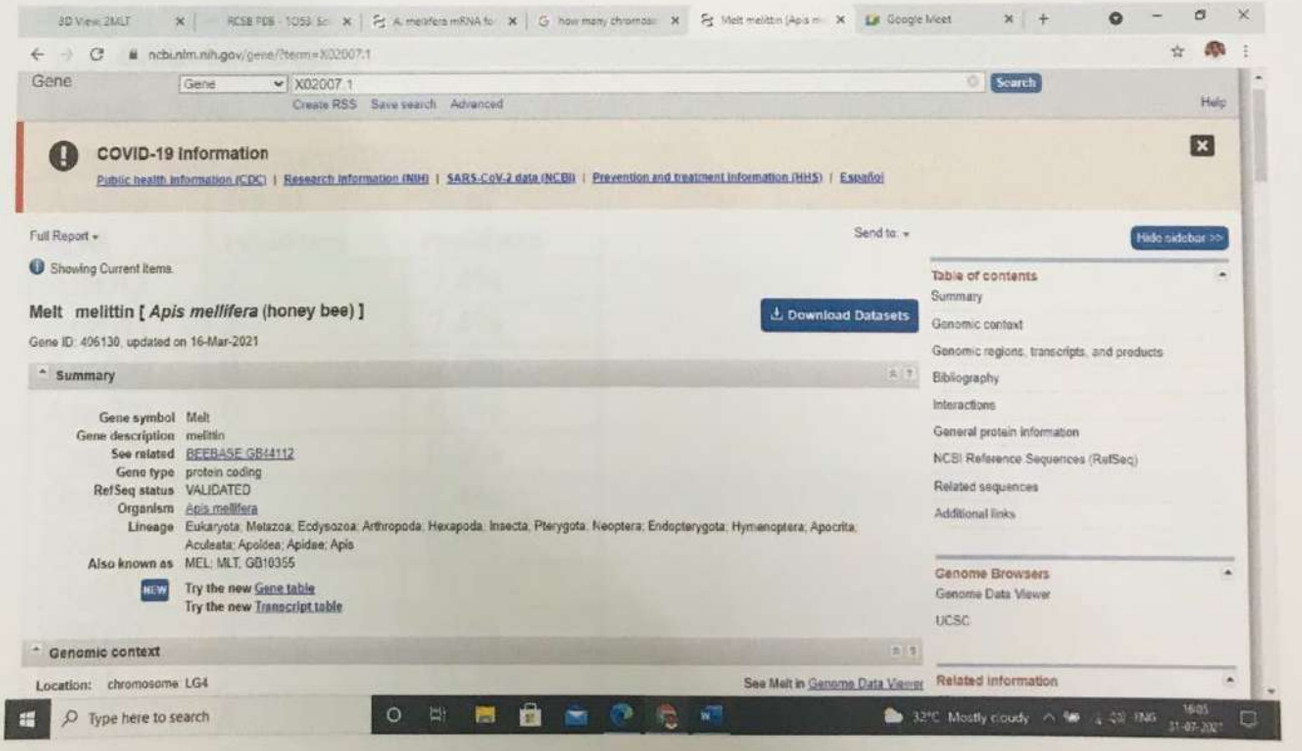
chromosomes: 16

Result:

The official gene symbol, alias name, chromosome number and ID has been retrieved from the NCBI

observation :





3. RETRIEVAL AND ANALYSIS OF PROTEIN SEQUENCE FROM PROTEIN DATABASE

Aim:

To analyse and retrieve the protein sequence of a protein from the Protein Data Bank (PDB) database.

Procedure:

1. Open the web browser and type the database addresshttps://www.rcsb.org/in the address bar.
2. Type the query (i.e., the protein name for which the amino acid sequence has to be retrieved) in the search bar of the home page.
3. Select the particular sequence which is required.
4. Download the file in FASTA format from the option "Download Files".
5. Report the result.

The retrieved sequence is in fasta format

>2MLT_1|Chains A, B |GIGAVLKVLTTGLPALISWIKRKRQQX

Organism: Apismellifera.
Protein: Melittin
Accession No: **2MLT**
Length: 27aa

Amino acid composition:

Amino acid	No of residues	% of residues
Ala(A)	2	7.4%
Arg(R)	2	7.4%
Asn(N)	0	0.0%
Asp(D)	0	0.0%
Cys(C)	0	0.0%
Gln(Q)	2	7.4%
Glu(E)	0	0.0%
Gly(G)	3	11.1%
His(H)	0	0.0%
Ile(I)	3	11.1%
Leu(L)	4	14.8%
Lys(K)	3	11.1%
Met(M)	0	0.0%
Phe(F)	0	0.0%
Pro(P)	1	3.7%

Ser(S)	1	3.7%
Thr(T)	2	7.4%
Trp(W)	1	3.7%
Tyr(Y)	0	0.0%
Val(V)	2	7.4%
Pyl(O)	0	0.0%
Sec (U)	0	0.0%

(B) 0 0.0%
(Z) 0 0.0%
(X) 1 3.7%

Result:

Thus, the required protein sequence is retrieved from the PDB database.

4. PRIMARY STRUCTURE ANALYSIS OF A PROTEIN

(i). Retrieval of Primary Protein Structure

Aim:

To retrieve the structure of a protein from Protein Data Bank (PDB) database.

Procedure:

1. Open the web browser and type the database address www.pdb.org in the address bar.
2. Type the query (the protein name for which structure has to be retrieved) in the search bar of the home page.
3. Select the protein of interest.
4. Download the file in FASTA format from the option "Download Files".
5. Report the result.

Organism: Apismellifera.
Protein: Melittin
Accession No: **2MLT**
Length: 27aa

Result:

The required primary protein structure is retrieved from the Protein Database in FASTA format

FASTA format protein file

>2MLT_1|Chains A, B|MELITTIN|Apismellifera (7460)

GIGAVLKVLTTGLPALISWIKRKRQQX

(ii). Analysis of primary protein structure

Aim:

To compute the physical and chemical parameters of the given protein.

Procedure:

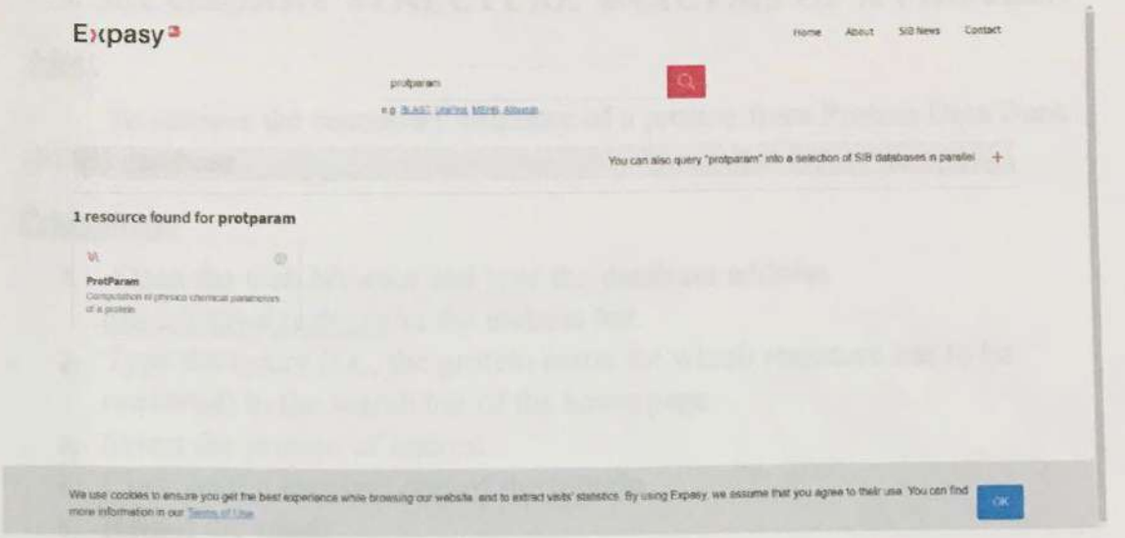
1. Open the web browser and type the database address www.expasy.org/tools in the address bar.
2. Select ProtParam from the list of displayed tools.
3. Paste the query sequence in the box given.
4. Click the option "Compute Parameters".
5. Report the result.

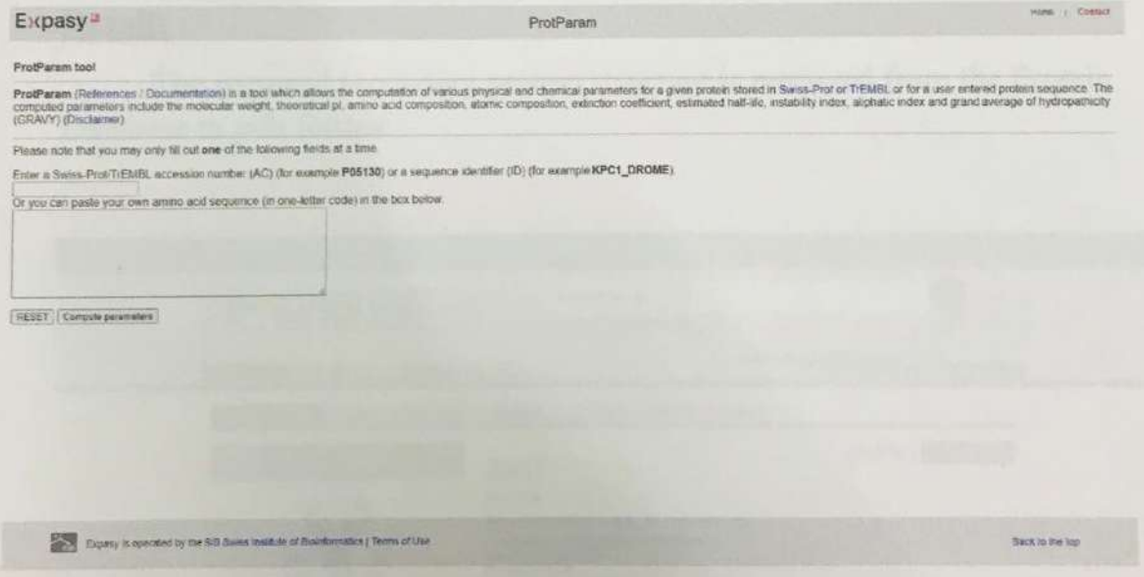
The computed parameters are,

Number of amino acids	**27**
Molecular Weight	2958.82
Theoretical pI	12.02
Total number of negatively charged residues (Asp + Glu)	0
Total number of positively charged residues (Arg + Lys)	5
Extinction Coefficient	85495
Number of atoms	437

Result:

Thus, protein parameters were computed using ProtParam tool.


Expasy
Home
About
SIB News
Contact
protparam
You can also query "protparam" into a selection of SIB databases in parallel
1 resource found for protparam
ProtParam
We use cookies to ensure you get the best experience while browsing our website, and to extract visits' statistics. By using Expasy, we assume that you agree to their use. You can find more information in our Terms of Use
OK


Expasy
ProtParam
ProtParam tool
ProtParam (References / Documentation) is a tool which allows the computation of various physical and chemical parameters for a given protein stored in Swiss-Prot or TrEMBL or for a user entered protein sequence. The computed parameters include the molecular weight, theoretical pI, amino acid composition, atomic composition, extinction coefficient, estimated half-life, instability index, aliphatic index and grand average of hydropathicity (GRAVY) (Disclaimer)
Please note that you may only fill out one of the following fields at a time.
Enter a Swiss-Prot/TrEMBL accession number (AC) (for example P05130) or a sequence identifier (ID) (for example KPC1_DROME).
Or you can paste your own amino acid sequence (in one-letter code) in the box below.
RESET
Compute parameters
Expasy is operated by the SIB Swiss Institute of Bioinformatics | Terms of Use
Back to the top

5. SECONDARY STRUCTURE ANALYSIS OF A PROTEIN

Aim:

To retrieve the secondary structure of a protein from Protein Data Bank (PDB) database

Procedure:

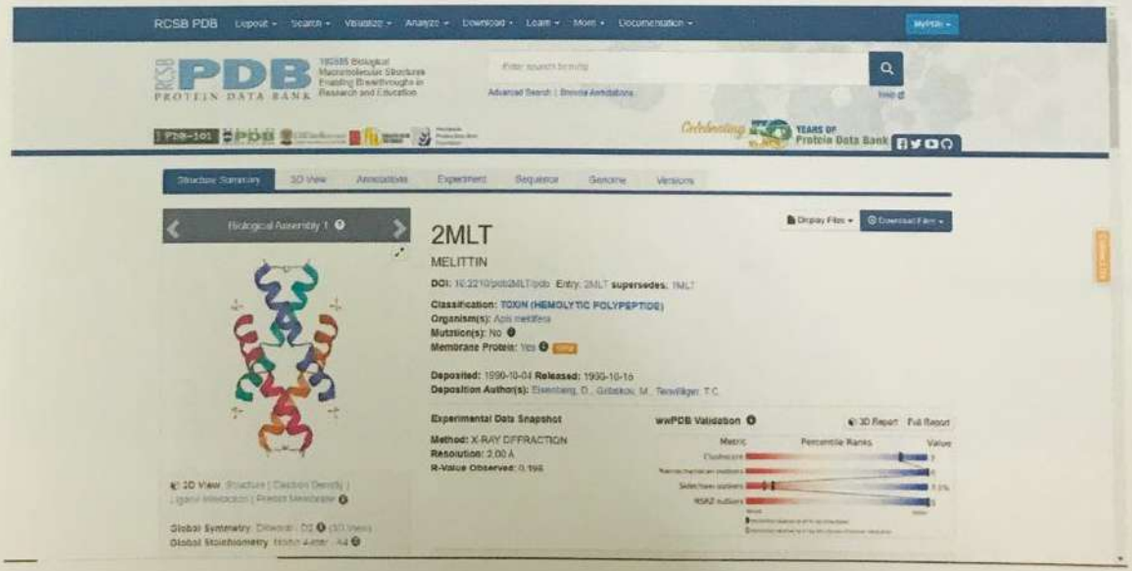
1. Open the web browser and type the database address https://www.rcsb.org/in the address bar.
2. Type the query (i.e., the protein name for which structure has to be retrieved) in the search bar of the home page.
3. Select the protein of interest.
4. Click on the required part of the protein
5. Report the result

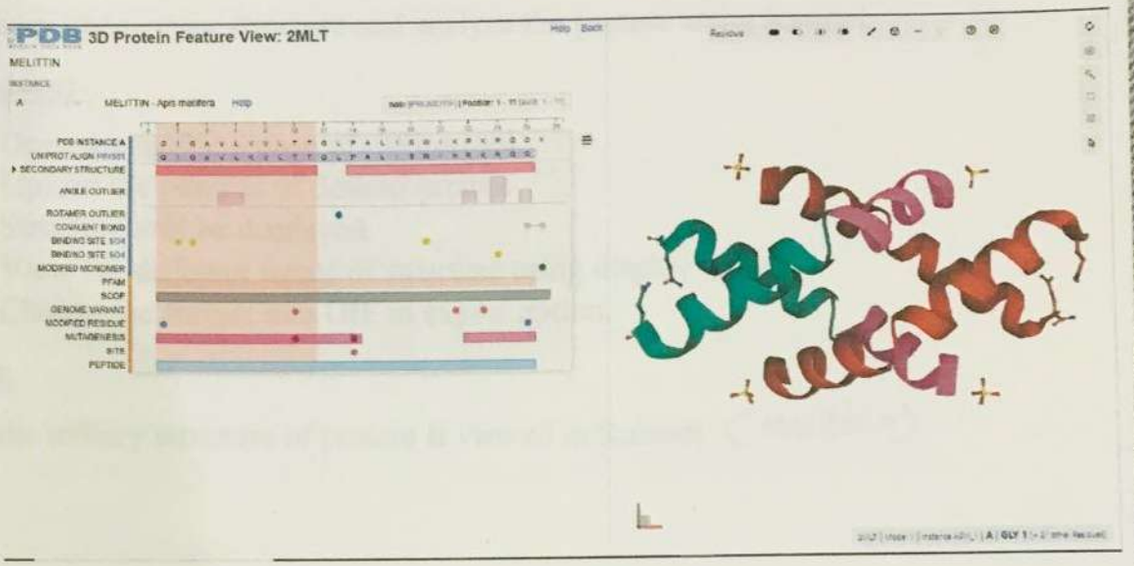
Output:

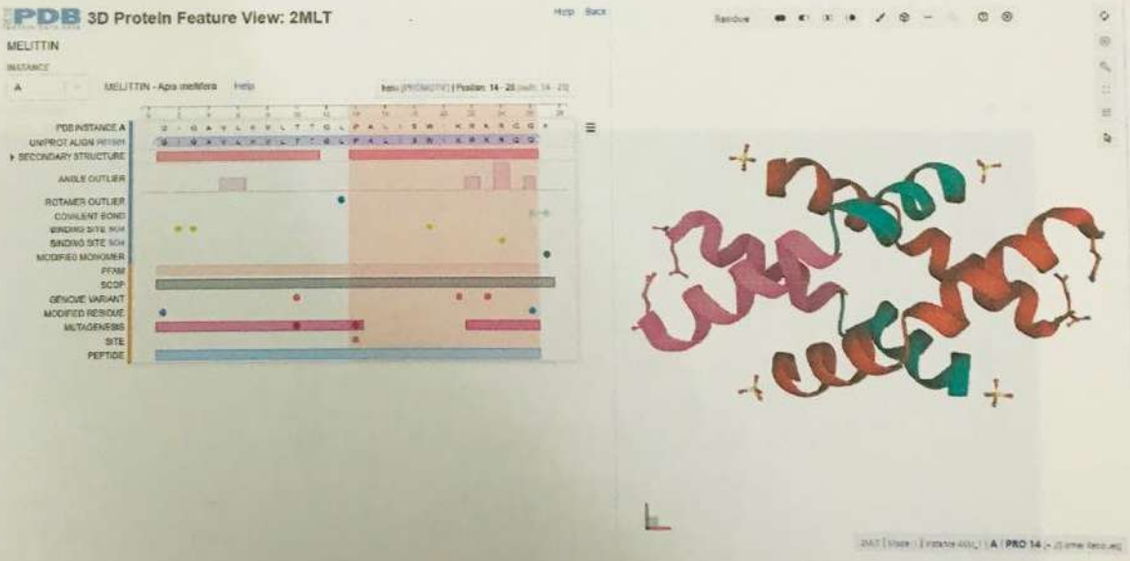
Organism: Apismellifera.
Protein: Melittin
Accession No: **2MLT**
Length: 27aa

Result:

The required secondary protein structure is retrieved from the Protein Database in pdb format




PDB 3D Protein Feature View: 2MLT
MELITTIN
A
MELITTIN - Apis mellifera
PDB INSTANCE A
UNIPROT ALIGN P01501
SECONDARY STRUCTURE
ANGLE OUTLIER
ROTAMER OUTLIER
COVALENT BOND
BINDING SITE 5O4
BINDING SITE 5O4
MODIFIED MONOMER
PFAM
SCOP
GENOME VARIANT
MODIFIED RESIDUE
MUTAGENESIS
SITE
PEPTIDE


PDB 3D Protein Feature View: 2MLT
MELITTIN
A
MELITTIN - Apis mellifera
PDB INSTANCE A
UNIPROT ALIGN P01501
SECONDARY STRUCTURE
ANGLE OUTLIER
ROTAMER OUTLIER
COVALENT BOND
BINDING SITE 5O4
BINDING SITE 5O4
MODIFIED MONOMER
PFAM
SCOP
GENOME VARIANT
MODIFIED RESIDUE
MUTAGENESIS
SITE
PEPTIDE

6. TERTIARY PROTEIN STRUCTURE ANALYSIS USING RASMOL

Aim:

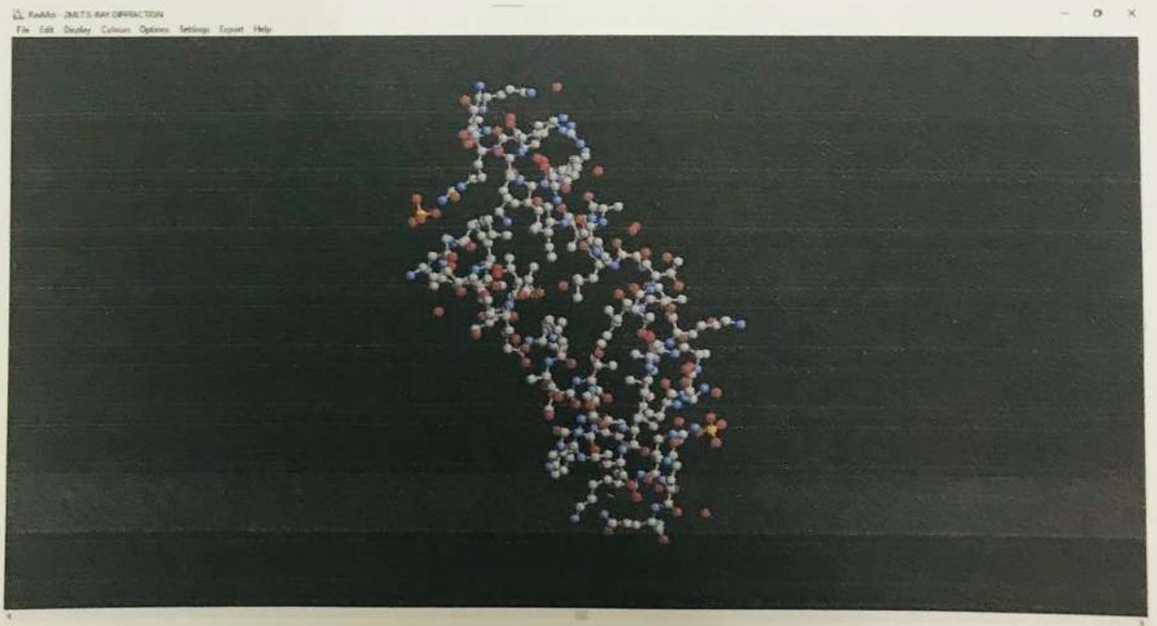
To view the tertiary structure and analyze the protein using Rasmol.

Procedure:

1. Open the working page of offline tool.
2. Upload the pdb file of desired protein.
3. Structure will be displayed.
4. View the different forms of structure using display option.
5. Change the format into GIF in export option.

Result:

Thus, the tertiary structure of protein is viewed in Rasmol. (Melittin)



7. PAIR-WISE SEQUENCE ALIGNMENT AND MULTIPLE SEQUENCE ALIGNMENT USING CLUSTALW

Aim:

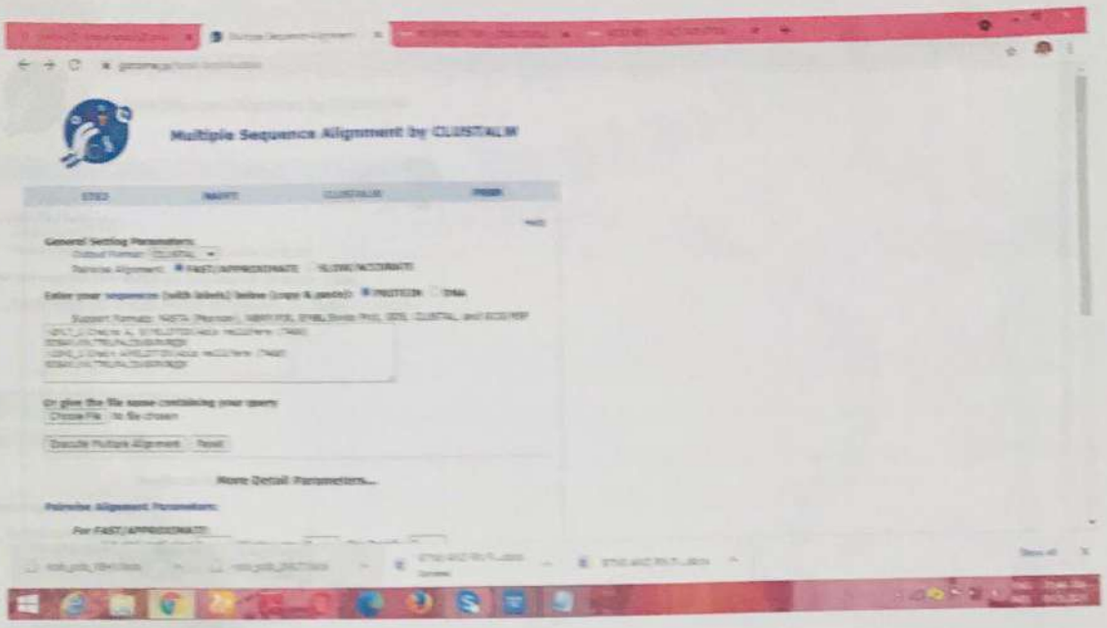
To perform pair wise and multiple sequence alignment using clustalw for given sequences.

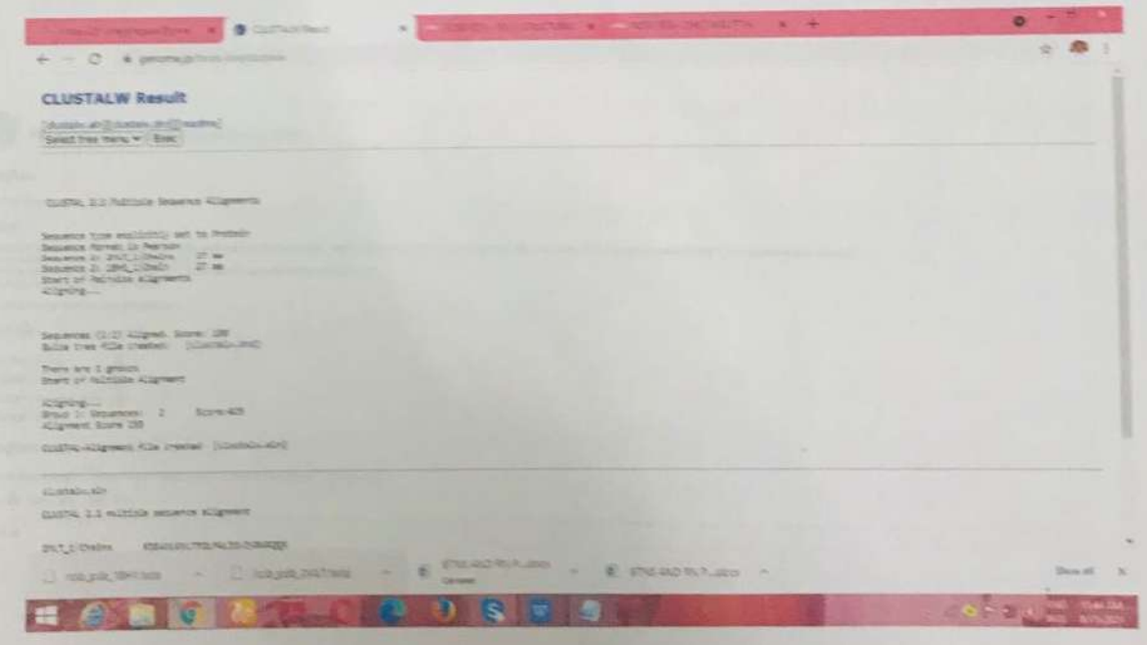
Procedure:

1. Open the web browser and type https://www.genome.jp/tools-bin/clustalw
2. Upload the sequences from the Notepad or paste the sequences in FASTA format.
3. Upload two sequences for pair-wise alignment or more than two sequences for multiple sequence alignment After uploading, choose the "Execute Multiple Alignment" option in the alignment icon.
4. Sequence alignment results will be appeared within few seconds after execution.
5. Report the result.

Result:

Thus the given sequences are aligned using ClustalW.


Multiple Sequence Alignment by CLUSTALW


CLUSTALW Result

8. PAIR WISE AND MULTIPLE SEQUENCE ALIGNMENT USING BLAST

Aim:

To perform pair wise and multiple sequence alignment using BLAST tool.

BLAST

BLAST is an acronym for Basic Local Alignment Search Tool. BLAST is a sophisticated software package that has become the single most important piece of software in the field of Bioinformatics. The following tools are different type of blast programs used with respect to the aim of the user.

Blast programs:

BLASTp – Compares an amino acid query sequence against a protein sequence database.

BLASTn – Compares a nucleotide query sequence against a nucleotide sequence database.

BLASTx – Compares a nucleotide query sequence translated in all reading frames against a protein sequence database.

tBLASTn – Compares a protein query sequence against a nucleotide sequence database dynamically translated in all reading frames.

tBLASTx – Compares the six frame translation of a nucleotide query sequence against the 6 frame translation of a nucleotide sequence database.

Megablast – For highly similar sequences.

Procedure:

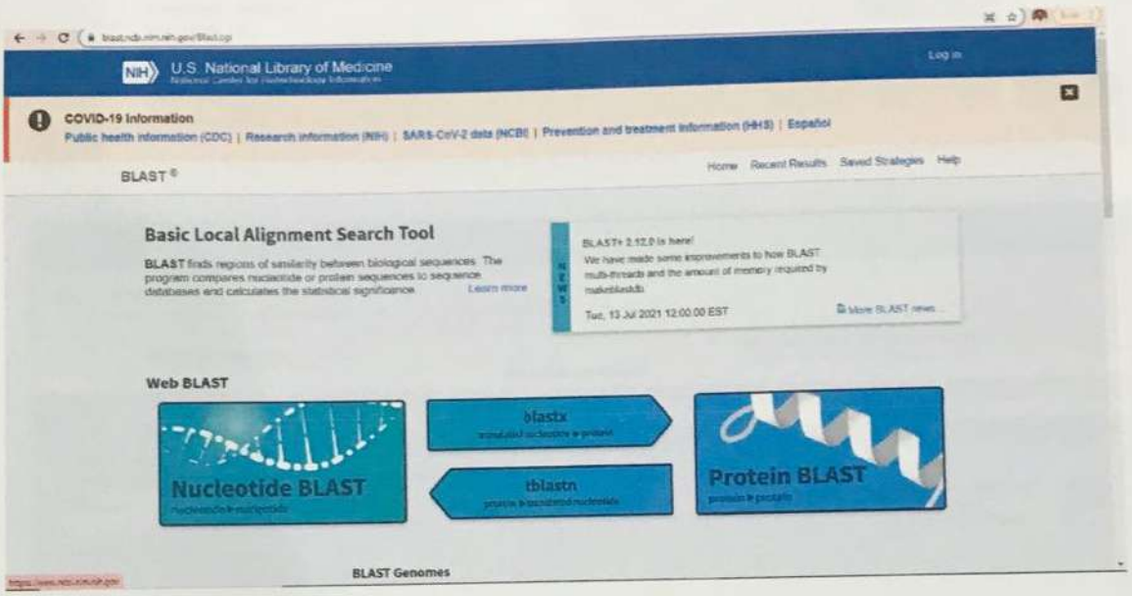
1. Open the web browser and type https://blast.ncbi.nlm.nih.gov/Blast.cgi
2. Click either nucleotide blast or protein blast icon according to the requirement
3. Select "Align two or more sequences" check box for opting multiple sequence alignment or deselect for pair wise alignment
4. Upload or paste a query sequence (in FASTA format) in the query box and execute BLAST for pair wise alignment. This will be identifying most similar sequences from the databank.
5. Upload or paste a query sequence (in FASTA format) in the query box and upload more than one sequences (in FASTA format) in the subject

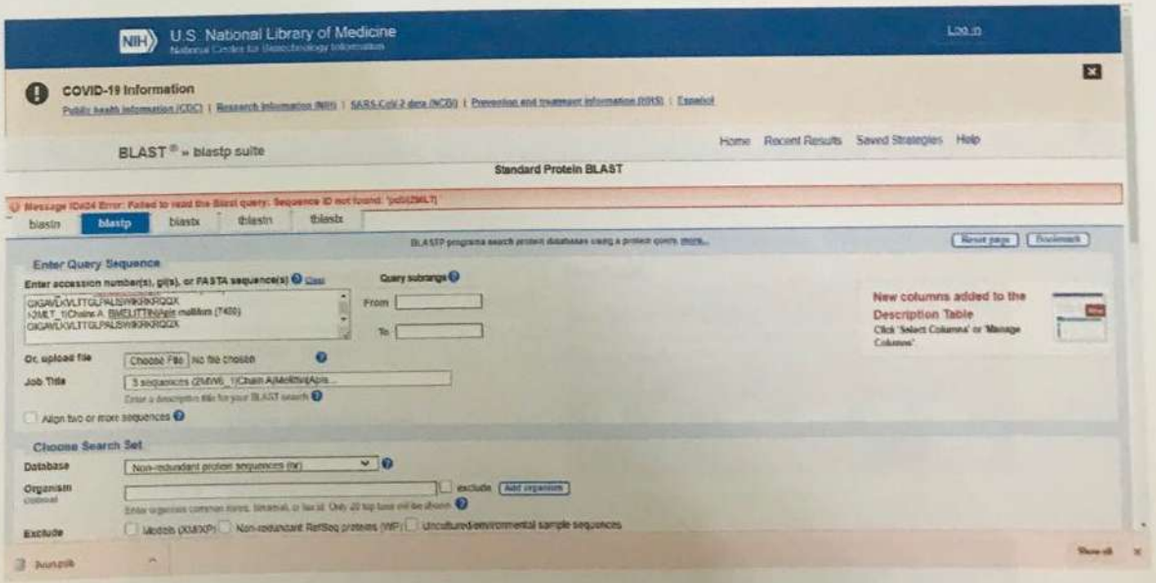
box and then execute BLAST for multiple sequence alignment. This will be identifying the similarity /dissimilarity among the sequences.

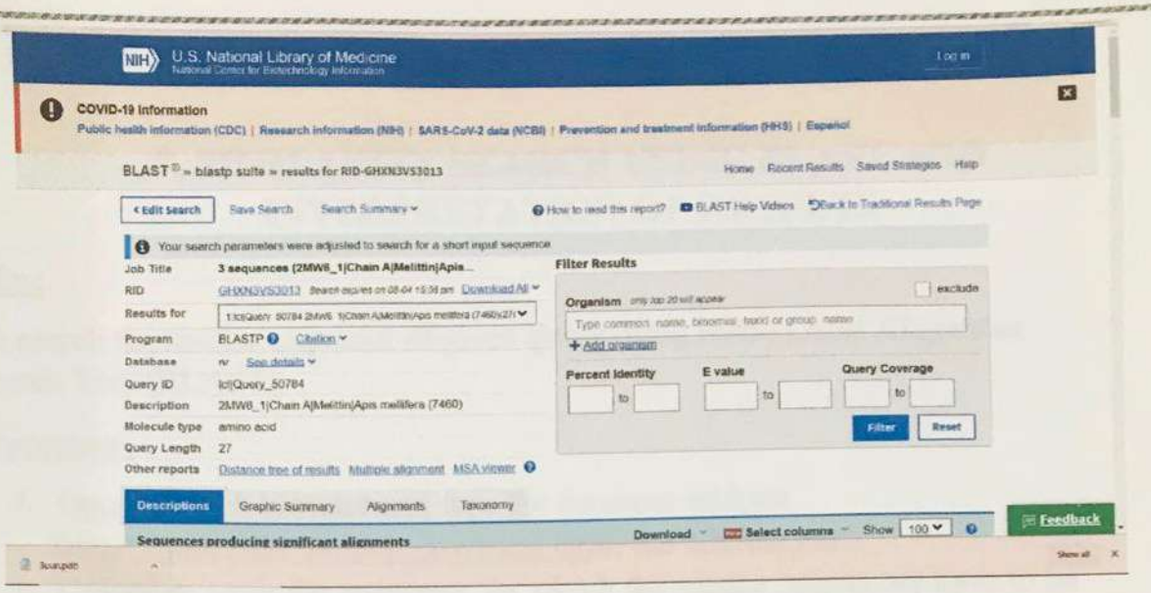
6. Report the result

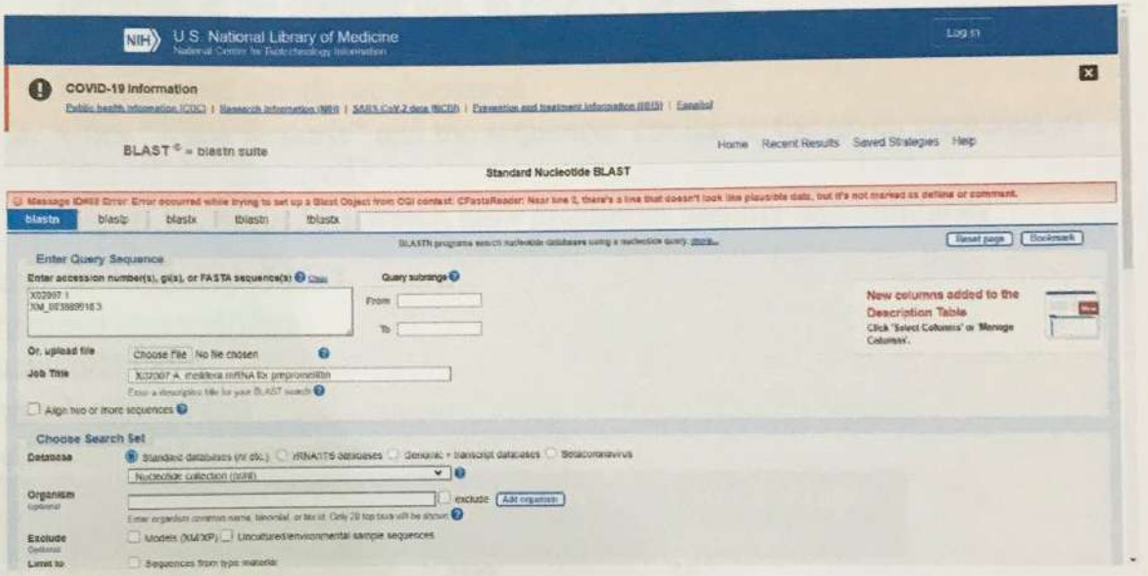
Result:

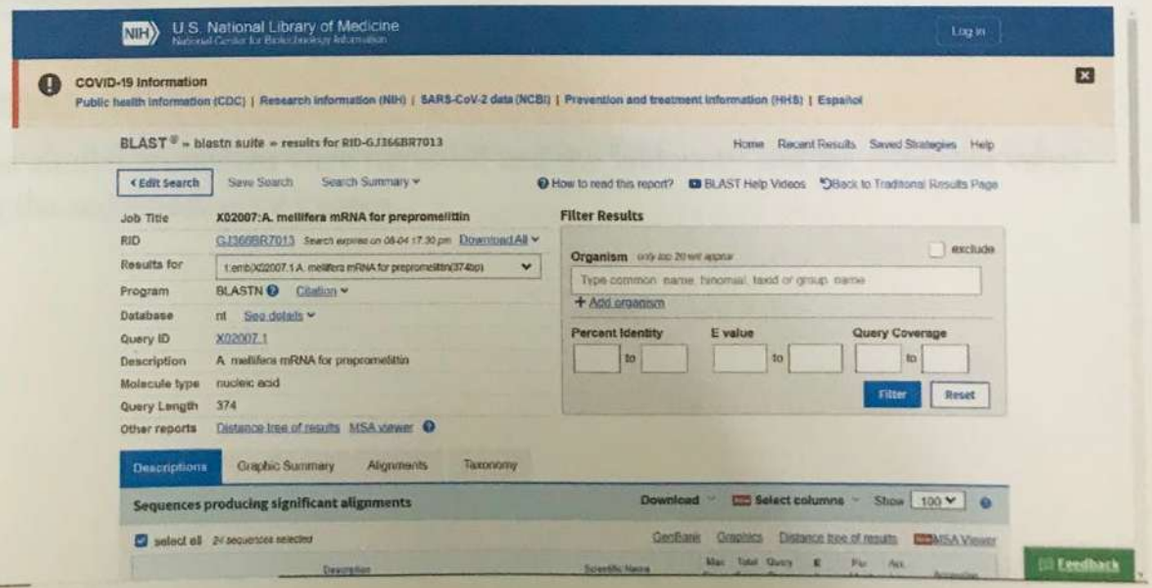
Thus the given sequences are aligned using BLAST






U.S. National Library of Medicine
COVID-19 Information
BLAST® » blastp suite » results for RID-GHXN3VS3013
Edit Search
Save Search
Search Summary
Job Title
3 sequences (2MW8_1|Chain A|Melittin|Apis...
RID
GHXN3VS3013
Results for
Program
BLASTP
Database
Query ID
lcl|Query_50784
Description
2MW8_1|Chain A|Melittin|Apis mellifera (7460)
Molecule type
amino acid
Query Length
27
Other reports
Distance tree of results
Multiple alignment
MSA viewer
Filter Results
Organism
Percent Identity
E value
Query Coverage
Filter
Reset
Descriptions
Graphic Summary
Alignments
Taxonomy
Sequences producing significant alignments
Download
Select columns
Show
Feedback


U.S. National Library of Medicine
COVID-19 Information
BLAST® » blastn suite
Standard Nucleotide BLAST
blastn
blastp
blastx
tblastn
tblastx
Enter Query Sequence
Enter accession number(s), gi(s), or FASTA sequence(s)
Query subrange
From
To
Or, upload file
Choose File
No file chosen
Job Title
Align two or more sequences
Choose Search Set
Database
Organism
Exclude
Limit to
New columns added to the Description Table


U.S. National Library of Medicine
COVID-19 Information
BLAST® » blastn suite » results for RID-GJ366BR7013
Edit Search
Save Search
Search Summary
Job Title
X02007:A. mellifera mRNA for prepromelittin
RID
GJ366BR7013
Results for
Program
BLASTN
Database
nt
Query ID
X02007.1
Description
A. mellifera mRNA for prepromelittin
Molecule type
nucleic acid
Query Length
374
Other reports
Distance tree of results
MSA viewer
Filter Results
Organism
Percent Identity
E value
Query Coverage
Filter
Reset
Descriptions
Graphic Summary
Alignments
Taxonomy
Sequences producing significant alignments
Download
Select columns
Show
select all
GenBank
Graphics
Distance tree of results
MSA Viewer
Feedback

9. SIMILARITY SEARCH USING BLAST AND INTERPRETATION OF RESULTS

Aim:

To search the similar sequence of given query using Basic Local Alignment Search Tool (BLAST).

Procedure:

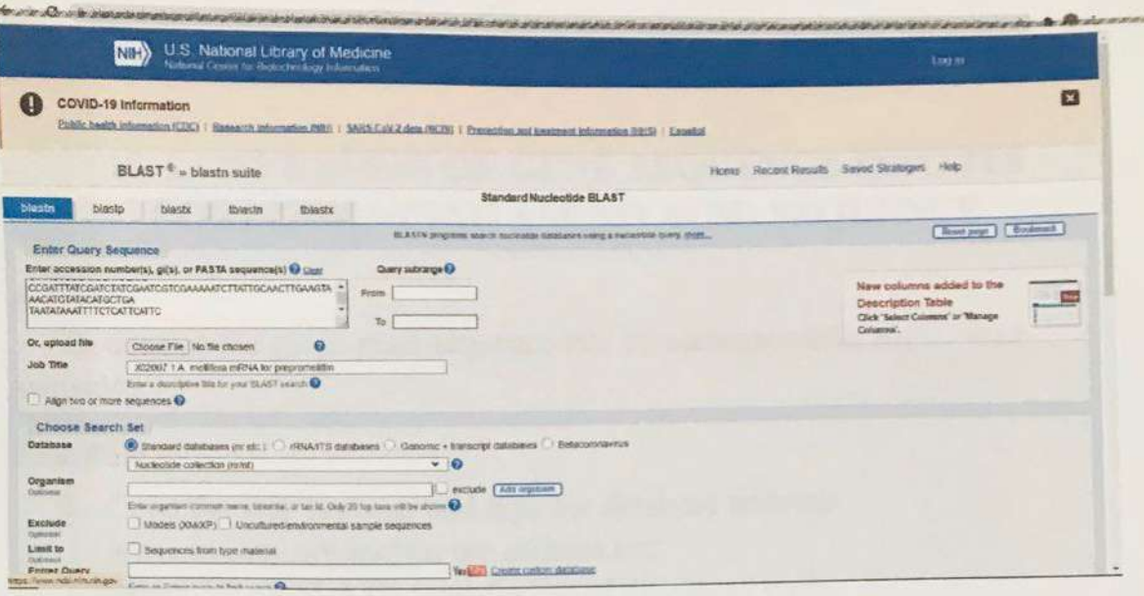
1. Open the web browser and type the database address https://blast.ncbi.nlm.nih.gov/Blast.cgiin the address bar.
1. Upload or paste the sequence, for which the similar sequences have to be retrieved, in the dialogue box, in FASTA format.
2. Click "Begin Search" option so that the page containing the organism's ID and other details are displayed.
3. Click "View Reports" and the sequences similar to the given sequence of interest are displayed.
4. Report the sequences which have maximum identity and similarity.

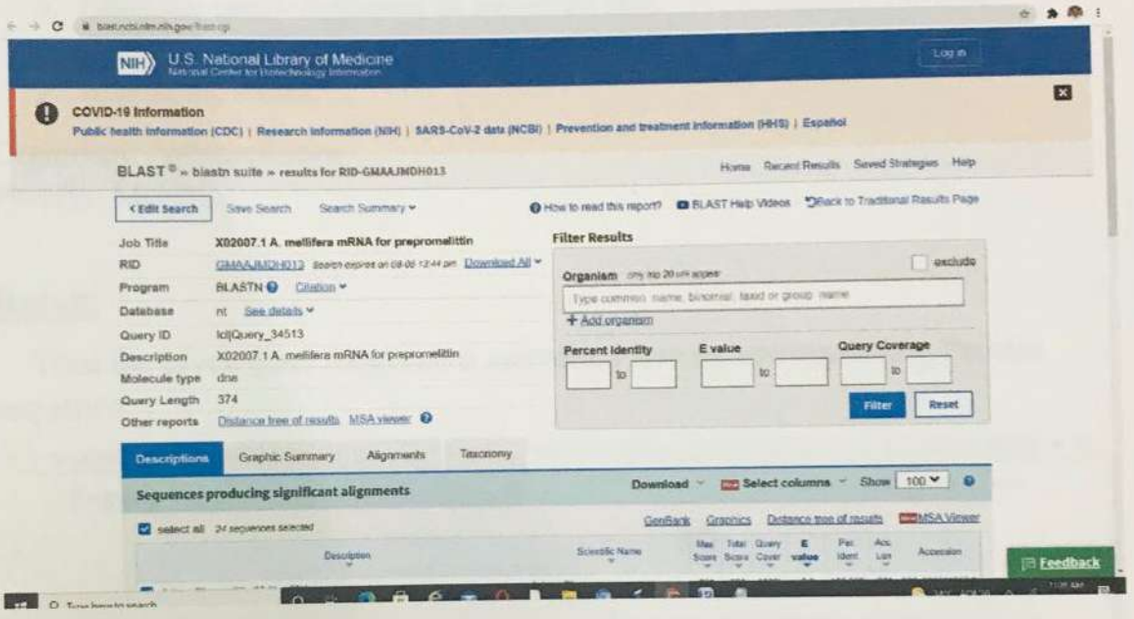
Query ID:lcl|Query_34513
Organism:apismellifera
Nucleotide:DNA
Query length:274bp
Retrieved similar sequences

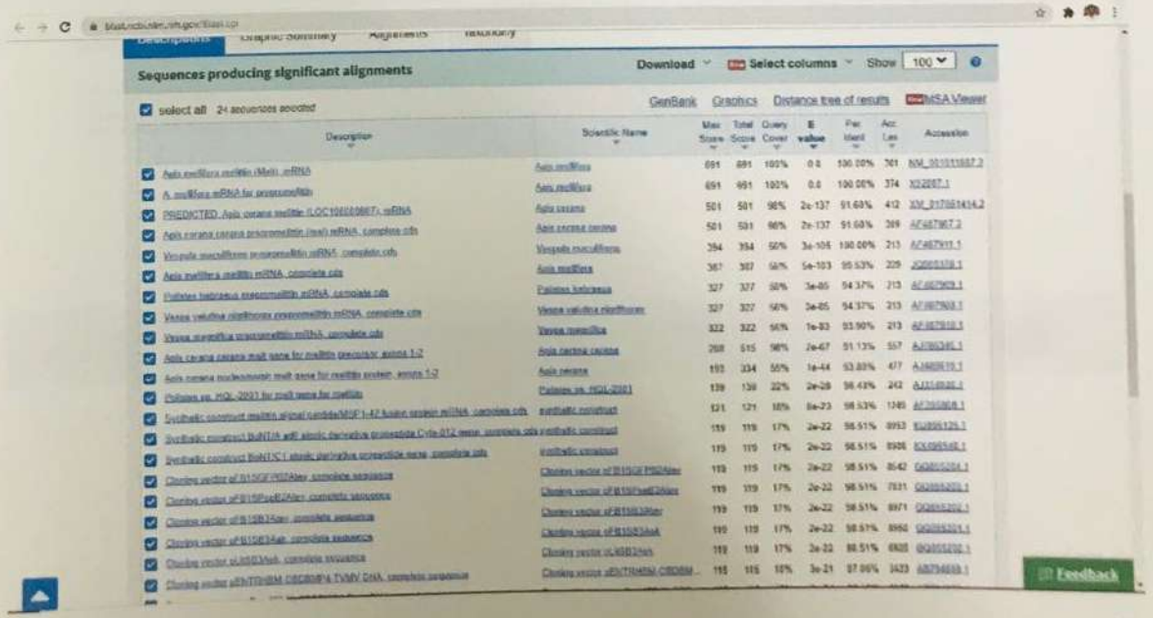
ID NO.	E-value	%identity	Length of the sequence
NM_001011607.2	0	100	381
AF487907.2	2e -137	91.60	389

Result:

The similar sequences were retrieved and the %identity and the expected value for the sequences were known


U.S. National Library of Medicine
COVID-19 Information
BLAST® » blastn suite
Standard Nucleotide BLAST
blastn
blastp
blastx
tblastn
tblastx
Enter Query Sequence
Enter accession number(s), gi(s), or FASTA sequence(s)
Query subrange
From
To
Or, upload file
Choose File
No file chosen
Job Title
X02007.1 A. mellifera mRNA for prepromelittin
Align two or more sequences
Choose Search Set
Database
Standard databases (nr etc.)
rRNA/ITS databases
Genomic + transcript databases
Betacoronavirus
Nucleotide collection (nr/nt)
Organism
exclude
Add organism
Exclude
Models (XM/XP)
Uncultured/environmental sample sequences
Limit to
Sequences from type material
Enter Query
Create custom database
New columns added to the Description Table
Click 'Select Columns' or 'Manage Columns'.


U.S. National Library of Medicine
COVID-19 Information
Public health information (CDC) | Research information (NIH) | SARS-CoV-2 data (NCBI) | Prevention and treatment information (HHS) | Español
BLAST® » blastn suite » results for RID-GMAAJMDH013
Home
Recent Results
Saved Strategies
Help
Edit Search
Save Search
Search Summary
How to read this report?
BLAST Help Videos
Back to Traditional Results Page
Job Title
X02007.1 A. mellifera mRNA for prepromelittin
RID
GMAAJMDH013
Download All
Program
BLASTN
Citation
Database
nt
See details
Query ID
lcl|Query_34513
Description
X02007.1 A. mellifera mRNA for prepromelittin
Molecule type
dna
Query Length
374
Other reports
Distance tree of results
MSA viewer
Filter Results
Organism
Add organism
Percent identity
E value
Query Coverage
Filter
Reset
Descriptions
Graphic Summary
Alignments
Taxonomy
Sequences producing significant alignments
Download
Select columns
Show
select all
GenBank
Graphics
Distance tree of results
MSA Viewer
Description
Scientific Name
Max Score
Total Score
Query Cover
E value
Per. Ident
Acc. Len
Accession
Feedback


Sequences producing significant alignments
Download
Select columns
Show
select all
GenBank
Graphics
Distance tree of results
MSA Viewer
Description
Scientific Name
Max Score
Total Score
Query Cover
E value
Per. Ident
Acc. Len
Accession
Feedback

10. CONVERSION OF GENE SEQUENCE INTO ITS CORRESPONDING AMINO ACID SEQUENCE

Aim:

To convert the given gene sequence into its corresponding amino acid sequence.

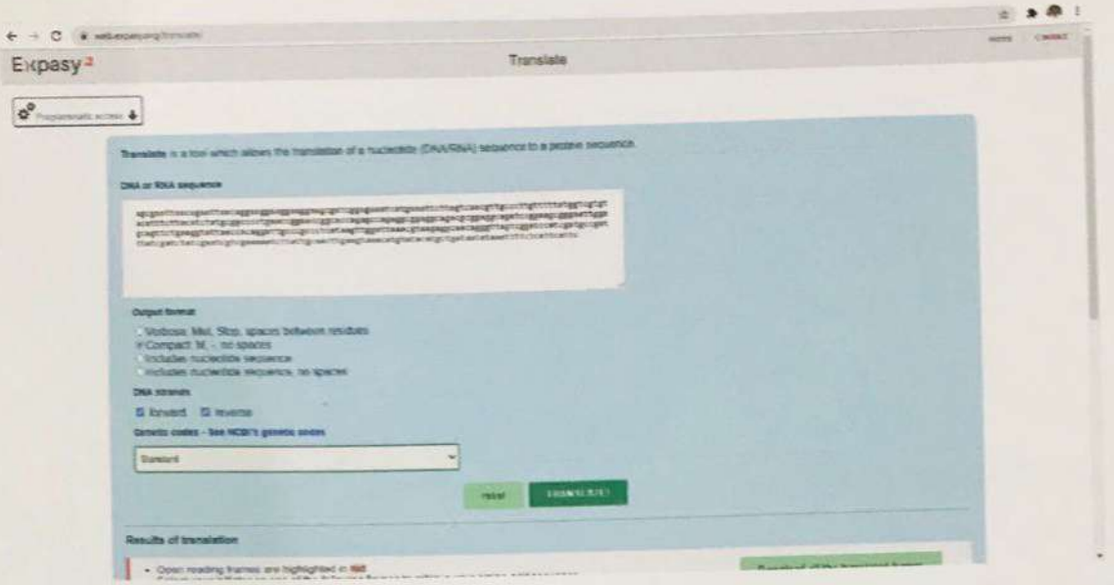
Procedure:

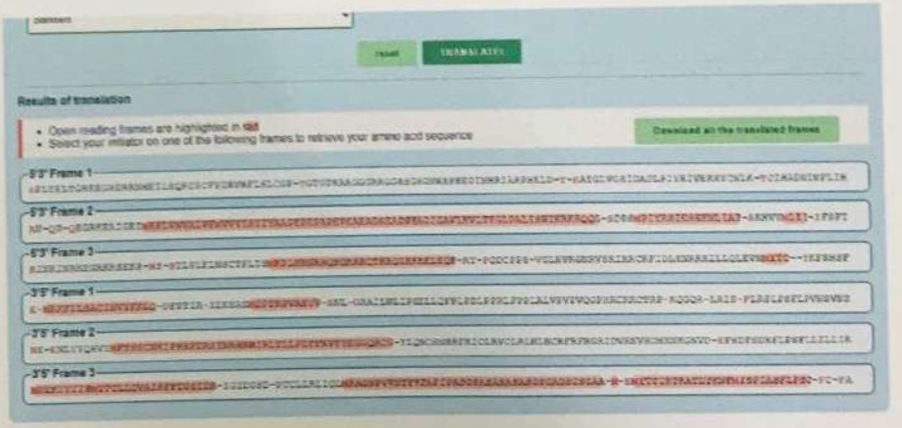
1. Open the web browser and type the database address www.expasy.org/toolsin the address bar.
2. Select TRANSLATE from the list of displayed tools.
3. Paste the query sequence in the given box.
4. Click TRANSLATE option.
5. Report the result.

Organism: Apismellifera.
Protein: Melittin

Result:

Thus the given gene sequence is converted into its corresponding Protein sequence.


Expasy
Translate
Translate is a tool which allows the translation of a nucleotide (DNA/RNA) sequence to a protein sequence.
DNA or RNA sequence
Output format:
Compact: M, - no spaces
DNA strands
forward
reverse
Genetic codes - See NCBI's genetic codes
Standard
reset
Results of translation
Open reading frames are highlighted in red


reset
Results of translation
Open reading frames are highlighted in red
Select your initiator on one of the following frames to retrieve your amino acid sequence
Download all the translated frames
5'3' Frame 1
5'3' Frame 2
5'3' Frame 3
3'5' Frame 1
3'5' Frame 2
3'5' Frame 3

SRR & CVR GOVERNMENT DEGREE COLLEGE (A) VIJAYAWADA

COMMUNITY SERVICE PROJECT

"Community service project report entitled ***"Farming Survey in Nehru Nagar Thanda*** *Government Degree College (A), Vijayawada in partial fulfilment for the award of the Degree of Bachelor science.*

Title of the Project: *Farming Survey in Jugguru Village*

Submitted

By

B.Muni Vagya Naik

Reg. No. 20311001

Project Supervisor

Dr.CH. SRINIVASA REDDY M.Sc., M.Phil., Ph.D
Department of Botany
SRR & CVR Government Degree College (A), Vijayawada

DECLARATION

I, **B.Muni Vagya Naik Regd.No: 20311001** hereby declare that the Community service project Entitled " **Farming Survey in Nehru Nagar Thanda**", Krishna District, Andhra Pradesh submitted to the SRR & CVR Government Degree College (A), Vijayawada for the award of the degree of Bachelor Science is the result of bonafide work carried out by me in the Department of Botany, SRR & CVR Government Degree College under the guidance of Dr. Ch. Srinivasa Reddy, Lecturer in Botany , Department of Botany in the Month of June 2022. I further declare that the work has not been previously submitted for any other degree or diploma of any other University.

By

B. muni vagya Naik

B.Muni Vagya Naik

Regd: 20311001

CERTIFICATE

Date: 28/6/2022

This is to certify that the Community Service Project report entitled **"Farming Survey in Nehru Nagar Thanda"** Krishna District, Andhra Pradesh is a bonafide work done and submitted by **B.Muni Vagya Naik** in partial fulfilment of the requirement for the award of Bachelor's Degree is record of work done by him in the month of June 2022 under my supervision.

Ch Srinivasa Reddy

CH.SRINIVASA REDDY

Dr. CH. SRINIVASA REDDY
LECTURER IN BOTANY
SRR & CVR GOVT. DEGREE COLLEGE
VIJAYAWADA-520 004.

ACKNOWLEDGEMENTS

Though words are seldom sufficient to express gratitude and feelings, it somehow gives me an opportunity to acknowledge those who helped me during my Community Service Project. The work of dissertation preparation was a daunting task and a fascinating experience. I take this opportunity to express my deep sense of gratitude, respect to my esteemed teacher and guide Dr. Ch. Srinivasa Reddy, Lecturer in Botany, SRR & CVR Government Degree College, Vijayawada *for his support, guidance and encouragement throughout this work. It has been honour and privilege for me to work under him for this Project. My special thanks to Our beloved Principal Dr. K.Bhagya Lakshmi , SRR & CVR Government Degree College, Vijayawada.* I would like to thank my family and friends for their cooperation and encouragement throughout my research work. *I am thankful to each and everyone who have given me constant support.*

B muni vagya Naik

[B. Muni Vagya Naik]

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SRR & CVR GOVERNMENT DEGREE COLLEGE (A) VIJAYAWADA

COMMUNITY SERVICE PROJECT

"Community service project report entitled ***"Farming Survey in Vutukuru Village"*** Krishna District, Andhra Pradesh *submitted to SRR &CVR Government Degree College (A), Vijayawada in partial fulfilment for the award of the Degree of Bachelor science.*

Title of the Project: *Farming Survey in Vutukuru Village*

Submitted

By

Bollepogu Kutumba Rani

Reg. No. 20311002

Project Supervisor

Dr.CH. SRINIVASA REDDY M.Sc., M.Phil., Ph.D
Department of Botany
SRR & CVR Government Degree College (A), Vijayawada

DECLARATION

I, *Bollepogu Kutumba Rani* **Regd.No: 20311002** hereby declare that the Community service project Entitled **" Farming Survey in** *Vutukuru Village* **"**, Krishna District, Andhra Pradesh submitted to the SRR & CVR Government Degree College (A), Vijayawada for the award of the degree of Bachelor Science is the result of bonafide work carried out by me in the Department of Botany, SRR & CVR Government Degree College under the guidance of Dr. Ch. Srinivasa Reddy, Lecturer in Botany , Department of Botany in the Month of June 2022. I further declare that the work has not been previously submitted for any other degree or diploma of any other University.

By

Bollepogu Kutumba Rani

Regd: 20311002

CERTIFICATE

Date: 28/06/2022

This is to certify that the Community Service Project report entitled **"Farming Survey in *Vutukuru Village*"** Krishna District, Andhra Pradesh is a bonafide work done and submitted by *Bollepogu Kutumba Rani* in partial fulfilment of the requirement for the award of Bachelor's Degree is record of work done by him in the month of June 2022 under my supervision.

Ch Srinivasa Reddy

CH.SRINIVASA REDDY

Dr. CH. SRINIVASA REDDY
LECTURER IN BOTANY
SRR & CVR GOVT. DEGREE COLLEGE
VIJAYAWADA-520 004.

ACKNOWLEDGEMENTS

Though words are seldom sufficient to express gratitude and feelings, it somehow gives me an opportunity to acknowledge those who helped me during my Community Service Project. The work of dissertation preparation was a daunting task and a fascinating experience. I take this opportunity to express my deep sense of gratitude, respect to my esteemed teacher and guide Dr. Ch. Srinivasa Reddy, Lecturer in Botany, SRR & CVR Government Degree College, Vijayawada *for his support, guidance and encouragement throughout this work. It has been honour and privilege for me to work under him for this Project. My special thanks to Our beloved Principal Dr. K.Bhagya Lakshmi , SRR & CVR Government Degree College, Vijayawada.* I would like to thank my family and friends for their cooperation and encouragement throughout my research work. *I am thankful to each and everyone who have given me constant support.*

[*Bollepogu Kutumba Rani*]

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SRR & CVR GOVERNMENT DEGREE COLLEGE (A) VIJAYAWADA

COMMUNITY SERVICE PROJECT

"Community service project report entitled ***"Farming Survey in Ganguru Village"*** Krishna District, Andhra Pradesh *submitted to SRR &CVR Government Degree College (A), Vijayawada in partial fulfilment for the award of the Degree of Bachelor science.*

Title of the Project: *Farming Survey in Ganguru Village*

Submitted

By

Gulivindala Rupa Vathi

Reg. No. 20311003

Project Supervisor

Dr.CH. SRINIVASA REDDY M.Sc., M.Phil., Ph.D
Department of Botany
SRR & CVR Government Degree College (A), Vijayawada

DECLARATION

I, **Gulivindala Rupa Vathi Regd.No: 20311003** hereby declare that the Community service project Entitled **" Farming Survey in Ganguru Village"**, Krishna District, Andhra Pradesh submitted to the SRR & CVR Government Degree College (A), Vijayawada for the award of the degree of Bachelor Science is the result of bonafide work carried out by me in the Department of Botany, SRR & CVR Government Degree College under the guidance of Dr. Ch. Srinivasa Reddy, Lecturer in Botany , Department of Botany in the Month of June 2022. I further declare that the work has not been previously submitted for any other degree or diploma of any other University.

By

G. Rupa

G. Rupa Vathi

Regd: 20311003

CERTIFICATE

Date: 28/06/2022

This is to certify that the Community Service Project report entitled **"Farming Survey in Ganguru Village"** Krishna District, Andhra Pradesh is a bonafide work done and submitted by **G. Rupa Vathi** in partial fulfilment of the requirement for the award of Bachelor's Degree is record of work done by him in the month of June 2022 under my supervision.

Ch Srinivasa Reddy

CH.SRINIVASA REDDY
Dr. CH. SRINIVASA REDDY
LECTURER IN BOTANY
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ACKNOWLEDGEMENTS

Though words are seldom sufficient to express gratitude and feelings, it somehow gives me an opportunity to acknowledge those who helped me during my Community Service Project. The work of dissertation preparation was a daunting task and a fascinating experience. I take this opportunity to express my deep sense of gratitude, respect to my esteemed teacher and guide Dr. Ch. Srinivasa Reddy, Lecturer in Botany, SRR & CVR Government Degree College, Vijayawada *for his support, guidance and encouragement throughout this work. It has been honour and privilege for me to work under him for this Project. My special thanks to Our beloved Principal Dr. K.Bhagya Lakshmi , SRR & CVR Government Degree College, Vijayawada.* I would like to thank my family and friends for their cooperation and encouragement throughout my research work. *I am thankful to each and everyone who have given me constant support.*

G. Rupa Vathi

[G. Rupa Vathi]

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SRR & CVR GOVERNMENT DEGREE COLLEGE (A)
VIJAYAWADA

COMMUNITY SERVICE PROJECT

"Community service project report entitled ***"Farming Survey in Jujjuru village*** *Government Degree College (A), Vijayawada in partial fulfilment for the award of the Degree of Bachelor science.*

Title of the Project: *Farming Survey in Jujjuru Village*

Submitted

By

K.Ravi Kumar

Reg. No. 20311004

Project Supervisor

Dr.CH. SRINIVASA REDDY M.Sc., M.Phil., Ph.D
Department of Botany
SRR & CVR Government Degree College (A), Vijayawada

DECLARATION

I, **K.Ravi Kumar Regd.No: 20311004** hereby declare that the Community service project Entitled **" Farming Survey in Jujjuru Village"**, Krishna District, Andhra Pradesh submitted to the SRR & CVR Government Degree College (A), Vijayawada for the award of the degree of Bachelor Science is the result of bonafide work carried out by me in the Department of Botany, SRR & CVR Government Degree College under the guidance of Dr. Ch. Srinivasa Reddy, Lecturer in Botany , Department of Botany in the Month of June 2022. I further declare that the work has not been previously submitted for any other degree or diploma of any other University.

By

K. Ravi kumar.

K.Ravi Kumar

Regd: 20311004

CERTIFICATE

Date: 28/06/2022

This is to certify that the Community Service Project report entitled **"Farming Survey in Jujjuru Village"** Krishna District, Andhra Pradesh is a bonafide work done and submitted by **K.Ravi Kumar** in partial fulfilment of the requirement for the award of Bachelor's Degree is record of work done by him in the month of June 2022 under my supervision.

Ch Srinivasa Reddy

CH.SRINIVASA REDDY

Dr. CH. SRINIVASA REDDY
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VIJAYAWADA-520 004

ACKNOWLEDGEMENTS

Though words are seldom sufficient to express gratitude and feelings, it somehow gives me an opportunity to acknowledge those who helped me during my Community Service Project. The work of dissertation preparation was a daunting task and a fascinating experience. I take this opportunity to express my deep sense of gratitude, respect to my esteemed teacher and guide Dr. Ch. Srinivasa Reddy, Lecturer in Botany, SRR & CVR Government Degree College, Vijayawada *for his support, guidance and encouragement throughout this work. It has been honour and privilege for me to work under him for this Project. My special thanks to Our beloved Principal Dr. K.Bhagya Lakshmi , SRR & CVR Government Degree College, Vijayawada.* I would like to thank my family and friends for their cooperation and encouragement throughout my research work. *I am thankful to each and everyone who have given me constant support.*

K. Ravi Kumar.

[K.Ravi Kumar]

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"Community service project report entitled ***"Farming Survey in Kavuluru village*** *Government Degree College (A), Vijayawada in partial fulfilment for the award of the Degree of Bachelor science.*

Title of the Project: *Farming Survey in Kavuluru Village*

Submitted

By

P.Lakshmi Bhavani

Reg. No. 20311006

Project Supervisor

Dr.CH. SRINIVASA REDDY M.Sc., M.Phil., Ph.D
Department of Botany
SRR & CVR Government Degree College (A), Vijayawada

DECLARATION

I, **P.Lakshmi Bhavani Regd.No: 20311006** hereby declare that the Community service project Entitled **" Farming Survey in Kavuluru Village"**, Krishna District, Andhra Pradesh submitted to the SRR & CVR Government Degree College (A), Vijayawada for the award of the degree of Bachelor Science is the result of bonafide work carried out by me in the Department of Botany, SRR & CVR Government Degree College under the guidance of Dr. Ch. Srinivasa Reddy, Lecturer in Botany , Department of Botany in the Month of June 2022. I further declare that the work has not been previously submitted for any other degree or diploma of any other University.

By

P.Lakshmi Bhavani

Regd: 20311006

CERTIFICATE

Date: 28/06/2022

This is to certify that the Community Service Project report entitled **"Farming Survey in Kavuluru Village"** Krishna District, Andhra Pradesh is a bonafide work done and submitted by **P.Lakshmi Bhavani** in partial fulfilment of the requirement for the award of Bachelor's Degree is record of work done by him in the month of June 2022 under my supervision.

Ch. Srinivasa Reddy

CH.SRINIVASA REDDY

Dr. CH. SRINIVASA REDDY
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VIJAYAWADA-520 004.

ACKNOWLEDGEMENTS

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[P.Lakshmi Bhavani]

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SRR & CVR GOVERNMENT DEGREE COLLEGE (A) VIJAYAWADA

COMMUNITY SERVICE PROJECT

"Community service project report entitled ***"Farming Survey in Boddpadu Village"*** Krishna District, Andhra Pradesh *submitted to SRR &CVR Government Degree College (A), Vijayawada in partial fulfilment for the award of the Degree of Bachelor science.*

Title of the Project: *Farming Survey in Boddapadu Village*

Submitted

By

Mr.Penugdona Sateesh

Reg. No. 20311008

Project Supervisor

Dr.CH. SRINIVASA REDDY M.Sc., M.Phil., Ph.D
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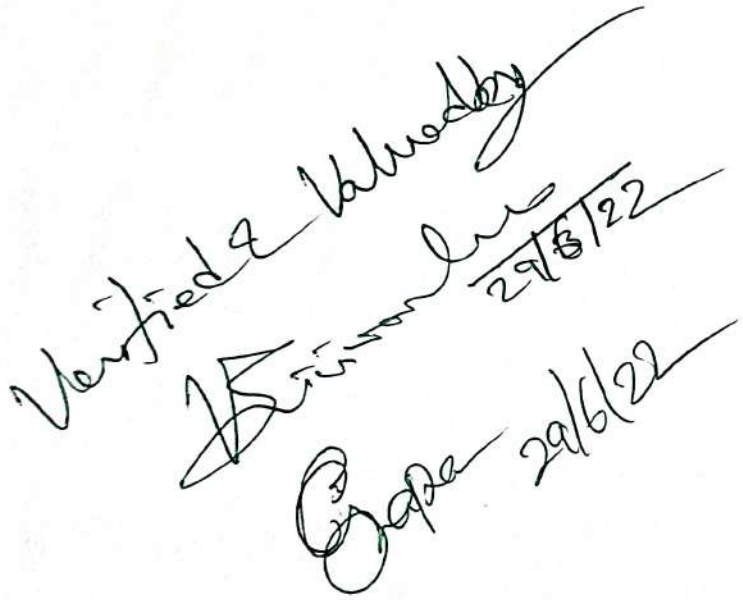
SRR & CVR Government Degree College (A), Vijayawada


Verified & Validated
29/6/22
29/6/22

DECLARATION

I, **Penugoda Sateesh Regd.No: 20311008** hereby declare that the Community service project Entitled **" Farming Survey in Boddpadu Village"**, Krishna District, Andhra Pradesh submitted to the SRR & CVR Government Degree College (A), Vijayawada for the award of the degree of Bachelor Science is the result of bonafide work carried out by me in the Department of Botany, SRR & CVR Government Degree College under the guidance of Dr. Ch. Srinivasa Reddy, Lecturer in Botany , Department of Botany in the Month of June 2022. I further declare that the work has not been previously submitted for any other degree or diploma of any other University.

By

P. Sateesh

Mr.Penugonda Sateesh

Regd: 20311008

CERTIFICATE

Date: 28/06/2022

This is to certify that the Community Service Project report entitled **"Farming Survey in Boddapadu Village"** Krishna District, Andhra Pradesh is a bonafide work done and submitted by **Penugonda Sateesh** in partial fulfilment of the requirement for the award of Bachelor's Degree is record of work done by him in the month of June 2022 under my supervision.

Ch Srinivasa Reddy

CH.SRINIVASA REDDY

Dr. CH. SRINIVASA REDDY
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SRR & CVR GOVT. DEGREE COLLEGE
VIJAYAWADA-520 004.

ACKNOWLEDGEMENTS

Though words are seldom sufficient to express gratitude and feelings, it somehow gives me an opportunity to acknowledge those who helped me during my Community Service Project. The work of dissertation preparation was a daunting task and a fascinating experience. I take this opportunity to express my deep sense of gratitude, respect to my esteemed teacher and guide Dr. Ch. Srinivasa Reddy, Lecturer in Botany, SRR & CVR Government Degree College, Vijayawada *for his support, guidance and encouragement throughout this work. It has been honour and privilege for me to work under him for this Project. My special thanks to Our beloved Principal Dr. K.Bhagya Lakshmi , SRR & CVR Government Degree College, Vijayawada.* I would like to thank my family and friends for their cooperation and encouragement throughout my research work. *I am thankful to each and everyone who have given me constant support.*

P. Sateesh.

[Penugonda Sateesh]

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SRR & CVR GOVERNMENT DEGREE COLLEGE (A) VIJAYAWADA

COMMUNITY SERVICE PROJECT

"Community service project report entitled ***"Farming Survey in Ganguru Village"*** Krishna District, Andhra Pradesh *submitted to SRR &CVR Government Degree College (A), Vijayawada in partial fulfilment for the award of the Degree of Bachelor science.*

Title of the Project: *Farming Survey in Ganguru Village*

Submitted

By

U. Suneetha

Reg. No. 20311010

Project Supervisor

Dr.CH. SRINIVASA REDDY M.Sc., M.Phil., Ph.D
Department of Botany
SRR & CVR Government Degree College (A), Vijayawada

DECLARATION

I, **U. Suneetha Regd.No: 20311010** hereby declare that the Community service project Entitled **" Farming Survey in Ganguru Village"**, Krishna District, Andhra Pradesh submitted to the SRR & CVR Government Degree College (A), Vijayawada for the award of the degree of Bachelor Science is the result of bonafide work carried out by me in the Department of Botany, SRR & CVR Government Degree College under the guidance of Dr. Ch. Srinivasa Reddy, Lecturer in Botany , Department of Botany in the Month of June 2022. I further declare that the work has not been previously submitted for any other degree or diploma of any other University.

By

U Suneetha

U.Suneetha

Regd: 20311010

CERTIFICATE

Date: 28/06/2022

This is to certify that the Community Service Project report entitled **"Farming Survey in Ganguru Village"** Krishna District, Andhra Pradesh is a bonafide work done and submitted by **U. Suneetha** in partial fulfilment of the requirement for the award of Bachelor's Degree is record of work done by him in the month of June 2022 under my supervision.

Ch. Srinivasa Reddy

CH.SRINIVASA REDDY

Dr. CH. SRINIVASA REDDY
LECTURER IN BOTANY
SRR & CVR GOVT. DEGREE COLLEGE
VIJAYAWADA-520 004.

ACKNOWLEDGEMENTS

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U. Suneetha

[U.Suneetha]

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SRR & CVR GOVERNMENT DEGREE COLLEGE (A)
VIJAYAWADA
COMMUNITY SERVICE PROJECT

"Community service project report entitled ***"Farming Survey in Vutukuru Village"*** Krishna District, Andhra Pradesh *submitted to SRR &CVR Government Degree College (A), Vijayawada in partial fulfilment for the award of the Degree of Bachelor science.*

Title of the Project: *Farming Survey in Vutukuru Village*

Submitted

By

Jutru Naga Malleswari

Reg. No. 20311011

Verified & evaluated by

Verified by

Project Supervisor

Dr.CH. SRINIVASA REDDY M.Sc., M.Phil., Ph.D
Department of Botany
SRR & CVR Government Degree College (A), Vijayawada

DECLARATION

I, **Jutru Naga Malleswari Regd.No: 20311011** hereby declare that the Community service project Entitled **" Farming Survey in** *Vutukuru Village"*, Krishna District, Andhra Pradesh submitted to the SRR & CVR Government Degree College (A), Vijayawada for the award of the degree of Bachelor Science is the result of bonafide work carried out by me in the Department of Botany, SRR & CVR Government Degree College under the guidance of Dr. Ch. Srinivasa Reddy, Lecturer in Botany , Department of Botany in the Month of June 2022. I further declare that the work has not been previously submitted for any other degree or diploma of any other University.

By

J. Naga malleswari

Jutru Naga Malleswari
Regd: 20311011

CERTIFICATE

Date:

This is to certify that the Community Service Project report entitled "**Farming Survey in *Vutukuru Village*** " Krishna District, Andhra Pradesh is a bonafide work done and submitted by **Jutru Naga Malleswari** in partial fulfilment of the requirement for the award of Bachelor's Degree is record of work done by him in the month of June 2022 under my supervision.

CH.SRINIVASA REDDY

ACKNOWLEDGEMENTS

Though words are seldom sufficient to express gratitude and feelings, it somehow gives me an opportunity to acknowledge those who helped me during my Community Service Project. The work of dissertation preparation was a daunting task and a fascinating experience. I take this opportunity to express my deep sense of gratitude, respect to my esteemed teacher and guide Dr. Ch. Srinivasa Reddy, Lecturer in Botany, SRR & CVR Government Degree College, Vijayawada *for his support, guidance and encouragement throughout this work. It has been honour and privilege for me to work under him for this Project. My special thanks to Our beloved Principal Dr. K.Bhagya Lakshmi , SRR & CVR Government Degree College, Vijayawada.* I would like to thank my family and friends for their cooperation and encouragement throughout my research work. *I am thankful to each and everyone who have given me constant support.*

J. Naga malleswari

[Jutru Naga Malleswari]

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SRR & CVR GOVERNMENT DEGREE COLLEGE (A)
VIJAYAWADA
COMMUNITY SERVICE PROJECT

"Community service project report entitled ***"Farming Survey in Dangabadra village*** *Government Degree College (A), Vijayawada in partial fulfilment for the award of the Degree of Bachelor science.*

Title of the Project: *Farming Survey in Dangabadra Village*

Submitted

By

V.Dhana Lakshmi

Reg. No. 20311012

Project Supervisor

Dr.CH. SRINIVASA REDDY M.Sc., M.Phil., Ph.D
Department of Botany
SRR & CVR Government Degree College (A), Vijayawada

DECLARATION

I, **V.Dhana Lakshmi Regd.No: 20311012** hereby declare that the Community service project Entitled **" Farming Survey in Danagabadra Village"**, Krishna District, Andhra Pradesh submitted to the SRR & CVR Government Degree College (A), Vijayawada for the award of the degree of Bachelor Science is the result of bonafide work carried out by me in the Department of Botany, SRR & CVR Government Degree College under the guidance of Dr. Ch. Srinivasa Reddy, Lecturer in Botany , Department of Botany in the Month of June 2022. I further declare that the work has not been previously submitted for any other degree or diploma of any other University.

By
V. Dhana lakshmi
V.Dhana Lakshmi

Regd: 20311012

CERTIFICATE

Date: 28-06-2022

This is to certify that the Community Service Project report entitled **"Farming Survey in Dangabadra Village"** Krishna District, Andhra Pradesh is a bonafide work done and submitted by **V.Dhana Lakshmi** in partial fulfilment of the requirement for the award of Bachelor's Degree is record of work done by him in the month of June 2022 under my supervision.

Ch Srinivasa Reddy

CH.SRINIVASA REDDY

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ACKNOWLEDGEMENTS

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V. Dhana Lakshmi

[V.Dhana Lakshmi]

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SRR & CVR GOVERNMENT DEGREE COLLEGE (A) VIJAYAWADA

COMMUNITY SERVICE PROJECT

"Community service project report entitled ***"Farming Survey in Manikonda Village"*** *Krishna District, Andhra Pradesh submitted to SRR &CVR Government Degree College (A), Vijayawada in partial fulfilment for the award of the Degree of Bachelor science.*

Title of the Project: *Farming Survey in Manikonda Village*

Submitted

By

Bonigala.Rechel Evangelin

Reg. No. 20311203

Project Supervisor

Dr.CH. SRINIVASA REDDY M.Sc., M.Phil., Ph.D
Department of Botany
SRR & CVR Government Degree College (A), Vijayawada

DECLARATION

I, *Bonigala.Rechel Evangelin* **Regd.No: 20311203** hereby declare that the Community service project Entitled **" Farming Survey in Manikonda Village"**, Krishna District, Andhra Pradesh submitted to the SRR & CVR Government Degree College (A), Vijayawada for the award of the degree of Bachelor Science is the result of bonafide work carried out by me in the Department of Botany, SRR & CVR Government Degree College under the guidance of Dr. Ch. Srinivasa Reddy, Lecturer in Botany , Department of Botany in the Month of June 2022. I further declare that the work has not been previously submitted for any other degree or diploma of any other University.

By

Bonigala.Rechel Evangelin

Regd: 20311203

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Date: 28/06/22

This is to certify that the Community Service Project report entitled **"Farming Survey in Manikonda Village"** Krishna District, Andhra Pradesh is a bonafide work done and submitted by *Bonigala.Rechel Evangelin* in partial fulfilment of the requirement for the award of Bachelor's Degree is record of work done by him in the month of June 2022 under my supervision.

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ACKNOWLEDGEMENTS

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[*Bonigala.Rechel Evangelin*]

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1. Green Synthesis and Characterization of Zinc Oxide (ZnO) Nano materials.
2. Eco friendly synthesis and Characterization of Copper Oxide (Cu O) Nano materials
3. Phytochemical Assisted Synthesis and Characterization of Mixture of CuO and ZnO Nano materials .

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9.	19306211	MCCS	P. LAKSHMI PRASANNA	Green Synthesis and Characterization of Zinc Oxide (ZnO) Nano materials	P. Lakshmi Prasanna
10.	19306212	MCCs	A.ARUN KUMAR	Green Synthesis and Characterization of Zinc Oxide (ZnO) Nano materials	A. Arun Kumar
11.	19306215	MCCs	G. BHARGAVI	Green Synthesis and Characterization of Zinc Oxide (ZnO) Nano materials	G. Bhargavi
12.	19306220	MCCs	K. VENKATA GOPI	Eco friendly synthesis and Characterization of Copper	K. Venkata Gopi

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19	19301007	MPC ™	R. CHANDRA SEKHAR	Phytochemical Assisted Synthesis and Characterization of Mixture of CuO and ZnO Nano materials	R. Chandra Sekhar
20	19301012	MPC ™	K.SURESH	Eco friendly synthesis and Characterization of Copper Oxide (Cu O) Nano materials	K. Suresh
21	19301013	MPC ™	B.GOPI KRISHNA NAIK	Eco friendly synthesis and Characterization of Copper Oxide (Cu O) Nano materials	B. Gopi Krishna Naik
22	19301015	MPC ™	S. GOPI KRISHNA	Eco friendly synthesis and Characterization of Copper Oxide (CuO)Nano materials	S. Gopi Krishna.

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26	19301209	MPC EM	SAKEENA BANU	Phytochemical Assisted Synthesis and Characterization of Mixture of CuO and ZnO Nano materials	Sakeena
27	19301212	MPC EM	R.DINESH SAI KUMAR	Phytochemical Assisted Synthesis and Characterization of Mixture of CuO and ZnO Nano materials	R. Sai
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29	19301218	MPC EM	CH. DURGA PRASAD	Eco friendly synthesis and Characterization of Copper Oxide (CuO) Nano materials	Ch. Durga Prasad
30	19301220	MPC EM	I.PARALOK	Eco friendly synthesis and Characterization of Copper Oxide (CuO) Nano materials	I. Paralok
31	19301225	MPC EM	E.SAI KRISHNA	Eco friendly synthesis and Characterization of Copper Oxide (CuO) Nano materials	E. Sai Krishna

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3	18304204	T Gnana Sruthi	A Sample survey on Covid-19 Pandamic year
4	18304205	G Rajeswari	A Sample survey on Covid-19 Pandamic year
5	18304206	B Surya Teja	A sample survey on Library Statisfaction
6	18304210	P Prabhu	Cyber Crime awareness for students
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10	18304219	K Leelavathi	A Sample survey on Covid-19 vaccination
11	18304223	K Bhagya Lakshmi	A Sample survey on Covid-19 vaccination
12	18304224	R Ramya Sri	performance of students
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15	18304233	A Indrani	A Sample survey on Covid-19 Pandamic year
16	18304234	P Sai Kumar	A sample survey on Library Statisfaction
17	18304235	P. Durga Prasad	A sample survey on Library Statisfaction
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19	18304241	A Krishnaveni	performance of students
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21	18304244	T Tejaswani	A Sample survey on Covid-19 Pandamic year
22	18304246	K Santhi	performance of students
23	18304247	S Bhavani	A Sample survey on Covid-19 vaccination
24	18304250	B Durga Bhavani	performance of students
25	18304252	K Ganesh	A sample survey on Library Statisfaction
26	18304254	T Bhavani	A Sample survey on Covid-19 vaccination

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Certified that this is a bonafied record of project work done in statistics entitled with "A SAMPLE SURVEY ON COVID-19" done by A.Tejaswini (18304244)) .For the partial fulfillment of the requirement for the award of the bachelor of science degree as a part of the curriculum during the academic year 2018-2021.

Head Of The Department
Head of the Dept. of Statistics
S.R.R. & C.V.P. GOVT. COLLEGE
VIJAYAWADA - 4.

Lecturer In charge

External Examiner

S.R.R.&C.V.R. DEGREE(A) & P. G COLLEGE
MACHAVARAM, VIJAYAWADA

CERTIFICATE

This is to certify that the project entitled ("**EFFECT OF COVID-19 ON ACADEMIC PERFORMANCE OF STUDENTS**") bonafide work done by **K.SANTHI-18304246** belongs to Department of Statistics in S.R.R.&C.V.R. GOVT DEGREE COLLEGE, VIJAYAWADA under my guidance in practical fulfilment of award for the degree in Bachelor of science.

P. RAMESH
(PROJECT GUIDE)

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MACHAVARAM, VIJAYAWADA

CERTIFICATE

This is to certify that the project entitled ("COVID-19 VACCINATION") bonafide work done by S. BHAVANI – 18304247 belongs to Department of Statistics S.R.R & C.V.R GOVT DEGREE COLLEGE, VIJAYAWADA under my guidance in partial fulfilment of award for the degree in Bachelor of science.

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Head of the Dept. of Statistics
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VIJAYAWADA

S.R.R.&C.V.R. DEGREE(A) & P. G COLLEGE
MACHAVARAM, VIJAYAWADA

CERTIFICATE

This is to certify that the project entitled ("**EFFECT OF COVID-19 ON ACADEMIC PERFORMANCE OF STUDENTS**") bonafide work done by **B.DURGA BHAVANI-18304250** belongs to Department of Statistics in S.R.R.&C.V.R. GOVT DEGREE COLLEGE, VIJAYAWADA under my guidance in practical fulfilment of award for the degree in Bachelor of science.

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(H.O.D OF DEPRTMENT OF STATISTICS)

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EXTERNAL EXAMINER

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VIJAYAWADA-520004

DEPARTMENT OF STATISTICS

CERTIFICATE

Certified that this is a bonafied record of project work done in statistics entitled with "A SAMPLE SURVEY ON LIBRARY SATISFICATION" done by A. GANESH (18304252) .For the partial fulfillment of the requirement for the award of the bachelor of science degree as a part of the curriculum during the academic year 2018-2021.

Head Of The Department

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Lecturer In charge

External Examiner

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CERTIFICATE

This is to certify that the project entitled ("COVID-19 VACCINATION") bonafide work done by T. BHAVANI – 18304254 belongs to Department of Statistics S.R.R & C.V.R GOVT DEGREE COLLEGE, VIJAYAWADA under my guidance in partial fulfilment of award for the degree in Bachelor of science.

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EXTERNAL EXAMINER

(H.O.D OF DEPARTMENT)

Head of the Dept. of Statistics
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Phytochemical Assisted Synthesis and Characterization of CuO and ZnO Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for

the Degree of Bachelor of Science

By

E. Divya Keerthana

[Regd.No: 19313217]

Under the Guidance of

Dr. G. NAGARJUNA

M.Sc., B.Ed., M.Ed., M.Phil., Ph.D.,

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JULY 2022

CERTIFICATE

DEPARTMENT OF CHEMISTRY

SRR & CVR GOVT. DEGREE COLLEGE (Autonomous)

NTR. DIST , VIJAYAWADA - 4, A.P.

This is to certify that the project work titled " **Phytochemical Assisted Synthesis and Characterization of Mixture of CuO and ZnO Nano materials"** is a bonafide record carried out by **E. Divya Keerthana [19313217]** under the supervision and the guidance of **Dr. G. Nagarjuna** as Partial fulfillment of requirements for the degree of bachelor of science from SRR &CVR GOVT. DEGREE COLLEGE (A) ,VIJAYAWADA .during the academic year 2021-2022.

Dr.G.Nagarjuna
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Dr. V.SRINIVASA RAO
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Machavaram, VIJAYAWADA-4

ACKNOWLEDGEMENT

I express my sincere thanks to my supervisor and HOD of Chemistry , Dr.V. Srinivasa Rao, M.Sc.,B.Ed., M.Phil., Ph.D., for his esteemed supervision, incessant support, inspiration throughout my project work, for ably guiding us throughout the course of the project.

I express our sincere appreciation to his guidance, kind co-operation and encouragement which enabled us to complete our work successfully.

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I would also thank to our Principal Dr. K. Bhagya Lakshmi for providing opportunity to complete this project and all my faculty members of the department.

I convey my thanks to all my class mates, and project group members

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Date :

DivyaKeerthana E

[E. DIVYA KEERTHANA]

Phytochemical Assisted Synthesis and Characterization of CuO and ZnO Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for

the Degree of Bachelor of Science

by

Ameena Kousar

[Regd.No : 19313226]

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This is to certify that the project work titled " **Phytochemical Assisted Synthesis and Characterization of Mixture of CuO and ZnO Nano materials"** is a bonafide record carried out by **Ameena Kousar[19313226]** under the supervision and the guidance of **Dr. G. Nagarjuna** as Partial fulfillment of requirements for the degree of bachelor of science from SRR &CVR GOVT. DEGREE COLLEGE (A) ,VIJAYAWADA .during the academic year 2021-2022.

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Dr. V.SRINIVASA RAO
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ACKNOWLEDGEMENT

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I would also thank to our Principal Dr. K. Bhagya Lakshmi for providing opportunity to complete this project and all my faculty members of the department.

I convey my thanks to all my class mates, and project group members

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Date :

[AMEENA KOUSAR]

Phytochemical Assisted Synthesis and Characterization of CuO and ZnO Nano Materials

Project Work submitted to

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In partial fulfillment of the requirement for

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by

K. PRIYA KRISHNA

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This is to certify that the project work titled " **Phytochemical Assisted Synthesis and Characterization of Mixture of CuO and ZnO Nano materials"** is a bonafide record carried out by **K.PRIYA KRISHNA [19313230]** under the supervision and the guidance of **Dr. G. Nagarjuna** as Partial fulfillment of requirements for the degree of bachelor of science from SRR &CVR GOVT. DEGREE COLLEGE (A) ,VIJAYAWADA .during the academic year 2021-2022.

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ACKNOWLEDGEMENT

I express my sincere thanks to my supervisor and HOD of Chemistry , Dr.V. Srinivasa Rao, M.Sc.,B.Ed., M.Phil., Ph.D., for his esteemed supervision, incessant support, inspiration throughout my project work, for ably guiding us throughout the course of the project.

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I would also thank to our Principal Dr. K. Bhagya Lakshmi for providing opportunity to complete this project and all my faculty members of the department.

I convey my thanks to all my class mates, and project group members

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Date :

K. Priya Krishna

[K. PRIYA KRISHNA]

Ecofriendly Synthesis and Characterization of Copper Oxide [CuO] Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for

the Degree of Bachelor of Science

by

A. CHAITANYA BHARATHI

[Regd .No: 19311204]

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M.Sc., B.Ed., M.Ed., M.Phil., Ph.D.,

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This is to certify that the project work titled "**Eco friendly synthesis and Characterization of Copper Oxide (Cu O) Nano materials** " is a bonafide record carried out by **A. CHAITANYA BHARATHI [19311204]** under the supervision and the guidance of **Dr.G.Nagarjuna** as Partial fulfillment of requirements for the degree of bachelor of science from SRR &CVR GOVT. DEGREE COLLEGE (A) ,VIJAYAWADA .during the academic year 2021-2022.

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ACKNOWLEDGEMENT

I express my sincere thanks to my supervisor and HOD of Chemistry , Dr.V. Srinivasa Rao, M.Sc.,B.Ed., M.Phil., Ph.D., for his esteemed supervision, incessant support, inspiration throughout my project work, for ably guiding us throughout the course of the project.

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I would also thank to our Principal Dr. K. Bhagya Lakshmi for providing opportunity to complete this project and all my faculty members of the department.

I convey my thanks to all my class mates, and project group members

A-chaitanya Bharathi

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Date :

[A. Chaitanya Bharathi]

Ecofriendly Synthesis and Characterization of Copper Oxide [CuO] Nano Materials

Project Work submitted to

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In partial fulfillment of the requirement for

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by

P.LIKITHA
[Regd.No : 19311208]

Under the Guidance of

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M.Sc., B.Ed., M.Ed., M.Phil., Ph.D.,

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NTR. DIST , VIJAYAWADA - 4, A.P.

This is to certify that the project work titled "**Eco friendly synthesis and Characterization of Copper Oxide (Cu O) Nano materials** " is a bonafide record carried out by **P. LIKITHA [19311208]** under the supervision and the guidance of **Dr.G.Nagarjuna** as Partial fulfillment of requirements for the degree of bachelor of science from SRR &CVR GOVT. DEGREE COLLEGE (A) ,VIJAYAWADA .during the academic year 2021-2022.

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ACKNOWLEDGEMENT

I express my sincere thanks to my supervisor and HOD of Chemistry , Dr.V. Srinivasa Rao, M.Sc.,B.Ed., M.Phil., Ph.D., for his esteemed supervision, incessant support, inspiration throughout my project work, for ably guiding us throughout the course of the project.

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I accord my thanks to Dr. G. Nagarjuna M.Sc.,B.Ed.,M.Ed., M.Phil., Ph.D. Lecturer in Chemistry for the encouragement and support for this project .

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I would also thank to our Principal Dr. K. Bhagya Lakshmi for providing opportunity to complete this project and all my faculty members of the department.

I convey my thanks to all my class mates, and project group members

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Date :

likitha pulivarthi

[Likitha. Pulivarthi]

Ecofriendly Synthesis and Characterization of Copper Oxide [CuO] Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for

the Degree of Bachelor of Science

by

P.JAYA LAKSHMI

[Regd.No: 19311233]

Under the Guidance of

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VIJAYAWADA –520004 , NTR DIST,A.P.

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DEPARTMENT OF CHEMISTRY

SRR & CVR GOVT. DEGREE COLLEGE (Autonomous)

NTR. DIST , VIJAYAWADA - 4, A.P.

This is to certify that the project work titled "**Eco friendly synthesis and Characterization of Copper Oxide (Cu O) Nano materials**" is a bonafide record carried out by **P. JAYA LAKSHMI [19311233]** under the supervision and the guidance of **Dr.G.Nagarjuna** as Partial fulfillment of requirements for the degree of bachelor of science from SRR &CVR GOVT. DEGREE COLLEGE (A) ,VIJAYAWADA .during the academic year 2021-2022.

Dr.G.Nagarjuna
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Dr. V. SRINIVASA RAO
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ACKNOWLEDGEMENT

I express my sincere thanks to my supervisor and HOD of Chemistry , Dr.V. Srinivasa Rao, M.Sc.,B.Ed., M.Phil., Ph.D., for his esteemed supervision, incessant support, inspiration throughout my project work, for ably guiding us throughout the course of the project.

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I accord my thanks to Dr. G. Nagarjuna M.Sc.,B.Ed.,M.Ed., M.Phil., Ph.D. Lecturer in Chemistry for the encouragement and support for this project .

I would like to take the opportunity to thank Dr.K.Rayapa Reddy, Dr.K T S Thamos Raju, Dr.Y.Subba Reddy, Department of Chemistry ,Andhra Loyola College,Vijayawada for providing me with all necessary characterization facilities during the project work.

I would also thank to our Principal Dr. K. Bhagya Lakshmi for providing opportunity to complete this project and all my faculty members of the department.

I convey my thanks to all my class mates, and project group members

Vijayawada

Date :

P. Jaya Lakshmi.

Green Synthesis and Characterization of Zinc Oxide (ZnO) Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for the

Degree of Bachelor of Science

By

K. BHAVANI

[Regd No: 19306204]

Under the Guidance of

Dr. G. Nagarjuna

M.Sc., B.Ed.,M.Ed., M.Phil., Ph.D.,

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VIJAYAWADA–520004, A.P.

JULY 2022

CERTIFICATE

DEPARTMENT OF CHEMISTRY

SRR & CVR GOVT. DEGREE COLLEGE (Autonomous)

NTR. DIST , VIJAYAWADA - 4, A.P.

This is to certify that the project work titled **"Green Synthesis and Characterization of Zinc Oxide (ZnO) Nano materials"** is a bonafide record carried out by **K. BHAVANI [19306204]** under the supervision and the guidance of **Dr.G.Nagarjuna** as Partial fulfillment of requirements for the degree of bachelor of science from SRR & CVR GOVT. DEGREE COLLEGE (A) ,VIJAYAWADA .during the academic year 2021-2022.

Dr. G. Nagarjuna
(project guide)

Dr. V. SRINIVASA RAO
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Examiner

Principal
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ACKNOWLEDGEMENT

I express my sincere thanks to my supervisor and HOD of Chemistry , Dr.V. Srinivasa Rao, M.Sc.,B.Ed., M.Phil., Ph.D., for his esteemed supervision, incessant support, inspiration throughout my project work, for ably guiding us throughout the course of the project.

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I would also thank to our Principal Dr. K. Bhagya Lakshmi for providing opportunity to complete this project and all my faculty members of the department.

I convey my thanks to all my class mates, and project group members

Vijayawada

Date : [ ]

Green Synthesis and Characterization of Zinc Oxide (ZnO) Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for the

Degree of Bachelor of Science

by

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Under the Guidance of

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JULY 2022

CERTIFICATE

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SRR & CVR GOVT. DEGREE COLLEGE (Autonomous)

NTR. DIST , VIJAYAWADA - 4, A.P.

This is to certify that the project work titled **"Green Synthesis and Characterization of Zinc Oxide (ZnO) Nano materials"** is a bonafide record carried out by **M. BHAVYA [19306208]** under the supervision and the guidance of **Dr. G.Nagarjuna** as Partial fulfillment of requirements for the degree of bachelor of science from SRR &CVR GOVT. DEGREE COLLEGE (A) ,VIJAYAWADA .during the academic year 2021-2022.

Dr. G. Nagarjuna
(project guide)

Dr. V. SRINIVASA RAO
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Machavaram, VIJAYAWADA-4

ACKNOWLEDGEMENT

I express my sincere thanks to my supervisor and HOD of Chemistry , Dr.V. Srinivasa Rao, M.Sc.,B.Ed., M.Phil., Ph.D., for his esteemed supervision, incessant support, inspiration throughout my project work, for ably guiding us throughout the course of the project.

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I convey my thanks to all my class mates, and project group members

Vijayawada

Date : [ ]

Green Synthesis and Characterization of Zinc Oxide (ZnO) Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for the

Degree of Bachelor of Science

by

P. LAKSHMI PRASANNA

[Regd .No : 19306211]

Under the Guidance of

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M.Sc., B.Ed., M.Ed., M.Phil., Ph.D.,

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DEPARTMENT OF CHEMISTRY

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JULY 2022

CERTIFICATE

DEPARTMENT OF CHEMISTRY

SRR & CVR GOVT. DEGREE COLLEGE (Autonomous)

NTR. DIST , VIJAYAWADA - 4, A.P.

This is to certify that the project work titled **"Green Synthesis and Characterization of Zinc Oxide (ZnO) Nano materials"** is a bonafide record carried out by **P. LAKSHMI PRASANNA [19306211]** under the supervision and the guidance of **Dr.G.Nagarjuna** as Partial fulfillment of requirements for the degree of bachelor of science from SRR &CVR GOVT. DEGREE COLLEGE (A) ,VIJAYAWADA .during the academic year 2021-2022.

Dr. G. Nagarjuna
(project guide)

Dr. V.SRINIVASA RAO
(Head of the department)

Examiner

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ACKNOWLEDGEMENT

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I convey my thanks to all my class mates, and project group members

Vijayawada

Date :

P. Lakshmi Prasanna

[P. LAKSHMI PRASANNA]

Green Synthesis and Characterization of Zinc Oxide (ZnO) Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for

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A. ARUN KUMAR

[Regd No: 19306212]

Under the Guidance of

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Lecturer in Chemistry

DEPARTMENT OF CHEMISTRY

SRR& CVR GOVT.DEGREE COLLEGE (AUTONOMOUS)

VIJAYAWADA –520004 , NTR DIST,A.P.

JULY 2022

CERTIFICATE

DEPARTMENT OF CHEMISTRY

SRR & CVR GOVT. DEGREE COLLEGE (Autonomous)

NTR. DIST , VIJAYAWADA - 4, A.P.

This is to certify that the project work titled **"Green Synthesis and Characterization of Zinc Oxide (ZnO) Nano materials"** is a bonafide record carried out by **A. ARUN KUMAR [19306212]** under the supervision and the guidance of **Dr.G.Nagarjuna** as Partial fulfillment of requirements for the degree of bachelor of science from SRR & CVR GOVT.DEGREE COLLEGE (A) ,VIJAYAWADA .during the academic year 2021-2022.

Dr. G. Nagarjuna
(project guide)

Dr. V.SRINIVASA RAO
(Head of the department)

Examiner

Principal
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Machavaram, VIJAYAWADA-4.

ACKNOWLEDGEMENT

I express my sincere thanks to my supervisor and HOD of Chemistry , Dr.V. Srinivasa Rao, M.Sc.,B.Ed., M.Phil., Ph.D., for his esteemed supervision, incessant support, inspiration throughout my project work, for ably guiding us throughout the course of the project.

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I would also thank to our Principal Dr. K. Bhagya Lakshmi for providing opportunity to complete this project and all my faculty members of the department.

I convey my thanks to all my class mates, and project group members

Vijayawada

Date :

A. Arun Kumar

[A. ARUN KUMAR]

Green Synthesis and Characterization of Zinc Oxide (ZnO) Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for

the Degree of Bachelor of Science

by

G. BHARGAVI

[Regd .No :19306215]

Under the Guidance of

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M.Sc., B.Ed., M.Ed.,M.Phil., Ph.D.,

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VIJAYAWADA –520004 , NTR DIST,A.P.

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Dr. G. Nagarjuna
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Dr. V.SRINIVASA RAO
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I convey my thanks to all my class mates, and project group members

Vijayawada

Date :

G. Bhargavi

[G. BHARGAVI]

Ecofriendly Synthesis and Characterization of Copper Oxide [CuO] Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for

the Degree of Bachelor of Science

by

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NTR. DIST , VIJAYAWADA - 4, A.P.

This is to certify that the project work titled "**Eco friendly synthesis and Characterization of Copper Oxide (Cu O) Nano materials** " is a bonafide record carried out by **K. VENKATA GOPI [19306220**]under the supervision and the guidance of **Dr.G.Nagarjuna** as Partial fulfillment of requirements for the degree of bachelor of science from SRR &CVR GOVT. DEGREE COLLEGE (A) ,VIJAYAWADA .during the academic year 2021-2022.

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Dr. V.SRINIVASA RAO
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I convey my thanks to all my class mates, and project group members

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Date :

K. Venkata Gopi

[K. VENKATA GOPI]

Ecofriendly Synthesis and Characterization of Copper Oxide [CuO] Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for

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by

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SRR & CVR GOVT. DEGREE COLLEGE (Autonomous)

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This is to certify that the project work titled "**Eco friendly synthesis and Characterization of Copper Oxide (Cu O) Nano materials** " is a bonafide record carried out by **B. VENKATAPATHI RAJU [19306227]** under the supervision and the guidance of **Dr.G.Nagarjuna** as Partial fulfillment of requirements for the degree of bachelor of science from SRR &CVR GOVT. DEGREE COLLEGE (A) ,VIJAYAWADA .during the academic year 2021-2022.

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Ecofriendly Synthesis and Characterization of Copper Oxide [CuO] Nano Materials

Project Work submitted to

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In partial fulfillment of the requirement for

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By

D.SUJATHA

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I convey my thanks to all my class mates, and project group members

Vijayawada

Date : [ ]

Ecofriendly Synthesis and Characterization of Copper Oxide [CuO] Nano Materials

Project Work submitted to

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In partial fulfillment of the requirement for

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by

G.PEDDANNA

[Regd.No: 19306230]

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G. peddanna

Vijayawada

Date : G. peddanna

Ecofriendly Synthesis and Characterization of Copper Oxide [CuO] Nano Materials

Project Work submitted to

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In partial fulfillment of the requirement for

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by

G.PRAVEEN

[Regd.No: 19306231]

Under the Guidance of

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NTR. DIST , VIJAYAWADA - 4, A.P.

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Dr.G.Nagarjuna
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Dr. V.SRINIVASA RAO
(Head of the department)

Examiner

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Machavaram, VIJAYAWADA-4.

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Vijayawada

Date :

G. Praveen.

[G. PRAVEEN]

Phytochemical Assisted Synthesis and Characterization of CuO and ZnO Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for

the Degree of Bachelor of Science

by

K. VINOD KUMAR

[Regd.No : 19301003]

Under the Guidance of

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VIJAYAWADA –520004 , NTR DIST,A.P.

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NTR. DIST , VIJAYAWADA - 4, A.P.

This is to certify that the project work titled " **Phytochemical Assisted Synthesis and Characterization of Mixture of CuO and ZnO Nano materials"** is a bonafide record carried out by **K.VINOD KUMAR [19301003]** under the supervision and the guidance of **Dr. V. SRINIVASA RAO** as Partial fulfillment of requirements for the degree of bachelor of science from SRR &CVR GOVT. DEGREE COLLEGE (A) ,VIJAYAWADA .during the academic year 2021-2022.

Dr.V. SRINIVASA RAO
(project guide)

Dr. V.SRINIVASA RAO
(Head of the department)

Examiner

Principal

PRINCIPAL
SRR & CVR GOVT. DEGREE COLLEGE
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Mochavaram, VIJAYAWADA-4.

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Vijayawada

Date :

[K. Vinod Kumar.]

Phytochemical Assisted Synthesis and Characterization of CuO and ZnO Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for

the Degree of Bachelor of Science

by

K. SRI KANTH

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Under the Guidance of

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CERTIFICATE

DEPARTMENT OF CHEMISTRY

SRR & CVR GOVT. DEGREE COLLEGE (Autonomous)

NTR. DIST , VIJAYAWADA - 4, A.P.

This is to certify that the project work titled " **Phytochemical Assisted Synthesis and Characterization of Mixture of CuO and ZnO Nano materials**" is a bonafide record carried out by **K.SRI KANTH [19301005]** under the supervision and the guidance of **Dr. V. SRINIVASA RAO** as Partial fulfillment of requirements for the degree of bachelor of science from SRR &CVR GOVT. DEGREE COLLEGE (A) ,VIJAYAWADA .during the academic year 2021-2022.

Dr.V. SRINIVASA RAO

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Dr. V.SRINIVASA RAO

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Principal

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Vijayawada

Date :

[K.SRIKANTH]

Phytochemical Assisted Synthesis and Characterization of CuO and ZnO Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for

the Degree of Bachelor of Science

by

R. CHANDRA SEKHAR

[Regd.No : 19301007]

Under the Guidance of

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M.Sc., B.Ed., M.Phil., Ph.D.

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VIJAYAWADA –520004 , NTR DIST,A.P.

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NTR. DIST , VIJAYAWADA - 4, A.P.

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Dr.V. SRINIVASA RAO
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Dr. V. SRINIVASA RAO
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Examiner

Principal

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Vijayawada

Date :

R. Chandra Sekhar.

[R. CHANDRA SEKHAR]

Ecofriendly Synthesis and Characterization of Copper Oxide [CuO] Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for

the Degree of Bachelor of Science

By

K.SURESH

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Dr.V. SRINIVASA RAO

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Dr. V.SRINIVASA RAO

(Head of the department)

Examiner

Principal

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K. Suresh.

Vijayawada

Date : [K. Suresh.]

Ecofriendly Synthesis and Characterization of Copper Oxide [CuO] Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for

the Degree of Bachelor of Science

by

B.GOPI KRISHNA NAIK

[Regd.No : 19301013]

Under the Guidance of

Dr. V. SRINIVASA RAO

M.Sc., B.Ed., M.Phil., Ph.D.,

Lecturer in Chemistry

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JULY 2022

CERTIFICATE

DEPARTMENT OF CHEMISTRY

SRR & CVR GOVT. DEGREE COLLEGE (Autonomous)

NTR. DIST , VIJAYAWADA - 4, A.P.

This is to certify that the project work titled "**Eco friendly synthesis and Characterization of Copper Oxide (Cu O) Nano materials** " is a bonafide record carried out by **B.GOPI KRISHNA NAIK [19301013]** under the supervision and the guidance of **Dr. V. SRINIVASA RAO** as Partial fulfillment of requirements for the degree of bachelor of science from SRR &CVR GOVT. DEGREE COLLEGE (A) ,VIJAYAWADA .during the academic year 2021-2022.

Dr.V. SRINIVASA RAO

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Dr. V.SRINIVASA RAO

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Principal

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ACKNOWLEDGEMENT

I express my sincere thanks to my supervisor and HOD of Chemistry , Dr.V. Srinivasa Rao, M.Sc.,B.Ed., M.Phil., Ph.D., for his esteemed supervision, incessant support, inspiration throughout my project work, for ably guiding us throughout the course of the project.

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I would also thank to our Principal Dr. K. Bhagya Lakshmi for providing opportunity to complete this project and all my faculty members of the department.

I convey my thanks to all my class mates, and project group members

Vijayawada

Date :

M. GOPI KRISHNA NAIK

Ecofriendly Synthesis and Characterization of Copper Oxide [CuO] Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for

the Degree of Bachelor of Science

by

S. GOPI KRISHNA

[Regd.No : 19301015]

Under the Guidance of

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DEPARTMENT OF CHEMISTRY

SRR &CVR GOVT. DEGREE COLLEGE (Autonomous)

NTR. DIST , VIJAYAWADA - 4, A.P.

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Dr.V. SRINIVASA RAO

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I convey my thanks to all my class mates, and project group members

Vijayawada

Date :

S. Gopikrishna.

[S. Gopi Krishna]

Phytochemical Assisted Synthesis and Characterization of CuO and ZnO Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for

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This is to certify that the project work titled " **Phytochemical Assisted Synthesis and Characterization of Mixture of CuO and ZnO Nano materials**" is a bonafide record carried out by **P. JAGADEESH [19301202]** under the supervision and the guidance of **Dr. V. SRINIVASA RAO** as Partial fulfillment of requirements for the degree of bachelor of science from SRR &CVR GOVT. DEGREE COLLEGE (A) ,VIJAYAWADA .during the academic year 2021-2022.

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ACKNOWLEDGEMENT

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I convey my thanks to all my class mates, and project group members

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Date :

P. Jagadeesh

[P. JAGADEESH]

Phytochemical Assisted Synthesis and Characterization of CuO and ZnO Nano Materials

Project Work submitted to

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In partial fulfillment of the requirement for

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This is to certify that the project work titled " **Phytochemical Assisted Synthesis and Characterization of Mixture of CuO and ZnO Nano materials"** is a bonafide record carried out by **V. LOKESH [19301207]** under the supervision and the guidance of **Dr. V. SRINIVASA RAO** as Partial fulfillment of requirements for the degree of bachelor of science from SRR &CVR GOVT. DEGREE COLLEGE (A) ,VIJAYAWADA .during the academic year 2021-2022.

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ACKNOWLEDGEMENT

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I convey my thanks to all my class mates, and project group members

Vijayawada

Date :

[V. Lokesh]

Phytochemical Assisted Synthesis and Characterization of CuO and ZnO Nano Materials

Project Work submitted to

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In partial fulfillment of the requirement for

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P.ADI

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ACKNOWLEDGEMENT

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I would also thank to our Principal Dr. K. Bhagya Lakshmi for providing opportunity to complete this project and all my faculty members of the department.

I convey my thanks to all my class mates, and project group members

P. Adi

Vijayawada

Date : [P. Adi]

Phytochemical Assisted Synthesis and Characterization of CuO and ZnO Nano Materials

Project Work submitted to

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In partial fulfillment of the requirement for

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This is to certify that the project work titled " **Phytochemical Assisted Synthesis and Characterization of Mixture of CuO and ZnO Nano materials"** is a bonafide record carried out by **SAKEENA BANU [19301209]** under the supervision and the guidance of **Dr.V.SRINIVASA RAO** as Partial fulfillment of requirements for the degree of bachelor of science from SRR &CVR GOVT. DEGREE COLLEGE (A) ,VIJAYAWADA .during the academic year 2021-2022.

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ACKNOWLEDGEMENT

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I convey my thanks to all my class mates, and project group members

Vijayawada

Date :

[Sakeena Banu]

Phytochemical Assisted Synthesis and Characterization of CuO and ZnO Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for

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Dr.V. SRINIVASA RAO
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Machavaram, VIJAYAWADA-4.

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R. Sai Kumar

Vijayawada

Date :

(R. Dinesh Sai Kumar)

Ecofriendly Synthesis and Characterization of Copper Oxide [CuO] Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for

the Degree of Bachelor of Science

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M.JYOTHI

[Regd.No : 19301216]

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SRR &CVR GOVT. DEGREE COLLEGE (Autonomous)

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Dr.V. SRINIVASA RAO
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Dr. V.SRINIVASA RAO
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I convey my thanks to all my class mates, and project group members

M. Nagajyothi

Vijayawada

Date :

M. Nagajyothi

Ecofriendly Synthesis and Characterization of Copper Oxide [CuO] Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for

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by

CH. DURGA PRASAD

[Regd. No : 19301218]

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Dr.V. SRINIVASA RAO
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Dr. V.SRINIVASA RAO
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Examiner

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I convey my thanks to all my class mates, and project group members

Vijayawada

Date :

Ch. Durga Prasad

[CH. DURGA PRASAD]

Ecofriendly Synthesis and Characterization of Copper Oxide [CuO] Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for

the Degree of Bachelor of Science

by

I.PARALOK

[Regd.No : 19301220]

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This is to certify that the project work titled "**Eco friendly synthesis and Characterization of Copper Oxide (Cu O) Nano materials** " is a bonafide record carried out by **I. PARALOK [19301220]** under the supervision and the guidance of **Dr. V. SRINIVASA RAO** as Partial fulfillment of requirements for the degree of bachelor of science from SRR &CVR GOVT. DEGREE COLLEGE (A) ,VIJAYAWADA .during the academic year 2021-2022.

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Dr. V.SRINIVASA RAO
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I convey my thanks to all my class mates, and project group members

I. Paralok

Vijayawada

Date :

[I. PARALOK]

Ecofriendly Synthesis and Characterization of Copper Oxide [CuO] Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for

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Dr.V. SRINIVASA RAO
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Examiner

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I would also thank to our Principal Dr. K. Bhagya Lakshmi for providing opportunity to complete this project and all my faculty members of the department.

I convey my thanks to all my class mates, and project group members

Vijayawada

Date :

E. Sai krishna

[E. SAI KRISHNA]

Ecofriendly Synthesis and Characterization of Copper Oxide [CuO] Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for

the Degree of Bachelor of Science

By

R.SAI KUMAR

[Regd.No : 19301234]

Under the Guidance of

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M.Sc., B.Ed., M.Phil., Ph.D.,

Lecturer in Chemistry

DEPARTMENT OF CHEMISTRY

SRR& CVR GOVT.DEGREE COLLEGE (AUTONOMOUS)

VIJAYAWADA –520004 , NTR DIST,A.P.

JULY 2022

CERTIFICATE

DEPARTMENT OF CHEMISTRY

SRR &CVR GOVT. DEGREE COLLEGE (Autonomous)

NTR. DIST , VIJAYAWADA - 4, A.P.

This is to certify that the project work titled "**Eco friendly synthesis and Characterization of Copper Oxide (Cu O) Nano materials** " is a bonafide record carried out by **R.SAI KUMAR [19301234]** under the supervision and the guidance of **Dr. V. SRINIVASA RAO** as Partial fulfillment of requirements for the degree of bachelor of science from SRR &CVR GOVT. DEGREE COLLEGE (A) ,VIJAYAWADA .during the academic year 2021-2022.

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Dr. V.SRINIVASA RAO

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Examiner

Principal

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ACKNOWLEDGEMENT

I express my sincere thanks to my supervisor and HOD of Chemistry , Dr.V. Srinivasa Rao, M.Sc.,B.Ed., M.Phil., Ph.D., for his esteemed supervision, incessant support, inspiration throughout my project work, for ably guiding us throughout the course of the project.

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I would also thank to our Principal Dr. K. Bhagya Lakshmi for providing opportunity to complete this project and all my faculty members of the department.

I convey my thanks to all my class mates, and project group members

Vijayawada

Date :

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Green Synthesis and Characterization of Zinc Oxide (ZnO) Nano Materials

Project Work submitted to

KRISHNA UNIVERSITY, MACHILIPATNAM

In partial fulfillment of the requirement for

the Degree of Bachelor of Science

by

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This is to certify that the project work titled **"Green Synthesis and Characterization of Zinc Oxide (ZnO) Nano materials"** is a bonafide record carried out by **D. SUBHANI [19301246]** under the supervision and the guidance of **Dr.V. SRINIVASA RAO** as Partial fulfillment of requirements for the degree of bachelor of science from SRR &CVR GOVT. DEGREE COLLEGE (A) ,VIJAYAWADA .during the academic year 2021-2022.

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ACKNOWLEDGEMENT

I express my sincere thanks to my supervisor and HOD of Chemistry , Dr.V. Srinivasa Rao, M.Sc.,B.Ed., M.Phil., Ph.D., for his esteemed supervision, incessant support, inspiration throughout my project work, for ably guiding us throughout the course of the project.

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I would also thank to our Principal Dr. K. Bhagya Lakshmi for providing opportunity to complete this project and all my faculty members of the department.

I convey my thanks to all my class mates, and project group members

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Date :

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Green Synthesis and Characterization of Zinc Oxide (ZnO) Nano Materials

Project Work submitted to

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In partial fulfillment of the requirement for

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ACKNOWLEDGEMENT

I express my sincere thanks to my supervisor and HOD of Chemistry , Dr.V. Srinivasa Rao, M.Sc.,B.Ed., M.Phil., Ph.D., for his esteemed supervision, incessant support, inspiration throughout my project work, for ably guiding us throughout the course of the project.

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I would also thank to our Principal Dr. K. Bhagya Lakshmi for providing opportunity to complete this project and all my faculty members of the department.

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Green Synthesis and Characterization of Zinc Oxide (ZnO) Nano Materials

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In partial fulfillment of the requirement for

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ACKNOWLEDGEMENT

I express my sincere thanks to my supervisor and HOD of Chemistry , Dr.V. Srinivasa Rao, M.Sc.,B.Ed., M.Phil., Ph.D., for his esteemed supervision, incessant support, inspiration throughout my project work, for ably guiding us throughout the course of the project.

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I convey my thanks to all my class mates, and project group members

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[G. Nani Babu]

Green Synthesis and Characterization of Zinc Oxide (ZnO) Nano Materials

Project Work submitted to

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In partial fulfillment of the requirement for

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ACKNOWLEDGEMENT

I express my sincere thanks to my supervisor and HOD of Chemistry , Dr.V. Srinivasa Rao, M.Sc.,B.Ed., M.Phil., Ph.D., for his esteemed supervision, incessant support, inspiration throughout my project work, for ably guiding us throughout the course of the project.

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Green Synthesis and Characterization of Zinc Oxide (ZnO) Nano Materials

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Vijayawada

Date :

J. Sandya.

| J. Sandya Bhavani |

A Study on the

CASH MANAGEMENT AT HELLO MOBILES PVT. LTD

Project Work Report

Submitted in partial fulfillment of the requirements
for the award of the Degree of
Master of Commerce (M.Com)
Under CBCS Framework for III Semester

Submitted by
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Submitted to

Krishna University
Machilipatnam
AY-2019-2021

A Study on the

AWARENESS AND UTILISATION OF ELECTRONIC BANKING

Project Work Report

Submitted in partial fulfillment of the requirements

for the award of the Degree of

Master of Commerce (M.Com)

Under CBCS Framework for III Semester

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Krishna University

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AY-2019-2021

A Study on the

A SALES PROMOTION THROUGH SOCIAL MEDIA

Project Work Report

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Master of Commerce (M.Com)

Under CBCS Framework for III Semester

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AY-2019-2021

A Study on

FINANCIAL PERFORMANCE THROUGH WORKING CAPITAL

With reference to

NEXGEN PVT. Limited

Vijayawada

Project Work Report

Submitted in partial fulfillment of the requirements
for the award of the
Degree of Master of Commerce (M.Com)
Under CBCS Framework for III Semester

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AY-2020-2021

A Study on

WORKING CAPITAL RATIOS

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A Study on the

AWARENESS AND UTILISATION OF ELECTRONIC BANKING

Project Work Report

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Master of Commerce (M.Com)

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A Study on the

CUSTOMERS PERCEPTION TOWARDS GST

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AY-2019-2021

A Study on the

A SALES PROMOTION THROUGH SOCIAL MEDIA

Project Work Report

Submitted in partial fulfillment of the requirements

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Master of Commerce (M.Com)

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AY-2019-2021

A Study on the

COMPARITIVE ANALYSIS OF MUTUAL FUNDS OVER EQUITY SHARES OF ICICI

Project Work Report

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Master of Commerce (M.Com)

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AY-2019-2021

A Study on

PERFORMANCE APPRAISAL

With reference to

VIJAYA DIARY IN KRISHNA DISTRICT

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AY-2020-2021

A Study on

IMPACT OF JOB SATISFACTION ON PERFORMANCE

With reference to

HINDUSTHAN COCO COLA COMPANY

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Project Work Report

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Master of Commerce (M.Com)

Under CBCS Framework for III Semester

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A Study on

CONSUMER BUYING BEHAVIOUR

With reference to

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Project Work Report

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Master of Commerce (M.Com)

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A Study on the

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A Study on

WORKING CAPITAL MANAGEMENT

reference to

Stock Holding Pvt. Ltd.

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AY-2020-2021

A Study on

CREDIT MANAGEMENT

With reference to

CHAITANYA GODAVARI GRAMEENA BANK

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A Study on

"SALES PROMOTION STRATEGIES AT HERO MOTOCORP LTD"

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AY-2020-2021

A Study on

PERFORMANCE APPRAISAL

With reference to

MODEL DIARY

Vijayawada

Project Work Report

Submitted in partial fulfillment of the requirements

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Master of Commerce (M.Com)

Under CBCS Framework for III Semester

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AY-2020-2021

A Study on

CONSUMER BUYING BEHAVIOUR

With reference to

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Project Work Report

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A Study on the

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A Study on

FUNDS FLOW STATEMENT

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A Study on the

AWARENESS AND UTILISATION OF ELECTRONIC BANKING

Project Work Report

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A Study on

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With reference to

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Vijayawada

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A Study on the

COMPARITIVE ANALYSIS OF MUTUAL FUNDS OVER EQUITY SHARES OF ICICI

Project Work Report

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AY-2019-2021

SRR & CVR Govt. Degree College (A), Vijayawada

DEPARTMENT OF FOOD TECHNOLOGY

Project Report

PLANNING LOW COST NUTRITIOUS RECIPES

SUBMITTED

BY

Y. MALLIKARJUNA (B.Sc. FMB)

Register No: 21315226

UNDER SUPERVISION

OF

Dr. K. Anuradha

Lecturer in Food Technology, Department of Food Technology

SRR & CVR Govt. Degree College (A), Vijayawada, NTR District, A.P

March 2022

SRR & CVR Govt. Degree College (A), Vijayawada

DECLARATION

I Y. MALLIKARJUNA, student of I B.Sc. MBF, Studying in SRR & CVR Govt. Degree College (A), Vijayawada with Register No: 21315226 hereby declare that "Planning Low Cost Nutritious Recipes" has been carried out and submitted by me. The work contained in this report is original.

Vijayawada
Dt. 16th March 2022

Y. Mallikarjuna
(Y. MALLIKARJUNA)

SIGNATURE OF THE LECTURER

SRR & CVR Govt. Degree College (A), Vijayawada

CERTIFICATE

This is to certify that **Project entitled "PLANNING LOW COST NUTRITIOUS RECIPES" submitted to SRR & CVR Govt. Degree College (A), Vijayawada in partial fulfillment of the requirements for the DEGREE OF BACHELOR OF SCIENCE is done by Y. Mallikarjuna B. Sc (MBF), with Register No: 21315226 during the academic year 2021-22 from SRR & CVR Govt. Degree College (A), Vijayawada, NTR District, Andhra Pradesh.**

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SIGNATURE OF THE LECTUTER

(Dr. K. Anuradha)

Jowar Upma

Ingredients:

- Jowar - 100 grams
- Oil - 2 tsp
- Mix vegtables - 1 cup
- Mustard seeds - ½ tsp
- Cumin seeds - ½ tsp
- Turmeric power - ½ tsp
- Hing - ¼ tsp
- Salt - 1 tsp
- Green chillies - 2
- Curry leaves - 3-4
- Cilantro - ¼ cup
- Chopped onion - ½ cup

Procedure:

- Take half cup of jowar in a bowl. wash it 2-3 times and soak it around 12 hrs.
- After 12 hrs wash them again & discard the water.
- Add it in the pressure cooker. Add one cup of water. Add ½ T spoon of salt. Now cook it up to get 7-8 whistles.
- Now a jowar has become soft and cooked well.
- Now in a pan add 2 tsp of oil. Once the oil is hot add ½ tsp mustard seeds, ½ tsp cumin seeds, ¼ tsp Hing, 4-5 curry leaves, 2 green chillies & half a cup of chopped onion
- Next add ¼ cup beens & one cup of mixed vegtables [carrot, soybean] and ½ tsp Turmeric power & ½ tsp salt and sauté it again
- cover with the lid go for around 1 to 2 min. Next we will add Boild & cooked jowar. 2 cups.

- Now mix it well.
- Cover it with lid let it cook for 1 min.
- After it squeeze around 1 tsp of lemon juice & 1/4 cup of coriander mix everything together.
- Finally High fibrous Rich protein jowar upma is ready to serve.

Nutritive values:-

Protein [gms]	Fat [gms]	Minerals [gms]	Fiber [gms]	Carbohydrates [gms]	Energy [cgms]
16.8	30.9	0.38	11.2	91.7	713

Why it is loco cost:-

- Jowar upma is a high protein containing Fibrous food.
- We can made it with only 30 Rupees.
- It is a Good replacement in the breakfast in the place of cornflakes & Oats upma [which are have high cost].

Soya Chunks curry:

INGREDIENTS:

- Big onion - 3
- Tomato - 3
- Garlic - 10pieces
- Green chilli - 2
- Ginger - 1/2 inch
- Water - 1 1/2 Glass
- Salt - 1/4 Tbspn
- Soya chunks - 200grms
- Oil - 3 Tbspn
- Cumin seeds - 1 Tbspn
- Turmeric - 1 Tbspn
- Red chilli powder - 1 Tbspn
- Coriander powder - 2 Tbspn
- Garam Masala - 1 Tbspn
- Coriander leaves - 30 grms. [Few]

→ Cooking Procedure:

- Finely choppe the onions & Tomatoes like very small speces [onion only].
- Next Grind the Chopped Tomatoes, Garlic, Green chilli, Ginger As a paste.
- Next take a bowl and add 1 1/2 glass water. boil It and add 1 Tbspn salt & Soya chunks 200gm.
- Close & cook for 5 mins in a medium flame.
- Next take an antoher bowl add 3 Tbspn oil, 1 Tbspn Cumin seeds & chopped onions. Saute in a medium flame to achieve golden brown colour
- After that Add the Grinded paste & saute for 3 mins. keep mix it well.
- Add Turmeric & Saute for 2 min. And next add the 1 Tbsp Red chilli powder, 2 Tbspn coriander power & Saute for 3 mins.

- Next Add the boiled soya chunks & pour water 2glass and mix it well.
- Close & cook for 3mins & add ½ Tbspn salt
- add 1 Tbspn Garam masala cook for 3 mins and finally add coriander leaves for garnish.
- And serve It.

Neutritive Values in Our curry

Protein [grms]	Fat [gms]	Mincrals [gms]	Fiber [gms]	Carbo hydrates [gms]	Energy [gms]
107	44.5	0.83	30,5	89.5	1186Cal.

- We can prepare this recipie at the cost of 30/-
- It is richsource of protein & Tasty healthy.
- Soya chanks are called by poormans meat.
- It many available protein at low cost.
- We coell spent 110/- to reach this protein content with chicken.

Soya chunks manchuria:

Ingradients:

- Soyachunks – 200gm
- All purpose flour - 1 Tbspn
- Cornstarch - 2 Tbspn.
- Onion big - 1
- Ginger garlic paste - 1 Tbspn
- Ginger - 1 inch [spikes]
- Garlic - 6 Pieces
- Red chilli paste - 1 Tbspn
- Sugar - 1/4 Tbspn
- MSG - 1/4 Tbspn
- Veniger - 2 Tbspn
- Green chillies - 4
- Oil - 5 Tbspns.

Cooking procedure:

- Take 200gms of soya chunks put it a little bit of salty hot water & soak in for 10 min. after that u can just squeeze out the water, not too much little bit. and take in a bowl.
- To that soaked soya chunks add 1 Tbspn Maida [All purpose flour], 1 Tbspn gingergalic paste, very little bit of salt & Some cornstarch [2 Tbspn] Mix it well.
- Next deep fry the soyachunks in a boiling oil. This coant take much time. Deep fry upto get golden brown colour.
- Next take the fried soya chunkes in a Tissue paper plate to remove addition oil.
- Next take a pan & Make a Slurry first take a bowl Take some veniger [2 Tbspns], soya sauce [2 Tbspns], 1/4 Tbspn MSG [Tastingsalt], 1/4 Tbspn Sugar, & Some red chilli paste [1 Tbspn] and add very little of water.

- & Mix them well.
- Now on that pan we are going to add some oil [1 Tbspn] add finely chopped ginger [5pieces] & Finely chopped garlic [1/4inch] saute a litt bit and add finely chopped 1 onion. add green chillies [4] add little bit salt. Saut that for a minute.
- When onions are slightly coloured and now we are going to add the mixture of that sauces.
- Next add the fried soya Mix it little bit.
- When It become little bit soft you can put of your stove.
- Our manchuria is ready to serve.

Nealritive values:

Protein [grms]	Fat [grms]	Minerals [grms]	Fiber [grms	Carbohydrate [grms]	Energy [grms
106	43	0.38	30	87	1100 kcal

Why It is lowcost!

- We can prepare this recipie at the cost of 45 rupees
- It is rich source of protein & Healthy.
- To reach this protein with chicken we should spent 110/-
- To reach this protein with redgram we should spent 55/-
- This is the reason why we are called Soya is a poors mans meat.

Hibiscs Juice

Ingredients:

Water - 1 litre

Sugar - 200 Gms

Red Hibiscus flower - 10-12 Nos.

Ginger juice - 1 Tsp

Lemon juice - 1 Tsp

Method:

- Boil water with sugar.
- When it boils add the washed Hibiscus flower and give a single boil.
- cool and remove the flower and ginger and lemon juice.
- Serve cold.

Nutritive value per serving.

Protein [gms]	Fat [gms]	Minerals [gms]	Fiber [gms]	Carbohydrate [gms]	Energy [Kcal]
0.07	0.02	0.06	0.03	41.06	165.86

Why it is low cost:

- This Hibiscus juice rich in minerals, vitamic' and carbohydrates.
- It is helpful to regulate over body temperature in summer.
- It is usefull to maintain the normal blood pressure.
- It Decreases heart disease by regulating colest rol.
- We can make this juices with the cost of 12 rupees.
- It is the good alternative for costly herbal juices & coconut water.

A PROJECT REPORT

On

A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of **BACHELOR OF SCIENCE**

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

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MACHAVARAM, VIJAYAWADA

2021-2022

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under my guidance and supervision in partial fulfillment of the requirement for the award of degree of **Bachelor of Science (MPC)**in **SRR&CVRGOVT.DEGREE COLLEGE(A),** Vijayawada, affiliated to **Krishna University, Machilipatnam,** during the academic year **2021-2022.**

Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

DECLARATION

I hereby declare that this project titled "A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"is a bonafide work done by me under the guidance of **Dr. G. LALITHA.** This project work is submitted to **SRR&CVR GOVT. DEGREE COLLEGE(A),** Vijayawada, in partial fulfillment of the requirements for the award of **BACHELOR OF SCIENCE** during the academic year **2021-2022.**

I also declare that this project is a result of my own effort and that it has not been submitted to any other University for the award of any degree.

Date: 30-07-2022

Place: VIJAYAWADA

ACKNOWLEDGEMENT

It is a great pleasure to take opportunity and express my gratitude to all those who helped me throughout my project work. First and foremost, I am thankful to **Dr.K.BHAGYALAKSHMI, Principal, SRR&CVR GOVT.DEGREE COLLEGE (A),** for giving me opportunity to take up the project work.

I also thank,**Dr. G. LALITHA,project guide** and**Head of the Department,K.V.N.LAKSHMI** for giving me opportunity to take up the project work and helping me through out and finally completing the project.

I would like to express my sincere and heartfelt thanks to all the teaching and non-teaching staff of the department for their continuous co-operation, which has given me the guidance to build up adamant aspiration over the completion of my project.

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Finally, I thank one and all who directly and indirectly helped me to complete my project successfully.

R. Bhuvana Suresh

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A PROJECT REPORT

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A STUDY ON NUMERICAL ANALYSIS

Introduction:

It is **the study of numerical methods that attempt at finding approximate solutions of problems rather than the exact ones.** Numerical analysis finds application in all fields of engineering and the physical sciences, and in the 21st century also the life and social sciences, medicine, business and even the arts.

Why Do We Need Numerical Analysis In Everyday Life?

The great advantage of using numerical analysis is that it investigates and provides accurate solutions to real-life problems from the field of science, engineering, biology, astrophysics and finance.

The word 'analysis' generally means to solve a problem through a set of equations and further reduce these equations using the methodologies of algebra, partial differential equations, calculus and other related fields of mathematics.

On similar grounds, numerical analysis implements arithmetic algorithms: addition, subtraction, multiplication, division and comparison to obtain numerical solutions.
A computer precisely performs these operations, meaning that numerical analysis and computers are intimately related.

The problem of continuous mathematics generally arises throughout the natural sciences, business management, engineering, astronomy and medicine. It is always more practical to carry out the tedious arithmetic operations using a computer.

Before the mid-20th century, until the advent of modern computers, all these repetitive operations had to be performed by manual interpolation. The overall agenda of numerical analysis is to give an *approximate*, but *accurate* solution to the advanced problem. The necessity for accuracy, is, of course, determined by the variety of applications.

Historical background:

Numerical algorithms predate the invention of modern and digital computers and are almost as sold as human civilization itself. Many renowned mathematicians of the past were engrossed in numerical analysis. *Archimedes of Syracuse* (287-211/212 BC), the most famous Greek mathematician, physicist, engineer and inventor, perfected new standards of contemporary geometry, including the 'method of exhaustion' to compute the area and volume of various geometric figures.
A famous document, *The Rhind Papyrus* (1650 BC) from the ancient Egyptian Middle Kingdom, bears a root-finding method to solve simple equations.

A PROJECT REPORT

On

A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

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A PROJECT REPORT

On

A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, *in partial fulfillment of the requirement for the award of Degree of* ***BACHELOR OF SCIENCE***

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

KANTETI SUBHASH (19306207)

Under the Guidance of

Dr. G. LALITHA GARU

DEPARTMENT OF MATHEMATICS

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2021-2022

CERTIFICATE

This is to certify that the project titled "A ***STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"*** *is a bonafide work done by* ***KANTETI SUBHASH (19306207)*** *under my guidance and supervision in partial fulfillment of the requirement for the award of degreeof* ***Bachelor ofScience (MCCS) EM*** *in* ***SRR & CVR GOVT. DEGREE COLLEGE (A),*** *Vijayawada, affiliated to* ***Krishna University, Machilipatnam,*** *during the academic year* ***2021-2022.***

Project Guide

EXAMINER

EXTERNAL

Dr. G. LALITHA

DECLARATION

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Date: 30/7/2022

Place: VIJAYAWADA

K. Subhasha.

ACKNOWLEDGEMENT

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

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2021-2022

CERTIFICATE

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Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

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Date: 30-July-2022

P. Chandana.

Place: VIJAYAWADA

ACKNOWLEDGEMENT

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A STUDY ON NUMERICAL ANALYSIS

Introduction:

It is **the study of numerical methods that attempt at finding approximate solutions of problems rather than the exact ones.** Numerical analysis finds application in all fields of engineering and the physical sciences, and in the 21st century also the life and social sciences, medicine, business and even the arts.

Why Do We Need Numerical Analysis In Everyday Life?

The great advantage of using numerical analysis is that it investigates and provides accurate solutions to real-life problems from the field of science, engineering, biology, astrophysics and finance.

The word 'analysis' generally means to solve a problem through a set of equations and further reduce these equations using the methodologies of algebra, partial differential equations, calculus and other related fields of mathematics.

On similar grounds, numerical analysis implements arithmetic algorithms: addition, subtraction, multiplication, division and comparison to obtain numerical solutions.
A computer precisely performs these operations, meaning that numerical analysis and computers are intimately related.

The problem of continuous mathematics generally arises throughout the natural sciences, business management, engineering, astronomy and medicine. It is always more practical to carry out the tedious arithmetic operations using a computer.

Before the mid-20th century, until the advent of modern computers, all these repetitive operations had to be performed by manual interpolation. The overall agenda of numerical analysis is to give an *approximate*, but *accurate* solution to the advanced problem. The necessity for accuracy, is, of course, determined by the variety of applications.

Historical background:

Numerical algorithms predate the invention of modern and digital computers and are almost as sold as human civilization itself. Many renowned mathematicians of the past were engrossed in numerical analysis. *Archimedes of Syracuse* (287-211/212 BC), the most famous Greek mathematician, physicist, engineer and inventor, perfected new standards of contemporary geometry, including the 'method of exhaustion' to compute the area and volume of various geometric figures.
A famous document, *The Rhind Papyrus* (1650 BC) from the ancient Egyptian Middle Kingdom, bears a root-finding method to solve simple equations.

A PROJECT REPORT

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A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

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2021-2022

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Project Guide

Dr. G. LALITHA

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Date: 30/7/2021

Place: VIJAYAWADA

K. Sujeeva Rao

ACKNOWLEDGEMENT

Acknowledgement

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UNIT-I LAPLACE TRANSFORM

A PROJECT REPORT

On

A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of BACHELOR OF SCIENCE

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

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Project Guide

Dr. G. LALITHA

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BACHELOR OF SCIENCE

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MACHAVARAM, VIJAYAWADA

2021-2022

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Project Guide ***EXTERNAL***

EXAMINER

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Date: 30-07-2022

Place: VIJAYAWADA

L. Vani

ACKNOWLEDGEMENT

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

C. NAGARJUNA (19306222)

Under the Guidance of

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DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

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Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

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Date: 30-07-2022

Place: VIJAYAWADA

C. Navya

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

SHAIK.MAHBUB BASHA(19304260)

Under the Guidance of

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DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

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SK. Rahab

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

SHAIK.EJAZ(19303203)

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SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

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Shaik. Ejaz

under my guidance and supervision in partial fulfillment of the requirement for the award of degree of Bachelor of Science (MECS)in SRR&CVRGOVT.DEGREE COLLEGE(A), Vijayawada, affiliated to Krishna University, Machilipatnam, during the academic year 2021-2022.

Project Guide

EXAMINER

Dr. G. LALITHA

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Sk. Ejaz

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A PROJECT REPORT

On

A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of BACHELOR OF SCIENCE

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

PERLA.YASWANTH KUMAR (19303204)

Under the Guidance of

Dr. G. LALITHA GARU

DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

This is to certify that the project titled "A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS" is a bonafide work done by

P. Yaswanth Kumar (19303204)

under my guidance and supervision in partial fulfillment of the requirement for the award of degree of Bachelor of Science (MECS)in SRR&CVRGOVT.DEGREE COLLEGE(A), Vijayawada, affiliated to Krishna University, Machilipatnam, during the academic year 2021-2022.

Project Guide

Dr. G. LALITHA

EXTERNAL

EXAMINER

DECLARATION

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Date: _____

Place: VIJAYAWADA

ACKNOWLEDGEMENT

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Finally, I thank one and all who directly and indirectly helped me to complete my project successfully.

P. Yashwanth Kumar.

$$na+b\Sigma x_i+c\Sigma x_i^2=\Sigma y_i$$

$$a\Sigma x_i+b\Sigma x_i^2+c\Sigma x_i^3=\Sigma x_i y_i$$

$$a\Sigma x_i^2+b\Sigma x_i^3+c\Sigma x_i^4=\Sigma x_i y_i^2$$

$$\left(\frac{dy}{dx}\right)_{x=x_n}=\frac{1}{h}\left\{\nabla y_n+\frac{1}{2}\nabla^2 y_n+\frac{1}{3}\nabla^3 y_n+\frac{7}{4}\nabla^4 y_n\right\}.$$

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

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MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

This is to certify that the project titled "A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS" is a bonafide work done by

Lingam. Subhashini

under my guidance and supervision in partial fulfillment of the requirement for the award of degree of Bachelor of Science (MECS)in SRR&CVRGOVT.DEGREE COLLEGE(A), Vijayawada, affiliated to Krishna University, Machilipatnam, during the academic year 2021-2022.

Project Guide

EXAMINER

Dr. G. LALITHA

EXTERNAL

DECLARATION

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Date: 30-07-2022

Place: VIJAYAWADA

L. Subhashini

ACKNOWLEDGEMENT

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

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CERTIFICATE

This is to certify that the project titled "A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS" is a bonafide work done by

A. Naga Divya kalyani

under my guidance and supervision in partial fulfillment of the requirement for the award of degree of Bachelor of Science (MECS)in SRR&CVRGOVT.DEGREE COLLEGE(A), Vijayawada, affiliated to Krishna University, Machilipatnam, during the academic year 2021-2022.

Project Guide

Dr. G. LALITHA

EXTERNAL

EXAMINER

DECLARATION

I hereby declare that this project titled "A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"is a bonafide work done by me under the guidance of **Dr. G. LALITHA.** This project work is submitted to **SRR&CVR GOVT. DEGREE COLLEGE(A),** Vijayawada, in partial fulfillment of the requirements for the award of **BACHELOR OF SCIENCE** during the academic year **2021-2022.**

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Date: 30/9/2022

Place: VIJAYAWADA

A. Naga Dipa Kalyani

ACKNOWLEDGEMENT

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SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of BACHELOR OF SCIENCE

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

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2021-2022

CERTIFICATE

This is to certify that the project titled "A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS" is a bonafide work done by CHEKKA. SWARNA SRI (193D3218)

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Project Guide

EXAMINER

Dr. G. LALITHA

EXTERNAL

DECLARATION

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Date: 30/7/2022

Ch. Surya Sri

Place: VIJAYAWADA

ACKNOWLEDGEMENT

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

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MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

This is to certify that the project titled **"A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"** is a bonafide work done by **JAGANNADHAM AVINASH(19303219)** under my guidance and supervision in partial fulfillment of the requirement for the award of degree of **Bachelor of Science (MECS)**in **SRR&CVRGOVT.DEGREE COLLEGE(A)**, Vijayawada, affiliated to **Krishna University, Machilipatnam**, during the academic year **2021-2022.**

Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

DECLARATION

I hereby declare that this project titled **"A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"** is a bonafide work done by me under the guidance of **Dr. G. LALITHA.** This project work is submitted to **SRR&CVR GOVT. DEGREE COLLEGE(A),** Vijayawada, in partial fulfillment of the requirements for the award of **BACHELOR OF SCIENCE** during the academic year **2021-2022.**

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Date: 30/07/2022

Place: VIJAYAWADA

J. Avinash

ACKNOWLEDGEMENT

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Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of **BACHELOR OF SCIENCE**

Submitted by

Gannavarapu Lokesh babu

Roll no: 19303221

III B.Sc (M.E.CS)

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Dr. G. LALITHA GARU

DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

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Project Guide:

Dr.G. Lalitha

EXAMINER

EXTERNAl

DECLARATION

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Kotesh Baby.

Date: _____

Place: VIJAYAWADA

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G. Lokesh Babu.

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In partial fulfilment of the requirement for the award of degree of

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SHAKI.SHAHANAZ(19303224)

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2021-2022

CERTIFICATE

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SHAIK. SHAHANAZ

under my guidance and supervision in partial fulfillment of the requirement for the award of degree of Bachelor of Science (MECS)in SRR&CVRGOVT.DEGREE COLLEGE(A), Vijayawada, affiliated to Krishna University, Machilipatnam, during the academic year 2021-2022.

Project Guide

Dr. G. LALITHA

EXTERNAL

EXAMINER

$\cos at = \frac{p}{p^2+a^2}$ If $p > 0$

$\sin at = \frac{p}{p^2+a^2}$ If $p > 0$

The normal equation $= na + b\Sigma xi = \Sigma yi$

$a\Sigma xi + b\Sigma xi^2 = \Sigma xiyi$

$\sinh x = \frac{e^x - e^{-x}}{2}$

$e^{at} = \frac{1}{p-a}$

$L(1) = \frac{1}{p} = 1 \Rightarrow L^{-1}\left[\frac{1}{p}\right]$

Inverse laplace Transform

$f(F) = L^{-1}[f(p)]$

$a\,L[y''] = a[p^2 L(y) - p(y(0) - y(0)]$

$bL[y] = b[pL(y) - y(0)]$

$cL[y] = cL(y)$

fourier sine Transform $\left[\frac{2}{\pi}\int_0^\infty f(0) \sin px\, dx\right.$

DECLARATION

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Date: 30/7/2022

Place: VIJAYAWADA

Shaik Shahanaz

( )

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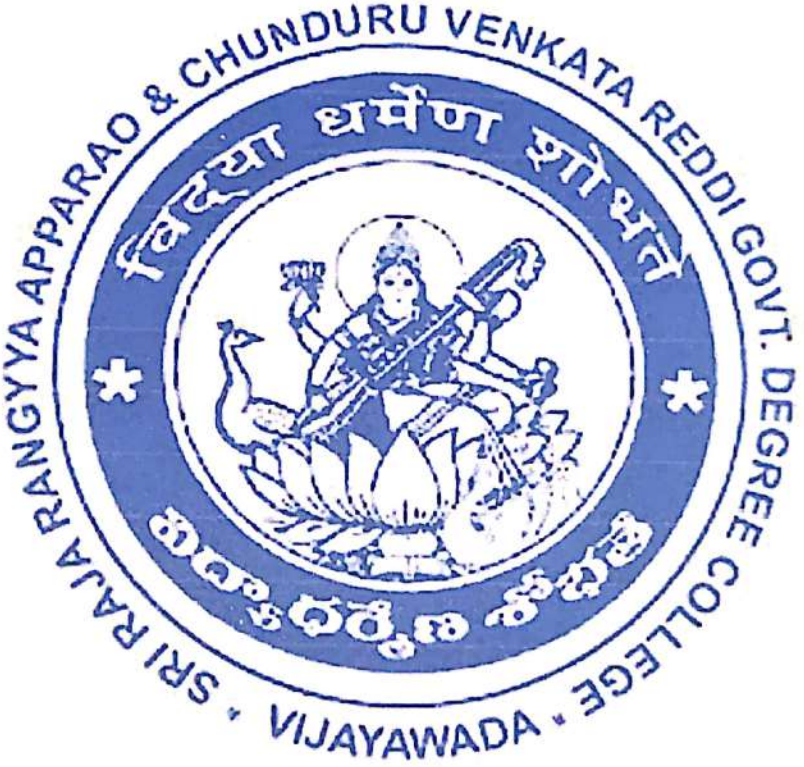
SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of **BACHELOR OF SCIENCE**



Submitted by

MAHANTHI SIDDHARTHA

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DEPARTMENT OF MATHEMATICS

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MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

This is to certify that the project titled "A **STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS**" is a bonafide work done by M. Siddhartha under my guidance and supervision in partial fulfillment of the requirement for the award of degree of **Bachelor of Science (MECS)**in **SRR&CVRGOVT.DEGREE COLLEGE(A)**, Vijayawada, affiliated to **Krishna University, Machilipatnam**, during the academic year **2021-2022.**

Project Guide:

Dr. G. Lalitha

EXAMINER

EXTERNAl

DECLARATION

I hereby declare that this project titled "A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"is a bonafide work done by me under the guidance of **Dr. G. LALITHA.** This project work is submitted to **SRR&CVR GOVT. DEGREE COLLEGE(A)**, Vijayawada, in partial fulfillment of the requirements for the award of **BACHELOR OF SCIENCE** during the academic year **2021-2022.**

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Date: 30/07/2022

Place: VIJAYAWADA

ACKNOWLEDGEMENT

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M. Siddhartha

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A PROJECT REPORT

On

A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of BACHELOR OF SCIENCE

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

SALADHI.MAHA LAKSHMI SIREESHA (19303228)

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MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

This is to certify that the project titled "A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS" is a bonafide work done by

S. MAHA LAKSHMI SIREESHA

under my guidance and supervision in partial fulfillment of the requirement for the award of degree of Bachelor of Science (MECS)in SRR&CVRGOVT.DEGREE COLLEGE(A), Vijayawada, affiliated to Krishna University, Machilipatnam, during the academic year 2021-2022.

Project Guide

Dr. G. LALITHA

EXTERNAL

EXAMINER

DECLARATION

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Date: 30/7/2022

Place: VIJAYAWADA

S. Hala Lakshmi Sireesha

ACKNOWLEDGEMENT

It is a great pleasure to take opportunity and express my gratitude to all those who helped me throughout my project work. First and foremost, I am thankful to **Dr.K.BHAGYALAKSHMI, Principal, SRR&CVR GOVT.DEGREE COLLEGE (A),** for giving me opportunity to take up the project work.

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SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA,

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

Kosuri Naveen (19303233)

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2021-2022

CERTIFICATE

This is to certify that the project titled **"A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"** is a bonafide work done by KOSURI NAVEEN(19303233) under my guidance and supervision in partial fulfillment of the requirement for the award of degree of **Bachelor of Science (MECS)**in **SRR&CVRGOVT.DEGREE COLLEGE(A)**, Vijayawada, affiliated to **Krishna University, Machilipatnam**, during the academic year **2021-2022.**

Project Guide

EXTERNALEXAMINER

Dr. G. LALITHA

DECLARATION

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Date: _____

Place: VIJAYAWADA

K. Naveen

( )

ACKNOWLEDGEMENT

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Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of **BACHELOR OF SCIENCE**

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CERTIFICATE

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Project Guide:

Dr.G. Lalitha

EXAMINER

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T. Manasa

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2021-2022

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Dr. G. LALITHA

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Date: 30.07.2022

Place: VIJAYAWADA

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V. Manjula Naik

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

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MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

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VAVILAPALLI. NAGAMANI

under my guidance and supervision in partial fulfillment of the requirement for the award of degree of Bachelor of Science (MECS)in SRR&CVRGOVT.DEGREE COLLEGE(A), Vijayawada, affiliated to Krishna University, Machilipatnam, during the academic year 2021-2022.

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Place: VIJAYAWADA

V. Nagamani

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In partial fulfilment of the requirement for the award of degree of

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Submitted by

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Project Guide

Dr. G. LALITHA

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Date: 30/7/2022

Place: VIJAYAWADA

L. Rani Teja

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

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2021-2022

CERTIFICATE

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Project Guide

Dr. G. LALITHA

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Y. Aravind

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

RAMAVATH RAVI NAIK

Regd: 19301206

Under the Guidance of

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SRR&CVR GOVT. DEGREE COLLEGE(A) MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

This is to certify that the project titled "A **STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"** is a bonafide work done by **RAMAVATH RAVI NAIK(19301206)** under my guidance and supervision in partial fulfillment of the requirement for the award of degree of **Bachelor of Science (MPC)**in **SRR&CVRGOVT.DEGREE COLLEGE(A)**, Vijayawada, affiliated to **Krishna University, Machilipatnam**, during the academic year **2021-2022.**

Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

DECLARATION

I hereby declare that this project titled **"A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"** is a bonafide work done by me under the guidance of **Dr. G. LALITHA.** This project work is submitted to **SRR&CVR GOVT. DEGREE COLLEGE(A)**, Vijayawada, in partial fulfillment of the requirements for the award of **BACHELOR OF SCIENCE** during the academic year **2021-2022.**

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Date: 30/07/2022

R. Rani Naik

Place: VIJAYAWADA

ACKNOWLEDGEMENT

It is a great pleasure to take opportunity and express my gratitude to all those who helped me throughout my project work. First and foremost, I am thankful to **Dr.K.BHAGYALAKSHMI, Principal, SRR&CVR GOVT.DEGREE COLLEGE (A),** for giving me opportunity to take up the project work.

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A PROJECT REPORT

On

A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

SRR & CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of BACHELOR OF SCIENCE

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

Malladi. Sweety(1932208)

Under the Guidance of

Dr. G. LALITHA GARU

DEPARTMENT OF MATHEMATICS

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MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

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Project Guide

Dr. G. LALITHA

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Date: 30-07-2022

M. Sweety

Place: VIJAYAWADA

ACKNOWLEDGEMENT

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

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MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

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Dr. G. LALITHA

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Date: 30/07/2022

Place: VIJAYAWADA

P. K. K. Reddy

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

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MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

This is to certify that the project titled **"A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"** is a bonafide work done by **P. Phanindra (19301214)** under my guidance and supervision in partial fulfillment of the requirement for the award of degree of **Bachelor of Science (MPC)** in **SRR & CVR GOVT. DEGREE COLLEGE (A)**, Vijayawada, affiliated to **Krishna University, Machilipatnam**, during the academic year **2021-2022.**

Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

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Date: 30-07-2022

Place: VIJAYAWADA

P. Phanindra

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

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CHELLE.SOWJANYA (19301215)

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MACHAVARAM, VIJAYAWADA

2021-2022

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Date: _____

Place: VIJAYAWADA

Ch. Soujanya

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

BANDI RAVI KIRAN **(19301224)**

3RD B.SC. (MPC) EM

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DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

This is to certify that the project titled "A **STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS**" is a bonafide work done by BANDI RAVI KIRAN **(19301224)** under my guidance and supervision in partial fulfillment of the requirement for the award of degree of **Bachelor of Science (MPC) EM** in **SRR & CVR GOVT. DEGREE COLLEGE(A)**, Vijayawada, affiliated to **Krishna University, Machilipatnam**, during the academic year **2021-2022.**

Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

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Place: VIJAYAWADA

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B. Ravi kiran.

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

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MACHAVARAM, VIJAYAWADA

2021-2022

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Date: _____

Place: VIJAYAWADA

B. Rajeswari

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A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

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2021-2022

CERTIFICATE

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Dr. G. LALITHA

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Date: 30-07-2022

Place: VIJAYAWADA

M. Anil

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

M. Sudheer (19301240) MPC-EM

Under the Guidance of

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SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

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Dr. G. LALITHA

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Date: 30-07-2022

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M. Sudheer

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9 S0LUTION OF SIMULATANEOUS LINEAR SYSTEM OF EQUATIONS 71-81

10 NUMERICAL SOLUTIONS OF ORDINARY DIFFERENTIAL EQUATIONS 82-86

A PROJECT REPORT

On

A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of BACHELOR OF SCIENCE

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

MUDAVATHU SUBBARAO NAIK (19301241)

3RD BSC MPC (E.M)

Under the Guidance of

Dr. G. LALITHA GARU

DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

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Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

DECLARATION

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Date: 30-07-2022

Place: VIJAYAWADA

Subba Rao D.K

ACKNOWLEDGEMENT

It is a great pleasure to take opportunity and express my gratitude to all those who helped me throughout my project work. First and foremost, I am thankful to **Dr.K.BHAGYALAKSHMI, Principal, SRR&CVR GOVT.DEGREE COLLEGE (A),** for giving me opportunity to take up the project work.

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M. Subbaraonaik

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

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CERTIFICATE

This is to certify that the project titled "A ***STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"*** *is a bonafide work done by* ***Kasukurthi Akhil*** *(19301243) under my guidance and supervision in partial fulfillment of the requirement for the award of degreeof* ***Bachelor of Science*** *(MPC) EM in* ***SRR & CVR GOVT. DEGREE COLLEGE (A),*** *Vijayawada, affiliated to* ***Krishna University, Machilipatnam,*** *duringthe academic year* ***2021-2022.***

Project Guide

EXAMINER

EXTERNAL

Dr. G. LALITHA

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Date: 30/07/22

Place: VIJAYAWADA

K. Akhil

ACKNOWLEDGEMENT

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BACHELOR OF SCIENCE

Submitted by

THAMMU GOPI CHAND (19301244)

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Project Guide

EXAMINER

EXTERNAL

Dr. G. LALITHA

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Date: 30-07-2022

Place: VIJAYAWADA

T. Gopi chand

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

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MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

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Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

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Date: 30/07/2022

Place: VIJAYAWADA

R. V. Devaraya

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Dr. G. LALITHA

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Date: 30/7/2022.

Place: VIJAYAWADA

V. Chandar Naik

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

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MACHAVARAM, VIJAYAWADA

2021-2022

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Date: _____

Place: VIJAYAWADA

B. Anjali

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In partial fulfilment of the requirement for the award of degree of

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Submitted by

Sk. Sameer (19301256) MPC-EM

Under the Guidance of

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SRR&CVR GOVT. DEGREE COLLEGE(A)

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2021-2022

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Date: 30-07-2022.

Place: VIJAYAWADA

Sk. Sameer.

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Submitted by

Y.SIVA SAI RAM (19302201)

Under the Guidance of

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MACHAVARAM, VIJAYAWADA

2021-2022

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Project Guide

Dr. G. LALITHA

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Date: 30/07/2022

Place: VIJAYAWADA

Y. Siva Sai Ram

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In partial fulfilment of the requirement for the award of degree of

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Submitted by

JAGANNADHAM PAVAN KUMAR (19302202)

Under the Guidance of

Dr. G. LALITHA GARU

DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

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Project Guide **EXTERNAL EXAMINER**

Dr. G. LALITHA

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DECLARATION

I hereby declare that this project titled **"A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"** is a bonafide work done by me under the guidance of **Dr. G. LALITHA.** This project work is submitted to **SRR&CVR GOVT. DEGREE COLLEGE(A),** Vijayawada, in partial fulfillment of the requirements for the award of **BACHELOR OF SCIENCE** during the academic year **2021-2022.**

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Date: 30/07/2022

Place: VIJAYAWADA

J. Pavan Kumar

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ACKNOWLEDGEMENT

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A PROJECT REPORT

On

A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of **BACHELOR OF SCIENCE**

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

PARAMASIVAN TARUN TEJA(19302204)

Under the Guidance of

Dr. G. LALITHA GARU

DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

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CERTIFICATE

This is to certify that the project titled **"A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"** is a bonafide work done by **PARAMASIVAN TASRUN TEJA (19302204)** under my guidance and supervision in partial fulfillment of the requirement for the award of degree of **Bachelor of Science (MPCS)**in **SRR&CVRGOVT.DEGREE COLLEGE(A)**, Vijayawada, affiliated to **Krishna University, Machilipatnam**, during the academic year **2021-2022.**

Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

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Date: 30/07/2022

P. Tarunija

Place: VIJAYAWADA

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SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of BACHELOR OF SCIENCE

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

MEKA KRUPA (19302206)

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DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

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CERTIFICATE

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Project Guide

Dr. G. LALITHA

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Date: 30-7-2022

Place: VIJAYAWADA

M. Keerthi.

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A PROJECT REPORT

On

A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of **BACHELOR OF SCIENCE**

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

AMPOLU. LAVANYA (19302209)

Under the Guidance of

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DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

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Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

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Date: 30/07/2022

Place: VIJAYAWADA

A. Lavanya

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A PROJECT REPORT

On

A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of BACHELOR OF SCIENCE

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

Sathivada jagadeesh (19302213)

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DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

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CERTIFICATE

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Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

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Date: _____

Place: VIJAYAWADA

S. Jagadeesh

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A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

Depalli.Krishnaveni (19302222)

Under the Guidance of

Dr. G. LALITHA GARU

DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

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Project Guide

Dr. G. LALITHA

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Date: 30/07/2022

D. Krishnaveni

Place: VIJAYAWADA

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A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

NUTHAKKI.LIKHITHA (19302225)

Under the Guidance of

Dr. G. LALITHA GARU

DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

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CERTIFICATE

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Project Guide

Dr. G. LALITHA

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Date: 30/07/2022

Place: VIJAYAWADA

N. Likhitha.

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A PROJECT REPORT

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A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

P.Deena (19302229)

Under the Guidance of

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DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

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Project Guide

Dr. G. LALITHA

EXTERNAL

EXAMINER

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P. Deena

Place: VIJAYAWADA

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A PROJECT REPORT

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SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of **BACHELOR OF SCIENCE**

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

P.rokesh 3rd bsc Mpcs (19302243)

Under the Guidance of

Dr. G. LALITHA GARU

DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

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CERTIFICATE

This is to certify that the project titled **"A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"** is a bonafide work done by P.ROKESH. 3rd bsc Mpcs (19302243) under my guidance and supervision in partial fulfillment of the requirement for the award of degree of **Bachelor of Science (MPCS)**in **SRR&CVRGOVT.DEGREE COLLEGE(A),** Vijayawada, affiliated to **Krishna University, Machilipatnam**, during the academic year **2021-2022.**

Project Guide

Dr. G. LALITHA

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Date: 30/07/2022

Place: VIJAYAWADA

Rokesh.P

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ACKNOWLEDGEMENT

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A PROJECT REPORT

On

A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of **BACHELOR OF SCIENCE**

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

POLAGANI. DHANYA SRI (19302251)

Under the Guidance of

Dr. G. LALITHA GARU

DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

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CERTIFICATE

This is to certify that the project titled **"A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"** is a bonafide work done by **POLAGANI. DHANYA SRI (19302251)** under my guidance and supervision in partial fulfillment of the requirement for the award of degree of **Bachelor of Science (MPCS)** in **SRR&CVRGOVT.DEGREE COLLEGE(A)**, Vijayawada, affiliated to **Krishna University, Machilipatnam**, during the academic year **2021-2022.**

Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

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DECLARATION

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Date: 30-7-2022

P. Dhanya Sri

Place: VIJAYAWADA

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ACKNOWLEDGEMENT

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A PROJECT REPORT

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SeA STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

A project submitted to **SRR & CVR GOVT DEGREE COLLEGE (A)** in partial fulfillment of the requirement for the award of Degree of **BACHELOR OF SCIENCE**

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

Kella . Durga Prasad (19302252)

Under the Guidance of

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SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

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CERTIFICATE

This is to certify that the project titled **"A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"** is a bonafide work done by **Kella. Durga Prasad (19302252)** under my guidance and supervision in partial fulfillment of the requirement for the award of degree of **Bachelor of Science (MPCS)** in **SRR&CVR GOVT. DEGREE COLLEGE(A)**, Vijayawada, affiliated to **Krishna University, Machilipatnam**, during the academic year **2021-2022**

Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

DECLARATION

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I hereby declare that this project titled "**MATHEMATICS**" is a bonafide work done by me under the guidance of **Dr. G. LALITHA.** This project work is submitted to **SRR&CVR GOVT. DEGREE COLLEGE(A)**, Vijayawada, in partial fulfillment of the requirements for the award of **BACHELOR OF SCIENCE** during the academic year **2021-2022.**

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Date: 30/07/2022

Place: VIJAYAWADA

V. Durgaprasad.

ACKNOWLEDGEMENT

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UNIT-I LAPLACE TRANSFORM

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A PROJECT REPORT

On

A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of **BACHELOR OF SCIENCE**

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

K.swami sekhar (19302257)

Under the Guidance of

Dr. G. LALITHA GARU

DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

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CERTIFICATE

This is to certify that the project titled **"A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"** is a bonafide work done by k.swami sekhar **(19302257)** under my guidance and supervision in partial fulfillment of the requirement for the award of degree of **Bachelor of Science (MPCS)**in **SRR&CVRGOVT.DEGREE COLLEGE(A)**, Vijayawada, affiliated to **Krishna University, Machilipatnam**, during the academic year **2021-2022.**

Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

DECLARATION

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Date: 30/07/2022

Place: VIJAYAWADA

Y. Swami Sekhar

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A PROJECT REPORT

On

A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of **BACHELOR OF SCIENCE**

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

B.venkatesh 3rd bsc Mpcs (19302258)

Under the Guidance of

Dr. G. LALITHA GARU

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SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

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CERTIFICATE

This is to certify that the project titled "**A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS**" is a bonafide work done by B.venkatesh 3rd bsc Mpcs (19302258) under my guidance and supervision in partial fulfillment of the requirement for the award of degree of **Bachelor of Science (MPCS)**in **SRR&CVRGOVT.DEGREE COLLEGE(A),** Vijayawada, affiliated to **Krishna University, Machilipatnam**, during the academic year **2021-2022.**

Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

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DECLARATION

I hereby declare that this project titled **"A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"** is a bonafide work done by me under the guidance of **Dr. G. LALITHA.** This project work is submitted to **SRR&CVR GOVT. DEGREE COLLEGE(A),** Vijayawada, in partial fulfillment of the requirements for the award of **BACHELOR OF SCIENCE** during the academic year **2021-2022.**

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Date: 30/07/2022

Place: VIJAYAWADA

B. Venkatesh

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ACKNOWLEDGEMENT

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A PROJECT REPORT

On

A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

A project submitted to **SRR & CVR GOVT DEGREE COLLEGE (A)** in partial fulfillment of the requirement for the award of Degree of **BACHELOR OF SCIENCE**

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

Sampari . Bhavitha (19302259)

Under the Guidance of

Dr. G. LALITHA GARU

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SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

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Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

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Date: 30/07/2022

S. Bhavitha

Place: VIJAYAWADA

ACKNOWLEDGEMENT

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UNIT-I LAPLACE TRANSFORM

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A PROJECT REPORT

On

A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

SRR & CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of BACHELOR OF SCIENCE

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

Kemburu. Sravanthi(19302269)

Under the Guidance of

Dr. G. LALITHA GARU

DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

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CERTIFICATE

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Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

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Date: 30-07-2022

K. Sowanthi

Place: VIJAYAWADA

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A PROJECT REPORT

On

A STUDY ON ADVANCED NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of **BACHELOR OF SCIENCE**

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

JALASUTRAM RAVITEJA(19304210)

Under the Guidance of

Dr. G. LALITHA GARU

DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

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Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

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Place: VIJAYAWADA

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T. Pavleja

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A PROJECT REPORT

On

A STUDY ON NUMERICAL ANALYSIS &INTEGRAL TRANSFORMS

Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of **BACHELOR OF SCIENCE**

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

CHUNDRU.MUKUNDA PRIYA (19304218)

Under the Guidance of

Dr. G. LALITHA MADAM

DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

This is to certify that the project titled **"A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"** is a bonafide work done by

Chundru Mukundapriya (19304218) under the guidance and supervision in partial fulfillment of the requirement for the award of degree of **Bachelor of Science (MSCS)**in **SRR&CVRGOVT.DEGREE COLLEGE(A)**, Vijayawada, affiliated to **Krishna University, Machilipatnam**, during the academic year **2021-2022.**

Project Guide

Dr. G. LALITHA

EXTERNALEXAMINER

DECLARATION

I hereby declare that this project titled **"A STUDY ON NUMERICAL ANALYSIS &INTEGRAL TRANSFORMS"** is a bonafide work done by me under the guidance of **Dr. G. LALITHA.** This project work is submitted to **SRR&CVR GOVT. DEGREE COLLEGE(A)**, Vijayawada, in partial fulfillment of the requirements for the award of **BACHELOR OF SCIENCE** during the academic year **2021-2022.**

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Date: 30/07/2022.

Place: VIJAYAWADA

Ch. Mukundapriya.

ACKNOWLEDGEMENT

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A PROJECT REPORT

On

A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of **BACHELOR OF SCIENCE**

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

VADISALA. BABU KUMAR(19304229)

Under the Guidance of

Dr. G. LALITHA GARU

DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

This is to certify that the project titled "A **STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"** is a bonafide work done by **VADISALA. BABU KUMAR (19304229)** under my guidance and supervision in partial fulfillment of the requirement for the award of degree of **Bachelor of Science (MSCS)**in **SRR&CVRGOVT.DEGREE COLLEGE(A)**, Vijayawada, affiliated to **Krishna University, Machilipatnam**, during the academic year **2021-2022.**

Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

DECLARATION

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Date: 30/07/2022

Place: VIJAYAWADA

V. Babu Kumar

ACKNOWLEDGEMENT

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

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CERTIFICATE

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Dr. G. LALITHA

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Date: 30-07-2020

Place: VIJAYAWADA

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B. Jagadish.

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BACHELOR OF SCIENCE

Submitted by

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Dr. G. LALITHA

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Date: 30/07/2022.

Place: VIJAYAWADA

Venna. Naga prasanna Abhilash

ACKNOWLEDGEMENT

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In partial fulfilment of the requirement for the award of degree of

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Project Guide

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Date: 30/07/2022

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G. Avinash

ACKNOWLEDGEMENT

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J. Avinash

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A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

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Project Guide

Dr. G. LALITHA

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Date: 30/7/2022

Place: VIJAYAWADA

P Prabha kumari

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J. Prabhakar

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On

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Submitted to

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

BEVARA LOKESH(19304239)

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CERTIFICATE

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Project Guide

Dr. G. LALITHA

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Date: 30-07-2022

Place: VIJAYAWADA

R. Lokesh

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

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Date: 30-07-2022

Place: VIJAYAWADA

A. Balaram Krishna

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In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted R MAHESH BABU (19304248)

Under the Guidance of

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SRR&CVR GOVT. DEGREE COLLEGE(A)

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2021-2022

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P. Mahesh Babu

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UNIT-I LAPLACE TRANSFORM

1).Find Laplace Transform of $[t^2+1]^2$

Sol: $[t^2+1]^2=t^4+2t^2+1$

A PROJECT REPORT

On

A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of **BACHELOR OF SCIENCE**

In partial fulfilment of the requirement for the award of degree of

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Submitted by

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MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

This is to certify that the project titled "A **STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"** is a bonafide work done by KANAKAPUDI BHAGAVAN **(19304251)** under my guidance and supervision in partial fulfillment of the requirement for the award of degree of **Bachelor of Science (MSCS)**in **SRR&CVRGOVT.DEGREE COLLEGE(A)**, Vijayawada, affiliated to **Krishna University, Machilipatnam**, during the academic year **2021-2022.**

Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

DECLARATION

I hereby declare that this project titled "A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"is a bonafide work done by me under the guidance of **Dr. G. LALITHA.** This project work is submitted to **SRR&CVR GOVT. DEGREE COLLEGE(A),** Vijayawada, in partial fulfillment of the requirements for the award of **BACHELOR OF SCIENCE** during the academic year **2021-2022.**

I also declare that this project is a result of my own effort and that it has not been submitted to any other University for the award of any degree.

Date: _____

Place: VIJAYAWADA

ACKNOWLEDGEMENT

It is a great pleasure to take opportunity and express my gratitude to all those who helped me throughout my project work. First and foremost, I am thankful to **Dr. K. BHAGYALAKSHMI, Principal, SRR&CVR GOVT.DEGREE COLLEGE (A),** for giving me opportunity to take up the project work.

I also thank, **Dr. G. LALITHA, project guide** and **Head of the Department, K.V.N.LAKSHMI** for giving me opportunity to take up the project work and helping me through out and finally completing the project.

I would like to express my sincere and heartfelt thanks to all the teaching and non-teaching staff of the department for their continuous co-operation, which has given me the guidance to build up adamant aspiration over the completion of my project.

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Finally, I thank one and all who directly and indirectly helped me to complete my project successfully.

K. Bhagavan

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A PROJECT REPORT

On

A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of **BACHELOR OF SCIENCE**

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

shaik ahammad vali (19304254)

Under the Guidance of

Dr. G. LALITHA GARU

DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

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Place: VIJAYAWADA

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Sk. Ahammad vali

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BACHELOR OF SCIENCE

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Nukulla. Vamsi Lakshmana Phanindra (19306218)

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MACHAVARAM, VIJAYAWADA

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A STUDY ON NUMERICAL ANALYSIS

Introduction:

It is **the study of numerical methods that attempt at finding approximate solutions of problems rather than the exact ones**. Numerical analysis finds application in all fields of engineering and the physical sciences, and in the 21st century also the life and social sciences, medicine, business and even the arts.

Why Do We Need Numerical Analysis In Everyday Life?

The great advantage of using numerical analysis is that it investigates and provides accurate solutions to real-life problems from the field of science, engineering, biology, astrophysics and finance.

The word 'analysis' generally means to solve a problem through a set of equations and further reduce these equations using the methodologies of algebra, partial differential equations, calculus and other related fields of mathematics.

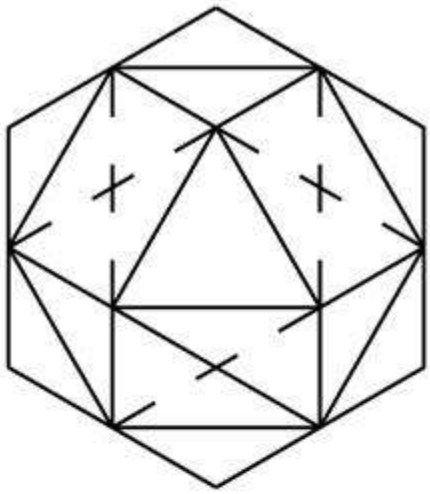
On similar grounds, numerical analysis implements arithmetic algorithms: addition, subtraction, multiplication, division and comparison to obtain numerical solutions.
A computer precisely performs these operations, meaning that numerical analysis and computers are intimately related.

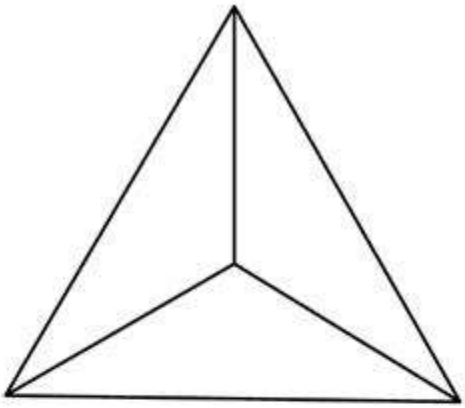
The problem of continuous mathematics generally arises throughout the natural sciences, business management, engineering, astronomy and medicine. It is always more practical to carry out the tedious arithmetic operations using a computer.

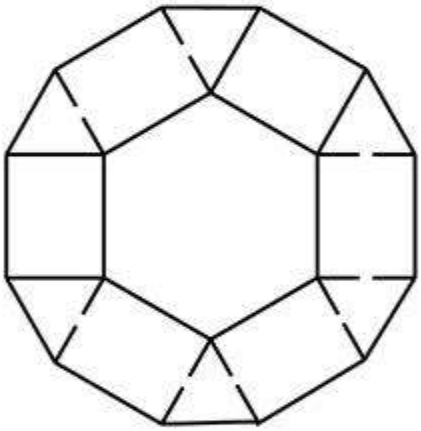
Before the mid-20th century, until the advent of modern computers, all these repetitive operations had to be performed by manual interpolation. The overall agenda of numerical analysis is to give an *approximate*, but *accurate* solution to the advanced problem. The necessity for accuracy, is, of course, determined by the variety of applications.

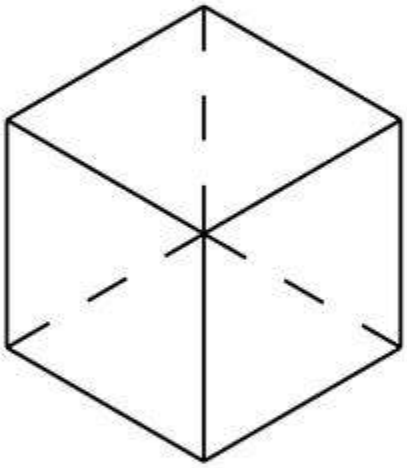
Historical background:

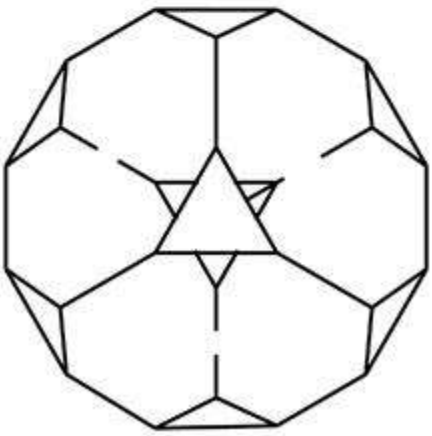
Numerical algorithms predate the invention of modern and digital computers and are almost as sold as human civilization itself. Many renowned mathematicians of the past were engrossed in numerical analysis. *Archimedes of Syracuse* (287-211/212 BC), the most famous Greek mathematician, physicist, engineer and inventor, perfected new standards of contemporary geometry, including the 'method of exhaustion' to compute the area and volume of various geometric figures.
A famous document, *The Rhind Papyrus* (1650 BC) from the ancient Egyptian Middle Kingdom, bears a root-finding method to solve simple equations.

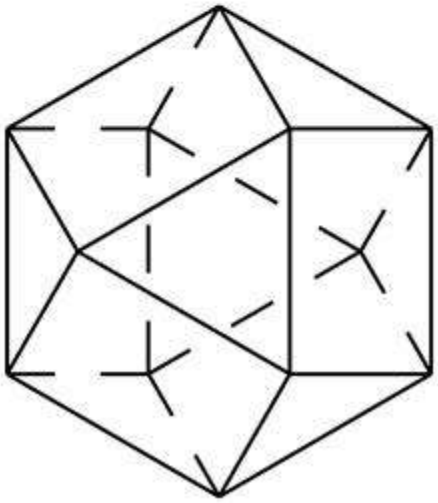












Archimedes shapes. He invented the 'method of exhaustion' to calculate the length, area and volume of such geometric figures (Photo Credit : ZASIMOV YURII/Shutterstock)

To facilitate computational solutions, *Newton* and *Leibnitz* invented integral and differential calculus. Various accurate mathematical models were developed, but they couldn't be solved explicitly.

To simplify this, several numerical methods were created by Isaac Newton. Following his work on root finding and interpolations, many other legendary mathematicians like Euler, Lagrange, and Gauss contributed to the field of numerical analysis. Another important aspect was the invention of logarithms by *John Naiper* (1614), replacing the tedious calculations of multiplication, division and exponentiation.

Different methods and areas under Numerical Analysis:

Direct methods lead us to the exact solution in a finite number of steps. For example, Gauss elimination is used to find the roots of the linear simultaneous equations immediately. On the contrary, *iterative methods* are most commonly used and are not expected to finish off in a certain number of steps. These are approximate methods starting from an initial guess and converging to an exact solution. *Gauss- Jacobi* and *Gauss- Seidel* are iterative methods used to solve a system of equations with a larger number of unknowns.

According to the problem to be solved, the field of numerical analysis is divided into various disciplines.

- *Interpolation* constructs a new set of data points within the range of the given function (or problem). This can be obtained through curve-fitting, and some examples include the Gaussian process, linear interpolation and polynomial interpolation.

- *Extrapolation* is identical to interpolation, but it has a higher risk of producing meaningless results. It is the process of roughly evaluating the value of the problem outside its range.

- *Regression* is a statistical process that helps in understanding the relationship between dependent and independent variables. It is widely used for forecasting and predicting in the field of machine learning.

- *Solving differential and integral equations:* Most mathematical models (particularly in engineering) are based on the solutions obtained by partial differential equations, ordinary differential equations and integral equations. Some popular techniques are Monte Carlo integration and the Newton-Cotes formula.

Modern applications of Numerical Analysis:

Sophisticated numerical analysis software has become indispensable in modern life. People are able to perform mathematical modeling even if they are unaware of the simulations involved. This can only be achieved through reliable, high-end and efficient software. Some of the major applications of numerical analysis are intriguing, yet easy to understand.

- *Car safety enhancements*: Car makers around the globe use numerical simulations to evaluate and enhance car safety. Pedestrian protection is also kept in mind while investigating car crash tests. The algorithms involved are partial differential equations and fed to the advanced computers to unravel optimal results.

Numerical analysis provides an integrated computer-aided engineering (CAE) solution for car safety and other physical parameters.

Airflow patterns in the Respiratory Tract: It is quite common for patients in ICUs to undergo respiratory failure. Mechanical ventilation is a treatment that helps in the sufficient exchange of oxygen and carbon dioxide for the normal functioning of the lungs. Various mathematical models use differential equations and computational algorithms to develop laminar airflow in the lungs using ventilators.

- *Onset and Progression of tumor cells:* Cancer is characterized by the accumulation of pre-malignant cells and tumor growth. In recent years, cost-effective statistical and probability models have been developed to detect cancer in the body. This allows for the computing of several parameters, such as population size, lifetime and mutations of the cancer cells.

- *Financial industry*: Modern businessmen make use of numerical techniques to allocate their resources efficiently. Some of the problems addressed by such applications are manufacturing,

storage, scheduling, investment and others. Quantitative analysts have expertise in this area, and use the algorithms in risk management and interest calculation.

- *Transportation of chemicals in the body:* Our body is constantly being exposed to various chemicals (or drugs). Not all are beneficial and the body must excrete them out into the environment. The diffusion and transport of such chemicals is studied with the help of ordinary and partial differential equations.

- *High hydrostatic pressure (HHP) processing:* This is a non-thermal process in which food and biotechnological substances are compressed under very high pressure of up to 1000 MPa to inactivate certain enzymes and micro-organisms. The treatment of fluid food is analyzed by means of numerical simulations. The enzyme is inactivated with the help of numerical equations.

- *Weather predictions:* Numerical weather predictions (NWP) are based on a set of differential equations known as hydro-thermodynamic equations. Very powerful and energy-efficient computers are used to process the bulk data and the information is extracted in the form of topographical charts.

- *Spacecraft Dynamics*: Increases in the size and complexity of spacecraft have demanded a complex mathematical model of its dynamics. To reduce the inconvenience and plan a smooth trajectory for the spacecraft, various open loop models are created because the dynamics in space behave very differently than they do on Earth.

In the past few decades, more attention has been given to the designing of spacecraft. Numerical analysis and simulations through user-friendly computer software make it simpler to modify the dynamics of the system .

Price estimation by airlines: Nowadays, airline ticket prices vary significantly, even for nearby seats within the same cabin. Airlines use computational techniques to increase their revenue, keeping a check on fuel, payroll, crew assignments and many other activities.

- *Machine learning*: The numerical algorithms of Newton's method and the Nestorov method are used in machine learning optimization. Artificial intelligence is another field where machine learning is applied with the help of numerical analysis.

Machine-learning algorithms are running behind a large number of applications in our daily lives.

Conclusion:

Numerical analysis is the branch of modern computation that finds applications in the field of engineering, life sciences and even arts. It has a remarkable ability to predict the world around us. The calculations are mostly made by the computers using MATLAB, FORTRAN 77 and other software programs to minimize errors. Clearly, numerical analysis has proved itself as a boon to humankind, from ancient times all the way to today, and they will surely help us move forward into the future!

Here we have made an imperative study on the following topics in Numerical Analysis:

- CURVE FITTING
- NUMERICAL DIFFERENTIATION
- NUMERICAL INTEGRATION
- SOLUTION OF SIMULATANEOUS LINEAR SYSTEM OF EQUATIONS
- NUMERICAL SOLUTIONS OF ORDINARY
- DIFFERENTIAL EQUATIONS

A STUDY ON LAPLACE TRANSFORMS

Introduction:

In mathematics, the Laplace transform, named after its discoveror Pierre-Simon Laplace (/ləˈplɑːs/), is **an integral transform that converts a function of a real variable (usually , in the time domain) to a function of a complex variable** (in the complex frequency domain, also known as s-domain, or s-plane). The Laplace transform is used **to solve differential equations**. It is accepted widely standard algebraic identities in many fields. We know that the Laplace transform simplifies a given LDE (linear differential equation) to an algebraic equation, which can later be solved using the standard algebraic identities.

What is Laplace transform and its properties?

Laplace transform is divided into **two types**, namely one-sided Laplace transformation and two-sided Laplace transformation.Laplace transforms have several properties for linear systems. The different properties are: Linearity, Differentiation, integration, multiplication, frequency shifting, time scaling, time shifting, convolution, conjugation, periodic function. There are two very important theorems associated with control systems.

Properties of Laplace transform:

The Laplace transform has a number of properties that make it useful for analyzing linear dynamical systems. The most significant advantage is that differentiation becomes multiplication, and integration becomes division, by *s* (reminiscent of the way logarithms change multiplication to addition of logarithms).
Because of this property, the Laplace variable *s* is also known as *operator variable* in the *L* domain: either *derivative operator* or (for s^{-1}) *integration operator*. The transform turns integral equations and differential equations to polynomial equations, which are much easier to solve. Once solved, use of the inverse Laplace transform reverts to the original domain.

What are the advantages of Laplace transform?

The advantage of using the Laplace transform is that **it converts an ODE into an algebraic equation of the same order that is simpler to solve, even though it is a function of a complex variable**.

Fourier transform

The Fourier transform is a special case (under certain conditions) of the bilateral Laplace transform. While the Fourier transform of a function is a complex function of a *real* variable (frequency), the Laplace transform of a function is a complex function of a *complex* variable. The Laplace transform is usually restricted to transformation of functions of t with $t \geq 0$. A consequence of this restriction is that the Laplace transform of a function is a holomorphic function of the variable s. Unlike the Fourier transform, the Laplace transform of a distribution is generally a well-behaved function. Techniques of complex variables can also be used to directly study Laplace transforms. As a holomorphic function, the Laplace transform has a power series representation. This power series expresses a function as a linear superposition of moments of the function. This perspective has applications in probability theory.

The Fourier transform is equivalent to evaluating the bilateral Laplace transform with imaginary

argument $s = i\omega$ or $s = 2\pi i\xi$[26] when the condition explained below is fulfilled,
This convention of the Fourier transform requires a factor of $1/2\pi$ on the inverse Fourier transform. This relationship between the Laplace and Fourier transforms is often used to determine the frequency spectrum of a signal or dynamical system.
The above relation is valid as stated if and only if the region of convergence (ROC) of $F(s)$ contains the imaginary axis, $\sigma = 0$.
For example, the function $f(t) = \cos(\omega_0 t)$ has a Laplace transform $F(s) = s/(s^2 + \omega_0{}^2)$ whose ROC is $\mathrm{Re}(s) > 0$. As $s = i\omega_0$ is a pole of $F(s)$, substituting $s = i\omega$ in $F(s)$ does not yield the Fourier transform of $f(t)u(t)$, which is proportional to the Dirac delta-function $\delta(\omega - \omega_0)$.
However, a relation of the form holds under much weaker conditions. For instance, this holds for the above example provided that the limit is understood as a weak limit of measures (see vague topology). General conditions relating the limit of the Laplace transform of a function on the boundary to the Fourier transform take the form of Paley–Wiener theorems.

In this project we made a detailed study about some of the applications of the following topics

- LAPLACE TRANSFORM
- INVERSE LAPLACE TRANSFORM
- APPLICATIONS OF LAPLACE TRANSFORM TO
- SOLUTIONS OF DIFFERENTIAL EQUATIONS
- FORIER TRANSFORMS
- FINITE FOURIER TRANSFORMS

LAPLACE TRANSFORM

1). Find Laplace Transform of $[t^2+1]^2$

Sol: $[t^2+1]^2 = t^4+2t^2+1$

$L[t^2+1]^2 = L[t^4] + L[t^2] + L[1]$

$= \frac{4!}{p5} + \frac{2!}{t^2} + L[1]$

$= \frac{24}{p5} + \frac{4}{p3} + \frac{1}{p}$

$= \frac{24+4p^2+p^4}{P^5}$

$= \frac{1}{p5}[24+4p^2+p^4]$

2) Find L[$\cosh^3 2t$]

Sol: $L[\cosh^3 2t]$

We know that $\cosh 3x = 4\cosh^3 x - 3\cosh x$

$\cosh^3 x = \frac{\cosh x + 3\cosh 2x}{4}$

$[\cosh^3 2t] = \frac{\cosh t + 3\cosh 2t}{4}$

$2[\cosh^3 2t] = 1/4\ L[\cosh 6t] + 3/4\ L[\cosh 2t]$

$= 1/4 \left[\frac{p}{P^2-36}\right] + 3/4\left[\frac{p}{P^2-4}\right]$

3)Statement If L[f(t)]=f(p) then

(i)$L[e^{-at}F(t)]=f(p+a)$

(ii)$L[e^{at}F(t)]=f(p-a)$

Sol:

Proof:- By the definition of laplace transformation

$$(i)L[f(t)] = \int_0^\infty e^{-pt}\, e^{-at}F(t)dt$$

$$=\int_0^\infty e^{-pt-at}F(t)$$

$$=\int_0^\infty e^{-t(p+a)}F(t)dt$$

$$=f(p+a)$$

$$(ii)L[e^{at}F(t)=\int_0^\infty e^{-pt}\, e^{at}F(t)dt$$

$$=\int_0^\infty e^{-pt+at}\, F(t)dt$$

$$=\int_0^\infty e^{t(-p+a)}\, F(t)dt$$

$$=\int_0^\infty e^{t(p-a)}\, F(t)dt$$

$$=f(p-a)$$

4) Find $L[e^{-t}\cos^2 t]$

Sol: $l[\cos^2 t]=l[1+\cos 2t]$

$$=1/2[L(1)+L(\cos 2t)]$$

$$= 1/2 \left[\frac{1}{p} + \frac{p}{p^2+4}\right]$$

By applying 1st shifting theorem

$$L[e^{-t}\cos^2 t]=1/2 \left[\frac{1}{p} + \frac{p}{P^2+4}\right]_{p\to p+1}$$

$$=1/2\left[\frac{1}{P+1}+\frac{P+1}{(P+1)^2+4}\right]$$

$$=1/2\left[\frac{1}{P}+\frac{P+1}{P^2+2p+5}\right]$$

5)Find Laplace Transform of $(sint-cost)^3$

Sol:($(sint-cost)^3=sin^3t-cos^3t-3sin^2cost+3sintcos^2t$

$=sin^3t-cos^3t-3(1-cos^2t)cost+3sint(1-sin^2t)$

$=sin^3t-cos^3t-3cost+3cos^3t+3sint-3sin^3t$

$=2cos^3t-2sin^3t-3cost+3sint$

$=2[cos3t+3cost\2]-2[3sint-sin3t/2]-3cost+3sint$

$=1/2[cos3t+3cost]-1/2[3sint-sin3t]-3cost+3sint$

$=cos3t/2+sin3t/2+sint(-3/2+3)+cost(3/2-3)$

$=cos3t/2+sin3t/2+sint(-3+6/2)+cost(3-6/2)$

$=1/2cos3t+1/2sin3t+3/2sint-3/2cost$

$=1/2[cos3t+sin3t+3sint-3cost]$

$=1/2\{L[cos3t+sin3t+3sint-3cost]\}$

$=1/2\{L(cos3t)+L(sin3t)+3L(sint)-3L(cost)\}$

$=1/2[(p/p^2+3^2)+(3/p^2+9)+3[1/p^2+1]-3[p/p^2+1]]$

$=1/2[(p/p^2+9)+(3/P^2+9)+(3/p^2+1)-(3/p^2+1)]$

Therefore $2(sint-cost)^3=1/2[(3+p/p^2+9)+(3-3p/p^2+1)]$

6.Find the laplace transform of f(t)=|t-1|+|t+1|,t ≥0

Sol: Given that f(t)=|t-1|+|t+1|,t ≥0 ----------eq(1)

When 0<t<1 =>|t-1|= -(t-1)= 1-t

|t+1|=t-1

Therefore f(t)=1-t+t+1=2t ----------eq(2)

When t >0 =>|t-1|=t-1

|t+1|=t+1

Therefore f(t)=t-1+t+1=2t------eq(3)

To Find L[f(t)]:-

$L[f(t)]=\int_0^1 e^{-pt}F(t)dt+\int_{\emptyset}^{\infty} e^{-pt}F(t)dt$

$=\int_0^1 e^{-pt}2dt+2\int_1^{\infty} e^{-pt}2tdt$

$=2\int_0^1 e^{-pt}dt+2\int_1^{\infty} e^{-pt}tdt$

$=2[e^{-pt/-p}]_{t=0}^{t=1}+2\int_1^{\infty} e^{-pt}tdt$

(.. $\int f(x)g(x)dx=f(x)\int g(x)dx-\int[d/dx\ f(x)\int g(x)dx]dx$)

$=2[e^{-pt/p}]_{t=o}^{t=1}+2[t\int_1^{\infty} e^{-pt}dt-\int[d/dx\ t\int_1^{\infty} e^{-pt}dt]\ ]dt$

$=2/-p[e^{-p}-e^{0}]+2\{t[e^{-pt/}-p]_{t=1}^{t=\infty}-\int_{t=1}^{t=\infty}[e^{-pt}/-p]dt\}$

$=-2/p[e^{p}-1]+2\{t[e^{-pt}/-p]_{t=1}^{t=\infty}-\int_{t=1}^{t=\infty}[e^{-pt}/-p]dt\}$

$=-2/p(e^{-p}-1)+2\{-1/p[e^{\infty}-e^{-p}]+1/p[e^{-pt/-p}]^{t=\infty}\}$

$=-2/p(e^{-p}-1)+2\{-1/p[0-e^{-p}+1/p^{2}[e^{-\infty}-e^{-p}]\}$

$=-2/p(e^{-p}-1)+2[e^{-p}/p+e^{-p}/p^{2}]$

$=2/p(1+e^{-p}/p)$

7) Find 2[sinhat cosat]

We know that sinhat=e^{at}-e^{-at}/2

Sinhat=cosat=1/2 e^{at}cosat-e^{-at}cosat-----eq(1)

2[cosat]=p/p^2+a^2

By applying first shifting theorem

2[e^{at}cosat]=p-a/$(p-a)^2$+a^2

=p-a/p^2+2a^2-2pa-------eq(2)

Similarly L[e^{-at}cosat]=p+a/p^2+2pa+2a^2-----eq(3)

Substitute (2)&(3) on (1)

2[sinhat cosat]=1/2 L[e^{at}cosat-e^{-at}cosat]

=1/2[p-a/p^2+2a^2-2pa – p+a/p^2+2pa+2a^2

8) Find L[$(1+t.e^{-t})]^3$

Sol: L[$(1+t.e^{-t})]^3$=L[1+t^3e^{-3t}+3te^{-t}+3t^2e^{-2t}]

L(t)=1!/p^2,L[1]=1/p,L[t^3]=3!/p^4,L[t^2]=2!\p^3

2[$(1+t.e^{t})]^3$=L(1+t^3e^{-3t}+3te^{-t}+3t^2e^{2t}]

=L[1]+L[$e^{-3}t^3$]+3L[e^{-t}t]+3L[$e^{-2t}t^2$]

=1/p+$[6/p^4]_{p->p+3}$ +3[1/p^2]+3$[2/p^3]_{p->p+2}$

=1/p+[6/$(p+3)^4$+3/$(p+1)^2$+6/$(p+2)^3$

=1/p +3/$(p+1^2)$+6/$(p+2)^3$+6/$(p+3)^4$

INVERSE LAPLACE TRANSFORM

9) Define the inverse Laplace transform and also find $L^{-1}\left[\frac{3P}{4P^2+25}\right]$.

Solution: - The inverse Laplace transform of function f (P) and is written as f (t) $=L^{-1}$ [F (P)]

$$L^{-1}\left[\frac{3P}{4P^2+25}\right]$$

$$= 3L^{-1}\left[\frac{P}{4P^2+\frac{25}{4}}\right]$$

$$= \frac{3}{4}L^{-1}\left[\frac{P}{P^2+\left(\frac{5}{2}\right)^2}\right]$$

$$= \frac{3}{4}\cos\left(\frac{5}{2}\right)t$$

10) Find the inverse Laplace transform of $\left\{\frac{3(P^2-2)^2}{2P^5}\right\}$

Solution: - $L^{-1}\left[\frac{3}{2}\frac{(p^2-2)^2}{P^5}\right]$

$$= \frac{3}{2}\,L^{-1}\left[\frac{P^4+4+P^2}{P^5}\right]$$

$$= \frac{3}{2}\left[L^{-1}\left[\frac{\cancel{P^4}}{\cancel{P^5}}\right] + 4L^{-1}\left[\frac{1}{P^5}\right] - 4L^{-1}\left[\frac{\cancel{P^2}}{\cancel{P^5}}\right]\right]$$

`

$$= \frac{3}{2}\left[L^{-1}\left[\frac{1}{P}\right] + 4L^{-1}\left[\frac{1}{P^5}\right] - 4L^{-1}\left[\frac{1}{P^3}\right]\right]$$

$$= \frac{3}{2}\left[1 + \frac{4}{4!} + L^{-1}\left[\frac{4!}{P^{4+1}}\right] - \frac{4}{2}L^{-1}\left[\frac{2!}{P^{2+1}}\right]\right]$$

$$= \frac{3}{2}\left[1 + \frac{1}{6} + t^4 - 2t^2\right]$$

$$= \frac{3}{2} + \frac{t^4}{4} - 3t^2$$

$$= \frac{3}{2} - 3t^2 + \frac{t^4}{4}$$

11) Find inverse Laplace transform of $\left\{\frac{3P-4}{P^2-4P+8}\right\}$ by using First shifting theorem.

Solution: - Given that $L^{-1}\left\{\frac{3P-4}{P^2-4P+8}\right\}$

$$= L^{-1}\left\{\frac{3P-4}{P^2-4P+4+4}\right\}$$

$$= L^{-1}\left\{\frac{3P-4}{(P^2-2)^2+4}\right\}$$

$$= L^{-1}\left\{\frac{3P-4}{(P^2-2)^2+2^2}\right\}$$

$$= L^{-1}\left\{\frac{3P-6+6-4}{(P^2-2)^2+2^2}\right\}$$

$$= L^{-1}\left\{\frac{3(P-2)+2}{(P-2)^2+2^2}\right\}$$

$$= 3L^{-1}\left\{\frac{P-2}{(P-2)^2+2^2}\right\} + L^{-1}\left\{\frac{2}{(P-2)^2+2}\right\}$$

By first shifting Theorem

$$= 3e^{2t}\cos 2t + e^{2t}\sin 2t$$

$$= e^{3t}[3\cos 2t + \sin 2t]$$

12) Find the inverse Laplace transform of $\left\{\frac{1}{(P+1)(P-2)}\right\}$ by using partial tractions.

Solutions: - $L^{-1}\left[\frac{1}{(P+1)(P-2)}\right]=\frac{A}{P+1}+\frac{B}{P-2}$

$A(P-2)+B(P+1)=1$

$AP-2A+BP+B=1$

Comparing coefficient on the both side $A+B=0$

$$\frac{-2A+B=-1}{3A=-1}$$

$A=-\frac{1}{3}$

$A+B=0$

$B=\frac{1}{3}$

$$\left[\frac{1}{(P+1)(P-2)}\right]=\frac{\frac{1}{3}}{P+1}+\frac{-\frac{1}{3}}{P-2}$$

$$L^{-1}\left[\frac{1}{(P+1)(P-2)}\right]=-\frac{1}{3}L^{-1}\left[\frac{1}{P+1}\right]+\frac{1}{3}L^{-1}\left[\frac{1}{P-2}\right]$$

$$=-\frac{1}{3}\left[e^{t}+\frac{1}{3}e^{2t}\right]$$

$$=-\frac{1}{3}\left[e^{-t}-e^{2t}\right]$$

13) Find the inverse Laplace transform of $\left\{\frac{6}{2P-3}-\frac{3+4P}{9P^2-16}+\frac{8-6P}{16P^2+9}\right\}$

Solution: - $= L^{-1}\left(\frac{6}{2P-3}\right)-L^{-1}\left(\frac{3}{9P^2-16}\right)-L^{-1}\left(\frac{4P}{9P^2-16}\right)+L^{-1}\left(\frac{8}{16P^2+9}\right)-L^{-1}\left(\frac{6P}{16P^2+9}\right)$

$= L^{-1}\left(\frac{6}{2\left(P-\frac{3}{2}\right)}\right)-L^{-1}\left(\frac{3}{9\left(P^2-\left(\frac{4}{3}\right)^2\right)}\right)-L^{-1}\left(\frac{4P}{9\left(P^2-\left(\frac{4}{3}\right)^2\right)}\right)+L^{-1}\left(\frac{8}{16\left(P^2+\left(\frac{3}{4}\right)^2\right)}\right)-\frac{6}{16}L^{-1}\left(\frac{P}{P^2+\frac{9}{16}}\right)$

$= 3L^{-1}\left[\frac{1}{P-\frac{3}{2}}\right]+\frac{1}{3*\frac{4}{3}}L^{-1}\left[\frac{\frac{4}{3}}{P^2-\left(\frac{4}{3}\right)^2}\right]-\frac{4}{9}L^{-1}\left(\frac{P}{\left(P^2-\left(\frac{4}{3}\right)^2\right)}\right)+\frac{1}{2}.\frac{1}{\frac{3}{4}}L^{-1}\left[\frac{\frac{3}{4}}{P^2+\left(\frac{3}{4}\right)^2}\right]-\frac{3}{8}L^{-1}\left(\frac{P}{P^2+\left(\frac{3}{4}\right)^2}\right)$

$= 3e^{3ht}-\frac{1}{4}\sin h\left(\frac{4}{3}\right)t-\frac{4}{9}\cos h\left(\frac{4}{3}\right)t+\frac{2}{3}\sin\left(\frac{3}{4}\right)t-\left[\frac{3}{8}\cos\left(\frac{3}{4}\right)t\right]$

14) State the find shifting theorem of inverse Laplace transforms and find $\mathbf{L^{-1}\left[\frac{P}{(P+1)^{\frac{5}{2}}}\right]}$

Solution:-

Statement: - If $L[f(t)]=f(p)\ then$

(I) $L^{-1}[f(p+a)]=e^{-at}L^{-1}f(p)$

(ii) $L^{-1}[f(p-a)]=e^{at}L^{-1}f(p)$

Proof: - If $L[f(t)]=$
$f(p)\ then\ first\ shifting\ therom\ of\ laplace\ transform\ L[e^{-at}f(t)]=f(p+a)\ and$

(I) $e^{-at}f(t) = L^{-1}[f(p+a)]$

$$L^{-1}[f(p+a)] = e^{-at}f(t)$$

$$= e^{-at}L^{-1}[f(p)]$$

(ii) $e^{-at}f(t) = L^{-1}[f(p-a)]$

$$L^{-1}[f(p-a)] = e^{-at}f(t)$$

$$= e^{-at}L^{-1}[f(p)]$$

Given that: $$\mathrm{L}^{-1}\left[\frac{\mathrm{P}}{(\mathrm{P}+1)\frac{5}{2}}\right] = \mathrm{L}^{-1}\left[\frac{\mathrm{p}+1-1}{(\mathrm{p}+1)\frac{5}{2}}\right]$$

$$= L^{-1}\left[\frac{p+1}{(p+1)\frac{5}{2}} - \frac{1}{(p+1)\frac{5}{2}}\right]$$

$$= L^{-1}\left[\frac{p+1}{(p+1)\frac{5}{2}}\right] - L^{-1}\left[\frac{1}{(p+1)\frac{5}{2}}\right]$$

By using first shifting therom

$$= e^{-t}L^{-1}\left[\frac{p}{p^{\frac{5}{2}}}\right] - e^{-t}L^{-1}\left[\frac{1}{p^{\frac{5}{2}}}\right]$$

$$= e^{-t}L^{-1}\left[\frac{1}{p^{\frac{5}{2}-1}}\right] - e^{-t}L^{-1}\left[\frac{1}{p^{\frac{5}{2}}}\right]$$

$$= e^{-t}L^{-1}\left[\frac{1}{p^{\frac{3}{2}}}\right] - e^{-t}L^{-1}\left[\frac{1}{p^{\frac{5}{2}}}\right]$$

$$= e^{-t}\left[\frac{t^{\frac{1}{2}}}{p^{\frac{3}{2}}}\right] + e^{-t}\left[\frac{t^{\frac{3}{2}}}{p^{\frac{5}{2}}}\right]$$

$$= e^{-t}\left[\frac{t^{\frac{1}{2}}}{p^{\frac{3}{2}}} + \frac{t^{\frac{3}{2}}}{p^{\frac{5}{2}}}\right]$$

15) State and prove second shifting theorem and inverse Laplace transform and also find $\frac{e^{-2p}}{(p-2)^2}$

Solution:-

Statement: - If $L[f(t)] = f(p)\ then\ L^{-1}[e^{-ap}\ f(p)] = G(t), where$

$$G(t) = \begin{cases} f(t-a), t > a \\ 0, t < a \end{cases}$$

Proof: - If $L[f(t)] = f(p)\ then$ second shifting property, we have

$$L[G(t)] = e^{-ap}L[f(t)]$$

Where $G(t) = \begin{cases} f(ta), t > a \\ 0, t < a \end{cases}$ $i.e,\ L[G(t)] = e^{-ap}f(p)$

$$\therefore G(t) = L^{-1}[e^{-ap}f(p)]$$

Now

$$L^{-1}\left[\frac{1}{(p-2)^2}\right] = e^{2t}L^{-1}\left[\frac{1}{p^2}\right] = e^{2t}t.$$

$By\ second\ shifting\ theorem, we\ have$

$$L^{-1}\left[\frac{e^{-2p}}{(p-2)^2}\right] = \begin{cases} e^{2(t-2)} & (t-2, t>2) \\ 0, & t<2 \end{cases}$$

16) Find $L^{-1}\left[\frac{4p+5}{(p-4)^2(p+3)}\right]$ *by using partial fraction*

Solution: - Let $\left[\frac{4p+5}{(p-4)^2(p+3)}\right] = \frac{A}{P-4} + \frac{B}{(P-4)^2} + \frac{C}{P+3}$

$$= \frac{A(p-4)(p+3)+B(p+3)+C(p-4)^2}{(p-4)^2(p+3)}$$

$$4p+5 = A(p-4)(p+3) + B(p+3) + C(p-4)^2 - (1)$$

Put $p = 4\ in\ (2)\ Then\ 21 = 7B$

$$B = 3$$

Put $p = -3\ in\ (2)\ Then\ -7 = C(-7)^2$

$$49C = -7$$

$$C = \frac{-1}{7}$$

$By\ comparing\ of coefficient\ of\ P^2\ in\ (1)\ we\ get$

$$A = \frac{1}{7}$$

$$A = \frac{1}{7}, B = 3, C = \frac{-1}{7}$$

$$L^{-1}\left[\frac{4p+5}{(p-4)^2(p+3)}\right]$$

$$= \frac{1}{7}L^{-1}\left[\frac{1}{p-4}\right] + 3L^{-1}\frac{1}{(P-4)^2}\frac{-1}{7}\frac{1}{P+3}$$

$$= \frac{1}{7}e^{4t} + 3e^{4t}L^{-1}\left[\frac{1}{p^2}\right]\frac{-1}{7}e^{4t} + 3e^{4t}(t)\frac{-1}{7}e^{-3t}$$

APPLICATIONS OF LAPLACE TRANSFORM TO SOLUTIONS OF DIFFERENTIAL EQUATIONS

17. Solve (D+1) y=1, if y=2, when t=0 by the method of Laplace transform.

Solution:

Given differential equation is $y' + y = 1 \rightarrow (1)$

Given conditions are $y = 2$ when $t = 0$

Taking Laplace tansform of both sides of (1), we get

$L\{y'\} + L\{y\} = L\{1\} => [p\,L\{y\} - y(0)] + L\{y\} = \frac{1}{p}$

$L\{y\}[p+1] = \frac{1}{p} + 2$, using given condition

$$L\{y\} = \frac{1}{p(p+1)} + \frac{2}{p+1} = \frac{1}{p} - \frac{1}{p+1} + \frac{2}{p+1} = \frac{1}{p} + \frac{1}{p+1}$$

Taking Inverse Laplace Transform, we get $y = L^{-1}\left[\frac{1}{p}\right] + L^{-1}\left[\frac{1}{p+1}\right] = 1 + e^{-t}$

∴The required solution is $y = 1 + e^{-t}$

18. Solve $(D^2 + 2D + 2)y = 0$ if y=Dy=1, When t=0 by the method of Laplace Transform.

Solution:

Taking the Laplace transform on both sides of the given equation, we have

$L\{y''\} - 2L\{y'\} + 2L\{y\} = 0$

$=> p^2L\{y\} - py(0) - y'(0) - 2[p\ L\{y\} = 0 => L\{y\} = \frac{p-1}{p^2-2p+2} = \frac{p-1}{(p-1)^2+1}$

$y = L^{-1}\{\frac{p-1}{(p-1)^2+1} = e^tL^{-1}\left\{\frac{p}{p^2+1}\right\} = e^t \cos t$

∴The required solution is $y = e^t\ cos\ t$.

19. Solve $(\boldsymbol{D^2 + 4D + 3})\boldsymbol{y} = \boldsymbol{e^{-t}}$, given $\boldsymbol{y = 1}, \boldsymbol{\frac{dy}{dx} = 1}$,when $\boldsymbol{t = 0}$ by the method of Laplace transform.

Solution:

Given equation is $(D^2 + 4D + 3) = e^{-t} \rightarrow (1)$

Applying Laplace transform on both sides, we get

$L\ \{\ D^2y\} + 4L\ \{Dy\} + L\{3y\} = L\{e^{-t}\}$

$p^2L[y] = py(0) - y'(0) + 4[pL[y] - y(0)] + 3L[y] = \frac{1}{p+1}$

$=> (p^2 + 4p + 3)L[y] - p - 5 = \frac{1}{p+1}$

$=> (p+1)(p+3)L[y] = \frac{1}{p+1} + p + 5$

$=> L[y] = \frac{1}{(p+1)^2(p+3)} + \frac{p+5}{(p+3)(p+1)} = \frac{p^2+6p+6}{(p+1)^2(p+3)}$

$$y(t) = L^{-1}\left\{\frac{p^2+6p+6}{(p+1)^2(p+3)}\right\} = L^{-1}\left\{\frac{7}{4}\frac{1}{(p+1)} + \frac{1}{2}\frac{1}{(p+1)^2} - \frac{3}{4}\frac{1}{(p+3)}\right\}$$

$$= \frac{7}{4}e^{-t} - \frac{1}{2}e^{-t}t - \frac{3}{4}e^{-3t}$$

$$\therefore The\ Solution\ is\ y == \frac{7}{4}e^{-t} - \frac{1}{2}e^{-t}t - \frac{3}{4}e^{-3t}$$

20. Solve $y''' + 2y'' - y' - 2y = 0$, given $y(0) = y'(0) = 0$ *and* $y''(0) = 6$ by the method of Laplace transform.

Solution:

Taking the Laplace transform of both sides, we get

$$[L(y) - p^2y(0) - py'(0) - y''(0)] + 2[p^2L(y) - py(0) - y'(0)] - [pL(y) - y(0)] - 2L(y) = 0$$

$$=> (p^3 + 2p^2 - p - 2)L(y) = 6 => L(y) = \frac{6}{p^3 + 2p^2 - p - 2}$$

$$=> L(y) = \frac{6}{(p-1)(p+1)(p+2)} = \frac{1}{p-1} - \frac{3}{p+1} + \frac{2}{p+2}$$

$$\therefore y = L^{-1}\left(\frac{1}{p-1}\right) - 3L^{-1}\left(\frac{1}{p+1}\right) + 2L^{-1}\left(\frac{1}{p+2}\right) = e^t - 3e^{-t} + 2e^{-2t}.$$

$$\therefore The\ solution\ is\ y = e^t - 3e^{-t} + 2e^{-2t}.$$

21. Solve $(D^2 - D - 2)y = 20\sin 2t$, if $y = -1, Dy = 2$, when $t = 0$ by method of Laplace Transform.

Solution:

Taking the Laplace Transform of both sides of the given equation, we have

$$L\{y''\} - L\{y'\} - 2L\{y\} = 20L\{\sin 2t\}$$

$$\Rightarrow p^2L\{y\} - py(0) - y'(0) - [pL\{y\} - y(0)] - 2L\{y\} = 20\frac{2}{p^2+4}$$

$$\Rightarrow (p^2 - p - 2)L\{y\} + p - 2 - 1 = \frac{40}{p^2+4} \Rightarrow (p-2)(p+1)L\{y\}$$

$$= \frac{40}{p^2+4} + (-p+3)$$

$$\Rightarrow L\{y\} = \frac{40}{(p-2)(p+1)(p^2+4)} + \frac{-p+3}{(p-2)(p+1)}$$

$$= \frac{5}{3(p-2)} - \frac{8}{3(p+1)} + \frac{p-6}{p^2+4} + \frac{1}{3(p-2)} - \frac{4}{3(p+1)}$$

$$= \frac{2}{p-2} - \frac{4}{p+1} + \frac{p}{p^2+4} - \frac{6}{p^2+4}$$

$$\therefore y = 2L^{-1}\left\{\frac{1}{p-2}\right\} - 4L^{-1}\left\{\frac{1}{p+1}\right\} + L^{-1}\left\{\frac{p}{p^2+4}\right\} - 6L^{-1}\left\{\frac{1}{p^2+4}\right\}$$

$$= 2e^{2t} - 4e^{-t} + \cos 2t - 3\sin 2t$$

$$\therefore \textit{The required solution is } y = 2e^{2t} - 4e^{-t} + \cos 2t - 3\sin 2t.$$

22.Solve$(D^2+D)y=t^2+2t$, where$y(0)=4, y'(0)=-2$, by method of Laplace Transform.

Solution:

Taking the Laplace Transform of both sides of the given equation, we have

$$L\{y''\}+L\{y'\}=L\{t^2\}+2L\{t\}$$

$$\Rightarrow p^2L\{y\}-py(0)-y'(0)+pL\{y\}-y(0)=\frac{2!}{p^3}+\frac{2}{p^2}$$

$$\Rightarrow (p^2+p)L\{y\}-4p+2-4=\frac{2}{p^3}+\frac{2}{p^2}$$

$$\Rightarrow p(p+1)L\{y\}=4p+2+\frac{2!}{p^3}+\frac{2}{p^2}$$

$$\Rightarrow L\{y\}=\frac{4p^4+2p^3+2p+2}{p^4(p+1)}=\frac{2}{p^4}+\frac{2}{p}+\frac{2}{p+1}$$

$$\therefore y=2L^{-1}\left\{\frac{1}{p^4}\right\}+2L^{-1}\left\{\frac{1}{p}\right\}+2L^{-1}\left\{\frac{1}{p+1}\right\}=\frac{1}{3}t^3+2+2e^{-t}$$

$$\therefore \textit{The required solution is } y=\frac{1}{3}t^3+2+2e^{-t}.$$

23. Solve $(D^2+2D+5)y=e^{-t}\sin t$,where $y(0)=0, y'(0)=-1$,by method of Laplace Transform.

Solution:

Given equation is $\frac{d^2y}{dt^2}+2\frac{dy}{dx}+5y=e^{-t}\sin t \Rightarrow y''+2y'+5y=e^{-t}\sin t \rightarrow (1)$

Taking Laplace transform on both sides of(1),We get

$$p^2L(y) - py(0) - y'(0) + 2[pL(y) - y(0)] + 5L(y) = \frac{1}{(p+1)^2+1}$$

$$\Rightarrow (p^2 + 2p + 5)L(y) = \frac{1}{p^2 + 2p + 2_1} = \frac{-(p^2 + 2p + 1)}{(p^2 + 2p + 2)}$$

$$\therefore L(y) = \frac{-(p^2 + 2p + 1)}{(p^2 + 2p + 2)(p^2 + 2p + 5)} = -\frac{4}{3}\frac{1}{p^2 + 2p + 5} + \frac{1}{3}\frac{1}{p^2 + 2p + 2}$$

∴Taking the inverse transform, we get

$$y = -\frac{4}{3}L^{-1}\left[\frac{1}{(p+1)^2 + 2^2}\right] + \frac{1}{3}L^{-1}\left[\frac{1}{(p+1)^2 + 1^2}\right]$$

$$= -\frac{4}{3}e^{-t}\sin 2t + \frac{1}{3}e^{-t}\sin t$$

$$= \frac{e^{-t}}{3}[\sin t - 4\sin 2t].$$

24. Solve $(D^2 + 1)y = \sin t \ \sin 2t\ ,\ t > 0$ if $y = 1, y' = 0$, when $t = 0$ by method of Laplace Transform.

Solution:

Given differential equation $y'' + y = \frac{1}{2}(\cos t - \cos 3t)$

$$\therefore L\{y''\} + L\{y\} = \frac{1}{2}L\{\cos t\} - \frac{1}{2}L\{\cos 3t\}$$

$$\Rightarrow p^2L(y) - py(0) - y'(0) + L\{y\} = \frac{p}{2(p^2+1)} - \frac{p}{2(p^2+9)}$$

$$=> (p^2+1)L\{y\} - p = \frac{p}{2(p^2+1)} - \frac{p}{2(p^2+9)}$$

$$\therefore L\{y\} = \frac{p}{(p^2+1)} + \frac{p}{2(p^2+1)^2} - \frac{p}{2(p^2+1)(p^2+9)}$$

$$= \frac{p}{(p^2+1)} - \frac{1}{2}\left(\frac{d}{dp}\frac{1}{(p^2+1)}\right) - \frac{p}{16(p^2+1)} + \frac{p}{16(p^2+9)}$$

$$=> y = L^{-1}\left\{\frac{p}{(p^2+1)}\right\} - \frac{1}{2}L^{-1}\left\{\frac{d}{dp}\frac{1}{(p^2+1)}\right\} - \frac{1}{16}L^{-1}\left\{\frac{p}{(p^2+1)}\right\} + \frac{1}{16}L^{-1}\frac{p}{(p^2+9)}$$

$$= \cos t + \frac{t}{4}L^{-1}\left\{\frac{1}{(p^2+1)}\right\} - \frac{1}{16}\cos t + \frac{1}{16}\cos 3t = \frac{15}{16}\cos t + \frac{t}{4}\sin t + \frac{1}{16}\cos 3t$$

$$\therefore \textit{The required equation is } y = \frac{15}{16}\cos t + \frac{t}{4}\sin t + \frac{1}{16}\cos 3t$$

FOURIER TRANSFORMS

25. Define (1) Fourier sine transformation (2) Fourier cosine transformation

Solution:

1.Fourier sine Transform :

Definition: The infinite Fourier sine Transform of f(x) in $(0, \infty)$ is defined as $\frac{\sqrt{2}}{\pi}\int_0^\infty f(x)sinpx\ dx$ and is denoted by F [f(x)] or $\bar{f}$(p), The function f(x) is called the inverse Fourier sine transform of $\bar{f}$(p)

i.e f(x) = $f^{-1}\left[\bar{f}(p)\right]$.

2.Fourier cosine Transform :

Definition: The infinite Fourier cosine Transform of f(x) cospx dx and is defined as $\frac{\sqrt{2}}{\pi}\int_0^\infty f(x)cospx\ dx$ and is denoted by F_c [f(x)] or $\overline{f}_c$(p). The function f(x) is called the inverse Fourier cosine Transform of $\overline{f}_c$(p)

i.e f(x) = $f^{-1}\{\ \overline{f}_c(p)\}$.

26. State and prove Linear Property of Fourier transform

Statement: If $\bar{f}$(p) and $\bar{g}$(p) are fourier transform of "f(x)" and "g(x)" respectively then

F[af(x)+bg(x)] = a$\bar{f}$(p)+b$\bar{g}$(p).

Proof:

F [f(x)] = $\bar{f}$(p) = $\frac{1}{\sqrt{2\pi}}$ $\int_{-\infty}^{+\infty} e^{ipx}f(x)dx$ …….. 1

F [g(x)] = $\bar{g}$(p) = $\frac{1}{\sqrt{2\pi}}$ $\int_{-\infty}^{+\infty} e^{ipx} g(x)dx$ 2

F [a f(x)+bg(x)] = $\frac{1}{\sqrt{2\pi}}$ $\int_{-\infty}^{+\infty} e^{ipx}(af(x) + bg(x))dx$

F [a f(x)+bg(x)] =$\frac{1}{\sqrt{2\pi}}$ $\int_{-\infty}^{+\infty} e^{ipx} af(x)dx + \frac{1}{\sqrt{2\pi}}$ $\int_{-\infty}^{+\infty} e^{ipx} bg(x)dx$

F [a f(x)+bg(x)] = a $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{ipx} f(x)dx$ + b $\frac{1}{\sqrt{2\pi}}$ $\int_{-\infty}^{+\infty} e^{ipx} g(x)dx$

F [a f(x)+bg(x)] = a$\bar{f}$(p)+b$\bar{g}$(p).

27. Find the cosine transform of the function of f(x) , if f(x) = cosx, when 0 < x < a ,If f(x) = 0 , when x>a .

Solution:

we know that, by the definition of fourier cosine transform

F_c [f(x)] = $\bar{f}_c$(p)= $\sqrt{2/\pi}$ $\int_0^{\infty} f(x)cospxdx$ 1

F_c [f(x)] = $\bar{f}_c$(p)= $\sqrt{2/\pi} \int_0^a cosx\ cospxdx + \sqrt{2/\pi}$ $\int_0^{\infty} 0\ cospxdx$

F_c [f(x)] = $\bar{f}_c$(p)=$\sqrt{2/\pi} \int_0^a cosx\ cospxdx$+ 0

F_c [f(x)] = $\bar{f}_c$(p)= $\sqrt{\frac{2}{\pi}}\ \frac{1}{2} \int_0^a 2cosx\ cospxdx$

F_c [f(x)] = $\bar{f}_c$(p)= $\frac{1}{\sqrt{2\pi}} \left[\int_0^a [\cos(1+p) + \cos(1-p)]dx\right]$

F_c [f(x)] = $\bar{f}_c$(p)=$\frac{1}{\sqrt{2\pi}}$ [$\int_0^a \cos(1+p)\,dx + \int_0^a \cos(1-p)]dx$]

F_c [f(x)] = $\bar{f}_c$(p)=$\frac{1}{\sqrt{2\pi}} \left[\left[\frac{sin(1+p)x}{(1+p)}\right]\right]_0^a + \left[\frac{\sin(1-p)x}{(1+p)}\right]_0^a$

F_c [f(x)] = $\overline{f_c}$(p)= $\frac{1}{\sqrt{2\pi}}$ [$\frac{\sin(1+p)a}{(1+p)}$ - $\frac{\sin(1+p)(0)}{(1+p)}$] + $\frac{\sin(1-p)a}{(1-p)}$ - $\frac{\sin(1-p)(0)}{(1-p)}$]

F_c [f(x)] = $\overline{f_c}$(p)= $\frac{1}{\sqrt{2\pi}}$ [$\frac{\sin(1+p)a}{(1+p)} - 0 + \frac{\sin(1-p)a}{(1-p)}$]

F_c [f(x)] = $\overline{f_c}$(p)= $\frac{1}{\sqrt{2\pi}}$ [$\frac{(1-p)\sin(1+p)a+(1+p)\sin(1-p)\sin(1-p)a}{1-p2}$]

28.Find Fourier sine and cosine transform of f(x) = e^{-x}.

Solution:

Given transform f(x) = e^{-x} ……(1)

We know that

F_c [f(x)] = $\overline{f_c}$(p)= $\sqrt{\frac{2}{\pi}} \int_0^\infty f(x) sinpxdx$

F_s [f(x)] = $\overline{f_s}$(p)= $\sqrt{\frac{2}{\pi}} \int_0^\infty e^{-x} sinpxdx$

F_s [f(x)] = $\overline{f_s}$(p)= $\sqrt{\frac{2}{\pi}} \left[\frac{e^{-x}}{(-1)^2+p^2}\{sinpx - pcospx\}\right]_0^\infty$

F_s [f(x)] = $\overline{f_s}$(p)= $\sqrt{\frac{2}{\pi}} \left[\frac{e^{-x}}{1+p^2}\{sinpx + pcospx\}\right]_0^\infty$

F_s [f(x)] = $\overline{f_s}$(p)= -$\sqrt{\frac{2}{\pi}} \frac{1}{(1+p^2)}$ $[e^{-x}\{sinpx + pcospx\}]_0^\infty$

F_s [f(x)] = $\overline{f_s}$(p)= -$\sqrt{\frac{2}{\pi}} \frac{1}{(1+p^2)} \left[\frac{1}{e^x} sinpx + \frac{1}{e^x} cospx\right]_0^\infty$

F_s [f(x)] = $\overline{f_s}$(p)= -$\sqrt{\frac{2}{\pi}} \frac{1}{(1+p^2)} \left[\frac{1}{e^\infty} sinp(\infty)\right] + \sqrt{\frac{2}{\pi}} \frac{1}{(1+p^2)}$ $\left[\frac{1}{e^0} pcosp(0)\right]$

F_s [f(x)] = $\bar{f}_s(p)$= 0-$\sqrt{\frac{2}{\pi}}\frac{1}{(1+p^2)}$ p(1)

F_s [f(x)] = $\bar{f}_s(p)$=$\sqrt{\frac{2}{\pi}}\frac{1}{(1+p^2)}$

29.Find the fourier transform of f(x) defined by f(x)=1, |x| < a and f(x)=0, |x| < a and hence evaluate 1. $\int_{-\infty}^{+\infty} \frac{\mathbf{sinpx\ cospx}}{\mathbf{p}} \boldsymbol{dp}$

2. $\int_{-\infty}^{+\infty} \frac{\mathbf{sinpx}}{\mathbf{p}} \boldsymbol{dp}$

Solution:

Given function $f(x) = \begin{cases} 1, & |\mathbf{x}| < a \\ 0, & |\mathbf{x}| > a \end{cases}$

By the definition of Fourier transform , we have

F[$f(x)$] = $\bar{f}$(p) = $\frac{1}{\sqrt{2\pi}}$ $\int_{-\infty}^{+\infty} e^{ipx} f(x)dx$

F[$f(x)$] = $\bar{f}$(p) = $\frac{1}{\sqrt{2\pi}}$ $\int_{-\infty}^{+\infty} e^{ipx} 1dx$

F[$f(x)$] = $\bar{f}$(p) = $\frac{1}{\sqrt{2\pi}} \left[\frac{e^{ipx}}{ip}\right]_{-a}^{+a}$

F[$f(x)$] = $\bar{f}$(p) = $\frac{1}{\sqrt{2\pi}} \frac{e^{ipa} - e^{-ipa}}{ip}$

F[$f(x)$] = $\bar{f}$(p) = $\frac{1}{\sqrt{2\pi}} \frac{1}{ip} [2ipsinpa] \left[\frac{e^{ix} - e^{-ix}}{2} = isinx\right]$

F[$f(x)$] = $\bar{f}$(p) = $\frac{1}{p\sqrt{2\pi}} [2sinpa]$

F(x) = $\overline{f(p)}$ = $\frac{2sinpa}{p\sqrt{2\pi}}$ i. $p \neq 0$

1) we know that , if $\bar{f}(p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{ipx} f(x)dx$

Then, $f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \bar{f}(p)\, e^{ipx} dp$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{2sinpa}{p\sqrt{2\pi}} e^{-ipx} dp = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$

$$f(x) = \frac{1}{\sqrt{2\pi}}\ 2\int_{-\infty}^{+\infty} \frac{sinpa}{p} e^{-ipx} dp = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$

$$f(x) = \frac{2}{2\pi}\ 2\int_{-\infty}^{+\infty} \frac{sinpa}{p} e^{-ipx} dp = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$

$$f(x) = \frac{1}{\pi}\ 2\int_{-\infty}^{+\infty} \frac{sinpa}{p} e^{-ipx} dp = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$

Now,

$\frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{sinpa}{p} cospxdp$ - $\frac{i}{\pi} \int_{-\infty}^{+\infty} \frac{sinpa}{p} sinpxdp = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{sinpacospx}{p} dp$ – i $\int_{-\infty}^{+\infty} \frac{sinpasinpx}{p} dp = \begin{cases} \pi, & |x| < a \\ 0, & |x| > a \end{cases}$

2) If x=0, a=1 is

We get $\int_{-\infty}^{+\infty} \frac{sinp}{p} dp = \pi$

$$2\int_{-\infty}^{+\infty} \frac{sinp}{p} dp = \pi$$

$$\int_{-\infty}^{+\infty} \frac{sinp}{p} dp = \frac{\pi}{2}$$

30. Find Fourier sine and cosine transform of f(x) = x.

Solution:

Given function f(x) = x …… 1

1.To find fourier sine transform of f(x)

F_s [f(x)] = $\overline{f_s}(p) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) sinpx dx$

F_s [f(x)] = $\overline{f_s}(p) = \sqrt{\frac{2}{\pi}} \int_0^\infty x\, sinpx dx$

By using integral by parts

F_s [f(x)] = $\overline{f_s}(p) = \sqrt{\frac{2}{\pi}} \left[x \int sinpx\, dx\right]_0^\infty -$

$\left[\left[\int \frac{d}{dx}(x) \int sinpx\, dx\right] dx\right]_0^\infty$

F_s [f(x)] = $\overline{f_s}(p) = \sqrt{\frac{2}{\pi}} \left[-x\frac{cospx}{p}\right]_0^\infty - \left[\int \frac{dx}{dx} - \frac{cospx}{p}\right]_0^\infty dx\Big]$

F_s [f(x)] = $\overline{f_s}(p) = \sqrt{\frac{2}{\pi}} \left[\frac{-xcospx}{p}\right]_0^\infty - \int \frac{cospx}{p}\Big]_0^\infty dx\Big]$

F_s [f(x)] = $\overline{f_s}(p) = \sqrt{\frac{2}{\pi}} \left[\left[\frac{-xcospx}{p}\right]_0^\infty - \left[\frac{\sin px}{p^2}\right]_0^\infty dx\right]$

F_s [f(x)] = $\overline{f_s}(p) = \sqrt{\frac{2}{\pi}} [0 - 0]$

F_s [f(x)] = 0

31.Find the Fourier sine transform of f(x)=$\frac{e^{-ax}}{x}$ and deduce that

$\int_0^{+\infty} \frac{e^{-ax}-e^{-bx}}{x}$**sin pxdx = tan$^{-1}\frac{p}{a}$ − tan$^{-1}\frac{p}{b}$.**

Solution:

Given that f(x)=$\frac{e^{-ax}}{x}$

By the definition of Fourier sine transform

F_s [f(x)] = $\overline{f}_s(p) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) sinpx dx$

F_s [f(x)] = $\overline{f}_s(p) = \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{e^{-ax}}{x} sinpx dx$ ……..(1)

Differentiating (1) with respect to 'p' on both sides

$\frac{d}{dp}\overline{f}_s(p) = \sqrt{\frac{2}{\pi}} \int_0^\infty \left[\frac{d}{dp} \frac{e^{-ax}}{x} sinpx\right] dx$

$\frac{d}{dp}\overline{f}_s(p) = \sqrt{\frac{2}{\pi}} \int_0^\infty \left[\frac{e^{-ax}}{x} xcopx\right] dx$

$\frac{d}{dp}\overline{f}_s(p) = \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-ax} xcospx\, dx$

$\frac{d}{dp}\overline{f}_s(p) = \sqrt{\frac{2}{\pi}} \left[\frac{e^{-ax}}{a^2+p^2}(-a\, cospx + psinpx)\right]_0^\infty$

$\frac{d}{dp}\overline{f}_s(p) = \sqrt{\frac{2}{\pi}} \left[\frac{e^{-a\infty}}{a^2+p^2}(-a\, cosp(\infty) + psinp(\infty))\right]$ - $\left[\frac{e^{-a(0)}}{a^2+p^2}(-a\, cosp(0) + psinp(0))\right]$

$\frac{d}{dp}\overline{f}_s(p) = \sqrt{\frac{2}{\pi}} \left[0 - \frac{(-a+0)}{a^2+p^2}\right]$

$\frac{d}{dp}\overline{f}_s(p) = \sqrt{\frac{2}{\pi}} \frac{a}{a^2+p^2}$

Integrating on both sides w.r.to 'p'

$\int \left(\frac{d}{dp}\overline{f}_s(p)\right) dp = \sqrt{\frac{2}{\pi}}\, a \int \frac{1}{a^2+p^2}\, dp + c$

$$\overline{f_s}(\mathrm{p}) = \sqrt{\frac{2}{\pi}}\, a \left[\frac{1}{a} \tan^{-1}\left(\frac{p}{a}\right)\right] + c$$

$$\overline{f_s}(\mathrm{p}) = \sqrt{\frac{2}{\pi}} \tan^{-1}\left(\frac{p}{a}\right) + c \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

But when p = 0 $\overline{\mathrm{Fs}}(\mathrm{p}) = 0$ c =0.

$$\overline{f_0}(\mathrm{p}) = \sqrt{\frac{2}{\pi}} \tan^{-1}\left(\frac{p}{a}\right)$$

To find $\int_0^{+\infty} \frac{e^{-ax} - e^{-bx}}{x} \sin px\, dx$

$$\overline{f_s}(\mathrm{p}) = \sqrt{\frac{2}{\pi}} \int_0^{+\infty} \frac{e^{-ax} sinpx}{x}\, dx = \sqrt{\frac{2}{\pi}} \tan^{-1}\left(\frac{p}{a}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Similarly,

$$\overline{f_s}(\mathrm{p}) = \sqrt{\frac{2}{\pi}} \int_0^{+\infty} \frac{e^{-bx} sinpx}{x}\, dx = \sqrt{\frac{2}{\pi}} \tan^{-1}\left(\frac{p}{b}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

From (3) & (4) we get ,

$$\sqrt{\frac{2}{\pi}} \int_0^{+\infty} \frac{e^{-ax} sinpx}{x}\, dx - \sqrt{\frac{2}{\pi}} \int_0^{+\infty} \frac{e^{-bx} sinpx}{x}\, dx = \sqrt{\frac{2}{\pi}} \left[\tan^{-1}\left(\frac{p}{a}\right) - \tan^{-1}\left(\frac{p}{b}\right)\right]$$

$$\sqrt{\frac{2}{\pi}} \left[\int_0^{+\infty} \frac{e^{-ax} - e^{-bx}}{x} \sin px\, dx\right] = \sqrt{\frac{2}{\pi}} \left[\tan^{-1}\left(\frac{p}{a}\right) - \tan^{-1}\left(\frac{p}{b}\right)\right]$$

$$\int_0^{+\infty} \frac{\boldsymbol{e^{-ax} - e^{-bx}}}{\boldsymbol{x}} \sin px\, dx = \tan^{-1}\left(\frac{p}{a}\right) - \tan^{-1}\left(\frac{p}{b}\right).$$

32. Find the Fourier cosine transformation of $\frac{e^{-ax}}{x}$ and deduce that,

$\int_0^{+\infty} \frac{e^{-ax}-e^{-bx}}{x} \cos pxdx = \frac{1}{\sqrt{2\pi}} \log \frac{p^2+b^2}{p^2+a^2}$

Solution: Given that , f(x) = $\frac{e^{-ax}}{x}$

To find Fourier cosine transformation on

$$F_c [f(x)] = \overline{f}_c(p) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \frac{e^{-ax}}{x} cospxdx \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Differentiating equation (1) w.r.to 'p' on both sides

$$\frac{d}{dp}\left(\overline{f}_c(p)\right) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \left[\frac{d}{dp} \frac{e^{-ax}}{x} cospx\right] dx$$

$$\frac{d}{dp}\left(\overline{f}_c(p)\right) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \frac{e^{-ax}}{x} (-x\, sinpx) dx$$

$$\frac{d}{dp}\left(\overline{f}_c(p)\right) = -\sqrt{\frac{2}{\pi}} \int_0^{\infty} e^{-ax} sinpx\, dx$$

$$\frac{d}{dp}\left(\overline{f}_c(p)\right) = -\sqrt{\frac{2}{\pi}} \left[\frac{e^{-ax}}{a^2+p^2} (-asinpx - pcospx)\right]_{x=0}^{x=\infty}$$

$$\frac{d}{dp}\left(\overline{f}_c(p)\right) = -\sqrt{\frac{2}{\pi}} \left[\frac{e^{-a(\infty)}}{a^2+p^2} \left(-asinp(\infty) - pcosp(\infty)\right) - \frac{e^{-a(0)}}{a^2+p^2} \left(-asinp(0) - pcosp(0)\right)\right]$$

$$\frac{d}{dp}\left(\overline{f}_c(p)\right) = -\sqrt{\frac{2}{\pi}} \left[\frac{0-(0-p(1))}{a^2+p^2}\right]$$

$$\frac{d}{dp}\left(\overline{f}_c(p)\right) = -\sqrt{\frac{2}{\pi}} \left[\frac{p}{a^2+p^2}\right]$$

$$\overline{f}_c(p) = -\sqrt{\frac{2}{\pi}} \int \frac{p}{a^2+p^2}\, dp + c$$

$\overline{f}_c(\mathrm{p}) = -\sqrt{\frac{2}{\pi}}\ \frac{1}{2}\ \int \frac{2p}{a^2+p^2}\ dp + c$

$\overline{f}_c(\mathrm{p}) = -\frac{1}{\sqrt{2\pi}}\ log(a^2 + p^2) + \mathrm{c}$

But where p = 0 , $\overline{f}_c(\mathrm{p}) = 0$, c = 0

$\overline{f}_c(\mathrm{p}) = -\frac{1}{\sqrt{2\pi}}\ log(a^2 + p^2)$…….(2)

$\overline{f}_c(\mathrm{p}) = \sqrt{\frac{2}{\pi}}\ \int_0^\infty \frac{e^{-ax}}{x}\ cospx\ dx = -\frac{1}{\sqrt{2\pi}}\ log(a^2 + p^2)$……..(3)

Similarly,

$\overline{f}_c(\mathrm{p}) = \sqrt{\frac{2}{\pi}}\ \int_0^\infty \frac{e^{-bx}}{x}\ cospx\ dx = -\frac{1}{\sqrt{2\pi}}\ log(b^2 + p^2)$…….(4)

From (3) & (4), $[3-4]$, we have

$\sqrt{\frac{2}{\pi}}\ \int_0^\infty \frac{e^{-ax}}{x}\ cospx\ dx - \sqrt{\frac{2}{\pi}}\ \int_0^\infty \frac{e^{-bx}}{x}\ cospx\ dx = -\frac{1}{\sqrt{2\pi}}\ log(a^2 + p^2) + \frac{1}{\sqrt{2\pi}}\ log(b^2 + p^2)$

$\sqrt{\frac{2}{\pi}}\ \left[\ \int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} cospx\ dx\right] = \frac{1}{\sqrt{2\pi}}\{log(b^2 + p^2) - log(a^2 + p^2)\}$

$\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} cospx\ dx = \sqrt{\frac{\pi}{2}}\ \frac{1}{\sqrt{2\pi}} \log \frac{b^2+p^2}{a^2+p^2}$

$\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} cospx\ dx = \frac{1}{2} \log \frac{b^2+p^2}{a^2+p^2}$

Finite Fourier Transforms

33. State and prove inversion Formula for Sine transformation.

Sol:

Theorem: If $\bar{f}_s(p)$ is the finite Fourier sine transform of f(x) over the interval (0,l) then the inversion formula for sine transform is given by f(x)=2/l $\sum_{p=1}^{\infty} \bar{f}s$Sin p$\pi x$/l i.e, f(x)=2/$\pi$ $\sum_{p=1}^{\infty} \bar{f}s(p)$ if (0,π) is the interval considered for $\bar{f}_s(p)$.

Proof: Defining f(x) in the interval (-l,0) such that f(x) is on odd function of x in (-l, l), by Fourier series ,we have

f(x)=$\sum_{p=1}^{\infty} bp$ Sin pπx/l where bp=2/l $\int_0^l f(x)$ Sin pπx/l dx=2/l $\bar{f}_s(p)$ where $\bar{f}s(p)$ is the finite Fourier Sine transform of f(x).

Hence f(x)=2/l $\sum_{p=1}^{\infty} \bar{f}_s(p)$ Sin pπx/l

f(x)= 2/l $\sum_{p=1}^{\infty} \bar{f}_s(p)$Sinpx

Where f(x) is an odd function of x in the interval (-π, π)

Such that $\bar{f}_s(p)$= $\int_0^{\pi} f(x)$ Sin px dx.

34. Find the finite Fourier and Cosine transform of f(x)=1.

Sol: We have $\bar{f}s(p)$=$\int_0^{\pi} f$(x) Sinpx dx=$\int_0^{\pi}$ 1 Sin px dx

=1-(-1)/p

And $\bar{f}c(p)$= $\int_0^{\pi} f(x)$cos px dx =$\int_0^{\pi}$ 1cos px dx=0 ifp=1,2,3⋯

And if p=0,then $\bar{f}c(p)$=$\int_0^{\pi}$ 1.1 dx=π.

35.Find the finite Cosine transform of $\left[1-\frac{x}{\pi}\right]$2,where 0<x<π.

sol: We have ,

$\bar{f}_s(p)=\int_0^{\pi}(1-\frac{x}{\pi})^2$ Cos px dx[

$=\frac{2}{\pi p}$ [-(1-$\frac{2}{\pi}$) $\frac{\cos px}{p}$]$_0^{\pi}$-$\frac{2}{p\pi}\frac{1}{p\pi}\int_0^{\pi}\cos px\,dx$

$=\frac{2}{\pi p\,2}$ -$\frac{2}{p2\pi2}$[$\frac{\sin px}{p}$]$_0^{\pi}$ =$\frac{2}{\pi p2}$ if p>0Type equation here.

And if p=0, then $f_c(p)=\int_0^{\pi}[1-\frac{x}{\pi}]^2dx=[-\frac{\pi}{3}(1-\frac{x}{\pi})^3]_0^{\pi}=\frac{\pi}{3}$

36. FIND THE SINE TRANSFORM OF F(X) IF $F(X)=X^3$ IN[O,π]

Sol: We have

$\bar{f}_s(p)=\int_0^{\pi}x^3\sin px\,dx = [-x3\frac{1}{p}cospx]_0^{\pi}+\frac{3}{p}\int_0^{\pi}xcospxdx$

=-$\frac{\pi3}{p}$ cospπ+ $\frac{3}{p}$[$\left(\frac{x2}{p}\ sinpx\right)_0^{\pi}$ -$\frac{2}{p}\int_0^{\pi}x\sin px\,dx$]

=-$\frac{\pi3}{p}$ cospπ-$\frac{6}{p2}$[$\frac{x}{p}$ cos px+ $\frac{sinpx}{p2}$]$_0^{\pi}$

=π$\left(\frac{6}{p3}-\frac{\pi2}{p}\right)$ cospπ

=π(-1)^p [$\frac{6}{p3}-\frac{\pi2}{p}$]

37.Find the finite Fourier sine and cosine transforms of f(x) if F(x)=1,0<x<$\frac{\pi}{2}$;f(x)=-1,$\frac{\pi}{2}$<x<π.

Sol: We have $\bar{f}_s(p)=\int_0^{\frac{\pi}{2}} 1 \operatorname{Sin} px\, dx+\int_{\frac{\pi}{2}}^{\pi}(-1)\operatorname{Sin} px dx$

$=-\frac{1}{p}\left(\cos\frac{p\pi}{2}-1\right)+\frac{1}{p}\left(cosp\pi-\frac{cosp\pi}{2}\right)$

$=-\frac{1}{p}\left(2\,cos\frac{p\pi}{2}-cosp\pi-1\right)$,p>0.But if( )

$\bar{F}_S(p)=\int_0^{\pi} f(x)\,0\,dx=0$

and $\bar{f}_c(p)=\int_0^{\pi} f(x)cospx\,dx=\int_0^{\frac{\pi}{2}} 1\,cospx\,dx+\int_{\frac{\pi}{2}}^{\pi}-1\cos pxdx$

$=\left(\frac{1}{p}\,sinpx\right)\pi\ \pi/2-\left(\frac{1}{p}\,Sin\,px\right)\pi\ \frac{\pi}{2}$

$=\frac{1}{p}\,Sin\frac{p\pi}{2}+\frac{1}{p}Sin\frac{p\pi}{2}$

$=\frac{2}{p}\sin\frac{p\pi}{2},p>0$

But if p=0,then $\bar{f}_c(p)=\int_0^{\pi} f(x)1dx$

$=\int_0^{\frac{\pi}{2}} 1\,dx+\int_{\frac{\pi}{2}}^{\pi}-1\,dx$

$=(x)_0\frac{\pi}{2}-(x)\pi\,\frac{\pi}{2}=0$

38.Find the finite Fourier Sine and cosine transforms of the function f(x)=2x; 0<x<4.

Sol: We have $\bar{f}_s(p)=\int_0^l f(x)\sin\frac{p\pi x}{l}dx$

$=\int_0^4 2x\sin\frac{p\pi x}{4}dx$. As

$l=4$ (given)$\left(-\frac{2x\frac{cosp\pi x}{4}}{p\pi/4}\right)0^4+2\left[\int_0^4\cos\frac{\frac{p\pi x}{4}}{p\pi/4}\,dx\right]$

$=-\frac{32}{\pi}\cos p\pi+\frac{8}{p\pi}\left(\sin\frac{\frac{p\pi x}{4}}{p\pi/4}\right)_0^4$

$=\frac{-32}{p\pi}\cos p\pi$

And $\bar{f}_c(p)=\int_0^l f(x)\cos\frac{p\pi x}{l}\,dx=\int_0^4 2x\cos\frac{p\pi x}{4}dx, as\ l=4$

$=\left[\frac{2x\sin\frac{p\pi x}{4}}{p\pi/4}\right]_0^4-2\int_0^4\sin(\frac{\frac{p\pi x}{4}}{\frac{p\pi}{4}})dx$

$=\frac{-8}{p\pi}[-\cos\frac{\frac{p\pi x}{4}}{p\pi/4}]_04$

$=\frac{32}{p2\pi2}(\cos(p\pi,-1),if\ p>0$

And if $p=0$,then $\bar{f}_s(p)=\int_0^4 2x.1\,dx=16$

39.Find the finite Fourier Cosine transform of f(x), if $f(x)=3x^2$

Sol: Given function $f(x)= 3x^2$

The finite Fourier cosine transformation of

$\bar{f}_c(p)=\int_0^{\pi} f(x)cospx\ dx$

$=\int_0^{\pi} 3x^2 \cos px\ dx$

$=3[x2\int \cos x\ dx]\ ^{\pi}_0 -\int_0^{\pi} \frac{d}{dx} x^2 \int cospx\ dx]]$

$=3\left\{\left[\frac{x\ sinpx}{p}\right]\pi 0\right\}-\int_0^{\pi}[2x\frac{\sin px}{p}]dx\}$

$=3[0-0]-\frac{6}{p}\int_0^{\pi} x\ sinpx\ dx$

$=\frac{-6}{p}\left[\frac{-x\ cospx}{p}\right]\pi 0-\int_0^{\pi}\frac{dx}{dx}\int \sin px\ dx$

$=\frac{-6}{p}\left[\left(\frac{-\pi\cos px}{p}\right)+\int_0^{\pi}\frac{cospx}{p}\ dx\right]$

$=\frac{6\pi}{p2}\cos p\pi-\frac{-6}{p}\ [\frac{sinpx}{p}]_0{}^{\pi}$

$=\frac{6\pi}{p2}\ (-1)^2-\frac{6}{p}\ [0-0]$

$=\frac{6\pi}{p2}\ (-1)^p.$

40.Find the finite Fourier Sine transform of f(x)=Coskx

Sol:we have $\bar{f}_s(p) = \int_0^\pi f(x) Sin\ px\ dx$

$\bar{f}_s(P) = \int_0^\pi Cos\ kx\ Sin\ px\ dx = \frac{1}{2} \int_0^\pi [Sin\ (p+k)x + Sin(p-k)x\ ]^{dx}$

$= \frac{1}{2} \left[-\frac{Cos(k+p)}{k+p} - Cos\frac{(p-k)x}{p-k} \right]_0^\pi$

$= \frac{1}{2} \left[-Cos\frac{(k+p)\pi}{k+p} - cos\frac{(p-k)\pi}{(p-k)} + \frac{1}{k+p} + \frac{1}{p-k} \right]$

$= \frac{1}{2} \left[\frac{-(p-k)\cos(k+p)\pi + (k+p)\cos(p-k)\pi}{p2-k2} + \frac{p-k+k+p}{p2-k2} \right]$

$= \frac{1}{2(p2-k2)} [-p\ \{Cos(k+p)\pi + Cos(p-k)\pi\} + k(Cos(k+p)\pi) - Cos(p-k)\pi + 2p)]$

$= \frac{1}{(p2-k2)} [-p\ Cosk\pi\ Cos\ p\pi - k\ Sink\pi\ Sinp\pi + p]$

$= \frac{p}{p2-k2}\ [1 - cosk\pi\ Cos\ p\pi]$

$= \frac{p}{p2-k2}\ [1 - (-1)p]$ Cos kπ].

ADVANCED NUMERICAL ANALYSIS

CURVE FITTING

1.Using the method of least square line fit a straight line to the following data?

x	**1**	**2**	**3**	**4**	**5**
Y	**2**	**4**	**6**	**8**	**10**

Sol):- Given that

xi	1	2	3	4	5
Yi	2	4	6	8	10

xi	xi	xi^2	xiyi
1	2	1	2
2	4	4	8
3	6	9	18
4	8	16	32
5	10	25	50

Here n=5; $\sum xi = 15$, $\sum yi = 30$, $\sum xi^2 = 55$; $\sum xi\, yi = 10$.

the normal equations,

$na + b\sum xi = \sum yi$ _____.(1)

$a\sum xi+b\sum xi^2=\sum xiyi$. .(2)

the equations are,

5a+15b=30. .(3)

15a+55b=110. (4)

solving (3)&(4),

15a+45b=90.

-15a-55b=-110.

-10b=-20 → b=2

From equation (3)

5a+30=30

5a=0

a=0

solving the above equation a=0, b=2

the least square straight line is y =a+bx

Y=a+2x Y=2x

2. By the method of least square line fit a straight line for the following data?

X	**1**	**2**	**3**	**4**	**5**
y	**14**	**27**	**40**	**55**	**68**

Sol):- Given that

xi	1	2	3	4	5
yi	14	27	40	55	68

xi	yi	xi^2	xiyi
1	14	1	14
2	27	4	54
3	40	9	120
4	55	16	220
5	68	25	340
15	204	55	748

Here n=5; $\sum xi=15$; $\sum yi=204$; $\sum xi^2=55$; $\sum xiyi=748$

the normal equations are

$na+b\sum xi=\sum yi$. .1

$a\sum xi+b\sum xi^2=\sum xiyi$. .2

the equations are

5a+15b=204. .3

15a+55b=748. .4

solving 3&4

15a+45b=612

-15a-55b=-748

-10b=-136

b=13.6

From equation 3

5a+13.6(15) = 204

5a+204=204

5a=0

a=0

solving the above equation a=0

b=13.6

the least square line is y= a+bx

Y=0+13.6x

Y=13.6x

3. Fit a stright line y=a+bx, by the method of least squares?

X	**0**	**1**	**2**	**3**
Y	**2**	**5**	**8**	**11**

Sol):- Given that

xi	yi	xi^2	xiyi
0	2	0	0
1	5	1	5
2	8	4	16

3	11	9	33
6	26	14	54

From the table we have n=4, $\sum x_i=6$, $\sum y_i=26$, $\sum x_i^2=14$, $\sum x_iy_i=54$

the normal equations are

$na+b\sum x_i=\sum y_i$. _____.1

$a\sum x_i+b\sum x_i^2=\sum x_iy_i$. _____.2

the equations are

4a+6b=26. _____.3

6a+14b=54. _____.4

solving these equations

we get a=2, b=3

the least square line is y=2+3x

4. using the method of least square fit a straight line to following the data?

x	**1**	**2**	**3**	**4**	**5**
y	**2**	**4**	**6**	**8**	**10**

Sol):- Given that

xi	1	2	3	4	5
Yi	2	4	6	8	10

xi	xi	xi^2	xiyi
1	2	1	2
2	4	4	8
3	6	9	18
4	8	16	32
5	10	25	50
15	30	55	110

Here n=5; $\sum xi$ =15, $\sum yi$=30, $\sum xi^2$=55; $\sum xi\ yi$=10

the normal equations are

$na+b\sum xi=\sum yi$. (1)

$a\sum xi+b\sum xi^2=\sum xiyi$. (2)

the equations are

5a+15b=30. (3)

15a+55b=100. (4)

Solving (3)&(4)

15a+45b=90

-15a-55b=-10

-10b=-20

b=2

From equation (3)

5a+30=30

5a=0

a=0

solving the above equation a=0

b=2

the least square line is y =a+bx

Y=a+2x

Y=2x

5. Fit a polynomial of the second degree of the following data?

x	**0**	**1**	**2**
y	**1**	**6**	**17**

Sol):- Consider the table

xi	Yi	xi^2	xi^3	xi^4	xiyi	xi^2yi
0	1	0	0	0	0	0
1	6	1	1	1	6	6
2	17	4	8	16	34	68

3	24	5	9	17	40	74

from the table we deserve that

$n=3; \sum xi=3; \sum xi^2=5; \sum xi^3=9; \sum xi^4=17; \sum yi=24; \sum xiyi=40; \sum xi^2yi=74$

the normal equations are

$na+b\sum xi+\sum xi^2=\sum yi$. _____.1

$a\sum xi+b\sum xi^2+c\sum xi^3=\sum xiyi$. _____.2

$a\sum xi^2+b\sum xi^3+c\sum xi^4=\sum xi^2yi$. _____.3

the equations are

$3a+3b+5c=24$. _____.4

$3a+5b+9c=40$. _____.5

$5a+9b+17c=74$. _____.6

solving equations 4,5&6

we get $a=1, b=2, c=3$

the required polynomial is $y= a+bx+cx^2$

$Y=1+2x+3x^2$

6. Fit parabola to following data?

x	**1**	**2**	**3**	**4**	**5**
y	**10**	**12**	**8**	**10**	**14**

Sol):- Consider the following the table data

xi	yi	xi^2	xi^3	xi^4	xiyi	xi^2yi
1	10	1	1	1	10	10
2	12	4	8	16	24	48
3	8	9	27	81	24	72
4	10	16	64	256	40	160
5	14	25	125	625	70	350
15	54	55	225	979	168	640

From the table we observe that

n=5, $\sum xi=15$, $\sum xi^2=55$, $\sum xi^3= 225$; $\sum xi^4 =979$; $\sum yi=54$ $\sum xiyi=168$; $\sum xi^2yi=60$

the normal equations are

$na+b\sum xi+c\sum xi^2=\sum yi$. .1

$a\sum xi+b\sum xi^2+c\sum xi^3=\sum xiyi$. .2

$a\sum xi^2+b\sum xi^3+c\sum xi^4 =\sum xi^2yi$. .3

the equations are

$5a+15b+55c=54$. .4

$15a+55b+225c=168$. .5

55a+225b+979c=640._____.6

solving above equations

We get a = 14, b=-3,6875, c=0.7142

the required eqation is

$Y=a+bx+cx^2=14-3.6875x+0.7142x^2$

It is a requires polynomial equation

7. Find a curve y=ae^bx to tha data?

x	**0**	**2**	**4**
y	**5.1**	**10**	**31.4**

Sol):- Consider the table

x	y	Y=log y	X^2	xy
0	5.1	1.6292	0	0
2	10	2.3026	4	4.6052
4	31.4	3.4468	16	13.772
6	46.5	7.3786	20	18.3924

Here n=3; $\sum x=6$; $\sum y=46.5$, $\sum y=7.3786$; $\sum x^2=20$, $\sum xy=18.3924$

the normal equations are

$na+b\sum xi=\sum yi$. _____.1

$a\sum xi+b\sum xi^2=\sum xiyi$. _____.2

the equations are

3a+6b=7.3786. _____.3

6a+20b=18.3924. _____.4

solving 1&2 we get

A=1.5507

B=0.4544

the required curve Is y=A+Bx

Y=1.5507+0.4544x

Log y=1.5507+0.4544x

8. Fit a second degree parabola to the following data?

x	**0**	**1**	**2**	**3**	**4**
y	**1**	**0**	**3**	**10**	**21**

Sol):- consider the following table for the given data.

xi	yi	xi^2	xi^3	xi^4	xi yi	xi^2 yi
0	1	0	0	0	0	0
1	0	1	1	1	0	0
2	3	4	8	16	6	12

3 4	10 21	9 19	27 64	81 256	30 84	90 336
10	35	30	100	354	120	438

From the table we observe that

n=5; $\sum x_i=10$; $\sum x_i^2=30$; $\sum x_i^3=100$; $\sum x_i^4=354$; $\sum y_i=35$; $\sum x_iy_i=120$; $\sum x_i^2y_i=438$

the normal equations are

$na+b\sum x_i+c\sum x_i^2=\sum y_i$. _____.1

$a\sum x_i+b\sum x_i^2+c\sum x_i^3=\sum x_iy_i$. _____.2

$a\sum x_i^2+b\sum x_i^3+c\sum x_i^4=\sum x_i^2y_i$. _____.3

the equations are

5a+10b+30c=35. _____.4

10a+30b+100c=120. _____.5

30a+100b+354c=438. _____.6

solving the above equations 4,5&6

we get

a=1, b=-3, c=2

the required equation is

$Y= a+bx+cx^2$

$Y=1-3x+2x^2$

NUMERICAL DIFFERENTIATION

9.Using the following table, compute $\frac{dy}{dx}$ at x=1.

x	**1**	**2**	**3**	**4**	**5**	**6**
y	**1**	**8**	**27**	**64**	**125**	**216**

Sol) Consider the forward interpolation formula table

x	Y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1	1	7	12	6	0	0
2	8	19	18	6	0	
3	27	37	24	6		
4	64	61	30			
5	125	91				
6	216					

By Applying Newtons forward interpolation formula $\frac{dy}{dx} = \frac{1}{h}[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0]$

$= \frac{1}{1}[7 - \frac{1}{2}\,12^{6} + \frac{1}{3}\,6^{2} - 1/4\,(0)]$

$= 1\,[7 - 6 + 2 - 0]$

$(\frac{d}{dx})_{x=1} = [3]$

$(d^2y/dx^2)_{x=1} = \frac{1}{h}[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0]$

$= 1/1^2\,[12 - 6 + \frac{11}{12}(0)]$

$= 1[12 - 6] = 6$

10.Find the order derivative of $\sqrt{x}$ at x=15 from the following table.

Sol) The difference table is as follows.

X	15	17	19	21	23	25
f(x)	3.8730	4.1231	4.3589	4.5826	4.7958	5.0000

x	Y =$\sqrt{x}$	Δ^1y	Δ^2y	Δ^3y	Δ^4y	Δ^5y
15	3.8730	0.2501	-0.0143	0.0022	-0.0006	-0.0205
17	4.1231	0.2350	-0.0121	0.0016	-0..211	
19	4.3589	0.2237	-0.0108	-0.0195		
21	4.5826	0.2132	-0.009			
23	4.7958	0.2042				
25	5.0000					

Here x_0 =15,

h=2,

Δy_0 =0.2501,

$\Delta^2 y_0$ = -0.0143

$\Delta^3 y_0$ =0.0022,

$\Delta^4 y_0$ = -0.0006,

$\Delta^5 y_0$ = -0.0143

$$\left(\frac{d}{dx}\right)x\text{-}x_0 = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 + \frac{1}{4}\Delta^4 y_0 + \ldots\right]$$

$$= \frac{1}{2}\left[0.2501 - \frac{1}{2}(-0.0143) + \frac{1}{3}(0.0022) + \frac{1}{4}(-0.0006)\right]$$

$$= \frac{1}{2}[0.2501+0.0072+0.0007-0.0002]$$

$= \frac{1}{2}[0.2578]$

$= 0.1289$

11. **Find $f^{1}(1.5)$from the following table**

x	**1.5**	**2.0**	**2.5**	**3.0**	**3.5**	**4.0**
f(x)	**3.375**	**7.000**	**13.025**	**24.000**	**38.875**	**59.000**

Sol) Consier the following table

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1.5	3.375	3.625	3.000	0.75	0
2.0	7.000	6.625	3.75	0.75	0
2.5	13.625	10.375	4.5	0.75	
3.0	24.000	14.875	5.25		
3.5	38.875	20.125			
4.0	59.000				

Here, $x_0 = 1.5$, $h = 0.5$

$f^{1}(1.5) = \frac{1}{h}[\Delta f(x) - \frac{1}{2}\Delta^2 f(x) + \frac{1}{3}\Delta^3 f(x) - \frac{1}{4}\Delta^4 f(x)]$

$= \frac{1}{0.5}[3.625 - \frac{1}{2}(3.000) + \frac{1}{3}(0.75)]$

$=2[3.625-1.5+0.25]$

$=4.75$

$f^{11\,(1.5)} = (\frac{1}{0.5})^2 [\Delta^2 f(x) - \Delta^3 f(x) + \frac{11}{12}\Delta^4 f(x)]$

= 4[3.000-0.75]

= 4[2.25]

= 9

12. Find $f^{1}(1)$ for $f(x)= 1/1+x^2$ using the following data.

x	**1.0**	**1.1**	**1.2**	**1.3**	**1.4**
f(x)	**0.5000**	**0.4524**	**0.4098**	**0.3717**	**0.3378**

Sol) consider the following table

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1.0	0.5000	-0.0476	0.0050	-0.0005	0.0002
1.1	0.4524	-0.0426	0.0045	-0.0003	
1.2	0.4098	-0.0381	0.0042		
1.3	0.3717	-0.0339			
1.4	0.3378				

Here, $x_0 = 1.0$, $h=0.1$

$$\left(\frac{dy}{dx}\right)x\text{-}x_0 = \frac{1}{h}[\Delta f(x) - \frac{1}{2}\Delta^2 f(x) + \frac{1}{3}\Delta^3 f(x) - \frac{1}{4}\Delta^4 f(x)]$$

$$f^{1}(1) = \frac{1}{0.1}[-0.0476 - \frac{1}{2}(0.0050) + \frac{1}{3}(-0.0005) - \frac{1}{4}(0.0002)]$$

= 10[-0.05031]

= -0.5031

13) Find ($\frac{dy}{dx}$)x = 1.76 from the following table

x	1.72	1.73	1.74	1.75	1.76
y	0.17907	0.17728	0.17552	0.17377	0.17204

Sol) Consider the following table

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1.72	0.17907	-0.00179	0.00003		
1.73	0.17728	-0.00176	0.00001	-0.00002	0.00003
1.74	0.17552	-0.00175	0.00002	0.00001	
1.75	0.17377	-0.00173			
1.76	0.17204				

($\frac{dy}{dx}$)x= x_n =

$\frac{1}{h}[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n]$

$= \frac{1}{0.01}[-0.00173 + \frac{1}{2}(0.0002) + \frac{1}{3}(0.00001) + \frac{1}{4}(0.00003)]$

$= \frac{1}{0.01}[-0.00173 + 0.00001 + 0.00003 + 0.000075]$

$= \frac{1}{0.01}[-0.0016825]$

$= -0.16825$

14) From the following table,find the value of $\frac{dy}{dx}$ at x = 2.03

x	**1.96**	**1.98**	**2.00**	**2.02**	**2.04**
y	**0.7825**	**0.7739**	**0.7657**	**0.7563**	**0.7563**

Sol) Consider the following table.

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1.96	0.7825	-0.0086	-0.0002	0.0002	-0.0004
1.98	0.7739	-0.0088	0	-0.0002	
2.00	0.7657	-0.088	-0.0002		
2.02	0.7563	-0.009			
2.04	0.7563				

Here, $x_n = 2.04$, h = 0.02, $u = x-x_n/h$

$U=\frac{2.03-2.04}{0.02} =0.5$

By applying Newtons interpolation formula, we have

$$y = y_n+u\nabla y_n+\frac{u(u+1)}{2!}\nabla^2 y_n+\frac{u(u+1)(u+2)}{3!}\nabla^3 y_n$$

$$+\ldots\ldots\ldots(1)$$

∴ Differentiating (1)w.r.to "r" we get,

$$\left(\frac{dy}{dx}\right)x = 2.03 = \frac{1}{0.02}[-0.0086+\frac{2(-0.5)+1}{2}(-0.0002)$$

$$+\frac{3(1)+6(-0.5)+2}{2}(-0.0002)]$$

$$=\frac{1}{0.02}[-0.0086+0-0.0000]$$

$$=\frac{1}{0.02}[-0.0086]$$

= -0.43

$(d^2y/dx^2)_{x=2.03} = \frac{1}{h*h}[\nabla^2 y_n + \frac{6u+6}{6} \nabla^3 y_n]$

$= \frac{1}{0.02*0.02}[-0.0002 + \frac{6(-0.5)+6}{6}(-0.0002)]$

$= \frac{1}{0.04}[-0.0002+0.5(-0.0002)]$

= 2.500[-0.0002-0.0001]

= -0.00075

15) Find ($\frac{dy}{dx}$) at x = 0.6 of the function y=f(x)

Tabulated below.

X	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
y	**1.5836494**	**1.7974426**	**2.0442376**	**2.3275054**	**2.6510818**

Sol) Consider the central differential table

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
0.4	1.5836494	0.2137932	0.0330018		
0.5	1.7974426	0.2467950	0.0364728	0.003471	0.000364
0.6	2.0442376	0.2832675	0.0403086	0.003835	
0.7	2.3275054	0.3235764			
0.8	2.6510818				

$(\frac{dy}{dx})x = x_0 =$

$\frac{1}{h}[\left(\frac{\Delta y0+\Delta y-1}{2}\right) - \frac{1}{6}(\frac{\Delta 3y+\Delta 3y-2}{3})]$

$= \frac{1}{0.1}[\frac{1}{2}(0.2832678+0.2467950)]$

$= \frac{1}{6}[0.0030358+0.0034210]$

$= 2.644225$

16) Find the maximum and minimum of a function from the following the table.

x	**0**	**1**	**2**	**3**	**4**
y	**0**	**0.25**	**0**	**2.25**	**1.6**

Sol) Consider the following table

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
0	0	0.25	-0.5	3	6
1	0.25	-0.25	2.50	9	
2	0	2.25	11.50		
3	2.25	13.75			
4	1.6				

By applying Newtons interpolation formula

$$\frac{dy}{dx} = \frac{1}{h}[\Delta y_0+\frac{2u-1}{2!}\nabla^2 y_0+\frac{3u*u-6u+2}{3!}\nabla^3 y_0+\frac{4u*u-18u*u-22u-6}{4!}\nabla^4 y_0]$$

$$= \frac{1}{1}[0.25+\frac{2u-1}{2}(-0.51)+\frac{3u*u-6u+2}{3!}(3)+\frac{4u*u-18u*u-22u-6}{24}(6)]$$

$= u^3-3u^2-2u$ ……..(1)

Maximum and minimum values

$u(u^2-3u-2) = 0$

$\Longrightarrow$ $u = 0, (u-1)(u-2) = 0$

$u = 0, 1, 2$

$(d^2y/dx^2) = \frac{1}{h*h}[\ \nabla^2 y_0+(u-1)\ \nabla^3 y_0+ \frac{6u*u-8u+11}{12}\nabla^4 y_0]$

$= \frac{1}{1}[-0.5+(u-1)3+\frac{6u*u-8u+11}{12}(6)]$

$= 1[-0.5+3u-3+\frac{6u*u-8u+11}{2}\ ]$

$= 3u^2-6u+2$

$u=0,$

$(d^2y/dx^2) = 2 >0$

$u = 1$

$(d^2y/dx^2) = -1 >0$

$u = 2$

$(d^2y/dx^2) = 3(4)-6(2)+2$

$= 12-12+2$

$= 2 >0$

The given function has maximum value at

$u = 0,2$

The given function has minimum value is -1.

NUMERICAL INTEGRATION

17. By using Simpsons 1/3 rule, Evaluate $\int_{-3}^{3} x^4$ by taking 7 equal subinterval

Sol: Given that f(x) = x^4

A = -3, b=3, n =7, h = $\frac{b-a}{n}$

x	-3	-2	-1	0	1	2	3
y	81	16	1	0	1	16	81

By applying Trapezoidal Rule,we have

$\int_{x0}^{xn} y\,dx = \frac{h}{3}$ ($y0 + yn$) + 4($y1 + y3$ + ---------- $yn-1$) + 2 ($y2 + y4$ + ---------- $yn-2$)

$\int_{-3}^{3} x^4 dx = \frac{h}{3}$ ($y0 + y6$) + 4($y1 + y3 + y5$) + 2 ($y2 + y4$)

$= \frac{0.85}{3}\ [(81 + 81) + 4(16 + 0 + 16) + 2(1 + 1)]$

$= \frac{0.85}{3}\ [(162) + 4(32) + 2(2)]$

$= \frac{0.85}{3}\ [162 + 128 + 4]$

$= \frac{0.85}{3}\ [294]$

$= 0.2833\ [294]$

$= 83.3$

18. Evaluate $\int_0^6 \frac{1}{1+x^2}$ dx by Simpsons 1/3 rule.

Sol: Here take (n)=6, a=0, b=6

Then $h = \frac{b-a}{n}$, $h = \frac{6.0}{6} = 1$

x	0	1	2	3	4	5	6
y	1	0.5	0.2	0.1	0.05	0.03	0.027

$y0 = 1/X_0^2 = 1/_0^2 = 1$

$y1 = 1/X_1^2 = 1/_1^2 = 1/1+1 = 1/2 = 0.5$

$y2 = 1/X_2^2 = 1/_2^2 = 1/1+4 = 1/5 = 0.2$

$y3 = 1/X_3^2 = 1/_3^2 = 1/1+9 = 1/10 = 0.1$

$y4 = 1/X_4^2 = 1/_4^2 = 1/1+16 = 1/17 = 0.05$

$y5 = 1/X_5^2 = 1/_5^2 = 1/1+25 = 1/26 = 0.03$

$y6 = 1/X_6^2 = 1/_6^2 = 1/1+36 = 1/37 = 0.027$

By applying simpson's 1/3 rule

$\int_{x0}^{xn} y\,dx = \frac{h}{3}$ ($y0 + yn$) + 4($y1 + y3$ + ---------- $yn-1$) + 2 ($y2 + y4$ + ---------- $yn-2$)

$\int_0^6 \frac{1}{1+x^2}\,dx = \frac{h}{3}$ ($y0 + yn$) + 4($y1 + y3 + y5$) + 2 ($y2 + y4$)

$= \frac{1}{3}$ (1+0.027) + 4 (0.5+0.1+0.63) + 2 (0.2+0.05)

$= \frac{1}{3}$ (1.027 +4(0.63)+ 2 (0.25)

$= \frac{1}{3}$ (1.027 + 2.25 + 0.5)

$= \frac{1}{3}$ (3.777) =0.33(3.777)

= 1.2464

19. **Evaluate $\int_{-3}^{3} x^4$ dx by applying Trapezoidal with h = 1**

SOL: Here y = f(x) = x^4

A = -3, b=3, h=1

x	-3	-2	-1	0	1	2	3
y	81	16	1	0	1	16	81

By applying Trapezoidal Rule

$\int_a^b \mathbf{f}(\mathbf{x})\, dx = \frac{h}{2}$ ($y_0 + y_n$) + 2 ($y_1 + y_2 +$ ---------- y_n -)

$\int_{-3}^{3} x^4$ **dx =** $\frac{1}{2}[(81 + 81) + 2(16 + 1 + 0 + 1 + 16)]$

$= \frac{1}{2}[162 + 2(34)]$

$= \frac{1}{2}[162 + 68]$

$= \frac{1}{2}[230]$

$= \frac{230}{2}$

= 115

20. **Evaluate I** $= \int_0^1 \frac{dx}{1+x}$ **Correct to 3 decimal places by applying trapezoidal rule with h= 0.2.**

Sol: Given that f(x) = $\frac{1}{1+x}$

Here a=0 and b=1, h=o.25

x	0	o.25	0.50	0.75	1.00
y	1	0.8	0.6	0.57	0.5

$y0 = \frac{1}{1+0.25} = \frac{1}{1.25} = 0.8$

$y1 = \frac{1}{1+0.50} = \frac{1}{1.5} = 0.6$

$y2 = \frac{1}{1+0.75} = \frac{1}{1.75} = 0.57$

$y3 = \frac{1}{1+1.00} = \frac{1}{2} = 0.5$

By applying trapezoidal Rule

$y = \frac{h}{2} (y0 + yn) + 2 (y1 + y2 + \text{----------} yn)$

$= \frac{0.25}{2}[(1.05) + 2(0.8 + 0.6 + 0.57)]$

$= 0.125[1.5 + 2(1.97)]$

$= 0.125[1.5 + 3.94]$

$= 0.125[5.44]$

Y = 0.680

21. By using Simpsons 3/8 rule, Evaluate integral of $\int_0^1 \frac{1}{1+x}$ **dx with h =** $\frac{1}{6}$ Sol): Given that

Here f(x) = $\frac{1}{1+x}$ and h = $\frac{1}{6}$

x	0	1/6	2/6	3/6	4/6	5/6	1
y	1	0.8571	0.7500	0.6666	0.6000	0.5454	0.5

$y_0 = \frac{1}{1+x_0} = \frac{1}{1+0} = 1$

$y_1 = \frac{1}{1+x_1} = \frac{1}{1+\frac{1}{6}} = 0.8571$

$y_2 = \frac{1}{1+x_2} = \frac{1}{1+\frac{2}{6}} = 0.7500$

$y_3 = \frac{1}{1+x_3} = \frac{1}{1+\frac{3}{6}} = 0.6666$

$y_4 = \frac{1}{1+x_4} = \frac{1}{1+\frac{4}{6}} = 0.6000$

$y_5 = \frac{1}{1+x_5} = \frac{1}{1+\frac{5}{6}} = 0.5454$

$y_6 = \frac{1}{1+x_6} = \frac{1}{1+1} = 0.5$

By using Simpsons 3/8 rule

$\int_{x0}^{xn} y\,dx = \frac{3h}{8}\ (y_0 + y_n) + 2(y_3 + y_6 + \text{----------} + y_{n-3}) + 3(y_1 + y_2 + \text{----------} + y_{n-1})$

$\int_0^1 \frac{1}{1+x}\,dx = \frac{3}{48}[(1+0.5) + 2(0.6666) + 3(0.8571 + 0.7500 + 0.6000 + 0.5454)]$

$= 0.0625\,[1.5 + 1.3332 + 3(2.7525)]$

$= 0.0625\,[1.5 + 1.3332 + 8.2575]$

$= 0.0625\,[11.0907] == 0.6931$

22. Evaluate $\int_0^4 e^x \, dx$ by applying Boole's rule with h = 1

Sol: Given that $\int_0^4 e^x \, dx$ with h = 1

x	0	1	2	3	4
y	1	2.718	7.389	20.085	54.598

$y0 = e^0 = 1$

$y1 = e^1 = 2.718$

$y2 = e^2 = 7.389$

$y3 = e^3 = 20.085$

$y4 = e^4 = 54.598$

$\int_0^n y \text{ dx} = \frac{2h}{45} [7y_0 + 32y_1 + 12y_2 + 32y_3 + 14y_4 + 32y_5 + - - - - - - - 7y_n]$

$\int_0^4 e^x \, dx = \frac{2(1)}{45} [7(1) + 32(2.7180) + 12(7.389) + 32(20.085) + 7(54.598)]$

$= \frac{2}{45} [7 + 86.976 + 88.668 + 642.72 + 382.186]$

$= \frac{2}{45} [1207.55]$

$= \frac{2415.1}{45}$

$= 53.668$

23. Divide the interval $[0, \pi/2]$ in to 6 equal parts with h =π/2 evalute $\int_0^{\pi/2} \sqrt{sinx}$ dx by Weddle's Rule

Sol:

X	**0**	**π/2**	**π/6**	**π/4**	**π/3**	**5π/2**	**π/2**
$\sqrt{sinx}$	**0**	**0.5087**	**0.7071**	**0.8408**	**0.9306**	**0.9828**	**1**

By using Weddle's rule

$\int_a^b y\, dx = \frac{3h}{10}$ [y0 + 5y1 + y2 + 6y3 + y4 + 5y5 + y6]

$\int_0^{\pi/2} \sqrt{sinx} = \frac{3(\pi/2)}{10}$ $[0 + 5(o.5087 + 0.7071 + 6(0.8408) + 0.9306 + 5(0.9828) + 1]$

$= 0.0785\ [2.5435 + 0.7071 + 5.0448 + 0.9306 + 4.914 + 1]$

$= 0.0785[15.14]$

$= 1.1885$

24.E**valute** $\int_0^{\pi/2} sinx\, dx$ **by using euler –macourins formula**

Sol: y f(x) = sinx and h=π/4

A NOW Y^1 = COSX

Y^{11} = - SINX

Y^{111} = -COSX

Y^{1111} = SINX

Y^1 (π/2) = 0; Y^1 (0) = 1; Y^{111} (π/2) = 0; Y^{111} (0) = -1

$\int_0^{\pi/2} \boldsymbol{sinx}\ \mathbf{dx} = \frac{h}{12}\ [y0 + 2y1 + y2] + \frac{h^2}{12}\left[\ \mathbf{Y1}\ \left(\frac{\pi}{2}\right) - \mathbf{Y1}\ (0)\right] + \frac{h^2}{120}$
$\left[\ y^{111}\left(\frac{\pi}{2}\right) - \ y^{111}\ (0)\right]$ + ----------

$= (\pi/8)\ [0 + 2 + 0]\ + \frac{\pi^2}{192} + \frac{\pi^4}{184320}$ +----------

$= 0.785398 + 0.05140 + 0.000528$

$= 0.83733$

$\int_0^{\pi/2} \boldsymbol{sinx}\ dx = 0.837330$

S0LUTION OF SIMULTANEOUS LINEAR SYSTEM OF EQUATIONS

25. SOLVE x + 2 y = 10, 2 x – y = 15 by matrix inversion method.

Sol). Let $A=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$;

$x = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix}1 & 0\\ 1 & 5\end{bmatrix}$

Given equation are equivalent to a $AX = B \Rightarrow X = A^{-1} B$

$A^{-1} = \frac{1}{-1-4}\begin{bmatrix}-1 & -2\\ -2 & 1\end{bmatrix}$

$= \frac{1}{5}\begin{bmatrix}-1 & -2\\ -2 & 1\end{bmatrix}$

$= \frac{1}{5}\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$

$\therefore X = A^{-1} B = \frac{1}{5}\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}\begin{bmatrix}10\\ 15\end{bmatrix} = \frac{1}{5}\begin{bmatrix}10 & 30\\ 20 & -15\end{bmatrix} = \frac{1}{5}\begin{bmatrix}40\\ 5\end{bmatrix} = \begin{bmatrix}8\\ 1\end{bmatrix}$

$\therefore$ X = 8 ; Y = 1 is the solution.

26). Solve equations x + y + z = 9, 2x + 5y + 7z = 52 ,2x + y – z =0 by using cramer's rule.

Sol). $\triangle = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{vmatrix}$

$= 1(-5-7)-1(-2-14)+1(2-10)$

$=-12+16-8$

$=-4$

$\triangle_1 = \begin{vmatrix} 9 & 1 & 1 \\ 52 & 5 & 7 \\ 0 & 1 & -1 \end{vmatrix} = 9(-5-7)-1(-52-0)+1(52-0)$

$=-108+52+52$

$=-4$

$\triangle_2 = \begin{vmatrix} 1 & 9 & 1 \\ 2 & 52 & 7 \\ 2 & 0 & -1 \end{vmatrix} = 1(-52-0)-9(-2-14)+1(0-104)$

$=-52+144-104$

$=-12$

$\triangle_3 = \begin{vmatrix} 1 & 1 & 9 \\ 2 & 5 & 52 \\ 2 & 1 & 0 \end{vmatrix} = 1(0-52)-1(0-104)+9(2-10)$

$=-52+104-72$

$=-20$

By cramer's rule

$x = \frac{\triangle 1}{\triangle} = \frac{-4}{-4} = 1,$

y $= \frac{\Delta 2}{\Delta} = \frac{-12}{-4} = 3$,

Z $= \frac{\Delta 3}{\Delta} = \frac{-20}{-4} = 5$.

∴ x = 1, y = 3, z = 5 is the solutions.

27). Solve the system of equations x+2y+z= 4, 2x-3y-z=-3,3x+y+2z=3 by Gauss elimination method.

Sol). let x+2y+z= 4–> (1)

2x-3y-z=-3–> (2)

3x+y+2z=3–> (3)

(2)-2(1) => -7y-3z=-11–> (4)

(3)-3(1) => -5y-z=-9–> (5)

(4)-3(5) => 8y =16–> (6)

(6) => y=2 ;(5) => -10 -z = -9 =>z= -1;

(1) => x+4-1 = 4=> x =1

∴ The solution is x = 1, y = 2, z = -1

28). Solve x-3y+2z=8, 3x+4y+z=5, -4x+2y-9z=2 by using cramer's rule.

Sol). cramer's rule

$\triangle = \begin{vmatrix} 1 & -3 & 2 \\ 3 & 4 & 1 \\ -4 & 2 & -9 \end{vmatrix}$

= 1(-36-2)+3(- 27+4)+2(6+16)

$\triangle_1 = \begin{vmatrix} 8 & -3 & 2 \\ 5 & 4 & 1 \\ 2 & 2 & -9 \end{vmatrix}$ = 8(-36-2)+3(-45-2)+2(10-8)

=-304-141+4

=-441

$\triangle_2 = \begin{vmatrix} 1 & 8 & 2 \\ 3 & 5 & 1 \\ -4 & 2 & -9 \end{vmatrix}$ = 1(45-2)-8(-27+4)+2(6+20)

= -47+184+52

=189

$\triangle_3 = \begin{vmatrix} 1 & -3 & 8 \\ 3 & 4 & 5 \\ -4 & 2 & 2 \end{vmatrix}$ = 1(8-10)+3(6+10)+8(6+16)

=-2+78+176

=252

By cramers rule

$x_1 = \frac{\triangle 1}{\triangle}$; $\frac{(-441)}{(-63)}$ 7;

$y = \frac{\triangle 2}{\triangle} = \frac{189}{-63}$ 3;

$Z = \frac{\triangle 3}{\triangle} = \frac{252}{-63}$ =-4

∴ x=7; y=-3 ; z=-4 is the solution.

29). Solve the equations 2x+y+z=10, 3x+2y+3z=18, x+4y+9z=16 by gauss elimination method.

Sol). Augumated matrix is

$$\begin{pmatrix} 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{pmatrix}$$

$$\begin{matrix} 2\mathrm{R}1 - 3\mathrm{R}1 \\ 2R3 - R1 \end{matrix} \begin{pmatrix} 2 & 1 & 1 & 10 \\ 0 & 1 & 3 & 6 \\ 0 & 7 & 17 & 22 \end{pmatrix}$$

$$\begin{matrix} \mathrm{R}1 - \mathrm{R}2 \\ R3 - 7R2 \sim \end{matrix} \begin{pmatrix} 2 & 0 & -2 & 4 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & -4 & -20 \end{pmatrix}$$

$$\sim \frac{1}{4}\mathrm{R}2 \quad \begin{pmatrix} 2 & 0 & -2 & 4 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 1 & 5 \end{pmatrix}$$

$$\sim \begin{matrix} \mathrm{R}1 + 2\mathrm{R}3 \\ R2 - 3R3 \end{matrix} \begin{pmatrix} 2 & 0 & 0 & 14 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & 5 \end{pmatrix}$$

$\frac{1}{2}R_1 \sim \begin{pmatrix} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & 5 \end{pmatrix}$

$\therefore$ x =7; y = -9; z = 5 is the soluting.

30). Solve by equations $2x_1 + x_2 + x_3 = 2$, $x_1+3x_2+2x_3 =2, 3x_1+x_2+2x_3=2$ by Lu decomposition method .

Sol). Given that equation are

$$2x_1 + x_2 + x_3 = 2 \rightarrow (1)$$

$$x_1+3x_2+2x_3 =2 \rightarrow (2)$$

$$3x_1+x_2+2x_3=2 \rightarrow (3)$$

Here the co-efficient matrix A is given by

$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 3 & 1 & 2 \end{pmatrix}$ and

$X = \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix}$; $B = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$ $\therefore Ax = B$

Now A= Lu = $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 3 & 1 & 2 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 & 0 \\ l21 & 1 & 0 \\ l31 & l32 & 1 \end{pmatrix} \quad \begin{pmatrix} u11 & u12 & u13 \\ 0 & u22 & u23 \\ 0 & 0 & u33 \end{pmatrix}$

$= \begin{pmatrix} u11 & u12 & u13 \\ l21u11 & l21u12 + u22 & l21u13u + 23 \\ l31u11 & l31u12 + l32u22 & l31u13 + l32u13u33 \end{pmatrix}$

On comparing two matericals , we get $u_{11}=2$, $u_{12} = 1$, $u_{13}=1$,

$l_{21}\, u_{11}=1 \Rightarrow l_{21}= 1/ u_{11}= 1/2$; $l_{31}\, u_{11}=3 \Rightarrow l_{31}=3/ u_{11}=3/2$

$l_{21}\, u_{12} + u_{22} =3 \Rightarrow u_{22}=3- l_{21}\, u_{12} =3-1/2\ (1) =3 -1/2 = 5/2$

$l_{21}\, u_{13} + u_{23} =2 \Rightarrow u_{23}=2- l_{21}\, u_{13} =2-1/2\ (1) =2 -1/2 = 3/2$

$l_{31}\, u_{12} + l_{32}\, u_{22} =1 \Rightarrow l_{32}= \frac{l - l31u12}{u22} = \frac{1-3/2\ (1)}{5/2} = -1/5$

$l_{31}\, u_{13} + l_{32} u_{23} + u_{33} = 2$

$\Rightarrow u_{33}=2-l_{31}u_{13}-l_{32}u_{23}=2-3/2\ (1)-(-1/5)(3/2)$

$=2-3/2 + 3/10 = 8/10 = 4/5$

Thus L = $\begin{pmatrix} 1 & 0 & 0 \\ l21 & 1 & 0 \\ l31 & l32 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -1/5 & 1 \end{pmatrix}$

And u = $\begin{pmatrix} u11 & u12 & u13 \\ 0 & u22 & u23 \\ 0 & 0 & u33 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 5/2 & 3/2 \\ 0 & 0 & 4/5 \end{pmatrix}$

Ly = B => $\begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -1/5 & 1 \end{pmatrix} \begin{pmatrix} y1 \\ y2 \\ y3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$

From this we get

$y_1=2$,

$\frac{1}{2} y_1 + y_2 = 2$;

$3/2\ y_1 - 1/5 y_2 + y_3 = 2$

On solving, we get

$y_1=2$;

$y_2= 1$;

$y_3= -4/5$

And since, we get have

Ux = y => $\begin{pmatrix} 2 & 1 & 1 \\ 0 & 5/2 & 3/2 \\ 0 & 0 & 4/5 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} y1 \\ y2 \\ y3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -4/5 \end{pmatrix}$

$\therefore 2x_1+x_2+x_3 = 2;$

$5/2\ x_2+3/2\ x_3 =1;$

$4/5\ x_3=-4/5$

By back substitution, we get $x_3 =-1;\ x_2 =1;$ x1=1

Hence, solution is $x_1 =1;\ x_2=1;\ x_3=-1$.

31. Solve the following equation by gauss –jacobi method 10x-y+z=12,

x-10y+z=12,x+y-10z =12. Correct to 3 decimals.

Sol). Given equation can be written by

$$\left.\begin{aligned} X &= \frac{1}{10}(12+y-z) \\ y &= \frac{1}{10}(x+z-12) \\ z &= \frac{1}{10}(x+y-12) \end{aligned}\right\} \rightarrow (1)$$

let us take the first approximation be

$x^{(1)}=0;(y)^{(1)}=0;(z)^{(1)}=0$

we abtain second approximation from (1), as

$x^{(2)}= \frac{1}{10}(12+0-0)=1.2;$

$y^{(2)}= \frac{1}{10}(0+0-12)=-1.2$;

$z^{(2)}=\frac{1}{10}(0+0-12)=-1.2$

now we obtain third approximation from (1)as

$X^{(3)}=\frac{1}{10}(12+y^{(2)}-z^{(2)})$

$=\frac{1}{10}(12-1.2+1.2)=1.2$

$Y^{(3)}=\frac{1}{10}(x^{(2)}+z^{(2)}-12)$

$=\frac{1}{10}(1.2-1.2-12)=-1.2$

$Z^{(3)}=\frac{1}{10}(x^{(2)}+z^{(2)}-12)$

$=\frac{1}{10}(1.2-1.2-12)=-1.2$

Second and third approximations are same.

$\therefore$ The solution is x = 1.2, y=-1.2, z=-1.2.

32). By using gauss – seidel iterative method solve the linear equations $x_1+10x_2+x_3=6$; $10x_1+x_2+x_3=6$ and $x_1+x_2+10x_3=6$.

Sol): Given equations can be written as

$$\left.\begin{aligned} x_1&=\frac{1}{10}(6-x_2-x_3)\\ x_2&=\frac{1}{10}(6-x_1-x_3)\\ x_3&=\frac{1}{10}(6-x_1-x_2)\end{aligned}\right\} \rightarrow (1)$$

Assuming first approximation $x_1^{(1)}=0, x_2^{(1)}=0, x_3^{(1)}=0$.

Substituting these values into the first equation of (1), we get

$x_1^{(2)} = \frac{3}{5} = 0.6$

Now substitute $x_1^{(2)} = 0.6$, $x_3^{(1)} = 0$ in the second equation of (1), we get

$x_2^{(2)} = \frac{1}{10}(6-0.6-0) = \frac{1}{10}(5.4) = 0.54$

substitute $x_1^{(2)} = 0.6$, $x_2^{(2)} = 0.54$ in the third equation of (1),we get

$x_3^{(2)} = \frac{1}{10}(6-0.6-0.54) = 0.486$

substitute the values $x_2^{(2)} = 0.54$, $x_3^{(2)} = 0.486$ in the first equation of (1),we get

$x_1^{(3)} = \frac{1}{10}(6-0.54-0.486) = 0.4974$

substitute the values $x_1^{(3)} = 0.4974$,

$x_3^{(2)} = 0.486$ in the second equation of (1), we get $x_2^{(2)} = \frac{1}{10}(6-0.4974-0.486) = 0.5017$

substitute the values $x_1^{(3)} = 0.4974$,

$x_2^{(3)} = 0.5017$ in the third equation of (1),

we get $x_3^{(3)} = \frac{1}{10}(6-0.4974-0.5017)$

$= 0.5001$

substitute the values $x_2^{(3)} = 0.5017$, $x_3^{(3)} = 0.5001$ in the first equation of (1),we get $x_1^{(4)} = \frac{1}{10}(6-0.5017-0.5001) = 0.4998$

substitute the values $x_1^{(4)} = 0.4998$, $x_3^{(3)} = 0.5001$ in the second equation of (1),we get $x_2^{(4)} = \frac{1}{10}(6-0.4998-0.5001) = 0.5001$

substitute the values $x_1^{(4)} = 0.4998$, $x_2^{(4)} = 0.5001$ in the third equation of (1), we get $x_3^{(3)} = \frac{1}{10}(6-0.4998-0.5001) = 0.5002$

∴ The solution is $x_1^{(4)}=0.4998=0.5, x_2^{(4)}=0.5001=0.5, x_3^{(4)}=0.5002=0.5$

NUMERICAL SOLUTIONS OF ORDINARY DIFFERENTIAL EQUATIONS.

33: Solve the differential equation $\frac{dy}{dx}$ = x + y , with

Y (0) = 1, x € [0,1] by talor series expansion to obtain y for

X = 0.1 . ?

Sol: the talor series for y (x) is given by $y(x)=1+xy_0'' + \frac{x^2}{2!}Y_0 + \frac{x^3}{3!} y_0'' + \ldots\ldots\ldots$

From given data, we have .

$y' = x+y = y'(0) = 1$

$y' = 1 + y' = y''(0)=2$

$y''' = 0+y'''(0) = 2$

take we get $y(x) = 1+x + \frac{x^2}{2!} + \frac{2x^3}{3!} + \frac{2x^4}{4!} + \ldots\ldots\ldots..$

putting x= 0.1 , we get $y(0.1) = 1=0.1+(0.1)^2 + \frac{2(0.1)^3}{3} + \frac{(0.1)^4}{12}$

=1+0.1+0.1+0.000333+0.0000083

=1.11033.

34: Solve the equation $y' = X+Y^2$ with $y_o=1$ when x=0 by picard 's method .?

Sol: we start with $y^0 = 1$ and obtain $y^{(1)} = 1+\int_0^x (x+1)\,dx$

$1+x+\frac{x^2}{2!}$ then the second approximation is $Y^{(2)}=1+$ ∫₁₀ᵀx≅[1+ [1+x+

$\mathbf{X} + \frac{x^2}{2}$]] dx = $1 + \frac{3}{2}x^2 + \frac{2}{3}x^3 + \frac{1}{4}x^4 + \frac{1}{20}x^5$

It is obvios that the integrations might become more and more diicult as,we proceed to higher approximation .

35: Using taylor series expansion solve the equation $\frac{dy}{dx} = x^2+y^2$ for x=0.4 given that y=0 when x=0.?

Sol: from the given data, we have $y' = x^2+y^2 = y'(0)=0$

$Y'' = 2x+2yy' = y''(0) =0.$

$Y''' = 2+ 2yy'' +2(y')^2 = y'''=2$

$Y'''' = 2yy''' + 2y'y'' + 4y'y'' = y''''(0)=0$

The taylor series for y(x) is given by

$Y(x) = 1+xy'_o + \frac{x^2}{3!}y''_o + \frac{x^3}{3!}y''_o + \ldots\ldots = 1+\frac{x^3}{3!}(2) = 1+$

$\frac{x^3}{3}e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots, \quad -\infty < x < \infty$

Putting x= 0.2, we get $y(0,4) = 1+\frac{(0.4)^3}{3} = 1.02133.$

36: Using picard's method to obtain y for x = 0.1 to 0.5 for the differential equation $\frac{dy}{dx}$ = 1 + xy with y(0)=1.?

Sol: As per given, we have f (x,y) = 1+ xy, x_o= 0 an y_o = 1, there fore, the first approximation y, of y is

$Y_1 = y_o + \int_0^x f(x, y)\,dx = 1 + \int_0^x [1 + x]\,dx = 1 + x + \frac{x^2}{2}$

The second approximation y_2 is given by

$Y_2 = y_0 + \int_{x_0}^{x} f(x.)\,dx = 1 + \int_0^x [1 + x_{[1+x+}\ ]\ ]\ x^{2}/2!\ ]\ ]$

$dx = 1 + \int_0^x [1 + x +]\ x^2 + x^{3/2}\ ]\,dx$.

$= 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8}$

Similarly, the third approximation y_3 is given by

$Y_3 = y_0 + \int_{x_0}^{x} f(x_1, y_2)dx = 1 + \int_0^4 [1 + x_{[1+}\ x^2/2 + x^3/3 + x^4/8]$]dx

$= 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8} + \frac{x^5}{15} + \frac{x^6}{18}$.

37: Solve the equations y= - y with the condition y(0)=1 for x= 0 0.04 in 4 steps by eulers method. ?

Sol: Here h= $\frac{0.04}{4}$ =0.01 given y (0)=1.

We know that $y_{n+1}=y_n+h\ f(x_x,y_n)$ for the n = 0,1,2,3…….

Y(0.01) = $y_1=y_0$ h f (x_0 ,y_0) = 1+ (0.01) (-1) = 0.99.

Y(0.02)=$y_2=y_1$+ h f (x_0,y_0)= 0.99+(0.01)(-0.99)=0.9801.

Y(0.03)=$y_3=y_2$ + h f (x_2,y_2)= 0.9801+ (0.01)(-0.9801)=0.9703.

Y(0.04)=y_3+ h f (x_3,y_3)= 0.9703+(0.01)(-0.9703)=0.9606..

38: Given $\frac{dy}{dx}$ = x^3+y , y (0) , compute y (0.2) by eulers method taking h= 0.01.?

Sol: Given $\frac{dy}{dx}$ = x^3+y , with the initial condition y (0)=1.

We have f (x, y) =x^3+$y^3$and x_0=0,y_0=0.01.

$X_1=x_0$+ h =0+0.01=0.01,$x_2=x_0$+2h=0+2(0.01)=0.02.

Applying eulers formula we get $y_1=y_0$+h f (x_0,y_0)

∴ y_1=1+(0.01)(x_0^3+y_0)=1+(0.01(0^3+1)=0.01

∴ y_1= y (0.01)=1.01,$y_2=y_1$=h f (x_1,y_1)=1.01+ (0.01)[x_1^3+y_1]

=1.01+$(0.01)^3$+1.011]=1.0201

∴ y_2=y(0.02)=1.0201.

39: Apply runge – kutta method find the solution of the differential equation y^1=3x+ $\frac{1}{2}$ y with y_0 = 1 at x= 0.1 .?

Sol: we have h = 0.1 given y_0=1,when x_0=0.

K_1=h f (x_0,y_0)=0.1 (3x_0+ $\frac{1}{2}$ y_0) 0.1 (3x_0+ $\frac{1}{2}$ x_1) = 0.05 => $k_1$0.05.

Similarly k_2= h f ($x_0+\frac{h}{2}, y_0+\frac{k_1}{2}$)= 0.1 f (0.05 ; 1.025)=0.06625=k_2=0.06625.

K_3=h f ($x_0+\frac{h}{2}, y_0+\frac{k_2}{2}$) 0.1 f (0.05, 1.033125) = 0.6665625=> k_2=0.06665625.

K_4=h f (x_0+h, y_0+k_3) = 0.1 f (0.1, 1. 0665625)= 0.0833328125 => k_4= 0.0833328125.

Obviously k = $\frac{1}{6}$ [k_1+2(k_2+k_3)+k_4=0.066652421875.

40: Solve $\frac{dy}{dx}$ = xy using runge – kutta method for

X= 0.2 given that y (0) =1 taking h = 0.2.?

Sol : Here f (x,y) =xy , x_0=0, y_0=1 and h= 0.2 .

K_1= h f (x_0,y_0) = (0.2) f (0,1) =0.

K_2= h f (x_0+ $\frac{1}{2}$ h , y_0 + $\frac{1}{2}$ k_1 = (0.2) f (0,1,1)

= (0.2) (1,1) = 0.0202.

K_3 =h f (x_0+ $\frac{1}{2}$h, y_0+ k_2) =(0.2) f (0.1, 1.01)

= (0.2) (0.1) (1.01) = 0.0202.

K_4 = h f (x_0 + h , y_0 + k_3) = (0.2) f (0.2) f (0.2,1.0202)

=(0.2)(0.2) (1.0202) = 0. 040808.

By runge – kutta method;

Y_1=y(0.2) = y_0+ $\frac{1}{6}$ [k_1 + 2 (k_2+k_3)+k_4

= 1+ $\frac{1}{6}$ [0 + 2(0.02 + 0.0202) +0.040808] = 1.0202.

1.Interpolation With Equal Intervals

1.1 INTRODUCTION :

Interpolation means insertion or filling up intermediate terms of the series. It is the technique of getting the value of a function for any intermediate value of the argument from the given set of values of function is called interpolation, while the process of computing the value of the function outside of the given range is called interpolation. The study of interpolation is based on the calculus of finite differences which

The following assumptions are to be kept in mind for interpolation.

(1) The values of the function should be either increasing or decreasing

(2) The rise or fall in the values should be uniform plays a vital role in numerical analysis manner.

1.2 INTERPOLATION METHODS :

(i) Graphical method

(i) Method of curve fitting

(iii) Applications of the calculus of finite differences formula.

1.2.1 Graphical Method :

Let y =f(x), then plot a graph for different values of x corresponding to the value of y. From the graph, we can find the value of y for given value of x.

For example

Year(x)	1891	1901	1911	1921	1931
Population (Y)(in 1000's)	46	66	81	93	101

Interpolate population for the year 1919.

The following steps are to be maintained

1. Take a suitable scale for x and y plot the different points on the graph for given data.

2. Join the point by hand curve.

3. Find the point on the curve corresponding to x = 1919 and find corresponding value of y

Remarks : This is simply an approximate method of evaluating the value of y and is not reliable in number of cases.

1.2.2 Method of Curve Fitting :

This method is applicable in some cases in which the form of the function is known. In such cases, by the method of least squares, we can fit the curve of known form to given set of values

and afterwards we can find the unknown value with the help of fitted curve.

Remarks :

(1) This is not exact method.

(2) In case of number of values are sufficiently large this method is complicated

(3) The form of the function in this method for given set of observations is assumed to be known.

1.2.3 Applications of the Calculus of Finite Differences :

(1) Newton-Gregory formula for interpolation. If n is a integer and h is the interval differencing, then $f(a+ nh) = f(a)+{}^{n}c_1\,\Delta f(a)+ {}^{n}c_2\Delta^2 f(a)+....+ {}^{n}c_n\,\Delta^{n}{}_{f(a)}$

We prove the theorem by using the method of induction.

we know that $\Delta f(a) = f(a + h) - f(a)$

$f(a+h)= f(a) +\Delta f(a)$

The theorem is true for n = 1.

Assume that the theorem is true for n=k.

$f(a+kh)=f(a)+{}^{k}c_1\,\Delta^2 f(a)+....+{}^{k}c_n\,\Delta^{k} f(a)$

We have $\Delta f(a+kh) = f(a+(k+ 1)h)-f(a+kh)$

$f(a + (k + 1)h) = f(a+kh) + h + \Delta f(a + kh)$

$= [f(a)+{}^{k}c_1\Delta f(a)+ {}^{k}c_2\Delta^{r} f(a)+....+ \Delta^{k} f(a)]$

$[\Delta^{r} f(a)+{}^{k}c_1\Delta^{r} f(a)+ {}^{k}c_1\Delta^{3} f(a)+.....+ \Delta^{k+1} f(a)]$

$= f(a)+({}^{k}c_1+1)\,\Delta f(a)+({}^{k}c_1+ {}^{k}c_2)\,\Delta^2 f(a)+.....+\Delta^{k+1} f(a)$

$= f(a)+{}^{k+1}c_1+\Delta f(a)+{}^{k+1}c_2\,\Delta^2 f(a)+.....+\Delta^{k+1} f(a)$

The theorem is true for n=k+1

By the method of induction, the theorem is true for any integer.

$f(a+ nh) = f(a)+{}^{n}c_1\,\Delta f(a)+ {}^{n}c_2\Delta^2 f(a)+....+ {}^{n}c_n\,\Delta^{n}{}_{f(a)}$

Note :

1. $f(a+nh) = f(a)+{}^{n}c_1\,\Delta f(a)+ {}^{n}c_2\Delta^2 f(a)+....+ {}^{n}c_n\,\Delta^{n}{}_{f(a)}$

2. Put a=0, h=1

$f(h)=f(0)+{}^{n}c_1\,\Delta f(0)+{}^{n}c_2+ {}^{n}c_2\,\Delta^2 f(0)+.....$

$f(x)=f(0)+\frac{x}{1!}\Delta f(0)+ \frac{x(x-1)}{2!}\Delta^2 f(0)+.....$

This formula is used only when the first term a =0 and h =1.

Alter : Let y=f(x) represent a function which assumes the values f(a),f(a+h)....f(a+nh) for (n +1) equidistant values a, a+ h, a +2h, a+nh of the independent variable x and let $P_n(x)$ be a polynomial in x of degree n.

This polynomial can be expressed in the form.

$P_n(x) = Bo +B (x-a) + B2(x-a)(x-a-h)+.....+ B_n(x-a)(x-a-h)(x-a-2h)....(x-a-(n-1)h)$ ....(1)

where B_o. B_1, B_n are constants and are choosen such that

$P_n(a) = f(a)$, $P_n(a + h) = f(a + h)$, . . . $P_n(a + nh) = f(a + nh)$

By substituting successively the values a, a+ h, a +2h . a+nh in equation(1) and writing the values of $P_n(a)$, $P_n(a+h)$... P(a + nh), we get $f(a)= B_0$ B1=f(a + h)-f(a)

$f(a) = B_0$, $f(a + h) = B_0+ hB_1$

$B_0 =f(a)$ or $B_1 = \frac{f(a+h)-f(a)}{h} = \frac{\Delta f(a)}{h}$

$f(a + 2h) = B_0 + 2hB_1 + h^rB_2$ (or)

A2 = f(a + 2h)-2{f(a+ h)-f(a)}-f(a)/$2h^2$

$= \frac{1}{2!h^r}\Delta^2 f(a)$

Similarly, we get $B_3 = \frac{1}{3!h^3}\Delta^3 f(a)$..... $B_n = \frac{1}{n!h^n}\Delta^n f(a)$

Substituting the values of B_0, B_1,....., B_n, we get

$P_n(x) = f(a) +\frac{\Delta f(a)}{h}$ (x-a)+ $\frac{\Delta^2 \mathbf{f(a)}}{\mathbf{2!}\boldsymbol{h^r}}$ **(**x-a)(x-a-h)+....+ $\frac{\Delta^n \mathbf{f(a)}}{\boldsymbol{n!h^n}}$ (x-a)(x-a-h)...(x-a-(n-1)h)

This is Newton-Gregory formula for forward interpolation putting $\frac{x-a}{h}$ = u or x=a+hu then(1) can be expressed as

$P_n(x)=P_n(a+nh) = f(a)+u\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+....+\frac{u(u-1).....(u-n+1)}{n!}\Delta^n f(a)$

Newton-Gregory Formula for Backward Interpolation

In this case , we write

$P_n(x) = C_0 +C_1$ (x-a-nh)+ C_2**(**x-a-nh)+....+C_n(x-a-nh)(x-a-nh+h).....(x-a-h) .....(1)

where Co, C_1 ,$C_{2...}$ C_n are constants and are choosen such that $f_{n(}a+ nh) = f(a+ nh)$,.., $P_n(a) = f(a)$

Substituting in (1), x=a + nh, a + nh-h, we have

$C_0 = f(a + nh)$, $f(a + nh-h) = C_o + C_1 (-h)$

$C_1 = \frac{f(a + nh) - f(a + nh - h)}{h} = \frac{\Delta f(a + nh)}{h}$

Similarly $C_2 = \frac{1}{2!h^r} \nabla^2 f(a+nh)$

$C_3 = \frac{1}{3!h^3} \nabla^3 f(a+nh)$

……

$C_n = \frac{1}{n!h^n} \nabla^n f(a+nh)$

Substituting these values of $C_o, C_1, \ldots, C_n$ in (1), we obtain

$P_n(x) = f(a+nh) + \frac{\nabla f(a+nh)}{1!h}(x-a+nh) + \frac{\nabla^2 f(a+nh))}{2!h}(x-a+nh)\ (x-(a+nh-h)) + \ldots + \frac{(\nabla^n f(a+nh)}{n!h^n}(x-a+nh)\ (x-(a+nh-h))\ldots\ldots(x-(a+h))$ ……(2)

This is Newton-Gregory formula for backward interpolation. Substituting $n = \frac{x-(a+ nh)}{h}$ or

x= a + nh + hl in eq. (2), then we obtain $P_n(x) = P_n(a + nh + h) = f(a + nh) + u\nabla f(a+nh) + \frac{u(u + 1)}{2!} \nabla^2 f(a+nh) + \ldots\ldots\ldots + \frac{u(u+1)\ldots\ldots(u+n-1)}{n!} \nabla^n f(a+nh)$ Where $P_n(x) = f(x)$.

1.3 SOLVED PROBLEMS

1. Given log x for x = 40, 45, 50, 55, 60, 65 for the following data :

x	40	45	50	55	60	65
logx	1.60206	1.65321	1.69877	1.74036	1.7785	1.81291

Estimate the value of log (58.75) by using Newton backward interpolation formula.

Solution.

X	10^5xlogx=f(x)	10^5x∇f(x)	10^5x∇^2f(x)	10^5x∇^3f(x)	10^5x∇^4f(x)
40	160206				
		5115			
45	165321		-539		
		4516		102	
50	169892		-437		-25
		4319		77	

55	174036		-360		-20
		3729		57	
60	177815		-330		
		3476			
65	181291				

(a + nh) + uh = 58.75

65 +5u = 58.75

u = 1.25

Newton backward interpolation formula

$$f(a+nh)+nh) = f(a+nh) + \frac{h}{1!}\Delta f(a+nh) + \frac{u(u+1)}{2!}\Delta^r f(a+nh) + \frac{u(u+1)(u+2)}{3!}\Delta^r f(a+nh)$$

$10^5 f(58.75) = 181291 - 1.25x(3476) + \frac{(-1.25)(-1.25u)x(-30.5)}{2!} + \frac{(-1.25)(-1.25+1)x(-1.25+2)}{6} x(5)$

= 181291-4345-47.3438+2.2267 = 176900.89

F(58.75) = 1.7690089

2. Find the cubic polynomial for the following data hence find f(u)

X	0	1	2	3
F(x)	-2	1	4	6

Solution :

x	**F(x)**	Δf(x)	Δ^2**f(x)**	Δ^3**f(x)**
0	-2			
		3		
1	1		0	-1
		3		
2	4		-1	
		2		
3	6			

Newton's Gregory's formula is

$$f(x) = f(0) + \frac{x}{1!}\Delta f(0) + \frac{x(x-1)}{2!}\Delta^2 f(0) + \frac{x(x-1)(x-2)}{3!}\Delta^3 f(0)$$

$$= -2 + \frac{x}{1!}(3) + \frac{x(x-1)}{2!}(0) + \frac{x(x-1)(x-2)}{3!}(-1)$$

$= -2+3x-\frac{(x^3+3x^2+2x)}{6} = \frac{-12-18x-x^3+3x^2-2x}{6}$

$= \frac{x^3+3x^2+16x-12}{6} = \frac{-(4)^3+3(4)^2+16(4)-12}{6}$

$= \frac{-64+48+64-12}{6} = \frac{36}{6} = 6$

f(4) = 6

3. Find f(1.5) by using Newton's advancing difference formula given that f(o)= +2, f(1)=4, f(2)=7, f(3)=8

x	F(x)	$\Delta f(x)$	Δ^2**f(x)**	Δ^3**f(x)**
0	**2**			
		2		
1	4		1	
		3		**-3**
2	7		-2	
		1		
3	8			

Solution. Newton's advancing formula is

$f(x) = f(0)+\frac{x}{1!}\Delta f(0)+\frac{x(x-1)}{2!}\Delta^2 f(0)+\frac{x(x-1)(x-2)}{3!}\Delta^3 f(0)$

$= 2+\frac{x}{1!}(2)+\frac{x(x-1)}{2!}(1)+\frac{x(x-1)(x-2)}{3!}(-3)$

$= 2+2x+\frac{x^2-x}{2!}+\frac{x^3-3x^2+2x}{2!}(-1)$

$= \frac{4+4x+x^2-x-x^3+3x^2-2x}{2}$

$F(x) = \frac{-(1.5)^3+4(1.5)^2+(1.5)+4}{2}$

$= \frac{-3.375+9+1.5+4}{2} = \frac{11-125}{2} = 5.5625$

f(1.5) = 5.5625

4. Find the polynomial and hence find f(5) from the following data by using Newton's Gregory's formula.

x	**0**	**1**	**2**	**3**	**4**

F(x)	0	7	26	63	124

Solution.

X	F(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	0				
		7			
1	7		12		
		19		6	
2	26		18		0
		37		6	
3	63		24		
		61			
4	124				

F(x) = f(0)+$\frac{x}{1!}\Delta f(0) + \frac{x(x-1)}{2!}\Delta^2 f(0) + \frac{x(x-1)(x-2)}{3!}\Delta^3 f(0)$

= 0+$\frac{x}{1!}(7) + \frac{x(x-1)}{2!}(12) + \frac{x(x-1)(x-2)}{3!}(6)$

= 7x+$6x^2$-6x+$x^3 - 3x^2 + 2x = x^3 + 3x^2 + 7x - 4x$

F(x) = $x^3 - 3x^2 + 2x$

F(5) = 5^3+$3(5)^2$+3(5) = 125+75+15 = 215

F(5) = 215

5. Find f from the given data by using Newton's Gregory's formula

X	10	20	30	40
F(x)	9	30	35	42

Solution .

X	F(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
10	**9**			
		21		
20	30		-16	
		5		18
30	35		2	
		7		
40	42			

a=10 , h= 10

Let a+uh=1.5

10+10u = 1.5

$u = \frac{5}{10} = \frac{1}{2}$

Newton's Gregory's formula is

$$F(a+uh) = f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)$$

$$F(52) = 0.7071+\frac{7}{5}(0.0589)+\frac{\frac{7}{5}(\frac{7}{5}-1)}{2!}(-0.0057)+\frac{\frac{7}{5}(\frac{7}{5}-1)(\frac{7}{5}-2)}{3!}(-0.0007)$$

= 7071+824.6-15.96+0.392 = 7880

10^4 f(52) = 7880

$F(52) = \frac{7880}{10^4} = 0.788$ $\sin 52^0 = 0.7880$

7. Using Newton's formula for interpolation estimate population for the year 1905 from the table.

Year (x)	Population(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1891	98752				
		33533			
1901	132285		2258		
		35791		-10435	
1911	168076		-8177		41358
		27614		30923	
1921	195690		22746		
		50360			
1931	246050				

Solution.

a = 1891, h = 10

Let a+ uh = 1905

u(10) = 1905-1891 = 14

$u = \frac{14}{10} = \frac{7}{6}$

Newton's Gregory's formula is

$$f(a+uh) = f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)+\frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(a)$$

f(1905) = 98752+$\frac{7}{5}(33533)+\frac{\frac{7}{5}(\frac{7}{5}-1)}{2}(2258)+\frac{\frac{7}{5}(\frac{7}{5}-1)(\frac{7}{5}-2)}{6}$(-10435)+$\frac{\frac{7}{5}(\frac{7}{5}-1)(\frac{7}{5}-2)(\frac{7}{5}-3)}{24}$(41358)

= 98,752 + 48946·2+ 632.24 + 584.36 + 926.4192

= 147841.2192

The population for the year 1905 is 14,784 approximately

8 . From the following table find the no. of students who obtained less than 45 marks.

x	30-40	40-50	50-60	60-70	70-80
F(x)	31	42	51	35	31

Solution .

Marks less than x	No.of students	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
40	31				
		42			
50	73		9		
		51		-25	
60	124		-16		37
		35		12	
70	159		-4		
		31			
80	190				

a=40,h=10

let a +uh = 45

u(10) = 45-40

u = $\frac{5}{10} = \frac{1}{2}$

Newton's advancing formula is

F(a+uh) = f(a)

$+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)+\frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(a)$

= $31+\frac{1}{2}(42)+\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(9)+\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(-25)+\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)(\frac{1}{2}-3)}{4!}(37)$

$= 31+21-\frac{9}{8}+(-\frac{75}{48})-\frac{15}{16}(\frac{37}{21})$

$= 52-1.125-1.5625-1.4453125$

$= 47.8671875 = 47$

: The no. of students getting less than 45 marks is 47 and the no. of students who getting more than 45 is 190-47-143

9. Find the no. of men getting the wages less than is rupees from the following :

0-10	10-20	20-30	30-40
9	30	35	42

Solution .

Wages x	No. of Men	$\Delta\mathbf{f(x)}$	$\Delta^2\mathbf{f(x)}$	$\Delta^3\mathbf{f(x)}$
0-10	9			
		21		
10-20	30		-16	
		5		18
20-30	35		2	
		7		
30-40	42			

Newton's advancing formula is

$f(a+uh) = f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)$

$f(15) = 9+\frac{1}{2}(21)+\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(-16)+\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(18)$

$= 9+\frac{21}{2}+2+\frac{3}{8}(\frac{18}{16}) = 11+\frac{21}{2}+\frac{9}{8}$

$= 11+10.5+1.125- 22.625 = 23$

11. Find the second degree polynomial from the data is then find f(5)

0	1	2	3	4
3	6	11	17	27

Solution .

X	F(x)	$\Delta\mathbf{f(x)}$	$\Delta^2\mathbf{f(x)}$	$\Delta^3\mathbf{f(x)}$
0	3			
		3		
1	6		2	

2	11	5	2	0
3	18	7	2	0
4	27	9		

Newton's Gregory's formula is

$f(x) = f(0)+\frac{x}{1!}\Delta f(0)+\frac{x(x-1)}{2!}\Delta^2 f(0)$

a=0 , h=1

a+uh = 5 u =5

$f(x) = 3+x(3)+\frac{x(x-1)}{2}(2)$

$= 3+3x + x^2-x = x^2 + 2x + 3$

$f(5) = (5)^2 + 2(5) + 3=25+10+ 3 = 38.$

12. Second degree polynomial passes through the points (o, 1), (1, 3), (2, 7),(3, 13) then find the polynomial or find f(x). Also find the value of f(1.5).

Solution.

X	Y = f(x)	$\Delta\mathbf{f}(\mathbf{x})$	$\Delta^{\mathbf{2}}\mathbf{f}(\mathbf{x})$	$\Delta^{\mathbf{3}}\mathbf{f}(\mathbf{x})$
0	1	2		
1	3	4	2	0
2	7	6	2	
3	13			

Newton's Gregory's formula is

$f(x) = f(0)+\frac{x}{1!}\Delta f(0)+\frac{x(x-1)}{2!}\Delta^2 f(0)$

$= 1+x(2)+\frac{x(x-1)}{2}(2)$

$f(x) = 1 + 2x + x^2-x$

$= x^2 + x + 1$

f(1.5) = $(1.5)^2+(1.5)+1$

= 2.25+1.5+1 = 4.75

f(1.5) = 4.75

13. Find the polynomial fits the data in the following table then also find f(13).

X	3	5	7	9	11
Y	6	24	58	109	174

solution. a=3, h = 2

Let a+uh = x = 13 3+2u = x

$$u = \frac{x-3}{2}$$

X	Y =f(x)	$\Delta\mathbf{f(x)}$	$\Delta^{\mathbf{2}}\mathbf{f(x)}$	$\Delta^{\mathbf{3}}\mathbf{f(x)}$	$\Delta^{\mathbf{4}}\mathbf{f(x)}$
3	6				
		18			
5	24		16		
		34		0	
7	58		16		0
		50		0	
9	109		16		
		66			
11	174				

Newton's Gregory's formula is

$$f(a+uh) = f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)$$

$$= 6+\frac{x-3}{1!}(18)+\frac{\left(\frac{x-3}{2}\right)\left(\frac{x-3}{2}-1\right)}{2!}(16)$$

f(a+uh) = $6+9x-27+2(x-3)(x-5)=9x-21+2x^2-16x+30$

f(x) = $2x^2-7x+9$

f(13) = $2(13)^2-7(13)+9=2(169)-91+9=338+9-91$

f(13) = 2.56

14. Find the value of f(35) from the following table :

X	Y=f(x)	$\Delta\mathbf{f(x)}$	$\Delta^{\mathbf{2}}\mathbf{f(x)}$	$\Delta^{\mathbf{3}}\mathbf{f(x)}$

20	512			
30	439	-73	-20	
40	346	-93	-10	10
50	243	-103		

Solution .

Newton's advancing formula is

F(a+uh) =

$f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)+\frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(a)$

a=20 , h=10 => a+uh=35a=30, h=10

20+10h=35 (or) 30+10u=35

h= $\frac{3}{2}$,u= $\frac{1}{2}$

$f(35)=439+\frac{1}{2}(-93)+\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(-10)+\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(-10)$

$=439-\frac{93}{2}+\frac{10}{8}+\frac{30}{48}$

=439-46.5+1,25+6.625=440.875-46.5=394.375

f(35)=394.

Central Difference Interpolation Formulae

1.3.1 INTRODUCTION :

In the previous chapter, we derived and discussed Newton forward and backward interpolation formula, which are applicable for interpolation, near the beginning and end respectively of tabulated data. We shall now discuss in the present chapter, about the central difference formula, which are suitable for interpolation near the

middle of a tabulated data.

The central difference operator (δ) is defined as $\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$

The first central difference operator δf(x) defined by

$$\delta f(x) = f(x+\frac{h}{2}) - f(x-\frac{h}{2})$$

$$\delta f(x) = (E^{\frac{1}{2}} - E^{-\frac{1}{2}}) f(x)$$

We have $E = 1 + \Delta$

$$\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}} = E^{-\frac{1}{2}}(E-1) = E^{-\frac{1}{2}}\Delta \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Again by the first backward difference of f(x)

We have $\nabla f(x) = f(x)-f(x-h) = f(x)-f(x-h) = E(x)- E^{-1}f(x)$

$$\nabla f(x) = (1- E^{-1})f(x)$$

$$\nabla = (1- E^{-1})$$

$$\nabla E^{\frac{1}{2}} = E^{\frac{1}{2}} - E^{-\frac{1}{2}} = \delta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From(1) and (2) $\delta = \Delta E^{-\frac{1}{2}} = \nabla E^{\frac{1}{2}}$ $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$

The average operator u is defined as

$$U = \frac{1}{2}[E^{\frac{1}{2}} + E^{-\frac{1}{2}}]$$

Results :

(1) $U^2 = 1+\frac{1}{4}\delta^2 u^2 f(x) = \frac{1}{2}(E^{\frac{1}{2}} + E^{-\frac{1}{2}})^2$

$= \frac{1}{4}(E^{\frac{1}{2}} + E^{-\frac{1}{2}})^2 = \frac{1}{4}(E+E^{-1}+2) = \frac{1}{4}(E+E^{-1}-2+4)$

$= \frac{(u+k-1)^{(2k+1)}}{(2k+1)!} = -\frac{(k-v)^{(2k+1)}}{(2k+1)!}$

$= \frac{v(v^2-1)(v^2-4)\ldots\ldots((v^2-k^2)}{(2k+1)!}$

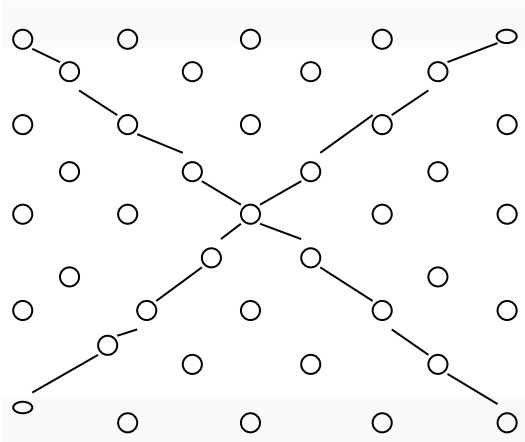
From (3), we obtain

$y_u = vy_o + \frac{v(v^2-1)}{3!}\Delta^2 y_{-1} + \frac{v(v^2-1)(v^2-4)}{5!}\Delta^4 y_{-2} + \ldots\ldots uy_1 + \frac{u(u^2-1)}{3!}\Delta^2 y_0 + \frac{u(u^2-1)(u^2-4)}{5!}\Delta^4 y_{-2} + \ldots\ldots\ldots + \frac{u(u^2-1)(u^2-4)(u^2-k^2)}{(2k+1)!}\Delta^{2k} y_{1-k} + \ldots\ldots\ldots\ldots$ ……..(5)

this can be written as

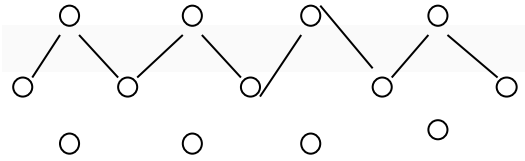
$f(u) = vf(0) + \frac{v(v^2-1)}{3!}\delta^2 f(0) + \frac{v(v^2-1)(v^2-4)}{5!}\delta^2 f(0) + \ldots\ldots + uf(1) + \frac{u(u^2-1)}{3!}\delta^2 f(1) + \frac{u(u^2-1)(u^2-4)}{5!}\delta^2 f(1) + \ldots$

this is known as Laplace-Everette's formula

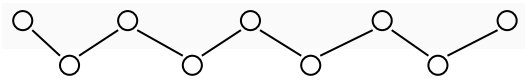


Newtons backward formula

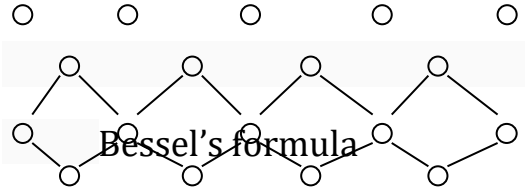
Newtons backward formula



Gauss backward formula

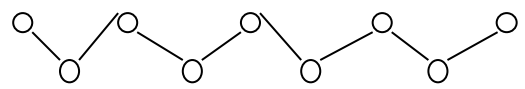


Gauss forward formula



Stirling's formula

Bessel's formula



Note 1: This formula is useful especially when using the tables in which only even order difference are tabulated.

All the difference formula can be explained with the help of following diagram.

Results : (1) $\delta = \Delta E^{-\frac{1}{2}} = \nabla E^{\frac{1}{2}}$

(2) $\Delta\nabla = \nabla\Delta = \Delta - \nabla = \delta^2$

(3) u= $\delta = \frac{1}{2}(\nabla + \Delta) = \frac{1}{2}\Delta E^{-1} + \frac{1}{2}\Delta$ (4) $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$)

Proof :

(1) $\delta = \Delta E^{-\frac{1}{2}} = \nabla E^{\frac{1}{2}}$

$\delta f(x) = f(x+\frac{h}{2}) - f(x-\frac{h}{2}) = \nabla f(x+\frac{h}{2}) = \nabla E^{\frac{1}{2}} f(x)$

$\delta = \nabla E^{\frac{1}{2}}$

(2) (a) $\Delta\nabla f(x) = \Delta\Delta E^{-1}f(x) = \Delta^2 E^{-1}f(x)$

(b) $\Delta\nabla f(x) = \Delta E^{-1}f(x) = \Delta^2 E^{-1}f(x)$

(c) $(\Delta - \nabla)f(x) = (\Delta - \nabla E^{-1})f(x) = \Delta(1-\frac{1}{E})f(x)$

$= \frac{\Delta(E-1)}{E}f(x) = \Delta.\Delta E^{-1}f(x) = \Delta^2 E^{-1}f(x)$

(d) $\delta^2 f(x) = (\Delta E^{-\frac{1}{2}})^2 f(x) = \Delta^2 E^{-1}f(x)$

$\Delta\nabla = \nabla\Delta = \Delta - \nabla = \delta^2$

(3) $u\delta f(x) = \frac{(E^{\frac{1}{2}} + E^{-\frac{1}{2}})}{2}(E^{\frac{1}{2}} - E^{-\frac{1}{2}})f(x)$

$= \frac{1}{2}(e - e^{-1})f(x) = \frac{1}{2}(e - \frac{1}{E})f(x) = \frac{1}{2}(E^2 - \frac{1}{E})f(x)$

$= \frac{(E-1)(E+1)}{2E}f(x) = \frac{\Delta}{2}(1+\frac{1}{E})f(x)$

We know that $\nabla = \Delta E^{-1} = \frac{1}{2}(\Delta + \nabla)f(x) = \frac{1}{2}(\Delta + \nabla E^{-1})f(x)$

$= \frac{1}{2}\nabla E^{-1} + \frac{1}{2}\Delta$

$$U\delta = \frac{1}{2}(\Delta + \nabla E^{-1})$$

(4) $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$

$(\frac{\Delta}{\nabla} - \frac{\nabla}{\Delta})f(x) = (\frac{\Delta}{\Delta E^{-1}} - \frac{\Delta E^{-1}}{\Delta})f(x) = (E - \frac{1}{E})f(x) = (\frac{E^2 1}{E})f(x)$

$= (E-1)\frac{(E+1)}{E} f(x) = \Delta(1+E^{-1})f(x) =$

$(\Delta + \nabla E^{-1})f(x)$

$= (\Delta + \nabla)f(x)$

$(\frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}) = \Delta + \nabla$

1.4 SOLVED PROBLEMS

1. Using Gauss forward formula find the value of y when x = 0.5 from the following table.

X	**-2**	**-1**	**0**	**1**	**2**
F(x)	**22**	**30**	**82**	**106**	**216**

Solution .

X	**F(x)**	$\delta f(x)$	$\delta^2 f(x)$	$\delta^3 f(x)$	$\delta^4 f(x)$
-2	22				
		8			
-1	30		44		
		52		-72	
0	82		-28		186
		24		114	
1	106		86		
		110			
2	216				

Gauss forward interpolation formula is

$f(x) = f(0) + \frac{x}{1!}\delta f(1/2) + \frac{x(x-1)}{2!}\delta^2 f(0) + \frac{x(x-1)(x+1)}{3!}\delta^3 f(1/2) + \frac{x(x-1)(x+1)(x-2)}{4!}\delta^4 f(0)$

$f(x) = 82 + \frac{x}{1!}(24) + \frac{x(x-1)}{2!}(-28) + \frac{x(x-1)(x+1)}{6}(114) + \frac{x(x-1)(x+1)(x-2)}{24}(186)$

$f(0.5) = 82 + (0.5)(24) + \frac{(0.5)(0.5-1)}{2}(-28)$

$$+\frac{(0.5)(0.5-1)(0.5+1)}{6}(114) + \frac{(0.5)(0.5-1)(0.5+1)(0.5-2)}{24}(186)$$

$$= 82 + 12+\frac{7}{2} - \frac{57}{24}+4.359 = 94+4.359+3.5-7.125$$

f(1/2) = 94.734

2. Using Gauss backward interpretation formula find the value at y when x=0.5[0,-1,+1,-2,+2]

Solution.

X	**F(x)**	$\delta f(x)$	$\delta^2 f(x)$	$\delta^3 f(x)$	$\delta^4 f(x)$
-2	22				
		8			
-1	30		44		
		52		-72	
0	82		-28		186
		24		114	
1	106		86		
		110			
2	216				

Gauss backward interpolation formula is

$$F(x) = f(0)+ \frac{x}{1!}\delta f(-1/2)+ \frac{x(x+1)}{2!}\delta^2 f(0) +\frac{x(x-1)(x+1)}{3!}\delta^2 f(-1/2)+ \frac{x(x-1)(x+1)(x+2)}{4!}(186)$$

$$f(0.5) = f(\tfrac{1}{2}) = 82+(0.5)(52) +\frac{(0.5)(0.5+1)}{2}(28) + \frac{(0.5)(0.5+1)(0.5-1)}{6}(-72)+ \frac{(0.5)(0.5+1)(0.5-1)(0.5+2)}{24}(186)$$

$$= 82 + 26-10.5+ 4.5-7.2656 = 112.5-17.7656$$

f(0.5) = 94.7344

a. Using Gauss backward interpolation formula and the value of f(-1/2) from the following data

X	**F(x)**	$\delta f(x)$	$\delta^2 f(x)$	$\delta^3 f(x)$	$\delta^4 f(x)$
-2	22				

-1		8	14		
	30			18	
0		22	32		6
	52			24	
1		54	56		
	106				
2		110			
	216				

Solution. Gauss backward interpolation formula is

$F(x) = f(0)+\frac{x}{1!}\delta f(-1/2)+\frac{x(x+1)}{2!}\delta^2 f(0)+\frac{x(x-1)(x+1)}{3!}\delta^2 f(-1/2)+\frac{x(x-1)(x+1)(x+2)}{4!}\delta^4 f(0)$

$= 52+\frac{x}{1!}(22)+\frac{x(x+1)}{2}(32)+\frac{x(x^2-1)}{6}(18)+\frac{x(x^2-1)(x+2)}{4!}(16)$

$= 52+11+\frac{\frac{1}{2}(\frac{1}{2}+1)}{2}(32)+\frac{\frac{1}{2}(\frac{1}{4}-1)}{6}(18)+\ )+\frac{\frac{1}{2}(\frac{1}{4}-1)(\frac{1}{2}+2)}{24}(16)$

$f(1/2) = 63+ 12-1.125-0.234375 = 75-1.359375 = 73.64$

4. Apply Gauss forward formula to obtain f(33) given that

X	**25**	**30**	**35**	**40**
F(x)	**0.2707**	**0.3027**	**0.3386**	**0.3794**

Solution .

X	10^4 **F(x)**	$10^4\delta f(x)$	$10^4\delta^2 f(x)$	$10^4\ \delta^3 f(x)$
-1	2707			
		320		
0	3027		39	
		359		10
1	3386		49	
		408		
2	3794			

Gauss forward interpolation formula is

$f(x) = f(0)+\frac{x}{1!}\delta f(1/2)+\frac{x(x-1)}{2!}\delta^2 f(0)+\frac{x(x-1)(x+1)}{3!}\delta^3 f(1/2)$

let us take a = 30 as origin h= 5

put a+uh = 32 $\quad 30+5u = 32 \Rightarrow u = \frac{2}{5}$

$$10^4 f(2/5) = 3027 + \frac{2}{5}(359) + \frac{\frac{2}{5}(\frac{2}{5}-1)}{2}(39) + \frac{\frac{2}{5}(\frac{2}{5}-1)(\frac{2}{5}+1)}{6}(10)$$

$$10^4 f(2/5) = 3027+143.6-4.68-0.045 = 3170.6-4.105 = 3165.36$$

$$f(\tfrac{2}{5}) = 0.3165$$

f(x) = 0.3165 when x =32

5. Use Gauss forward formula to find the value of y when y=3.75 from the following table.

X	2.5	3.0	3.5	4.0	4.5	5.0
Y	24.145	22.043	20.225	18.644	17.262	16.047

Solution. Taking 3.2 as the origin and 0.5 as the unit, the value of y required will be value for

$$u = \frac{3.75-3.5}{0.5} = 0.5$$

U	$\mathbf{Y_u}$	$\boldsymbol{\delta Y_u}$	$\boldsymbol{\delta^2 Y_u}$	$\boldsymbol{\delta^3 Y_u}$	$\boldsymbol{\delta^4 Y_u}$	$\boldsymbol{\delta^5 Y_u}$
-2	24.145					
		-2.102				
-1	22.043		0.284			
		-1.818		-0.047		
0	20.225		0.237		0.009	
		-1.581		-0.038		-0.003
1	18.644		0.99		0.006	
		-1.382		-0.032		
2	17.262		0.167			
		-1.215				
3	16.047					

Using the Gauss forward formula we have

$$F(u) = f(0) + u\delta f(0) + \frac{u(u-1)}{2!}\delta f(1) + \frac{u(u-1)(u+1)}{6}\delta^2 f(-1/2) + \frac{u(u-1)(u+1)(u-2)}{24}\delta^4 f(1/2)$$

$$Y_5 = 20.225+5(-1.5881)+\frac{0.5(0.5-1)}{2}(0.237)+\frac{0.5(0.5-1)(0.5+1)}{6}(-0.38)+\frac{0.5(0.5-1)(0.5+1)(0.5-2)}{24}(0.009)$$

$$+\frac{0.5(0.5-1)(0.5+1)(0.5-2)(0.5+2)}{120}(-0.003)$$

= 20.225-0.7905-0.029625+ 0.00238 + 0.9023750 + 0.0002106

= 19.40 approx

Thus value of y for x = 3.75 is 19.40 approximately.

6. Use Gauss forward formula to find y_{30} given that y_{21} = 18.4708, y_{25} = 17.8144, y_{29} = 17.1070, y_{33} = 16.3432, y_{37} = 15.5154.

X	21	25	29	33	37
Y	18.4708	17.8144	17.1070	16.3432	15.5154

Solution. Arrange the above data in the following form :

Take the origin x = 29 and h = u as the unit; then the value of y for u = $\frac{30-29}{4}$ = 0.25 is to be found. Construct the following difference table.

X	$u = \frac{x-29}{4}$	Y_u	δY_u	$\delta^2 Y_u$	$\delta^3 Y_u$	$\delta^4 Y_u$
21	-2	18.4708				
			-0.6564			
25	-1	17.8144		-0.0510		
			-0.7074		-0.0054	
29	0	17.1070		-0.0564		-
			-0.7638		-00076	0.0022
33	1	16.3432		-0.640		
			-0.8278			
37	2	15.5154`				

Now applying the gauss forward interpolation formula.

We get

f(0.4) = 17.1070+ (0.25) (-0.7638)+$\frac{(0.25)(-0.75)}{2}(-0.0564)+\frac{(1.25)(0.25)(-0.75)}{6}(-0.0076)+$

$\frac{(1.25)(0.25)(-0.75)(-1.75)}{24}(-0.0022)$

= 17.1070-0.19095+0.0052875+0.00002968-0.00000375

= 16.9216 approx.

7. Given that $\sqrt{12500}$ = 111.803399; $\sqrt{12510}$ = 111.848111; $\sqrt{12520}$ = 111.892806;

$\sqrt{12530}$ = 111.937483 Show by Gauss's backward formula that $\sqrt{12516}$ = 111.874930

Solution. Taking 12520 as the origin h 10 as the unit, the value of y required is for

$$u = \frac{12516-12520}{10} = -0.4$$

U	Y_u	δY_u	$\delta^2 Y_u$	$\delta^3 Y_u$
-2	111.803399			
		0.044712		
-1	111.848111		-0.00017	
		0.044695		-0.000001
0	111.892806		-0.000018	
		0.044677		
1	111.937483			

Now by using Gauss backward formula, we have

$y_{-0.4}$ = 111.8928+ (-0.4)(0.044695)-+$\frac{(0.6)(-0.4)}{2!}$(-0.000018) +

$\frac{(0.6)(-0.4)(-1.4)}{3!}$(-0.000001)

= 111.8928-0.017878 + negligible quantities = 111.874930

i.e., $\sqrt{12516}$ = 111.874930.

8. Apply Gauss forward formula to find the value of f(x) at x = 3.75 from the following table.

X	2.5	3.0	3.5	4.0	4.5
F(x)	24.145	22.043	20.225	18.644	17.262

Solution. Let us take a = 3.5 as origin, h = 0.5

Put a + uh = 3.75 => 3.5+ 0.5u = 3.75

0.5u = 0.25

u = 0.5 = ½

u	10^3 f(u)	$10^3\ \delta f(u)$	$10^3\ \delta^2$ f(u)	$10^3\ \delta^3$f(u)	$10^3\ \delta^4$ f(u)
-2	24145	-2102			
			284		
-1	22043	-1818		-47	

			237		9
0	20225	-1581		-38	
			199		6
1	18644	-1382		-32	
			167		
2	17262	-1215			

$10^3 f(\frac{1}{2}) = 20225+\frac{1}{2}(-1581)+\frac{\frac{1}{2}(\frac{1}{2}-1)}{2}(237)+\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}+1)}{6}(-38)+$

$$\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}+1)((\frac{1}{2}-2)}{24}(9)$$

$= 20225-790.5-29.625+2.375+0.2709 = 19407.46$

$f(\frac{1}{2}) = 19.408$

$f(x) = 19.408$ when x= 375.

9**. Find the value of cos 51^0. 42' by Gauss backward formula given that**

X	**50^0**	**51^0**	**52^0**	**53^0**	**54^0**
Cosx	**0.6428**	**0.6293**	**0.6157**	**0.6018**	**0.5878**

Solution : Let us take a =50^0 as origin h =1^0

Put a+uh = 51^042^1,52^0 +u = 51^042^I => u= $-18^I = \frac{-18}{60}$ = 0.3, u = -0.3

u	**10^4 f(u)**	**$10^4\ \delta f(u)$**	**$10^4\ \delta^2$ f(u)**	**$10^4\ \delta$f(u)**	**$10^4\ \delta^4$ f(u)**
-2	6428	-135			
-1	6293	-136	-1	-2	
					4
0	6157	-139	-3	2	
1	6018	-140	-1		
2	5878				

Gauss backward interpolation formula is

f(u) = f(0)+$\frac{u}{1!}$ $\boldsymbol{\delta}$ f($\frac{1}{2}$) +$\frac{u(u+1)}{2}$ $\boldsymbol{\delta^2}$ f(0)+ $\frac{u(u+1)(u-1)}{3!}$ δ^3(-$\frac{1}{2}$)+

$\frac{u(u+1)(u-1)(u+2)}{4!}$ δ^4f(0)

10^4f(-0.3) = 6157 + (-0.3)(-136)+$\frac{(-0.3)(0.7)}{2}$(-3) + $\frac{(-0.3)(0.7)(-1.3)}{6}$(-2) +

$\frac{(-0.3)(0.7)(-1.3)(1.7)}{24}$(2)

= 6157 + 40.8 + 0.315 -0,091 + 0.07735 = 6198.1013

f(-0.3) = 0.61981013

cos $57^0 42^{|}$ = 0.6198.

10. Find by Gauss backward formula the sales of the conceived by the year 1936.

Given that

Year	**1901**	**1911**	**1921**	**1931**	**1941**	**1951**
Sales	**12**	**15**	**20**	**27**	**29**	**52**

(in thousand of rupees)

Solution : let us take a = 1931 as origin

H = 10

a + uh = 1936 => 1931 + 10u = 1936 => 10u = 5

=> U=$\frac{1}{2}$[0,-1,1,-2,2,-3]

U	**f(u)**	$\boldsymbol{\delta f(u)}$	$\boldsymbol{\delta^2 f(u)}$	$\boldsymbol{\delta^3 f(u)}$	$\boldsymbol{\delta^4 f(u)}$
-3	12				
		3			
-2	15		2		
		5		0	
-1	20		2		3
		7		3	
0	27		5		-7
		12		-4	
1	39		1		
		13			

2	52				

Gauss backward interpolation formula is

$f(u = f(0) + \frac{u}{1!}\delta f(\frac{1}{2}) + \frac{u(u+1)}{2}\delta^{2}f(0) + \frac{u(u+1)(u-1)}{3!}\delta^{3}f(\frac{1}{2}) +$

$\frac{u(u+1)(u-1)(u+2)}{4!}\delta^{4}f(0$

$f(\frac{1}{2}) = 27 + (0.5)7 + \frac{(0.5)(0.5+1)}{2}(5) + \frac{(-0.5)(0.5+1)(0.5-1)}{6}(3) +$

$\frac{(-0.5)(0.5+1)(0.5-1)(0.5-1)}{24}(-7)$

= 27 + 3.5 + 1.875 -0.1875 + 0.2734 = 32.46 thousand 32.7

In the year 1936 = 32.7 thousands. Difference table:

X	$U = \frac{x-30}{5}$	Y_u	δy_u	$\delta^2 y_u$	$\delta^3 y_u$	$\delta^4 y_u$
20	-2	49225				
			-909			
25	-1	48316		-171		
			-1080		-59	
30	0	47236		-230		-21
			-1310		-80	
35	1	45926		-310		
			-1620			
40	2	44306				

Using Stirlings formula

Putting u = -0.4 we get difference

$y_{-0.4} = 47236 + (-0.4)\frac{-1310-1080}{2} + \frac{0.16}{2}(-230) + \frac{(-0.4)(0.16-1)}{6}(\frac{80-59}{2})$

$+ \frac{(0.16)(0.16-1)}{24}(-21)$

= 47326+ 478 -18.4 – 3.8920 + 1176

= 47692

Y_{28} = 47692

12. Apply Bessel's formula to obtain y_{25}, given y_{20} = 2854, y_{24} = 3162, y_{32} = 3992

Solution. take 24 as origin and 4 $u = \frac{25-24}{4} = \frac{1}{4} = 0.25$

Difference table is

X	$u = \frac{x-24}{4}$	Y_u	δy_u	$\delta^2 y_u$	$\delta^3 y_u$
20	-1	2854			
			308		
24	0	3162		74	
			382		-8
28	1	3544		64	
			448		
32	2	3992			

Using bessel's formula, we get

$Y_{1/4} = \frac{1}{2}(3162+3544)+(\frac{1}{4}-\frac{1}{2})382+\frac{\frac{1}{4}(\frac{1}{4}-\frac{1}{2})}{2!}(\frac{14+66}{2})+\frac{(\frac{1}{4}-\frac{1}{2})(\frac{1}{4})(\frac{1}{4}-1)}{3!}(-8)$

$= 3353 - 95.5 - 6.5625 - 0.0625 = 3250.875.$

13.Given y_{20} =24 , y_{24} =32, y_{28} =35, y_{32} =40 find y_{25} by Bessel's formula.

Solution : Taking origin at x =24 and u as the unit.

Find y_u for $u = \frac{25-24}{4} = \frac{1}{4} = 0.25$

The difference table is

X	$U = \frac{x-24}{4}$	Y_u	δy_u	$\delta^2 y_u$	$\delta^3 y_u$
20	-1	24			
			8		
24	0	32		-5	
			3		7
28	1	35		2	
			5		
32	2	40			

By using Bessel's formula we have

$$y_{\frac{1}{4}} = \frac{32+35}{2} + (\frac{1}{4} - \frac{1}{2})3 + \frac{\frac{1}{4}\left(\frac{1}{4}-1\right)\left(\frac{-5+2}{2}\right)}{2} + \frac{(\frac{1}{4}-\frac{1}{2})\frac{1}{4}\left(\frac{1}{4}-1\right)}{6}(7)$$

= 33.5-0.75+ 0.1406+0.54687 = 32.945287.

14.Given that

θ	0^0	5^0	10^0	15^0	20^0	25^0	30^0
$\tan\theta$	**0.0000**	**0.0875**	**0.1763**	**0.2679**	**0.3640**	**0.4663**	**0.5774**

Show that tan 16^0 = 0.2867 (use Stirilling's formula)

Solution : Taking 15^0 as origin and 5^0 as the unit , the value required will be for

$$U = \frac{16^0-15^0}{15^0} = 0.2^0$$

θ	U	Y_u	δy_u	$\delta^2 y_u$	$\delta^3 y_u$	$\delta^4 y_u$	$\delta^5 y_u$	$\delta^6 y_u$
0^0	-3	0.0000						
5^0	-2	0.0875	0.0872					
10^0	-1	0.1763	0.0888	0.0013	0.0015			
15^0	0	0.2679	0.916	0.0028	0.0017	0.0002	-0.0002	
20^0	1	0.3640	0.961	0.0045	0.0017	0.0000	0.0009	0.0011
25^0	2	0.4663	0.1023	0.0062	0.0026	0.0009		
30^0	3	0.5774	0.1111	0.0088				

By using Stirlings formula, we have

Tan 16^0 = $y_{0.2}$ = 0.2679+ $\frac{0.2}{2}$ (0.0961+0.0916){+ $\frac{0.04}{2}$ (0.0045)+ $\frac{0.2(0.4-1)}{2}$ $\frac{1}{2}$(0.0017+0.0017) +

$\frac{0.4(0.4-1)}{24}$ (0.0000) + $\frac{0.2(0.4-1)(0.04-1)}{120}$ $\frac{1}{2}$ (-0.0002+0.0009) + $\frac{0.2(0.04-1)(0.04-4)}{720}$(0.0011)

= 0.2679 + 0.01877 + 0.00009 + negligible

= 0.2867.

1.5 Applications in engineering :

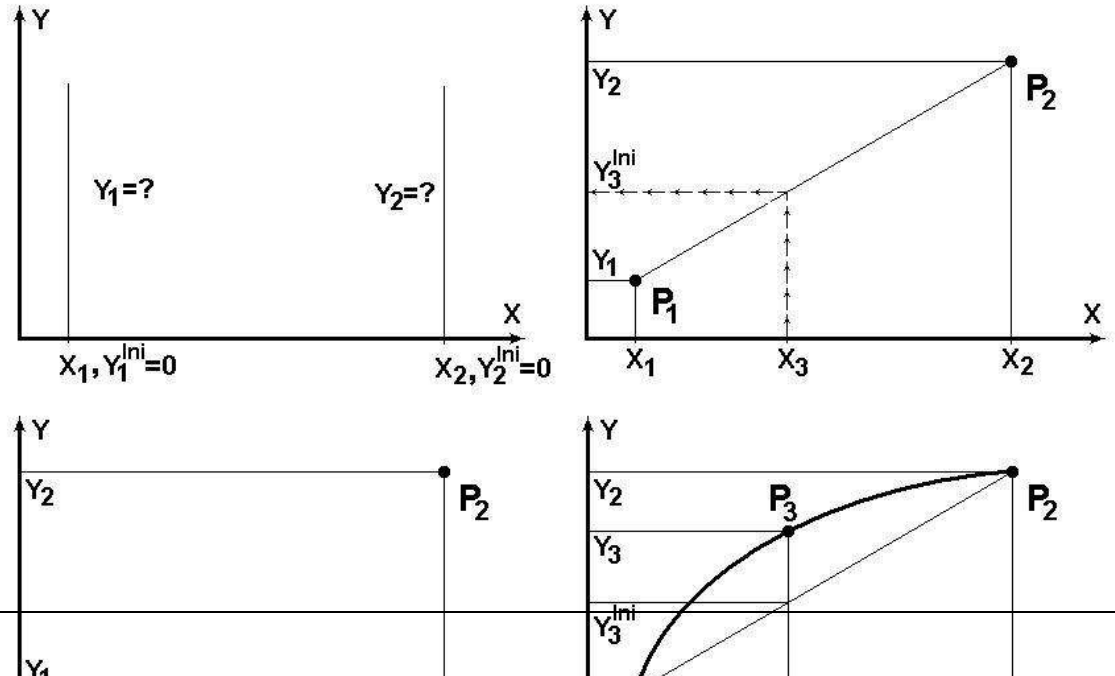
General Ideas Regarding the Use of the Interpolation in the Numerical Analysis

There are certain cases when approximation solutions are useful within the numerical methods field itself.

Some of these cases are presented in the next sections.

Approximation of the Initial Solution for Iterative Methods

Let us consider a theoretical problem where the formulation of the problem depends on some parameters and an iterative solution is possible. It is well known that the final solution depends on the initial approximation in terms of accuracy and of computer runtime.



2. NUMERICAL INTEGRATION

Numerical integration is the approximate computation of an integral using numerical techniques. The numerical computation of an integral is sometimes called quadrature. Ueberhuber (1997, p. 71) uses the word "quadrature" to mean numerical computation of a univariate integral, and "cubature" to mean numerical computation of a multiple integral.

There are a wide range of methods available for numerical integration. A good source for such techniques is Press et al. (1992). Numerical integration is implemented in the Wolfram Language as NIntegrate[f, {x, xmin, xmax}].

The most straightforward numerical integration technique uses the Newton-Cotes formulas (also called quadrature formulas), which approximate a function tabulated at a sequence of regularly spaced intervals by various degree polynomials. If the endpoints are tabulated, then the 2- and 3-point formulas are called the trapezoidal rule

and Simpson's rule, respectively. The 5-point formula is called Boole's rule. A generalization of the trapezoidal rule is Romberg integration, which can yield accurate results for many fewer function evaluations.

If the functions are known analytically instead of being tabulated at equally spaced intervals, the best numerical method of integration is called Gaussian quadrature. By picking the abscissas at which to evaluate the function, Gaussian quadrature produces the most accurate approximations possible. However, given the speed of modern computers, the additional complication of the Gaussian

quadrature formalism often makes it less desirable than simply brute-force calculating twice as many points on a regular grid (which also permits the already computed values of the function to be re-used). An excellent reference for Gaussian quadrature is Hildebrand (1956).

Modern numerical integrations methods based on information theory have been developed to simulate information systems such as computer controlled systems, communication systems, and control systems since in these cases, the classical methods (which are based on

approximation theory) are not as efficient (Smith 1974).

2.1 INTRODUCTION :

The numerical integration is the process of evaluating the approximate Value of the integration $\int_a^b f(x)dx$ from a set of numerical values of f(x). If the problem is to be applied to the integration of function of a single variable, then the process is called as quadrature.

The problem of numerical integration is evolved by first approximating the f(x) by a polynomial with the help of interpolation formula and then integrating this expression between the limits.

$\therefore$ To find the value of $\int_a^b f(x)\,dx$, first express the function f(x) by interpolation formula, say p(x) and then integrate between the limits.

$$\therefore \int_a^b f(x)dx = \int_a^b p(x)dx$$

The error is given by $\int_a^b f(x)dx - \int_a^b p(x)dx$

2.2 A GENERAL QUADRATURE FORMULA FOR NUMERICAL INTEGRATION

Consider the integral $\int_a^b f(x)dx$. Let f(x) be given for some equally spaced values of the orguments say ,x_0 , x_0+h ,x_0+2h,…. Let the range (b-a) be divided into n equal parts, each of which is of width h.

$$b-a = nh$$

$$h = \frac{b-a}{n}$$

let $x_0 = a, x_1 = x_0+h = a+h$, $x_2 = a+2h$ ….. $x_n = a+nh = b$

$$I = \int_a^{a+nh} f(x)dx$$

Putting $x = a+uh$

dx = hdu

x -> a -> u ->0

x ->a+uh -> u->n

I = $\int_a^n f(a+uh)du$ = h $\int_a^n f(a+uh)du$

No write as f(a+uh) on the Newton's interpolation formula as

= h $\int_0^n [f(a) + \frac{u}{1!}\Delta f(a) + \frac{u(u-1)}{2!}\Delta^2 f(a) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(a) + \cdots +$ $\frac{u(u-1)\ldots.(u-n+1)}{n!}\Delta^n f(a)\ ]\ du$

= h [f(a).u + $\frac{u^2}{2}\Delta f(a)$+($\frac{u^3}{3} - \frac{u^2}{2}$) $\Delta^2 f(a)$+ $\frac{(\frac{u^4}{4} - \frac{3u^3}{3} + \frac{2u^2}{2})}{6}\Delta^3 f(a) + \cdots +]$

= h[nf(a) + $\frac{n^2}{2}\Delta f(a)$ + ($\frac{n^2}{6} - \frac{n^2}{4}$) $\Delta^2 f(a)$ +($\frac{n^4}{24} - \frac{n^3}{6} + \frac{n^2}{6}$)$\Delta^3 f(a)$ +]

$\int_a^b f(x)dx$ = h[ny_0 + $\frac{n^2}{2}$(y_1-y_0)+ ($\frac{n^3}{6} - \frac{n^2}{4}$)($y_2-2y_1$+$y_0$)+$\left(\frac{n^4}{24} - \frac{n^3}{6} + \frac{n^2}{6}\right)$

(y_3-3y_2+3y_1-y_0)+……..] ……..(1)

$\int_a^b f(x)dx$ = $\int_a^{a+nh} f(x)dx$

This is known as general quadrature formula we deduce the following important quadratic formulas by taking n= 1,2,3….

2.3 TRAPEZOIDAL RULE :

Put n=1 eq(1) and neglect the second and higher order differences when n=1 , the argument are a, a+h and its corresponding functional values are y_0 and y_1 thus we have

$\int_a^{a+nh} f(x)dx$ = h[y_0 +$\frac{1}{2}$(y_1-y_0)] = $\frac{h}{2}$ (y_0+y_1)

I_1 = $\int_a^{a+nh} f(x)dx$ = $\frac{h}{2}$ (y_0+y_1)

I_2 = $\int_a^{a+nh} f(x)dx$ = $\frac{h}{2}$ (y_0+y_1)

I_3 = $\int_{a+2h}^{a+3h} f(x)dx$ = $\frac{h}{2}$ (y_2+y_3)

……..

……..

$I_n = \int_{a+(n-1)h}^{a+nh} f(x)dx = \frac{h}{2}(y_{n-1}+y_n)$

Adding these n integrals, we have

$\int_a^{a+nh=b} f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

This rule is called the trapezoidal rule.

2.4 SIMPSON'S $\frac{1}{3}$ RULE :

Put n=2 in the equation (1) and neglect the third and higher order difference. In this case the arguments are a, a+h, a+2h the corresponding entries are y_0,y_1 and y_2, thus we have

$\int_a^{a+2h} f(x)dx = h[2y_0 + 2(y_1-y_0)+(\frac{y}{3}-1)(y_2-2y_1+y_0)]$

$= \frac{h}{3}[y_0+4y_1+y_2]$

Similarly $I_2 = \int_a^{a+2h} f(x)dx = \frac{h}{3}(y_0+4y_1+y_2)$

$I_3 = \int_{a+4h}^{a+6h} f(x)dx = \frac{h}{3}(y_4+4y_5+y_6)$

……..

……..

$I_n = \int_{a+(n-2)h}^{a+6h} f(x)dx = \frac{h}{3}(y_{n-2}+4y_{n-1}+y_n)$

Adding these integrals, we have $I = \int_a^b f(x)dx$

$= \frac{h}{3}[(y_0+y_n)+(y_1+y_3+y_5+\ldots+y_{n-1})+2(y_2+y_4+\ldots.+y_{n-2})]$

This is known as SIMPSON'S $\frac{1}{3}$ RULE

2.5 SIMPSON'S $\frac{3}{8}$ RULE :

Put n=3 in equation(1) and neglect fourth and higher order differences. In this case, the arguats are a, a+h, a+2h and a+3h and the corresponding fractional values are y_0, y_1, y_2 and y_3, thus we have

$$\int_a^{a+3h} f(x)dx = n[3y_0+\frac{9}{2}(y_1-y_0)+(\frac{9}{2}-\frac{9}{4})(y_2-2y_1+y_0) + (\frac{27}{8}-\frac{27}{6}-\frac{9}{6})(y_3-3y_2+3y_1-y_0)]$$

$$= h[3y_0+\frac{9}{2}(y_1-y_0)+(\frac{9}{2}-\frac{9}{4})(y_2-2y_1+y_0) + (\frac{27}{8}-\frac{27}{6}-\frac{9}{6})(y_3-3y_2+3y_1-y_0)]$$

$$= h[3y_0+\frac{9}{2}(y_1-y_0)+(\frac{9}{4})(y_2-2y_1+y_0) + (\frac{3}{8})(y_3-3y_2+3y_1-y_0)]$$

$$I_1 = \int_a^{a+3h} f(x)dx = \frac{3h}{8}(y_0+3y_1+3y_2+y_3)$$

$$I_2 = \int_{a+4h}^{a+6h} f(x)dx = \frac{3h}{8}(y_3+3y_4+3y_5+y_6)$$

……..

……..

$$I_n = \int_{a+(n-2)h}^{a+6h} f(x)dx = \frac{3h}{8}(y_{n-3}+3y_{n-2}+3y_{n-1}+y_n)$$

Adding all these integrals we get

$$\int_a^{a+nh} f(x)dx = \frac{3h}{8}[(y_0+y_n)+3(y_1+y_2+y_4+\ldots+y_{n-1})+2(y_3+y_6+\ldots.+y_{n-3})]$$

This formulas is called the simpson's rule.

Note : Applying this formula, the number of sub-internals should be taken as multiple of three

2.6 WEDDLE'S RULE :

put n=6 in equation (1) and neglect the seventh and higher order differences. in this case the arguments are from a to a+6h and there will be only seven functional values, thus we get

$$\int_a^{a+6h} f(x)dx = h[6y_0+18\Delta y_0+27\Delta^2 y_0+24\Delta^3 y_0+\frac{123}{10}\Delta^4 y_0+\frac{33}{10}\Delta^5 y_0+\frac{41}{140}\Delta^6 y_0]$$

Replacing $\frac{41}{140}\Delta^6 y_0$ by $\frac{3}{10}\Delta^6 y_0$,

Since the error involved is negligible.

Thus above equation can be expressed as

$$\int_a^{a+6h} f(x)dx = h[6y_0+18\Delta y_0+27\Delta^2 y_0+24\Delta^3 y_0+\frac{123}{10}\Delta^4 y_0+\frac{33}{10}\Delta^5 y_0+\frac{3}{10}\Delta^6 y_0]$$

$= \frac{3h}{10}(y_0+5y_1+y_2+ 6y_3+y_4+5y_5+y_6)$

Similarly $\int_{a+6h}^{a+12h} f(x)dx = \frac{3h}{10}(y_6+5y_7+y_8+ 6y_9+y_{10}+5y_{11}+y_{12})$

…….

…….

$\int_{a+(n-6)h}^{a+nh} f(x)dx = \frac{3h}{10}(y_{n-6}+5y_{n-5}+y_{n-4}+ 6y_{n-3}+y_{n-2}+5y_{n-1}+y_n)$

Adding these integrals, we have

$\int_a^{a+nh} f(x)dx = \frac{3h}{10}(y_0+5y_1+y_2+ 6y_3+y_4+5y_5+2y_6+ 5y_7+y_8 +\ldots\ldots\ldots)$

This formula is known as Weddle's rule.

Note:

1. This rule is more accurate than the Simpson's rule, but it requires atleast seven consecutive values of the function
2. This method is applicable when n is a multiple of 6.

2.7 ERROR IN QUADRATURE FORMULA :

The error terms in the quadrature formula is given by

$$E = \int_a^b y dx - \int_a^b p(x)dx$$

where P(x) is the polynomial representing the function y = f(x) in the interval [a, b].

(i) Error in Trapezoidal Rule : Expanding y = f(x), about $x = x_o$ by Taylor's series, we obtain

$$y = y_0 + (x-x_0)y_0 + \frac{(x-x0)^2}{2!} y_0^{11} + \ldots\ldots\ldots\ldots(1)$$

$$\int_{x_0}^{x_0+h} y\,dx = \int_{x_0}^{x_0+h} [y_0 + (x - x_0)y_0 + \frac{(x-x_0)^2}{2!} y_0 + \cdots]dx$$

$$= y_0 n + \frac{h^2}{2!} y_0 + \frac{h^2}{3!} y_0 + \ldots\ldots\ldots\ldots(2)$$

But area of the first trapezium in the interval $[x_o, x_1] = \frac{1}{2}h(y_0 + y_1)$ ….(3)

Then putting $x = x_o + h$ and $y = y_1$ in (1), we obtain

$$y_1 = y_0 + h\,y_0 + \frac{h^2}{2!}y_0 + \ldots\ldots\ldots\ldots$$

Putting this value of y, in (3), we have

$$B = \frac{1}{2}h[y_0 + y_0 + h\,y_0 + \frac{h^2}{2!}y_0 + \ldots\ldots\ldots] = h\,y_0 + \frac{h^2}{2!}y_0 + \frac{h^3}{2.2!}y_0 + \ldots\ldots \quad \ldots\ldots(4)$$

Hence the error in the interval $[x_o, x_1]$

$$= \int_{x_0}^{x_1} y\,dx - B_1 = [\frac{1}{3!} - \frac{1}{2.2!}]\,h^3 y_0 + \ldots\ldots\ldots\ldots = -\frac{h^3}{12}y_0 + \ldots\ldots$$

Principal part of the error in $[x_o, x_1]$ = $-\frac{h^3}{12}y_0$

And similarly principal part of the error in $[x_1, x_2]$ = $-\frac{h^3}{12}y_1$ etc.,

Thus, the total error = $-\frac{h^3}{12}[y_0 + y_1 + y_{n-1}]$ = E , say

Then assuming that y''(x) is the largest of the n quantities y_o , y_1 ,.. . ., y_{n-1} ,

we can say that

$$E < -\frac{nh^3}{12}y^{11}(x) = -\frac{(b-a)h^2}{12}y^{11}(x) \text{ as } b\text{-}a = nh \quad \ldots\ldots\ldots\ldots\ldots(5)$$

Thus the error in the trapezoidal rule is of the order h^2.

(ii) Error in Simpson's $\frac{1}{3}$ Rule : Expanding y = f(x) about $x = x_o$ by Taylor's series, we have

$$y = y_0 + (x\text{-}x_0)y_0 + \frac{(x-x0)^2}{2!}y_0 + \ldots\ldots\ldots\ldots$$

$$\int_{x_0}^{x^2} y\,dx = \int_{x_0}^{x_0+2h}[y_0 - (x - x_0) + \frac{(x-x_0)^2}{2!}\,y_{0+\ldots}]\,dr$$

$$2hy_0 + \frac{4h^2}{2!}y_0 + \frac{8h^3}{3!}y_0 + \frac{16h^4}{4!}y_0 + \ldots + \frac{32h^5}{5!}y_0 + \ldots\ldots\ldots \quad \ldots\ldots\ldots\ldots(2)$$

and area over the first double strip by Simpson's 1/3 rule

$$= \frac{1}{3}h(y_0 + 4y_1 + y_2) = A_1 \text{ say}$$

Then putting $x = x_0 + h$ and $y = y_1$ in (1), we get

$$y_2 = y_0 + 2hy_0 + \frac{4h^2}{2!}y_0 + \frac{8h^3}{3!}y_0 + \ldots\ldots\ldots\ldots$$

Further putting $x = x_o + 2h$ and y = 1/2 in (1), we get

$y_2 = y_0 + 2hy_0 + \frac{4h^2}{2!}y_0 + \frac{8h^3}{3!}y_0 + \ldots\ldots\ldots$

(3) → $A_1 = \frac{h}{3}[y_0+4(y_0+ hy_0+ \frac{h^2}{2!} y_{0+\ldots\ldots\ldots})+(y_0+2h\, y_0+ \frac{4h^2}{2!} y_0 + \frac{8h^3}{3!} y_0 + \ldots\ldots\ldots)]$

$= 2hy_0 +2h^2y_0 + \frac{4h^3}{3} y_0 + \frac{2h^2}{8} y_0 + \frac{5h^5}{18} y_0 +\ldots\ldots\ldots$ ……(4)

Hence the error in the interval $[x_o, x_2] = \int_{x_0}^{x} y\, dx - A_1$

$= [\frac{4}{15} - \frac{5}{18}] h^5y_0^5 +\ldots\ldots$ …….(5)

and the principal part of the error in $[x_o, x_2]$

$= [\frac{4}{15} - \frac{5}{18}] h^5y_0^5 = - -\frac{h^5}{90}y_0^5$

On the same lines principal part of the error in $[x_2, x_4]$

$= \frac{h^5}{90}y_2^5$, etc.,

Total error = $- -\frac{x^5}{90}[y_0^4 + y_2^4 +\ldots\ldots\ldots.+ y_2^4 (n-1)] = E$, Say ……(6)

The assuming $y_{1v}(x)$ as the largest of y_0^{1V} , y_2^{1V} …… , y_{2n-2}^{1V}

We readily obtain $E<-\frac{n/5}{90} y_0^{1V}(x) = - \frac{(b-a)h^y y^5(x)}{180}$ …….(7) [2nh = b-a]

(iii) Error in Simpson's 3/8 rule : Proceeding as above, we have the principal part of the error in the interval $[x_o, x_3]$ as $= -\frac{3h^5}{80} y^5$

(iv) Error in Weddle's rule: Principal part of the error in the interval$[x_0,x_6] = -(h^7/140)y_0^5$

2.8 SOLVED PROBLEMS ::

1.Evalaute the value of the integral using $\int_2^{1.4}(sinx - logx + e^x)$dx

(a) The Trapoidal rule, (b) Simpson's one-third rule,

(c) Simpson's three eight rule.

Solution. In this case divide the range of integration into twelve equal parts by taking h = -1 and then the values of the function y = sin x -$\log_e x$ +e are computed for each point of subdivision which are as under

x	Sinx	$\text{Log}_e x=\log_{10} x x 2.30258$	e^x	$Y=\sin x-\log_e x+e^x$

0.2	0.19867	-1.60943	1.22140	3.02950
0.3	0.29552	-1.20397	1.34986	2.84935
0.4	0.38942	-0.91629	1.49182	2.79753
0.5	0.47943	-0.69315	1.64872	2.82130
0.6	0.5646	-0.51083	1.82212	2.89759
0.7	0.64422	-0.35667	2.01375	3.01464
0.8	0.71736	-0.22314	2.22554	3.16604
0.9	0.78333	-0.10537	2.45960	3.34830
1.0	0.84147	+0.00000	2.71828	3.55975
1.1	0.89121	0.09530	3.00417	3.80008
1.2	0.93204	0.18232	3.32012	4.06984
1.3	0.96356	0.26236	3.66930	4.37050
1.4	0.98545	0.33647	4.05520	4.70418

(a)By Trapezoidal rule, we have

$\int_2^{1.4}$(sin x -$\log_e{}^x$ + e^x)dx = $\int_2^{2+12x\ 0.1} y\ dx$

Here n = 12; h = 0.1, x_o = 0.2, y = sin x -$\log_e{}^r$+ e^x

Again w.k.t. $\int_{x0}^{xo\ +nh} y\ dx$ = $\frac{h}{2}$[y_0+y_{n+2}(y_1+y_2+…+y_{n-1})]

Substituting these values of, h_1, y_o, y_n…, in the above formula we get

$\int_2^{1.4}$(sin x − logex + e^x) dx = $\frac{1}{20}$[3.02950+ 4.70418+2(2.8493+…+4.37050)]

= $\frac{1}{20}$[7.73368+2(36.69492)]

= 4.05617

(b) By Simpson's rule $\frac{1}{3}$ we have

$\int_{x0}^{xo\ +nh} y\ dx$ = $\frac{h}{3}$[y_0+y_6+4(y_1+y_3+…)+2(y_2+y_4+…)]

Put the values of h, y_o, $y_{1..}$... from the table, we have

= $\int_2^{1.4}$(sin x − logex + e^x) dx

= $\frac{1}{3}$[7.73368+ 4(20.20417)-3(16.49075)]

= $\frac{1}{3}$ [121.53186] = 4.05106

(c) By Simpson's 3/8 rule we have

$\int_{x0}^{xo\ +nh} y\ dx$ = $\frac{3h}{8}$[(y_0+ y_n)+3(y_1+y_2+y_4+…+$y_{n-1)}$+2(y_3+y_6+….+y_{n-3})]

$= \frac{3(0.1)}{8}[7.73368+ (26.90750)+ 2(9.78742)] = 4.05116$

2. A curve is drawn to pass through the points given by the following table.

x	1	1.5	2	2.5	3	3.5	4
y	2	2.4	2.7	2.8	3	2.6	2.1

Estimate the area bounded by the curve, the x-axis and the lines x=1,x=4.

Solution. By Simpson's rule, we have required area

$= \frac{h}{3}[y_0+4y_1+2y_2+4y_3+.....+4y_{n-1}+y_n]$

$= \frac{0.5}{3}$ [2+ 9.6+5.4+11.2+10.4+ 2.1]

$= \frac{50}{30}$x46.7 $= \frac{23.35}{3}$ = 7.783 unit of area

By Weddle's rule, we have required area

$= \frac{3h}{10}[y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6]$

$= \frac{3x0.5}{10}[2+12.0+2.7+16.8+3.13.0+2.1]$ = 7.74 units of area

3. Calculate by Simpson's rule an approximate value of by taking seven equidistant ordinates. Compare it with the exact value and the value obtained by using the Trapezoidal rule.

Solution. Let us divide the range of integration (-3, 3) into six equal parts each of width $\frac{3(-3)}{6}$ = 1 unit. Hence h-1, value of the function for each point of sub division are as given below.

x	$y= x^4$
-3	$(-3)^4 = 81$
-2	$(-2)^4 = 16$
-1	$(-1)^4 = 1$
0	$(0)^4 = 0$
1	$(1)^4 = 1$
2	$(2)^4 = 16$
3	$(3)^4 = 81$

Now by Simpson's rule, we obtain

$\int_{-3}^{3} x^4$ dx = $\frac{h}{3}$[y_0+y_6+4(y_1+y_3+y_5)+2(y_2+y_4)]

= $\frac{1}{3}$[162+4x32+ 2x2] = $\frac{1}{3}$x294 = 98

But the exact value of

$\int_{-3}^{3} x^4$ dx = $\left[\frac{x^5}{5}\right]_{-3}^{3}$ = $\frac{1}{5}$[$(3)^5$ –$(-3)^5$] = $\frac{1}{5}$x486 = 97.2

Also by Trapezoidal rule, we have

$\int_{-3}^{3} x^4$ dx = $\frac{h}{2}$[y_0+y_6+2(y_1+ y_2+y_3+ y_4+y_5)] = $\frac{1}{2}$[162+2x34]

Thus it is obvious that the Trapezoidal rule does not give an accurate result.

4.Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rule. Hence obtain the approximate value of π in each case.

Solution. Divide the range into six equal parts each of with h = $\frac{1}{6}$ and compute

The value of y = $\frac{1}{1+x^2}$ at each point of sub division.

These values are given below.

X	$Y_x = 1/(1+x^2)$
0	1.0000
1/6	36/37 = 0.97297
2/6	36/40 = 0.90000
3/6	36/45 = -0.80000
4/6	36/52 = 0.69231
5/6	36/61 = 0.59016
1	1 / 2 = 0.5000

By Simpson's rule, we have

$\int_{x_0}^{x_0+6h} y\,dx$ = $\frac{h}{3}$[y_0+y_6+4(y_1+y_3+y_5)+2(y_2+y_4)]

= $\frac{1}{18}$ [1.00 + 0.500 + 4(0.97292 + 0.50000 + 0.59016) +

2(0.900+ 0.69231)] ……(1)

= 0.785395

By Simpson's 3/8 rule we have

$\int_{x_0}^{x_0+6h} y\,dx = \frac{3h}{8}[y_0+y_6+3(y_1+y_2+ y_4+y_5)+2(y_3)]$

$= \frac{1}{16}$ [1.5000 + 3 (0.97297 + 0.900 + 0.69231+0.59016 +

2 x (0.800))] = 0.785395 ……(2)

Again, $\int_0^1 \frac{dx}{1+x^2} = [\tan^{-1} x]^1_0 = \pi/4$ …….(3)

From (1) and (2), we get π = 3.14188 and from (2) and (3) we get

π = 3.141580

4.Evaluate $\int_2^{10} \frac{dx}{x}$ by dividing the range into '8' equal parts by using Trapezoidal rule.

Solution. Given integral, $\int_2^{10} \frac{dx}{x}$

Let f(x) = $\frac{1}{4}$

a = 2, b = 10;n = 8

h = $\frac{b-a}{n} = \frac{10-2}{8} = 1$

x	y = f(x) = $\frac{1}{x}$
2	1/2 = 0.5000
3	1/3 = 0.3333
4	1/4 = 0.2500
5	1/5 = 0.2000
6	1/6 = 0.1666
7	1/7 = 0.1428
8	1/8 = 0.1250
9	1/9 = 0.1000
10	1/10 = 0.1000

Trapezoidal rule is $\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

$\int_2^{10} \frac{dx}{x} = \frac{h}{2}[(y_0+y_8)+2(y_1+y_2+y_3+ y_{4+} y_{5+} y_6+y_7)]$

$= \frac{1}{2}[(0.5+ 0.1)+ 2(0.333+ 0.25)+ 0.2+ 0.125+ 0.166 + 0.1428 +0.1111]$

$= \frac{1}{2}[(0.6)+2(1.3288)]$

$= 0.3+1.3288$

$= 1.62880$

5.. **Evaluate $\int_4^{5.2} f(x)dx$ by using Trapezoidal rule from the following.**

X	4	4.2	4.4	4.6	4.8	5.0	5.2
Y	1.4	1.4	1.5	1.5	1.6	1.6	1.6

Where h = 0.2

Solution. Trapezoidal rule is, $\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

$\int_4^{5.2} f(x)dx = \frac{0.2}{2}[(1.4+ 1.6)+ 2(1.4+1.5+ 1.5+1.6+ 1.6)]$

$= 0.1[3+2(7.6)]$

$= 0.1[18.2] = 1.82$

6. Evaluate $\log_e 7$ from the integral $\int_1^7 \frac{dx}{x}$

Solution. Given $f(x) = \frac{1}{x}$ a = 1, b = 7, n = 6, $h = \frac{b-a}{n} = \frac{7-1}{6} = \frac{6}{6} = 1$

x	y= f(x)
1	1.0000
2	0.5000
3	0.333
4	0.2500
5	0.2000
6	0.1666

7	0.1428

Trapezoidal rule is, $\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

$= \frac{h}{2}[(1+ 0.1428)+ 20.5+ 0.333+0.25+ 0.2+ 0.1666)]$

$= \frac{1}{2}[1.1428+ 2(1.4499)]$

$= \frac{1}{2}[4.0426]$ and $\int_1^7 \frac{dx}{x} = [\log_e x]_1^7$

$= \log_e 7 = \log_e 1 = \log_e 7$

$\log_e 7 = 2.0113$

7. A curve is drawn to passes through the point given by the following table.

X	1	1.5	2	2.5	3	3.5	4
Y	2	2.4	2.7	2.8	3	2.6	2

Solution. Estimate the area bounded by the curve, the axis-x and the lines x=1,x=4by using Trapezoidal rule.

Trapezoidal rule is h= $\frac{4-1}{6}$=0.5

$\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+y_4+y_5)]$

$= \frac{0.5}{2}[2+2.1+2+ (2.4 +2.7+2.8+2.6+3)]$

$\frac{0.5}{2}(4.1+27.0) = \frac{15.35}{2} = 7.775$ units of area

8.**. calculate e $\int_2^{10} \frac{dx}{1+x}$ by dividing the arrange into 8 equal parts by using Simpson's 1/3 rule**.

Solution.: Simpson's 1/3 rule is

$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_5+\ldots..)+2(y_2+ y_4+\ldots....)]$

$\int_2^{10} \frac{dx}{1+x} = \frac{h}{3}[(O.333+ 0.0909)+ 4(0.25+ 0.1666+ 0.125+0.1)$

$+ 200.2+ 0.142+ 0.111)]$

$= \frac{1}{3}[0.4242 + 4(0.6416) + 2(0.4539)) = \frac{1}{3}[3.8984] = 1.2994666$

9.**calculate $\int_a^b f(x)dx$ by using Simpson's $\frac{1}{3}$ rule**

X	4	4.2	4.4	4.6	4.8	5.0	5.2
Y	1.4	1.4	1.5	1.5	1.6	1.6	1.6

where h = 0.2

Solution: Simpson's $\frac{1}{3}$ rule is

$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_5+.....)+2(y_2+ y_4+.....)]$

$\int_4^{5.2} f(x)dx = \frac{0.2}{3}[(1.4+1.6)+ 4(1.4+1.5+ 1.6)+ 2(1.5+ 1.6)]$

$= \frac{0.2}{3}[3+4(4.5)+2(3.1)] = \frac{0.2}{3}[27.2]$

$= 1.813333.....$

10.Evaluate $\int_2^{10} \frac{dx}{1+x}$ by dividing the range iato 8 equal parts by using Simpson's $\frac{1}{3}$ rule.

Solution:. Let f(x)= $\frac{1}{x}$

a = 2, b = 10, n =number of equal parts = 8

h = $\frac{b-a}{n} = \frac{8}{8} = 1$

X	Y = f(x) = $\frac{1}{x}$
2	0.5000
3	0.3333
4	0.2500
5	0.2000
6	0.1666
7	0.1428
8	0.1250

9	0.1111
10	0.1000

Simpson's $\frac{1}{3}$ rule is

$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_5+.....+y_{n-1})+2(y_2+ y_4+y_6.....+y_{n-2})]$

$= \frac{1}{3}$ [I0.6)+ 4(0.3333+ 0.2+ 0.1428+ 0.1111)] +2(0.25+0.1666+0.125)]

$=\frac{1}{3}$[0.6+4(0.7872+ 1.0832)]

$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_6+.....)+2(y_2+ y_4+.....)]$

$\int_0^1 \frac{dx}{1+x^2} = \frac{1}{6*3}$ [(1 +0.5)+ 4(0. 9729+0.8+0.3+ 2(09+0.6923)]

$= \frac{1}{18}$[1.5+9.452+3.1846] = $\frac{1}{18}$[14.1366]

= 0.7853666

On the other hand $\int_0^1 \frac{1}{1+x^2}dx = (\tan^{-1})^1_0$

$= \tan^{-1}(1)-\tan^{-1}(0)$

$= \frac{\pi}{4}-0 = \frac{\pi}{4}$

$= \frac{\pi}{4} = 0.7853$

$= \pi = 3142$

11.. **Evaluate $\int_0^4 e^x\,dx$ by using Simpson's rule using the data e = 2°.72, e2= 739, e3=20.09, e4=54.60 and compare it will the actual value.**

Solution.: Divide the whole range (0,4) into four equal parts by taking h=1. By

Simpson's rule 1/3 we have

$\int_0^4 e^x$ dx = $\int_{x_0}^{x_0+4h} y\,dx$ = $\frac{h}{3}$(y_0+y_4+4(y_1+y_3)+242)

= $\frac{1}{3}$[1+54.60+4(2.72+20.09)+2(7.39)]

= $\frac{1}{3}$[55.60+91.24+14.78] = $\frac{1}{3}$[161.62]

= 553.373

The actual value of the integral is

$\int_0^4 e^x$ dx = e^4 - e^0 = 54.60 – 1.0 = 53.60

12.Evaluate $\int_4^{5.2} \log, x$ dx by (a) Simpson's $\frac{1}{3}$ (b) $\frac{3}{8}$ rule . After finding the true value of the integral.

Solution. Divide the whole range of integration ie., (4, 5, 2) into six equal parts taking h 0.2 and then compute the values of the function f(x)= $\log^x_e$ for each of sub division. These computed values are as follows.

X	Y = $\log^x_e$ x	X	Y = $\log^x_e$ x
Xo = 4.0	1.38629436	Xo+ 4h= 4.8	1.56861592
Xo+ h =4.2	1.43508453		
		Xo+5h= 5.0	1.60943791
X_0 + 2h= 4.4	1.48160454		
		Xo+ 6h= 5.2	1.64865863
Xo +3h= 4.6	1.52605630		

(a) By Simpson's $\frac{1}{3}$ rule, we have

$\int_4^{5.2}$ log, x dx = $\frac{h}{3}$ [y_0+y_6+(y_1+y_3++y_5)+2(y_2+y_4)]

= $\frac{2}{8}$ [3.03495299+ 4(4.57056874)+ 2(3.05022046)]

=1.82784726.

(b) By Simpson's 3/8 rule, we have

$\int_4^{5.2} \log_e x \, dx = \frac{3h}{8}[y_0+y_2+3(y_1+y_2++y_4+ y_5)+2 y_3]$

$= \frac{3(0.2)}{8}[03495299+ 3(6.09424290)+ 2 (1.52605630)]$

$= \frac{0.3}{4}[24.371294291]$

= = 1.82784707

Again exact value of the integral is

$\int_4^{5.2} \log_e x \, dx = x(\log_e x-1)\int_{4.0}^{5.4} 1.82784741$

13. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ rule.

Solution : let f(x) = $\frac{1}{1+x^2}$ let n =6 , a= -0 , b = 1 ,h = $\frac{b-a}{n} = \frac{1-0}{6} = \frac{1-0}{6} = \frac{1}{6}$ = 0.1666

X	f(x) = $\frac{1}{1+x^2}$
0	1
1/6	0.9729
2/6	0.9000
3/6	0.8000
4/6	0.6923
5/6	0.5901
$\frac{6}{6}$=1	0.5000

Simpson's $\frac{1}{3}$ rule is

a = 0, b = 6, n= 6

$h = \frac{b-a}{n} = \frac{6-0}{6} = 1$

$y_0 = f(0) -= \frac{1}{1+0^2} = \frac{1}{1} = 1$

$y_1 = f(1) = \frac{1}{1+1^2} = \frac{1}{2} = 0.5000$

$y_2 = f(2) = \frac{1}{1+2^2} = \frac{1}{5} = 0.2000$

$y_3 = f(3) = \frac{1}{1+3^2} = \frac{1}{10} = 0.1000$

$y_4 = f(4) = \frac{1}{1+4^2} = \frac{1}{17} = 0.0588$

$y_5 = f(5) = \frac{1}{1+5^2} = \frac{1}{26} = 0.0385$

$y_6 = f(6) = \frac{1}{1+6^2} = \frac{1}{37} = 0.0270$

Simpson's $\frac{1}{3}$ rule is

$$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_5+.....)+2(y_2+ y_4+.....)]$$

$$= \frac{1}{3}[(1 + 0.027)+ 4(0.5+ 0.1 + 0.0385)+ 2(0.2+ 0.0588)]$$

$$= \frac{1}{3} [3.7104] = 1.2368$$

14. Find the value of $\int_0^1 \frac{x^2}{1+x^2}$ dx by using Simpson's $\frac{1}{3}$ rule and also find the error

Solution. : Dividing the whole range (o, 1) into 4 equal parts nd making use of Simpson's $\frac{1}{3}$ rule we have

$$\int_0^1 \frac{x^2}{1+x^2} dx = \frac{h}{3}[(y_0+y_4)+4(y_1+ y_3+)+2y_2)]$$

$h = \frac{1}{4}$, a =0 , b= 1 ,$n = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4}$

X	F(x)

0	0
0.25	0.0615
0.5	0.2222
0.75	0.3956
1.0	-0.5000

$\int_0^1 \frac{x^2}{1+x^2}$ dx = $\frac{1}{12}$ [(0+ 0.5)+ 4C0.0615+ 0.3956)+ 2(0.222)]

= $\frac{1}{12}$ [0.5+ 1.8284+ 0.444]

=$\frac{2.7728}{12}$ = 0.2310

15. Find $\int_4^{5.2} \log x_e$ dx by using Simpson's $\frac{1}{3}$ rule.

solution.: Given integral is $\int_4^{5.2} \log x_e$ dx

A = 4, b = 52 let n= 6 = $\frac{b-a}{n}$ = $\frac{5.2-2}{6}$ = 0.2

X	F(x)
4	1.3862
4.2	1.4350
4.4	1.4816
4.6	1.5260
4.8	1.5686
5.0	1.6094
5.2	1.6486

$\int_4^{5.2} \log x_e$ = $\frac{h}{3}$[(y_0+y_6)+4(y_1+ y_3+ y_5)+2(y_2+ y_4)]]

= $\frac{0.2}{3}$[(1.3862+1.6481+4(1.435071 +1.5260+1.6094)

+2(1.4816+1.5686)]

= $\frac{0.2}{3}$ [(3.0348)+ 4 (4.5704)+ 2(3.05027)]

= $\frac{0.2}{3}$[3.0348+18.2816+ 6.1004] = $\frac{5.48336}{3}$ = 1.8277866.

16. Evaluate $\int_{-3}^{3} x^4$dx by using Simpson's $\frac{1}{3}$ rule.

Solution. We know the integral is $\int_{-3}^{3} x^4$dx

Where h = $\frac{b-a}{h}$ = $\frac{3-(-3)}{6}$ = $\frac{6}{6}$ = 1

x	F(x)
-3	81
-2	16
-1	1
0	0
1	1
2	16
3	81

Simpson's $\frac{1}{3}$ rule is

$\int_b^a f(x)dx$ = $\frac{h}{3}$ [(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)]

= $\frac{1}{3}$[(81+81)+4(16+0+16)+2(1+1)]

a = 3.b = 3 , n = 6

$$h = \frac{b-a}{h} = \frac{3-(-3)}{6} = 1$$

let $f(x) = x^2$

$y_0 = f(-3) = (-3)^2 = 9$

$y_1 = f(-2) = (-2)^2 = 4$

$y_2 = f(-1) = (-1)^2 = 1$

$y_3 = f(0) = (0)^2 = 0$

$y_4 = f(1) = (1)^2 = 1$

$y_5 = f(2) = (2)^2 = 4$

$y_6 = f(3) = (3)^2 = 9$

Simpson's $\frac{1}{3}$ rule

$$\int_a^b f(x) = \frac{h}{3}[(y_0+y_n)+4(y_1+y_{3+\ldots})+2(y_2+y_{4+\ldots})]$$

$$= \frac{1}{3}[(9+9)+4(4+0+4)+2(1+1)]$$

$$= \frac{54}{3} = 18 = \frac{1}{3}[162+128+4] = \frac{1}{3}[294] = 98$$

17. calculate an approximate value of $\int_a^{\pi/2} sinx\, dx$ by (a) Trapezoidal rule (b) Simpson's rule using 11 ordinates

Solution : Divide the range of integration into a ten equal parts taking h = $\pi/20$ and then compute the values of the function f(x) = sin x for each point of sub-division. The computed values are as follows :

X	Y=sin x

0	0.00000
$\pi/20$	0.15663
$2\pi/20$	0.30902
$3\,\pi/20$	0.45399
$4\pi/20$	0.58778
$5\pi/20$	0.707111
$6\,\pi/20$	0.80900
$7\,\pi/20$	0.89101
$8\,\pi/20$	0.95106
$9\pi/20$	0.98769
$10\,\pi/20$	1.00000

(a) By the trapezoidal rule, we have

$$\int_0^{\pi/2} sinx\ \mathrm{dx} = \frac{h}{3}[(y_0+y_{10})+2(y_1+y_{2+\ldots})]$$
$$= \frac{\pi/20}{2}[1.0000+2(5.85311)]$$
$$= \frac{\pi}{40}(12.70622) = 0.9981$$

(b) By Simpson's $\frac{1}{3}$ rule , we have

$$\int_0^{\pi/2} sinx\ \mathrm{dx} = \frac{h}{3}[(y_0+y_n)+4(y_1+y_{3+\ldots})+2(y_2+y_{4+\ldots})]$$
$$= \frac{\pi}{60}[1.00000+4(3.19623)+2(2.65688)]$$
$$= \frac{\pi}{60}[1.00000+12.78492+5.31376]$$
$$= \frac{\pi}{60}[19.09868] = 1.0006$$

Again the exact value of the integral

$$\int_0^{\pi/2} sinx\ \mathrm{dx} = [-\cos x]_0^{\pi/2} = 1.0000$$

The error (i) Due to Simpson's $\frac{1}{3}$ rule , we have = -0.0006

(ii) Due to trapezoidal rule = 0.0019

18.Evaluate $\int_a^b f(x)dx$,by Simpson's $\frac{3}{8}$ rule from the following

X	4	4.2	4.4	4.6	4.8	5.0	5.2
Y	1.4	1.4	1.5	1.5	1.6	1.6	1.6

Where

Solution : Simpson's $\frac{3}{8}$ rule is

$\int_a^b f(x)dx = \frac{3h}{8}[y_0+3y_1+2y_2+3y_2+2y_3+3y_4+3y_5+y_6]$

$= \frac{3(0.2)}{8}[1.4+3(1.4)+3(1.5)+2(1.5)+3(1.6)+3(1.6)+1.6]$

$= \frac{0.6}{8}[1.4+4.2+4.5+3.0+4.8+4.8+1.6]$

$= \frac{0.6}{8}(24.3) = 1.8225$

19. Evaluate $\int_{-3}^{3} x^4 dx$,by using Simpson's $\frac{3}{8}$ rule

Solution. we know by the above problem.

Let $f(x) = x^4$

X	F(X)
-3	81
-2	16
-1	1
0	0
1	1
2	16
3	81

Where h=1

Simpson's 3/8 rule is

$\int_a^b f(x)dx = \frac{3h}{8}[(y_0+ y_6)+3y_1+2y_3+3y_4+3y_5]$

$= \frac{3}{8}[81+81+3(16+1+1+16)]$

$= \frac{3}{8}[162+102] = \frac{3}{8}(264) = 99$

23. A curve is drawn to passes through the part given by the following .

X	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
Y	**2**	**2.4**	**2.7**	**2.8**	**3**	**2.6**	**2.1**

Estimate the area bounded by the curve the x- axis and the lines x =1, x=4 by using Simpson's $\frac{3}{8}$rule.

Solution. h $= \frac{4-1}{6} = 0.5$

Simpson's 3/8 rule is

$$\int_a^b f(x)dx = \frac{3h}{8}[(y_0+ y_6)+3y_1+2y_3+3y_4+3y_5]$$

$$= \frac{3(0.5)}{8}[2+2.1+3(2.4+2.7+3+2.6)+2(2.8)]$$

$$= \frac{6.35}{8} = 8.04375 \text{ units of area}$$

20. Evaluate $\int_a^b f(x)$ dx by using weddles formula from the following.

x	4	4.2	4.4	4.6	4.8	5.0	5.2
F(x)	1.4	1.4	1.5	1.5	1.6	1.6	1.6

Where h=0.4

Weddle's formula is

$$\int_a^b f(x)dx = \frac{3h}{10}[y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6]$$

$$= \frac{3(0.2)}{10}[(1.4+1.6)+5(1.4+1.6)+(1.5+1.6)+6(1.5)]$$

$$= \frac{0.6}{10}[3+15+3.1+9]$$

$$= \frac{0.6}{10}(30.1)$$

$$= 1.806$$

21. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using weddle's formula.

Solution. a =0 , b= 1 ,h = $\frac{1-0}{6}$ = $\frac{1}{6}$ = 0.1666

X	f(x)
0	1
1/6	0.9729
2/6	0.9000
3/6	0.8000
4/6	0.6923
5/6	0.5901
$\frac{6}{6}$=1	0.5000

Weddle's formula is

$\int_a^b f(x)dx$ = $\frac{3h}{10}$[y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6]

= $\frac{3}{6x10}$[(1+0.5)+5(0.972+0.590)+(0.9+0.6923)+6(0.8)]

= $\frac{1}{20}$[1.5+7.815+1.5923+4.8]

= $\frac{1}{20}$[15.703]

= 0.785365

2.9 Applications of Numerical Methods

Considering the following model equation:

$$(\mu u_y)_y + y^n u_y = C, \qquad (1)$$

defined in a domain $\Omega = [0\ 1]$.

Where $\mu = (1 - exp(-y/\varepsilon) + \delta)/Re$, $\varepsilon << 1$, $\delta << 1$, $Re >> 1$, $n > 0$.

The first term simulates dissipative terms in the Navier-Stokes equations, whereas the second term models the contribution of convective terms and the right hand side represents the pressure gradient term or source in the transport equations. The "viscosity" coefficient μ corresponds to the effective viscosity coefficient. The coefficient is rapidly changed from a relatively small value $\mu_l = \delta /Re$ (laminar viscosity) to a "turbulent" viscosity $\mu_t \approx 1/Re$.

The equation simulates the low-Reynolds-number effects and can be considered as a model equation for the LR model. The left hand side point in the domain Ω will be treated as a "wall". The low-Reynolds-number effects occur nearby this point. If we set $\mu = (1+ \delta)/Re$ in (1), we have the HR approach.

Furthermore, we will consider the following values for the constants: $Re = 10^2$, $\varepsilon = 3*10^{-2}$, $\delta = 10^{-2}$. For simplicity the right-hand side will be considered as the constant $C = -1$

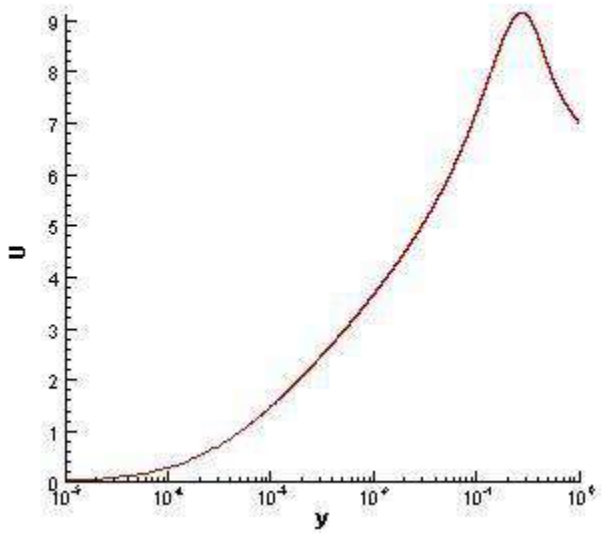
although this assumption is not important for our consideration.

Consider the following boundary-value Dirichlet problem:

$$(\mu u_{y)y} + y^n u_y = C \qquad (2)$$

$$u(0) = u_0 \quad u(1) = u_1$$

In the case of $n = 2$, $u_0 = 0$ and $u_1 = 7$, the solution is shown in Figure 1. The profile includes both the linear near-wall and logarithmic parts. Near the wall, $u = \varepsilon\, ln(1+y/(\varepsilon\delta))$. The thickness of the viscous "sub-layer", where u is a linear function, can be approximated as $y_v \approx \varepsilon\delta$.



In the calculations, the compact scheme [4] is used. The scheme allows us to calculate both the function and its derivative with a fourth-order of approximation. In Table 1, the results of calculations of the "friction" $\tau_w = \mu_w du/dy(0)$ (here $\mu_w = \mu(0)$) are given on different meshes. To exclude questions concerned with a mesh adaptation, a uniform mesh is used

Grid	10^2	$5*10^2$	10^3	$5*10^3$	10^4
τ_w	$3.1*10^{-1}$	$3.2*10^{-1}$	$3.3*10^{-1}$	$3.36*10^{-1}$	$3.36*10^{-1}$

Table 1. Calculation of τ_w on different meshes.o

If we use the HR model with the same boundary conditions, then $\tau_w = 4.9*10^{-1}$, which shows the importance of the sub-layer.

3. Linear System of Equations

Solutions of simultaneous linear equations:

Rate, Distance and Time:

You can calculate the best routes for your running or cycling schedule by creating a mathematical expression that takes into account the distance and your average speed for various parts of the route. You can use the equations to set different goals, such as to maximize time for build endurance, or to maximize speed for performance.

Planes, Trains and Automobiles:

The same formula used to calculate running times can be used to determine speed, distances and time duration when traveling by car, plane or train and you want to know the values for the unknown variables in your travel situations.

The Best Deal:

You want to find out the better deal when renting a car, and you're comparing two rental companies. By putting the variable and fixed costs, such as the per - mile and daily rate, into an algebraic expression, then solving for the total cost, you ca see which company saves you money for different amounts of driving.

The Best Plan:

You can use this same process with a system of equations when trying to decide on the best call phone plan, determining at how many minutes both companies charge the same amount and deciding from there which is the best plan for you and your intended usage.

Deciding on a Loan:

Simultaneous equations can be used to determine the best loan

choice to make when buying a car or a house when you consider the duration of the loan, the interest rate and the monthly payment of the loan. Other variables may be involved as well, with the information at hand, you can calculate which loan is the best choice for you.

Cost and Demand:

Simultaneous equations can be used when considering the relationship between the price of a commodity and the quantities of the commodity people want to buy at a certain price. An equation can be written that describes the relationship between quantity, price and other variables, such as income. These relationship equations can be solved simultaneously to determine the best way to price the commodity and sell it.

In the Air:

An air traffic controller can use Simultaneous equations to ensure two airplanes don't intersect at the same time.

The Best job for the Money:

System of equations can be used when trying to determine if you'll make more money at one job or another, taking multiple variables into account, such as salary, benefits and commissions.

Investing Wisely:

You can use Simultaneous equations to decide on your best investment option, taking into account the duration of the investment, the interest it will accrue, as well as other variables that will affect the end result. If you know the amount you'd like to accrue, you can set the options equal to each other and figure out which option is best for your future.

Mixing it up:

With respect to mixtures Simultaneous equations can be used for achieving a certain consistency in a resultant product, which is dependent on the consistency of the compounds mixed together to produce it.

Rates :

Linear equations can be useful tool for comparing rates of pay. For example, if one company offers to pay you $450 per week and other offers $10 per hour, and both ask you to work 40 hours per week, which company is offering the better rate of pay? A linear equation can help you figure it out! The first company's offer expressed as 450=40x. the second company's offer is expressed as y=10(40). After computing the two offers, the equations tell you that the first company is offering the better rate of pay at $11.25 per hour.

Variable costs:

Imagine that you taking a taxi while on vacation. You know that the taxi service charges $9 to pick your family up form your hotel and another $0.15 per mile for the trip. Without knowing how many miles it will be to each destination, you can set up a linear equation that can be used to find the cost of any taxi trip you take on your trip. By using "x" to represent the number of miles to your destination and "y" to represent the cost of that taxi ride, the linear equation would be: y= 0.15x+9.

3.1 MATRIX INVERSION METHOD:

* Solve x + 2y = 10, 2x - y = 15 by matrix inverse method?

Solution:-

Let $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 10 \\ 15 \end{bmatrix}$

Given equations are equivalent to AX = B

$\Rightarrow X = A^{-1} B$.

$$A^{-1} = \frac{1}{-1-4}\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix} \frac{1}{-1-4} = -\frac{-1}{5}\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix}$$

$$\therefore X = A^{-1} B = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} \frac{1}{5}\begin{bmatrix} 10 \\ 15 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 10 + 30 \\ 20 - 15 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 40 \\ 5 \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

$\therefore$ x = 18, y = 1 is the solution.

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* Solve the equations 3x + 2y + 4z = 7; 2x + y + z = 7; x + 3y + 5z = 2. by matrix inversion method ?

Solution:-

Let $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 7 \\ 7 \\ 2 \end{bmatrix}$

Given equations are equivalent to AX = B

det A = 3(5 - 3) - 2(10 - 1) +4(6 - 1) = 6 - 18 + 20 = 8 ≠ 0.

$\therefore$ X = A^{-1} B is the solution.

Now adj A = $\begin{bmatrix} 2 & 2 & -2 \\ -9 & 11 & 5 \\ 5 & -7 & -1 \end{bmatrix}$

=> $A^{-1} = \frac{AdjA}{\det A} = \frac{1}{8}\begin{bmatrix} 2 & 2 & -2 \\ -9 & 11 & 5 \\ 5 & -7 & -1 \end{bmatrix}$

$\therefore$ $X = A^{-1} B = \frac{1}{8}\begin{bmatrix} 2 & 2 & -2 \\ -9 & 11 & 5 \\ 5 & -7 & -1 \end{bmatrix}\begin{bmatrix} 7 \\ 7 \\ 2 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ 24 \\ -16 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ -2 \end{bmatrix}$

$\therefore$ x = 3, y = 3, z = -2 is the solution.

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3.2 GAUSS ELIMINATION METHOD:

* Solve the equations 2x - y + 4z = 12; 8x - 3y =2z = 23; 4x + 11y - z = 33 by Gauss Elimination Method ?

Solution:-

Given equations are $2x - y + 4z = 12$, $8x - 3y + 2z = 23$, $4x + 11y - z = 33$.

Dividing first equation by 2, we get $x - \frac{1}{2}y + 2z = 6$ ----------(1)

Now subtract (1) after multiplying by 8 and 4 respectively from second and third equations we get $y - 14z = -25$----->(2), $13y - 9z = 9$ ------(3)

Subtract (2) after multiplied by 13 from (3) we get $173z = 334$

$\Rightarrow z = \frac{334}{173} = 1.931$

From (3); $13y - 9\left(\frac{334}{173}\right) = 9 \Rightarrow 13y = 9 + \frac{3006}{173} \Rightarrow y = \frac{4563}{2249} = 2.029$

From (1); $x = 6 + \frac{1}{2}\left(\frac{4563}{2249}\right) - 2\left(\frac{334}{173}\right) = 6 + \frac{1}{2}(2.029) - 2(1.931)$

$= 6 + 1.0145 - 3.862 = 3.153.$

$\therefore$ The solution is $x = 3.153$, $y = 2.029$, $z = 1.931$.

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3.3GAUSS JORDAN METHOD:

* Solve the system of equations $2x + y + z = 1$, $x - 2y - 3z = 1$, $3x + 2y + 4z = 5$ by Gauss Jordan Method.

Solution:-

The augmented matrix is $\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & -2 & -3 & 1 \\ 3 & 2 & 4 & 5 \end{bmatrix}$

$2R_2 - R_1, 2R_3 - 3R_1 \sim \begin{bmatrix} 2 & 1 & 1 & 1 \\ 0 & -5 & -7 & 1 \\ 0 & 1 & 5 & 7 \end{bmatrix}$

$5R_1 + R_2, 5R_3 + R_2 \sim \begin{bmatrix} 10 & 0 & -2 & 6 \\ 0 & -5 & -7 & 1 \\ 0 & 0 & 18 & 36 \end{bmatrix}$

$9R_1 + R_3, 18R_2 + 7R_3 \sim \begin{bmatrix} 90 & 0 & 0 & 90 \\ 0 & -90 & 0 & 270 \\ 0 & 0 & 18 & 36 \end{bmatrix}$ $\frac{1}{90}$

$R_1, \frac{1}{90}R_2, \frac{1}{18}R_3 \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

$\therefore$ **By** Gauss Jordan method, $x = 1$, $y = -3$, $z = 2$.

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* Solve the equations by Gauss Jordan Method . $5x + 2y + z = 12$, $x + 4y + 2z = 15$, $x + 2y + 5z = 20$.

Solution:-

The augmented matrix is $\begin{bmatrix} 5 & 2 & 1 & 12 \\ 1 & 4 & 2 & 11 \\ 1 & 2 & 5 & 20 \end{bmatrix}$

$5R_2 - R_1, 5R_3 - R_1 \sim \begin{bmatrix} 5 & 2 & 1 & 12 \\ 0 & 18 & 9 & 63 \\ 0 & 8 & 24 & 88 \end{bmatrix}$

$9R_1 - R_2, 9R_3 - 4R_1 \sim \begin{bmatrix} 45 & 0 & 0 & 45 \\ 0 & 18 & 9 & 63 \\ 0 & 0 & 180 & 540 \end{bmatrix}$

$180R_3 \sim \begin{bmatrix} 45 & 0 & 0 & 45 \\ 0 & 18 & 9 & 63 \\ 0 & 0 & 1 & 3 \end{bmatrix}$ $R_2 - 9R_3 \sim \begin{bmatrix} 45 & 0 & 0 & 45 \\ 0 & 18 & 0 & 36 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

$\frac{1}{45}R_1, \frac{1}{18}R_2 \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

$\therefore$ **By** Gauss Jordan method, $x = 1$, $y = 2$, $z = 3$.

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3.4 LU - DECOMPOSITION METHOD:

* Solve the system of equations $2x_1 + x_2 + x_3 = 2$, $x_1 + 3x_2 + 2x_3 = 2$, $3x_1 + x_2 + 2x_3 = 2$ by LU Decomposition Method?

Solution:-

Given equations are $2x_1 + x_2 + x_3 = 2$----------(1)

$x_1 + 3x_2 + 2x_3 = 2$---------(2)

$3x_1 + x_2 + 2x_3 = 2$---------(3)

Here the coefficient matrix A is given by $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 3 & 1 & 2 \end{bmatrix}$ and

$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$

$\therefore AX = B$

Now $A = LU \Rightarrow \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$

$$= \begin{bmatrix} u_{11} & u_{12} & u_{12} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{bmatrix}$$

On comparing two matrices, we get $U_{11} = 2$, $U_{12} = 1$, $U_{13} = 1$;

$L_{21}U_{11} = 1 \Rightarrow L_{21} = \frac{1}{U_{11}} = \frac{1}{2}$; $L_{31}U_{11} = 3 \Rightarrow L_{31} = \frac{3}{U_{11}} = \frac{3}{2}$;

$L_{21}U_{12}+U_{22} = 3 \Rightarrow 3 - L_{21}U_{12} = 3 - \frac{1}{2}(1) = \frac{5}{2}$

$L_{21}U_{13}+U_{2} = 3 \Rightarrow 2 - L_{21}U_{12} = 2 - \frac{1}{2}(1) = \frac{3}{2}$

$L_{31}U_{12}+L_{32}U_{22} = 1 \Rightarrow L_{32} = \frac{1-L_{31}U_{12}}{U_{22}} = \frac{1-\frac{3}{2}}{\frac{5}{2}}(1) = \frac{-1}{5}$

$L_{31}U_{13}+L_{32}U_{23}+U_{33} = 2$

$\Rightarrow U_{33} = 2 - L_{31}U_{13} - L_{32}U_{23} = 2 - \frac{3}{2} + \frac{1}{5}\left(\frac{3}{2}\right) = 2 - \frac{3}{2} + \frac{3}{10} = \frac{8}{10} = \frac{4}{5}$

Thus $L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{3}{2} & \frac{-1}{5} & 1 \end{bmatrix}$ and

$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 0 & \frac{5}{2} & \frac{3}{2} \\ 0 & 0 & \frac{4}{5} \end{bmatrix}$

LY = B => $\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{3}{2} & \frac{-1}{5} & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$

From this we get $y_1 = 1$, $\frac{1}{2}y_1 + y_2 = 2$, $\frac{3}{2}y_1 - \frac{1}{5}y_2 + y_3 = 2$.

On solving, we get $y_1 = 2$, $y_2 = 1$, $y_3 = \frac{-4}{5}$.

And since, we have UX =Y => $\begin{bmatrix} 2 & 1 & 1 \\ 0 & \frac{5}{2} & \frac{3}{2} \\ 0 & 0 & \frac{4}{5} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ \frac{-4}{5} \end{bmatrix}$

$2x_1 + x_2 + x_3 = 2$, $\frac{5}{2}x_2 + \frac{3}{2}x_3 = 1$, $\frac{4}{5}x_3 = \frac{-4}{5}$

By back substitution, we get $x_1 = 1$, $x_2 = 1$, $x_3 = -1$.

$\therefore$ Hence, solution is $x_1 = 1$, $x_2 = 1$, $x_3 = -1$

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* Solve the equations 2x + 3y + z = 9, x + 2y + 3z = 6, 3x + y 2z = 8 by LU Decomposition method.

Solution:-

Given equations can be written as AX= B where A = $\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$, X= $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, B = $\begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$

Let A = LU

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{12} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{bmatrix}$$

Comparing, we get U_{11}= 2, U_{12}= 3, U_{13}= 1 and $L_{21}U_{11}$= 1 => L_{21}= $\frac{1}{2}$;

$L_{21}U_{12}+U_{22}$= 2 => U_{22}= 2 - $L_{21}U_{12}$ = 2 - $\frac{3}{2}$ = $\frac{1}{2}$;

$L_{21}U_{13}+U_{23}$= 3 => U_{23}= 3 - $L_{21}U_{13}$ = 3 - $\frac{1}{2}$ = $\frac{5}{2}$;

$L_{31}U_{11}$ = 3 => L_{31}= $\frac{3}{U_{11}}$ = $\frac{3}{2}$ and

$L_{31}U_{12}$ +$L_{32}U_{22}$ = 1 => L_{32}= $\frac{1-L_{31}U_{12}}{U_{22}}$ = $\frac{1-\frac{9}{2}}{\frac{1}{2}}$ = -7

$L_{31}U_{13} + L_{32}U_{23} = 2 \Rightarrow U_{33} = 1 - L_{31}U_{13} - L_{32}U_{23} = 2 - \frac{3}{2} - (-7)\left(\frac{5}{2}\right) = 2 - \frac{3}{2} + \frac{35}{2} = \frac{36}{2} = 18.$

Thus L and U are given by $L = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -7 & 1 \end{bmatrix}$, $U = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1/2 & 5/2 \\ 0 & 0 & 18 \end{bmatrix}$

$LY = B \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -7 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$

From this we get $y_1 = 9$, $\frac{1}{2}y_1 + y_2 = 6$, $\frac{3}{2}y_1 - 7y_2 + y_3 = 8$.

On solving, we get $y_1 = 9$, $y_2 = \frac{3}{2}$, $y_3 = 5$.

Further since $UX = Y \Rightarrow \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1/2 & 5/2 \\ 0 & 0 & 18 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 3/2 \\ 8 \end{bmatrix}$

$2x + 3y + z = 9$, $1/2y + 5/2z = 3/2$, $18z = 5$.

By back substitution, we get $z = 5/18$ and $1/2y + 5/2(5/18) = 3/2$

$\Rightarrow y = 3 - 25/18 = 29/18$

And $2x + 3(29/18) + (5/18) = 9 \Rightarrow 2x = 9 - 29/6 - 5/18 = 9 - 92/18$

$\Rightarrow 2x = 70/18 \Rightarrow x = 35/18$

Hence, the solution is $x = 35/18$, $y = 29/18$, $z = 5/18$.

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3.5 ILL CONDITIONED OR WELL CONDITIONED:

* Establish whether the following matrix is singular, ill conditioned or well conditioned: $\begin{bmatrix} 3 & -1 & 3 \\ 1 & 0 & 4 \\ 2 & 2 & 1 \end{bmatrix}$?

Solution:-

Let A = $\begin{bmatrix} 3 & -1 & 3 \\ 1 & 0 & 4 \\ 2 & 2 & 1 \end{bmatrix}$

det A = 3(0 - 8) + 1(1 - 8) + 3(2 - 0) = -24 - 7 + 6 = -25

$\therefore$ A is not singular.

$\begin{vmatrix} 3.01 & -1 & 3 \\ 1 & 0 & 4 \\ 2 & 2 & 1 \end{vmatrix}$ = (3.01)(0 - 8) + 1(1 - 8) + 3(2 - 0)= -24.08-7+6 =-25.08.

$\therefore$ The matrix is well conditioned.

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* Show that A = $\begin{bmatrix} -73 & 78 & 24 \\ 92 & 66 & 25 \\ -80 & 37 & 10 \end{bmatrix}$ is ill- conditioned?

Solution :-

The ,matrix A = $\begin{bmatrix} -73 & 78 & 24 \\ 92 & 66 & 25 \\ -80 & 37 & 10 \end{bmatrix}$ is well conditioned, since $|A|$ = 1 whereas

$\begin{vmatrix} -73 & 78 & 24 \\ 92.01 & 66 & 25 \\ -80 & 37 & 10 \end{vmatrix}$ = 2.08, $\begin{vmatrix} -73 & 78.01 & 24 \\ 92 & 66 & 25 \\ -80 & 37 & 10 \end{vmatrix}$ = -28.20

and $\begin{vmatrix} -73 & 78 & 24 \\ 92 & 66 & 25 \\ -80 & 37 & 10.01 \end{vmatrix}$ = -118.94

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3.6 GAUSS - JACOBI METHOD:

* Solve the following equations by Jacobi method 20x + y -2z = 17, 3x + 20y - z = -18, 2x - 3y + 20z = 25?

Solution:-

Given equations can be written as $x = \frac{1}{20}(17 - y + 2z)$---------(1)

$y = \frac{1}{20}(-18 - 3x + z)$--------(2)

$z = \frac{1}{20}(25- 2x + 3y)$--------(3)

Let us take first approximation $x^{(1)} = 0$, $y^{(1)} = 0$, $z^{(1)} = 0$.

Then the second approximation using (1), we get

$x^{(2)} = \frac{1}{20}(17 - y^{(1)} + 2z^{(1)}) = \frac{17}{20} = 0.85$

$y^{(2)} = \frac{1}{20}(-18 - 3x^{(1)} + z^{(1)}) = \frac{1}{20}(-18 - 0 + 0) = -\frac{18}{20} = -0.9.$

$z^{(2)} = \frac{1}{20}(25- 2x^{(1)} + 3y^{(1)}) = \frac{1}{20}(25 - 0+ 0) = \frac{25}{20} = 1.25.$

Substituting these values in R.H.S of (1), we get third approximation

$x^{(3)} = \frac{1}{20}(17 - y^{(2)} + 2z^{(2)}) = \frac{1}{20}(17 + 0.9 + 2(1.25)) = \frac{20.4}{20} = 1.02.$

$y^{(3)} = \frac{1}{20}(-18 - 3x^{(2)} + z^{(2)}) = \frac{1}{20}(-18 - 3(0.85) + 1.25) = \frac{-19.3}{20} = -0.965.$

$z^{(3)} = \frac{1}{20}(25 - 2x^{(2)} + 3y^{(2)}) = \frac{1}{20}(25 - 2(0.85) + 3(-0.9)) = \frac{20.6}{20} = 1.03.$

Now Substitute these values in (1), we get fourth approximation

$x^{(4)} = \frac{1}{20}(17 - y^{(3)} + 2z^{(3)}) = \frac{1}{20}(17 + 0.965 + 2(1.03)) = \frac{20.025}{20} = 1.0013.$

$y^{(4)} = \frac{1}{20}(-18 - 3x^{(3)} + z^{(3)}) = \frac{1}{20}(-18 - 3(1.02) + 1.03) = -\frac{20.03}{20} = -1.0015.$

$z^{(4)} = \frac{1}{20}(25 - 2x^{(3)} + 3y^{(3)}) = \frac{1}{20}(25 - 2(1.02) + 3(-0.965)) = \frac{20.065}{20} = 1.0032.$

Substitute these values in (1), we get fifth approximation

$x^{(5)} = \frac{1}{20}(17 - y^{(4)} + 2z^{(4)}) = \frac{1}{20}(17 + 1.0015 + 2(1.0032)) = \frac{20.0079}{20} =$

1.0004.

$y^{(5)} = \frac{1}{20}(-18 - 3x^{(4)} + z^{(4)}) = \frac{1}{20}(-18 - 3(1.0013) + 1.0032) =$

$- = -\frac{20.0007}{20} = -1.00003.$

$z^{(5)} = \frac{1}{20}(25 - 2x^{(4)} + 3y^{(4)}) = \frac{1}{20}(25 - 2(1.0013) + 3(1.0015)) = \frac{19.9929}{20} =$

1.9996.

Substitute these values in (1), we get sixth approximation

$x^{(6)} = \frac{1}{20}(17 - y^{(5)} + 2z^{(5)}) = \frac{1}{20}(17 + 1.00003 + 2(0.9996)) = \frac{19.99923}{20} =$

0.99996.

$y^{(6)} = \frac{1}{20}(-18 - 3x^{(5)} + z^{(3)}) = \frac{1}{20}(-18 - 3(1.0004) + 0.9996) = \frac{-20.0016}{20} =$

$-1.00008.$

$z^{(6)} = \frac{1}{20}(25 - 2x^{(5)} + 3y^{(5)}) = \frac{1}{20}(25 - 2(1.0004) + 3(-1.00003)) = \frac{19.99911}{20} =$

$0.99995.$

The values in the fifth and sixth approximation are nearly same.

$\therefore$ Hence the solution is $x = 1, y = -1, z = 1$.

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* Solve the equations by Jacobi Method $83x + 11y - 4z = 95$; $7x + 52y + 13z = 104$; $3x + 8y + 29z = 71$?

Solution:- Given equation can be written as

$x = \frac{1}{83}(95 - 11y + 4z)$, $y = \frac{1}{52}(104 - 7x - 13z)$, $z = \frac{1}{83}(71 - 3x - 8y)$------(1).

Let us take the first approximation to be $x^{(1)} = 0, y^{(1)} = 0, z^{(1)} = 0$.

Then the second approximation using (1), we get

$x^{(2)} = \frac{1}{83}(95 - 11y^{(1)} + 4z^{(1)}) = \frac{1}{83}(95 - 0 + 0) = 1.14$

$y^{(2)} = \frac{1}{52}(104 - 7x^{(1)} - 13z^{(1)}) = \frac{1}{52}(104 - 0 - 0) = \frac{104}{52} = 2.$

$z^{(2)} = \frac{1}{29}(71 - 3x^{(1)} - 8y^{(1)}) = \frac{1}{29}(71 - 0 - 0) = \frac{71}{23} = 2.4.$

Substituting these values in R.H.S of (1), we get third approximation

$x^{(3)} = \frac{1}{83}(95 - 11y^{(2)} + 4z^{(2)}) = \frac{1}{83}(95 - 11(2) + 4(2.4)) = \frac{82.6}{83} = 0.995 \sim 1.$

$y^{(3)} = \frac{1}{52}(104 - 7x^{(2)} - 13z^{(2)}) = \frac{1}{52}(104 - 7(1.14) - 13(2.4)) = \frac{64.82}{52} = 1.25.$

$z^{(3)} = \frac{1}{29}(71- 3x^{(2)} - 8y^{(2)}) = \frac{1}{29}(71 - 3(1.14) - 8(2)) = \frac{51.58}{29} = 1.78.$

Now Substitute these values in (1), we get fourth approximation

$x^{(4)} = \frac{1}{83}(95 - 11y^{(3)} + 4z^{(3)}) = \frac{1}{83}(95 -11(1.25)+ 4(1.78)) = \frac{88.37}{83} = 1.1.$

$y^{(4)} = \frac{1}{52}(104 - 7x^{(3)} - 13z^{(3)}) = \frac{1}{52}(104 - 7(1) - 13(1.78)) = \frac{73.86}{52} = 1.42.$

$z^{(4)} = \frac{1}{29}(71- 3x^{(3)} - 8y^{(3)}) = \frac{1}{29}(71 - 3(1) - 8(1.25)) = \frac{58}{29} = 2.$

Substitute these values in (1), we get fifth approximation

$x^{(5)} = \frac{1}{83}(95 - 11y^{(4)} + 4z^{(4)}) = \frac{1}{83}(95 -11(1.42)+ 4(2)) = \frac{-87.38}{83} = 1.1.$

$y^{(5)} = \frac{1}{52}(104 - 7x^{(4)} - 13z^{(4)}) = \frac{1}{52}(104 - 7(1.1) - 13(2)) = \frac{70.3}{52} = 1.4.$

$z^{(5)} = \frac{1}{29}(71- 3x^{(4)} - 8y^{(4)}) = \frac{1}{29}(71 - 3(1.1) - 8(1.42)) = \frac{51.7}{29} = 1.78.$

$\therefore$ Hence the solution is x = 1.1, y = 1.4, z = 1.78.

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3.7 GAUSS SEIDEL METHOD

* Solve by Gauss Seidel method $10x_1 + x_2 + x_3 = 12$, $2x_1 + 10x_2 + x_3 = 13$, $2x_1 + 2x_2 + 10x_3 = 14$.

Solution:-

From the given equations we have

$$X_1 = \frac{1}{10}[12 - x_1 - x_2]\text{---------(1)}$$

$$X_2 = \frac{1}{10}[13 - 2x_1 - x_3]\text{---------(2)}$$

$$X_3 = \frac{1}{10}[14 - 2x_1 - 2x_2]\text{---------(3)}$$

Putting x = 0 in right side of the equation (1), we get $x_1^{(1)} = \frac{12}{10} = 1.2$

Putting $x_1 = x_1^{(1)} = 1.2$, $x_3 = 0$ in (2), we get $x_2^{(1)} = \frac{1}{10}(13 - 2.4 - 0) = 10.6/10 = 1.06$

Putting $x_1 = x_1^{(1)} = 1.2$, and $x_2 = x_2^{(1)} = 1.06$ in (3), we get

$x_3^{(2)} = \frac{1}{10}[14 - 2.4 - 2.12] = 0.948.$

For the second iteration we have $x_1^{(2)} = \frac{1}{10}[12 - 1.06 - 0.948] = 0.9992.$

$x_2^{(2)} = \frac{1}{10}[13 - 2(0.9992) - 0.948] = 1.00536.$

$x_3^{(2)} = \frac{1}{10}[14 - 2(0.9992) - 2(1.00536)] = 0.999098.$

Thus the iteration process is continued

The results are tabulated as follows correcting to four decimal places.

i	$x_1^{(i)}$	$x_2^{(i)}$	$x_3^{(i)}$
0	1.2000	0.0000	0.0000
1	1.2000	1.0600	0.9480

2	0.9992	1.0054	0.9991
3	0.9996	1.0000	1.0000
4	1.0000	1.0000	1.0000
5	1.0000	1.0000	1.0000

$\therefore$ The exact value of the roots are $x_1 = 1$, $x_2 = 1$, $x_3 = 1$.

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Network Analysis in linear system of Equations:

Set up a system of linear equations to represent the network shown in the figure. Then solve the system.

Solution:-

Each of the network's five junctions give rise to a linear equations as follows:

$X_1 + X_2 = 20$ Junction 1

$X_3 - X_4 = -20$ Junction

$X_2 + X_3 = 20$ Junction 3

$X_1 - X_5 = -10$ Junction 4

$-X_4 + X_5 = -10$ Junction 5

The augmented matrix for this system is

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 20 \\ 0 & 0 & 1 & -1 & 0 & -20 \\ 0 & 1 & 1 & 0 & 0 & 20 \\ 1 & 0 & 0 & 0 & -1 & -10 \\ 0 & 0 & 0 & -1 & 1 & -10 \end{bmatrix}$$

Gauss Jordan elimination produces the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & -10 \\ 0 & 1 & 0 & 0 & 1 & 30 \\ 0 & 0 & 1 & 0 & -1 & -10 \\ 0 & 0 & 0 & 1 & -1 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

From the matrix above, you can see that

$X_1 - X_5 = -10$, $X_2 - X_5 = 30$,

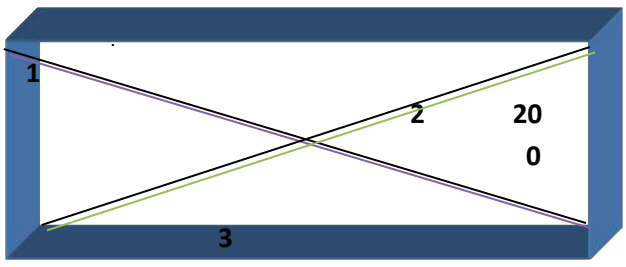
$X_3 - X_5 = -10$ and $X_4 - X_5 = -10$

Letting $t = X_5$, we have

$X_1 = t - 10$, $X_2 = -t + 30$, $X_3 = t + 30$, $X_4 = t + 10$, $X_5 = t$

Where t is the real number, so this system has infinitely many solutions.

1



2

4

5

Figure

In example 5, suppose you could control the mount of flow along the branch labeled X_5. using the solution of example 5 you could then control the flow represented by each of the other variables. For instance, letting t =10 would reduces the flow of X_1 and X_3 to zero, as shown In the figure.

4. LAPLACE TRANSFORM

4.1 Definition and properties of Laplace Transform, piecewise continuous functions, the Laplace Transform method of solving initial value problems

The method of Laplace transforms is a system that relies on algebra (rather than calculus-based methods) to solve linear differential equations. While it might seem to be a somewhat cumbersome method at times, it is a very powerful tool that enables us to readily deal with linear differential equations with discontinuous forcing functions.

***Definition*:** Let $f(t)$ be defined for $t \geq 0$. The Laplace transform of $f(t)$, denoted by $F(s)$ or $\mathcal{L}\{f(t)\}$, is an *integral transform* given by the *Laplace integral*:

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\, dt .$$

Provided that this (improper) integral exists, i.e. that the integral is convergent.

The Laplace transform is an operation that transforms a function of t (i.e., a function of *time domain*), defined on [0, ∞), to a function of s (i.e., of *frequency domain*)*. $F(s)$ is the *Laplace transform*, or simply *transform*, of $f(t)$. Together the two functions $f(t)$ and $F(s)$ are called a *Laplace transform pair*.

For functions of t continuous on [0, ∞), the above transformation to the frequency domain is *one-to-one*. That is, different continuous functions will have different transforms.

The integral is divergent whenever $s \le 0$. However, when $s > 0$, it converges to

$$\frac{1}{}(0 - e^{0}) = \frac{-1}{}(-1) = \frac{1}{} = F(s)$$

Example: Let $f(t) = t$, then $F(s) = \dfrac{1}{s^2}, \quad s > 0.$

[This is left to you as an exercise.]

The integral is divergent whenever $s \le a$. However, when $s > a$, it converges to

$$\frac{1}{}(0 - e^{0}) = \frac{1}{}(-1) = \frac{1}{} = F(s).$$

$$a - s \qquad a - s \qquad s - a$$

Definition: A function $f(t)$ is called *piecewise continuous* if it only has finitely many (or none whatsoever – a continuous function is considered to be "piecewise continuous"!) discontinuities on any interval $[a, b]$, and that both one-sided limits exist as t approaches each of those discontinuity from within the interval. The last part of the definition means that f could have removable and/or jump discontinuities only; it cannot have any infinity discontinuity.

Theorem: Suppose that

1. f is piecewise continuous on the interval $0 \le t \le A$ for any $A > 0$.

2. $|f(t)| \le K e^{at}$ when $t \ge M$, for any real constant a, and some positive constants K and M. (This means that f is "of *exponential order*", i.e. its rate of growth is no faster than that of exponential functions.)

3.Then the Laplace transform, $F(s) = \mathcal{L}\{f(t)\}$, exists for $s > a$.

***Note*:** The above theorem gives a sufficient condition for the existence of Laplace transforms. It is not a necessary condition. A function does not need to satisfy the two conditions in order to have a Laplace transform.

Examples of such functions that nevertheless have Laplace transforms are logarithmic functions and the unit impulse function.

Some properties of the Laplace Transform:

1. $\mathcal{L}\{0\} = 0$
2. $\mathcal{L}\{f(t) \pm g(t)\} = \mathcal{L}\{f(t)\} \pm \mathcal{L}\{g(t)\}$
3. $\mathcal{L}\{c\, f(t)\} = c\, \mathcal{L}\{f(t)\}$, for any constant c.

Properties 2 and 3 together means that the Laplace transform is *linear*.

[The derivative of Laplace transforms]

$\mathcal{L}\{(-t)f(t)\} = F'(s)$ or, equivalently $\mathcal{L}\{t\, f(t)\} = -F'(s)$

Example: $\mathcal{L}\{t^2\} = -(\mathcal{L}\{t\})' = -\frac{d}{ds}\left(\frac{1}{s}\right)^2 = -\left(-\frac{2}{s^3}\right) = \frac{2}{s^3}$

In general, the derivatives of Laplace transforms satisfy

$\mathcal{L}\{(-t)^n f(t)\} = F^{(n)}(s)$ or, equivalently $\mathcal{L}\{t^n f(t)\} = (-1)^n F^{(n)}(s)$

Warning: The Laplace transform, while a linear operation, is **not** *multiplicative*. That is, in general $\mathcal{L}\{f(t)\, g(t)\} \neq \mathcal{L}\{f(t)\}\, \mathcal{L}\{g(t)\}$.

Exercise: (a) Use property 4 above, and the fact that $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$,

to deduce that $\mathcal{L}\{t\, e^{at}\} = \frac{1}{(s-a)^2}$. (b) What will $\mathcal{L}\{t^2 e^{at}\}$ be?

Exercises C-1.1:

1 – 5 Use the (integral transformation) definition of the Laplace transform to find the Laplace transform of each function below.

1. t^2 2. te^{6t}

3. $\cos 3t$ 4. $e^{-t}\sin 2t$

5.* $e^{i\alpha t}$, where i and α are constants, $i = \sqrt{-1}$.

6 – 8 Each function $F(s)$ below is defined by a definite integral. Without integrating, find an explicit expression for each $F(s)$.

[Hint: each expression is the Laplace transform of a certain function. Use your knowledge of Laplace Transformation, or with the help of a table of common Laplace transforms to find the answer.]

6. $\int_{+0}^{\infty} e^{-(s+7)t}\,dt$

7. $\int_{+0}^{\infty} t^2 e^{-(s-3)t}\,dt$

8. $\int_{+0}^{\infty} 4e^{-st}\sin 6t\,dt$

9. Let $f(t)$ be a continuous function and $F(s)$ be its Laplace transform. Suppose $f(t) \geq 0$ for $t \geq 0$. (a) Show that $F(0)$ is equal to the area of the region under the curve $y = f(t)$, $0 \leq t \leq \infty$. (b) Verify this fact by comparing the value of $F(0)$ with the area under the curve obtained the usual way (i.e. by integration) for each of the functions e^{-t}, te^{-2t}, and t^2e^{-4t}.

Answers C-1.1:

1. $\dfrac{2}{s^3}$ 2. $\dfrac{1}{(s-6)^2}$

3. $\dfrac{s}{s^2 + 9}$ 4. $\dfrac{2}{s^2 + 2s + 5}$

5. $\dfrac{s}{s^2 + \alpha^2} + i\dfrac{\alpha}{s^2 + \alpha^2}$

Note: Since the Euler's formula says that $e^{i\alpha t} = \cos \alpha t + i \sin \alpha t$, therefore, $\mathcal{L}\{e^{i\alpha t}\} = \mathcal{L}\{\cos \alpha t + i \sin \alpha t\}$. That is, the real part of its Laplace transform corresponds to that of $\cos \alpha t$, the imaginary part corresponds to that of $\sin \alpha t$. (Check it for yourself!)

6. $\dfrac{1}{s + 7}$ 7. $\dfrac{2}{(s - 3)^3}$ 8. $\dfrac{24}{s^2 + 36}$

9. Set $s = 0$ in the definition $F(s) = \int_0^\infty e^{-st} f(t)\, dt$ to show that

$F(0) = \int_0^\infty f(t) dt = A$.

Solution of Initial Value Problems :

We now shall meet "the new System": how the Laplace transforms can be used to solve linear differential equations algebraically.

Theorem: [**Laplace transform of derivatives**] Suppose f is of exponential order, and that f is continuous and f' is piecewise continuous on any interval $0 \le t \le A$. Then $\mathcal{L}\{f'(t)\} = s\,\mathcal{L}\{f(t)\} - f(0)$

Applying the theorem multiple times yields:

$$\mathcal{L}\{f''(t)\} = s^2\,\mathcal{L}\{f(t)\} - s f(0) - f'(0),$$

$$\mathcal{L}\{f'''(t)\} = s^3\,\mathcal{L}\{f(t)\} - s^2 f(0) - s f'(0) - f''(0),$$

$$\vdots \qquad \mathcal{L}\{f^{(n)}(t)\} = s^n\,\mathcal{L}\{f(t)\} - s^{n-1} - s^2 f^{(n-3)}(0) - s f^{(n-2)}(0) - f$$

$$\vdots$$

This is an extremely useful aspect of the Laplace transform: that it changes differentiation with respect to *t* into multiplication by *s* (and, as seen a little earlier, differentiation with respect to *s* into multiplication by −*t*, on the other hand). Equally importantly, it says that the Laplace transform, when applied to a differential equation, would change derivatives into algebraic expressions in terms of *s* and (the transform of) the dependent variable itself. Thus, it can transform a differential equation into an algebraic equation.

We are now ready to see how the Laplace transform can be used to solve differentiation equations.

Solving initial value problems using the method of Laplace transforms

To solve a linear differential equation using Laplace transforms, there are only 3 basic steps:

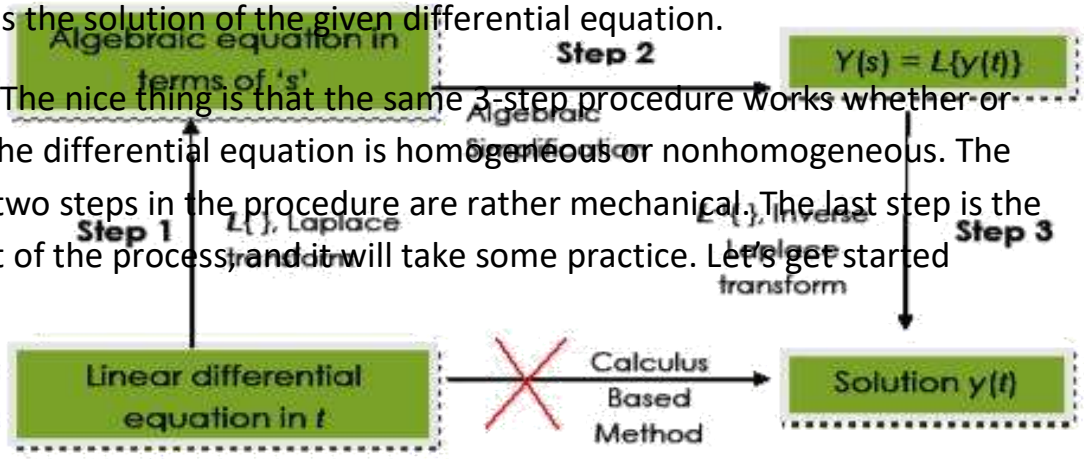
1.Take the Laplace transforms of both sides of an equation.

2.Simplify algebraically the result to solve for $\mathcal{L}\{y\} = Y(s)$ in terms of *s*.

3.Find the inverse transform of *Y*(*s*). (Or, rather, find a function *y*(*t*) whose Laplace transform matches the expression of *Y*(*s*).) This inverse transform, *y*(*t*), is the solution of the given differential equation.

The Laplace Transform Method of Solving an Initial Value Problem

Flow Chart



The nice thing is that the same 3-step procedure works whether or not the differential equation is homogeneous or nonhomogeneous. The first two steps in the procedure are rather mechanical. The last step is the heart of the process, and it will take some practice. Let's get started

***Example*:** $y'' - 6y' + 5y = 0, \qquad y(0) = 1, y'(0) = -3$

[Step 1] Transform both sides

$\mathcal{L}\{y'' - 6y' + 5y\} = \mathcal{L}\{0\}$

$(s^2\,\mathcal{L}\{y\} - s\,y(0) - y'(0)) - 6(s\mathcal{L}\{y\} - y(0)) + 5\mathcal{L}\{y\} = 0$

[Step 2] Simplify to find $Y(s) = \mathcal{L}\{y\}$

$(s^2\,\mathcal{L}\{y\} - s - (-3)) - 6(s\,\mathcal{L}\{y\} - 1) + 5\mathcal{L}\{y\} = 0\ (s^2 - 6\,s + 5)$

$\mathcal{L}\{y\} - s + 9 = 0(s^2 - 6\,s + 5)\ \mathcal{L}\{y\} = s - 9$

[Step 3] Find the inverse transform $y(t)$

Use partial fractions to simplify,

$$\mathcal{L}\{y\} = \frac{s-9}{s^2-6s+5} = \frac{a}{s-1} + \frac{b}{s-5}$$

$$\frac{s-9}{s^2-6s+5} = \frac{a(s-5)}{(s-1)(s-5)} + \frac{b(s-1)}{(s-5)(s-1)}$$

$s - 9 = a(s - 5) + b(s - 1) = (a + b)s + (-5a - b)$

Equating the corresponding coefficients:

$1 = a + b \qquad a = 2$

$-9 = -5a - b \qquad b = -1$

Hence,

$$\mathcal{L}\{y\} = \frac{s-9}{s^2-6s+5} = \frac{2}{s-1} - \frac{1}{s-5}\ .$$

The last expression corresponds to the Laplace transform of $2e^{t} - e^{5t}$. Therefore, it must be that, $y(t) = 2e^{t} - e^{5t}$.

Many of the observant students no doubt have noticed an interesting aspect (out of many) of the method of Laplace transform: that it finds the particular solution of an initial value problem directly, without solving for the general solution first. Indeed, it usually takes more effort to find the general solution of an equation than it takes to find a particular solution!

The Laplace Transform method can be used to solve linear differential equations of any order, rather than just second order equations as in the previous example. The method will also solve a nonhomogeneous linear differential equation directly, using the exact same three basic steps, without having to separately solve for the complementary and particular solutions. These points are illustrated in the next two examples.

Example: $y' + 2y = 4t\,e^{-2t}$, $y(0) = -3$.

[Step 1] Transform both sides

$$\mathcal{L}\{y' + 2y\} = \mathcal{L}\{4t\,e^{-2t}\}$$

$$(s\mathcal{L}\{y\} - y(0)) + 2\mathcal{L}\{y\} = \mathcal{L}\{4t\,e^{-2t}\} = \frac{4}{(s+2)^2}$$

$$(s\mathcal{L}\{y\} - (-3)) + 2\mathcal{L}\{y\} = \frac{4}{(s+2)^2}$$

$$(s+2)\,\mathcal{L}\{y\} + 3 = \frac{4}{(s+2)^2}$$

$$(s+2)\,\mathcal{L}\{y\} = \frac{4}{(s+2)^2} - 3$$

$$(s+2)^2$$

$$\mathcal{L}\{y\} = \frac{4}{3} - \frac{3}{s+2} = \frac{4-3(s+2)}{3} \quad 2 = \frac{-3s^2-12s-8}{3}$$
$$(s+2) \qquad (s+2) \qquad (s+2) \qquad (s+2)$$

[Step 3] Find the inverse transform $y(t)$

By partial fractions,

$$\mathcal{L}\{y\} = \frac{-3s^2 \quad -12s-8}{3} = \frac{a}{3} + \frac{b}{2} + \frac{c}{s+2}.$$
$$(s+2) \qquad (s+2) \qquad (s+2) \qquad s+2$$

$$\frac{-3s^{\,2}-12s-8}{(s+2)^3} = \frac{a}{(s+\ 2)^3} + \frac{b(s+2)}{(s+2)^3} + \frac{c(s+2)^2}{(s+2)^3}$$

$$= \frac{a+bs+2b+cs^2 \quad +4cs+4c}{(s+2)^3} = \frac{cs^2+(b+4c)s+(a+2b+4c)}{(s+2)^3}$$

$-3 = c$ $\qquad$ $a = 4$

$-12 = b + 4c$ $\qquad$ $b = 0$

$-8 = a + 2b + 4c$ $\qquad$ $c = -3$

$$\mathcal{L}\{y\} = \frac{-3s^2 - 12s - 8}{(s+2)^3} = \frac{4}{(s+2)^3} - \frac{3}{s+2}.$$

This expression corresponds to the Laplace transform of $2t^2 e^{-2t} - 3e^{-2t}$. Therefore,

$$y(t) = 2t^2 e^{-2t} - 3e^{-2t}.$$

Note: $\mathcal{L}\{t^n e^{at}\} = \frac{}{(s-a)^{n+1}}$

Example: $y'' - 3y' + 2y = e^{3t}$, $y(0) = 1, y'(0) = 0$

[Step 1] Transform both sides

$(s^2 \mathcal{L}\{y\} - s\,y(0) - y'(0)) - 3(s\mathcal{L}\{y\} - y(0)) + 2\mathcal{L}\{y\} = \mathcal{L}\{e^{3t}\}$

[Step 2] Simplify to find $Y(s) = \mathcal{L}\{y\}$

$(s^2 \mathcal{L}\{y\} - s - 0) - 3(s\mathcal{L}\{y\} - 1) + 2\mathcal{L}\{y\} = 1 / (s - 3)$

$(s^2 - 3s + 2)\,\mathcal{L}\{y\} - s + 3 = 1 / (s - 3)$

$(s^2 - 3$

s

$$+ 2)\,\mathcal{L}\{y\} = s - 3 + \frac{1}{s-3} = \frac{(s-3)^2 + 1}{s-3}$$

$$\mathcal{L}\{y\} = \frac{s^2 - 6s + 10}{(s^2 - 3s + 2)(s-3)} = \frac{s^2 - 6s + 10}{(s-1)(s-2)(s-3)}$$

[Step3] Find the inverse transform $y(t)$

By partial fractions,

$$\mathcal{L}\{y\} = \frac{s^2 - 6s + 10}{(s-1)(s-2)(s-3)} = \frac{5}{2}\cdot\frac{1}{s-1} - 2\cdot\frac{1}{s-2} + \frac{1}{2}\cdot\frac{1}{s-3}.$$

Therefore, $y(t) = \frac{5}{2}e^{t} - 2e^{2t} + \frac{1}{2}e^{3t}$.

For the next example, we will need the following Laplace transforms:

$$\mathcal{L}\{\cos bt\} = \frac{s}{s^2 + b^2}, \quad s > 0$$

$$\mathcal{L}\{\sin bt\} = \frac{b}{s^2 + b^2}, \quad s > 0$$

$$\mathcal{L}\{e^{at}\cos bt\} = \frac{s-a}{(s-a)^2 + b^2}, \quad s > a$$

$$\mathcal{L}\{e^{at}\sin bt\} = \frac{b}{(s-a)^2 + b^2}, \quad s > a$$

$$\frac{}{(s-a)^2+b^2}$$

***Note*:** The values of a and b in the last two expressions' denominators can be determined without using the method of completing the squares. Any irreducible quadratic polynomial $s^2 + Bs + C$ can always be written in the

required from of $(s-a)^2 + b^2$ by using the quadratic formula to find (necessarily complex-valued roots) s . The value a is the real part of s, and the value b is just the absolute value of the imaginary part of s. That is, if $s = \lambda \pm \mu\, i$, then $a = \lambda$ and $b = \mu$.

Example: $y'' - 2y' + 2y = \cos(t)$, $y(0) = 1, y'(0) = 0$

[Step 1] Transform both sides

$(s^2\, \mathcal{L}\{y\} - s\, y(0) - y'(0)) - 2(s\mathcal{L}\{y\} - y(0)) + 2\mathcal{L}\{y\} = \mathcal{L}\{\cos(t)\}$

[Step 2] Simplify to find $Y(s) = \mathcal{L}\{y\}$

$(s^2\, \mathcal{L}\{y\} - s - 0) - 2(s\mathcal{L}\{y\} - 1) + 2\mathcal{L}\{y\} = s\,/\,(s^2 + 1)$

$(s^2 - 2\,s + 2)\, \mathcal{L}\{y\} - s + 2 = s\,/\,(s^2 + 1)$

$$(s^2 - 2\,s + 2)\, \mathcal{L}\{y\} = s - 2 + \frac{s}{s^2+1} = \frac{(s-2)(s^2+1) + s}{s^2+1}$$

$$\mathcal{L}\{y\} = \frac{s^3 - 2s^2 + s - 2 + s}{(s^2+1)(s^2 - 2s + 2)} = \frac{s^3 - 2s^2 + 2s - 2}{(s^2+1)(s^2 - 2s + 2)}$$

[Step 3] Find the inverse transform $y(t)$

By partial fractions,

$$\mathcal{L}\{y\} = \frac{s^3 - 2s^2 + 2s - 2}{} = \frac{1\,s - 2}{} + \frac{4s - 6}{}$$

$$\frac{}{(s^2+1)(s^2-2s+2)} = \frac{}{5(s^2+1)} + \frac{}{5(s^2-2s+2)}$$

$$= \frac{1}{5}\left[\frac{s}{s^2+1} - \frac{2}{s^2+1} + \frac{4(s-1)}{(s-1)^2+1} - \frac{2}{(s-1)^2+1}\right]$$

which corresponds to

$$y(t) = \frac{1}{5}\left[\cos(t) - 2\sin(t) + 4e^t\cos(t) - 2e^t\sin(t)\right]$$

Examples: Find the inverse Laplace transform of each

(i) $F(s) = \dfrac{2s-5}{s^2+4s+8}$

Rewrite $F(s)$ as:

$$F(s) = \frac{2s-5}{(s+2)^2+2^2} = \frac{2(s+2)}{(s+2)^2+2^2} - \frac{9}{2}\cdot\frac{2}{(s+2)^2+2^2}$$

Answer: $f(t) = 2e^{-2t}\cos(2t) - \frac{9}{2}e^{-2t}\sin(2t)$

(ii) $F(s) = \frac{s+4}{(s-2)^3}$

Use partial fractions to rewrite $F(s)$ as:

$$F(s) = \frac{1}{(s-2)^2} + \frac{6}{(s-2)^3}$$

Answer: $f(t) = te^{2t} + 3t^2e^{2t}$

Appendix A

4.2 Some Additional Properties of Laplace Transforms

I. Suppose $f(t)$ is discontinuous at $t = 0$, then the Laplace transform of its derivative becomes

$$\mathcal{L}\{f'(t)\} = s\,\mathcal{L}\{f(t)\} - \lim_{t \to 0-} f(t).$$

Suppose $f(t)$ is a periodic function of period T, that is, $f(t + T) = f(t)$, for all t in its domain, then

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt.$$

***Comment*:** This property is convenient when finding the Laplace transform of a discontinuous periodic function, whose discontinuities (necessarily there are infinitely many, due to the periodic nature of the function) would make the usual approach of integrating from 0 to ∞ unwieldy.

Let c > 0 be a constant, the *time-scaling* property of Laplace transform states that

$$\mathcal{L}\{f(ct)\} = \frac{1}{c} F\left(\frac{s}{c}\right).$$

IV. We have known that $\mathcal{L}\{-t\,f(t)\} = F'(s)$. Taking the inverse transforms on both sides yields $-t\,f(t) = \mathcal{L}^{-1}\{F'(s)\}$. Therefore, $f(t) = -\frac{1}{t}\mathcal{L}^{-1}\{F'(s)\}$. That is, if we know how to invert the function $F'(s)$, then we also know how to find the inverse of its anti-derivative $F(s)$. Formally,

$$\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} F(u)\,du.$$

V. Similarly, dividing $F(s)$ by s corresponding an integral with respect to t.

$$\mathcal{L}\left\{\int_0^t f(u)\,du\right\} = \frac{F(s)}{s}$$

Comment: Therefore, dividing a function by its independent variable has the effect of anti-differentiation with respect to the other independent variable for Laplace transforms. These two properties (IV and V) are the f

$$\mathcal{L}\left\{\int_0^t f(\tau)g(t-\tau)d\tau\right\} = \mathcal{L}\{f(t)\}\mathcal{L}\{g(t)\}.$$

In addition, observe that, by taking the inverse Laplace transform of both sides the property VI, we have

$$\int_0^{\tau} f(\tau)g(t-\tau)d\tau = \mathcal{L}^{-1}\{\mathcal{L}\{f(t)\}\mathcal{L}\{g(t)\}\}.$$

In other words, we can obtain the inverse Laplace transform of a (simple) function of s that is itself a product of Laplace transforms of two known functions of t by taking the convolution of the two functions of t.

Example: Find the inverse Laplace transform of

$$F(s) = \frac{1}{s(s^2 + 4)} .$$

First, note that

$$F(s) = \frac{1}{s} \times \frac{1}{s^2 + 4} = \frac{1}{2} \quad \mathcal{L}\,\mathcal{L} \quad \{1\}\,\{\sin 2t\}$$

Therefore, its inverse can be found by the convolution integral (letting $f(t) = 1$ and $g(t) = \sin 2t$):

4.3 Applications of Laplace transform

This section describes the applications of laplace transform in the domain of engineering. Laplace transform has several applications in almost all Engineering disciplines such as System Modelling, Analysis of Electrical and Electronic Circuits, Digital Signal Processing and Process Controls. Laplace transforms are critical for process controls. It helps analyze the variables, which when altered, produces

desired manipulations in the result. For example, while study heat experiments, Laplace transform is used to find out to what extent the given input can be altered by changing temperature, hence one can alter temperature to get desired output for a while. This is an efficient and easier way to control processes that are guided by differential equations

Electric Circuit theory

Example 1 :: The switching transient phenomenon in the RL , RC or RLC circuits can be solved by laplace transform. Let us consider a series RLC circuit as shown figure 1 to which a dc voltage V_0 is suddenly applied.

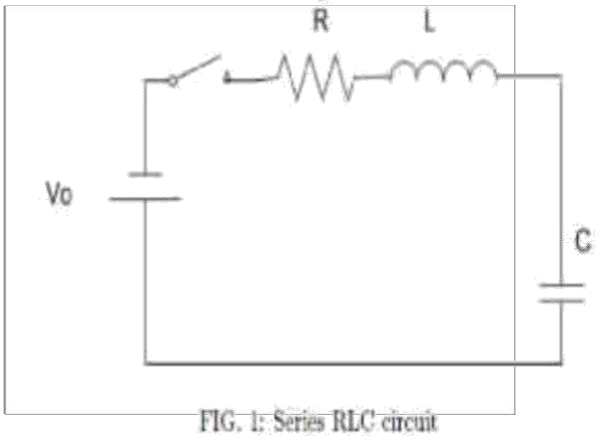


FIG. 1: Series RLC circuit

Now applying kirchoff's voltage law (KVL) to the circuit , we have

Differentiating bothsides, $R_i + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = V_0$

$$L\frac{d^2i}{dt^2} + \frac{1}{Ci} + R\frac{di}{dt} = 0 \text{ or}$$

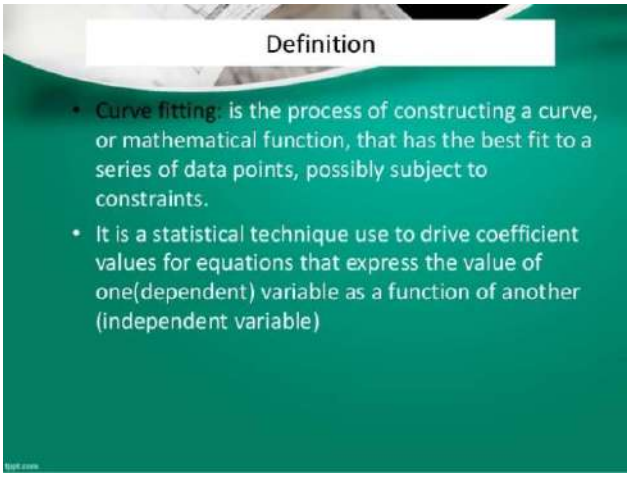
$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0 \quad \rightarrow (1)$$

Applying laplace transform to this equation, let us assume that the solution of this equation is

$$i(t) = K e^{sT}$$

$$L K s^2 e^{sT} + R K e^{sT} + \frac{1}{C K} e^{sT} = 0$$

5.CURVE FITTING



5.1 INTRODUCTION :

Curve and Surface Fitting

Curve fitting is one of the most powerful and most widely used analysis tools in Origin. Curve fitting examines the relationship between one or more predictors (independent variables) and a response variable (dependent variable), with the goal of defining a "best fit" model of the relationship.

Origin provides tools for linear, polynomial, and nonlinear curve fitting along with validation and goodness-of-fit tests. You can summarize and present your results with customized fitting reports. There are many time-saving options such as a copy-and-paste-operation feature which allows you to "paste" a just-completed fitting operation to another curve or data column. Curve fitting operations can also be part of an Analysis Template™, allowing you to perform batch fitting operations on any number of data files or data columns.

In this chapter we are concerned with the problem of fitting

an equation or a curve to data involving period values. An approximate non-mathematical relationship between the two variables , can be established by a diagram called **scatter diagram .** The exact mathematical relationship between the

two variables is given by simple algebra expression called **curve fitting .** Though there are infinite variety of curve in mathematics , the curve used for the purpose of curve fitting are relatively limited in type. The straight line is the simple and one of the most important curve used.Application of Curve Fitting in Indian Structure

The straight line :

The equation $y = a + bx$…. (1) Is an equation of the degree in x and y .It represents a straight line .

The difference $y_i - (a + bx_i)$….. (2)

Is zero if and only if the point (x_i, y_i) lies on the line given by (1)

Fitting a straight line:

Usually fitting a straight line means finding the value of the parameter a and b of the straight line given by (1) , as well as actually constructing the line itself .The Graphic method and the method of least square are two useful method for fitting a straight line .

The graphical method:

The method can be used whenever the given formula can be plotted as a straight line either directly or after suitable transformation. The straight line drawn after a careful visual estimate of its position has been made with the aid of a ruler. The coordinated of any two points on the measures and substitute in equation (1). The resulting equations in a and b are then solved for their *parameters.* Graphical method , whatever its theoretical attractions , suffers from the disadvantage that it is difficult apply in practice except for the straight line .This method will give fairly good results when finely divided coordinate paper is used , but in general it is recommended .

An alternative method, which id in almost universal use at present time, is known as the of **Least Squares** and we proceed to discuss it at length.

The method of lest squares:

This method of curve fitting was suggested early in the nineteenth century by the French mathematician Adrian Legendre. The method of least squares assumes that the best fitting line in the curve for which the sum of the squares of the vertical distances of the points (x_r , y_r) from the line is minimum. This method is more accurate that the graphical method.

Let $y = a + bx$ Refer (1)

Be the equation of the line. The ordinate of any point Q_r on the line vertically above or below a given point P_r , can be found by substituting the abscissa x_r , in the right-hand side of (1) . The two coordinates of Q_r will be $(x_r , a + bx_r)$ (see in the above figure). The vertical distances e_r from the line of any point p_r with coordinates (x_r , y_r) will therefore be given by the equation

$$e_r = y_r - (a + x_r) \ldots\ldots \quad (3)$$

We may say that e_r represents the difference between the actual ordinate y_r , of a pint and its theoretical ordinate $a + bx_r$.

Let $q = \sum_{r=1}^{n} e_r^2 = \sum_{r=1}^{n} (y_r - (a + bx_r))^2$ ……. (4)

The best fitting line is that line for which the sum of the square, $q = \sum e_r^2$ is a minimum. We find the value of a and b which make q minimum as follows :Differentiating q partially with respect to a and b equally these partially to zero , we obtain

$$\therefore \frac{\partial_q}{\partial_a} = \sum_{r=1}^{n} (-2)[y_r - (a + bx_r)] = 0 \ ,$$

And $$\frac{\partial_q}{\partial_o} = \sum_{r=1}^{n} (-2) x_r [y_r - (a + bx_r)] = 0 \ ,$$

Which yield the so – called **System of normal equations,**

$$\sum_{r=1}^{n} y_r = an + b \sum_{r=1}^{n} x_r \quad ,\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

And $\sum_{r=1}^{n} x_r \, y_r = a \sum_{r=1}^{n} x_r + b \sum_{r=1}^{n} x_r^2$ ……..(6)

Solving this system of equations we get

$$a = x = \frac{\sum_1^n x_r^2 \ \sum_1^n y_r - \sum_1^n x_r \sum x_r y_r}{n\sum_1^n x_r^2 - [\sum_1^n x_r} \quad \text{..... (7)}$$

$$b = \frac{n[\sum_{r=1}^n x_r y_r] - [\sum_{r=1}^n x_r][\sum_{r=1}^n x_r^2]}{n[\sum_{r=1}^n x_r^2] - [\sum_{r=1}^n x_r]^2} \quad \text{,.....(8)}$$

Where n is the number of points (x_r , y_r).

To simplify the for formulae, we get

$$t_r = \frac{x_r - \bar{x}}{h} \ , \qquad (9)$$

Where h is the interval between successive value of x assumed to be equally spaced. Shifting the origin to the mean $\bar{x}$ and compressing the intervals to unity we get transformed equation of (2) referred to the new (t, y) axes as

$$Y = \bar{a} + \bar{b}t \ , \qquad (10)$$

Where $\bar{a}\ and\ \bar{b}$ are the parameter in the new equation . In terms of the new coordinate formula (7) and (8) becomes

$$\bar{a} = \frac{\sum_1^n t_r^2 \sum_1^n y_r - \sum_1^n t_r \sum t_r y_r}{n\sum t_r^2 - (\sum t_r)^2} \ , \qquad (11)$$

$$\bar{b} = \frac{n\sum_1^n t_r y_1 - \sum_1^n t_r \sum t_r y_r}{n\sum_1^n t_r^2 - [\sum_1^n t_r]^2} \ , \qquad \text{.....} \qquad (12)$$

Transformation (9)

replace the $x_r\ \grave{s}$ by unit deviations t_r from the mean $\bar{x}$.

Since

$$\sum_{1}^{n} t_r = 0$$

The formula (11)and (12) reduce to

$$\bar{a} = \frac{\sum_1^n t_r^2 \sum_1^n y_r - 0}{n \sum t_r^2 - 0}$$

And

$$\bar{b} = \frac{n \sum_1^n t_r y_r - 0}{n \sum t_r^2 - 0}$$

Which further simplify to the form

5.2 Fitting a parabola:

When a set of point exhibits a parabolic trend , the fitting of a quadratic functions

$$y = a + bx + cx^2 ,$$

To the data may be carried out by the method of least square , Which leads to the three equations

$$na + b \sum_1^n x_r + c \sum x_r^2 = \sum_1^n y_r$$

$$a \sum_{1}^{n} x_r + b \sum_{1}^{n} x_r^2 + c \sum_{1}^{n} x_r^3 = \sum_{1}^{n} x_r y_r .$$

$$a \sum_{1}^{n} x_r^2 + b \sum_{1}^{n} x_r^3 + c \sum_{1}^{n} x_r^4 = \sum_{1}^{n} x_r^2 y_r .$$

The above equations can be solve for a , b, c.

5.3 EXPONETIAL FUNCTION Y= Ae^{bx}

Transforming the exponential equation $y = ae^{bx}$ by taking logarithms on both sides we get

$$\log y = \log a + bx \log e .$$

If we replace $\log y$ by y and the constant $\log a$ and b $\log e$ by $\grave{a}$ and $\acute{b}$

Respectively , we obtain

$$y = \acute{a} + b\acute{x} ,$$

Which defines a straight line .

Example:

1.Find the lest square line fir the data points (-1,10) ,(0,9),(1,7),(,4,5), (,2,5),(3,4),(4,3),(5,0) and (6,-1).

Solution: Here

x_r	y_r	x_r^2	y_r^2
-1	10	1	-10
0	9	0	0
1	7	1	7
2	5	4	10
3	4	9	12
4	3	9	12
5	0	25	0
6	-1	36	-6

20	37	92	25

From the table we have

$$n = 8\,, \sum x_r = 20\,, \sum y_r = 37, \sum x_r^2 = 92\,, and\ \sum x_r y_r = 25\,.$$

The normal equation are

$$na + b\sum x_r = \sum y_r\ ,$$

$$a\sum x_r + \sum x_r^2 = \sum x_r y_r\ .$$

Putting the value in normal equations

$$8a + 20b = 37\ ,$$

And

$$20a + 97b + 25\quad ,$$

And solving these equations we get

$$a = 8.6428571\ ,$$

$$b = -1.6071429\ \ .$$

Therefore the least square line is

$$y = 8.6428571 + (-1)1.6071429x\ ,$$

i.e.,

$$y = -1.6071429x + 8.6428571\,.$$

Example 2: **Find the least square line $y = a + bx$ for the data**

X	**-2**	**-1**	**0**	**1**	**2**
Y	**1**	**2**	**3**	**3**	**4**

Solution:

x_r	y_r	x_r^2	$x_r y_r$
-2	1	4	-2
-1	2	1	-2
0	3	0	0
1	3	1	3
2	4	4	8
0	13	10	7

$\therefore$ *In this case* $n = 5 , \sum x_r = 0 , \sum y_r = 13 , \sum x_{r2} = 10 , \sum x_r y_r = 7.$

The normal equation are

$$na + b \sum x_r = \sum y_r \ ,$$

$$a \sum x_r + b \sum x_r^2 = \sum x_r y_r \ .$$

Putting the value of $n , \sum x_r , \sum y_r , \sum x_r^2 \ and \ \sum x_r y_r$ in the above equation we get

$$5a = 13 , 10b = 7 \ .$$

Solving

$$a = 2.6 , b = 0.7 \ .$$

$\therefore$The required line of fit is

$$y = 2.6 + b = (0.7)x \ .$$

Example 3:

Find a formula for the line of the form $y = a + bx + cx^2$ Which will fit the following data

X	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Y	3.1950	3.2299	3.2532	3.2516	3.2516	3.2282	3.1807	3.1266	3.0594	2.9759

Solution :

The normal equation are

$$na = b\sum x_r + c\sum x_r^2 = \sum y_r^2 \ ,$$

$$a\sum x_r + b\sum x_r^2 + c\sum x_r^3 = \sum x_r y_r$$

$$a\sum x_r^2 + b\sum x_r^3 + c\sum x_r^4 = \sum x_r^2 y_r \ .$$

Substituting the value of $\sum x_r \ , \sum x_r^2 \ , \sum x_r^3 , \sum x_r y_r , \sum x_r^2 \ y_r , \sum y_r ,$ *and* n, *we get*

$$10a + 4.5b + 2.85c = 31.7616 \ ,$$

$$4.5a + 2.85b + 2.025c = 14.0896 \ ,$$

$$2.85a + 2.025b + 1.5333 = 8.82881 \ ,$$

And solving these equations we obtain

$$a = 3.1951 \ , b = 0.44254 \ , c = -0.76531.$$

$\therefore$ *The requied equation is*

$$y = 3.1951 + 0.44254x - 0.76531x^2.$$

Example 4:

Fit second degree parabola to the following data

X	0	1	2	3	4

y	0	1.8	1.3	2.5	6.3

Solution : The values of x are 0,1,2,3,4

The number of value is odd. Shifting the origin to the middle value 2 of x, and making the substituting

$$u = x - 2, v = y ,$$

The curve of fit as

$$v = a + bu + cu^2 \ ,$$

We obtain

x	v	u	v	uv	u^2	u^2v	u^3	u^4
0	1	-1	1	-2	4	4	-8	16
1	1.8	-2	1.8	1.6	1	1.8	-1	1
2	1.2	0	1.3	0	0	0	0	0
3	2.5	1	2.5	2.5	1	2.5	1	1
4	6.3	2	6.3	12.6	4	25.2	8	16
		0	12.9	11.3	10	33.5	0	34

The normal equation are

$$\sum u = na + b\sum u + c\sum u_r^2 \ ,$$

$$\sum uv = a\sum u + bu^2 + c\sum u^3 \ ,$$

$$\sum u^2 v = a\sum u^2 + b\sum u^3 + c\sum u^4 \ .$$

Putting the values of n ,$\sum u$, $\sum v$, $\sum uv$, … … … . , etc,

12.9=5a+40c,

$$11.3 = 10b \ ,$$

33.5=10a+34c,

Solving these normal equations we get

$$a = 1.48, b = 1.13, c = 0.550.$$

∴The required equation is

$$u = 1.48 + 1.13u + 1.55u^2 \quad ,$$

Substituting $u = x - 2$, $v = y$ in the above equation we get,

$$y = 1.42 + 1.13(x - 2) + (0.55)(x - 2)^2 \quad .$$

$$\therefore The\ requird\ curve\ of\ fit\ is\ the\ parabola$$

$$y = 1.42 - 1.07x + 0.55x^2.$$

5.4 Solved problems:

1)Find the lest square line $y = a + bx$ for the data

X	-4	-2	0	2	4
Y	1.2	2.8	6.2	7.8	13.2

A)	X	y	x^2	xy
	-4	1.2	16	-4.8
	-2	2.8	4	-5.6
	0	6.2	0	0
	2	7.8	4	15.6
	4	13.2	16	52.8
	0	31.2	4.	58

The normal equations are

$$\sum y = na + b\sum xi$$

$$\sum xy = a\sum x_i + b\sum x_i^2$$

31.2=5a

$$a = \frac{31.2}{5}$$

= 6.24

$$58 = b(40)$$

$$\therefore y = 6.24 + 1.45 \ .$$

2) **Fit a straight line to the following data regarding x as the independent variable**

X	0	1	2	3	4
Y	1	1.8	3.3	4.5	6.3

A)	x	y	x^2	xy
	0	1	0	0
	1	1.8	1	1.8
	2	3.3	4	6.6
	3	4.5	4	13.5
	4	6.3	16	25.2
	10	16.9	30	47.1

$$na + b\sum xi = \sum y$$

$$5a + b(10) = 16.9 \qquad (1)$$

$$a\sum x + b\sum x^2 = \sum xy$$

$$a(10) + b(30) = 47.1$$

$$1*2\rightarrow\ 10a + 20b = 33.8$$

$$\underline{2\rightarrow\ 10a + 30b = 47.1}$$

$$-10b = -11.3$$

$$b = \frac{13.3}{10}$$

=1.33

Substituting b in (1)

$$5a + (1.33)10 = 16.9$$

$$5a = 16.9 - 13.3$$

$$a = \frac{3.6}{5}$$

=0.72

$$\therefore y = 0.72 + 1.33x$$

3) Find the least square line $y = a + bx$

	X	-2	0	2	4	6
	Y	1	3	6	8	13

A)	X	y	x^2	xy
	-2	1	4	-2

0	3	0	-2
2	6	4	12
4	8	16	12
6	13	36	78
10	31	60	78

$$na + b\sum x_i = \sum xy$$

$$5a + 10b = 31 \quad (1)$$

$$a\sum x + b\sum x^2 = \sum xy$$

$$10a + 60b = 102 \quad (2)$$

$$1*2 \rightarrow 10a + 20b = 62$$

10a+60b=120

$$-40b = -58$$

$$b = 1.45$$

Substituting b in (1)

$$5a + (1.45)10 = 31$$

$$5a = 31 - 14.5$$

$$a = \frac{16.5}{5}$$

=3.3

$$a = 3.3 \ , b = 1.45$$

$$\therefore y = 3.3 + 1.45x$$

4) Find the least squares parabolic lift $y = a + bx + cx^2$

A)	x	y	x^2	x^3	x^4	xy	x^2y

-3	15	9	-27	81	-45	135
-1	5	1	-1	1	-5	5
1	1	1	1	1	1	1
3	5	9	27	81	15	45
0	26	20	0	164	-34	186

$$\sum y = na + b\sum x + c\sum x^2$$

$$26 = 4a + 0 + 20c$$

$$4a + 20c = 26 \qquad (1)$$

$$\sum xy = a\sum ax + b\sum x^2 + c\sum x^3$$

$$-34 = 0 + b(20) + c(0)$$

$$20b = -34$$

$$b = -\frac{34}{20}$$

$$= -1.7$$

$$\sum x^2y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

$$186 = 20a + 164c \qquad (2)$$

$$1*5 \quad \rightarrow \quad 20a + 100c = 130$$

$$\underline{2 \quad \rightarrow \quad 20a + 164c = 186}$$

$$-64c = -56$$

$$c = 0.875$$

$$4a + 20(0.875) = 26$$

$$4a = 36 - 17.5$$

$$a = \frac{8.5}{4}$$

$$= 2.125$$

$$y = a + bx + cx^2$$

$$\therefore y = 2.215 - 1.70x + 0.875x^2$$

5.5 APPLICATIONS OF CURVE FITTING

Fitting plane curves to data points :

If a function of the form cannot be postulated, one can still try to fit a plane curve.

Other types of curves, such as conic sections (circular, elliptical, parabolic, and hyperbolic arcs) or trigonometric functions (such as sine and cosine), may also be used, in certain cases. For example, trajectories of objects under the influence of gravity follow a parabolic path, when air resistance is ignored. Hence, matching trajectory data points to a parabolic curve would make sense. Tides follow sinusoidal patterns, hence tidal data points should be matched to a sine wave, or the sum of two sine waves of different periods, if the effects of the Moon and Sun are both considered.

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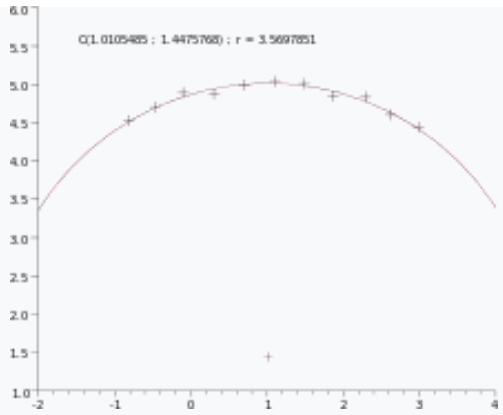
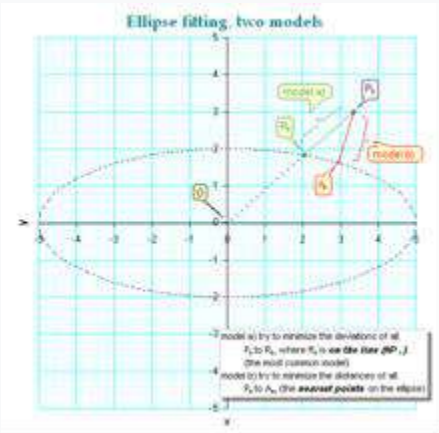
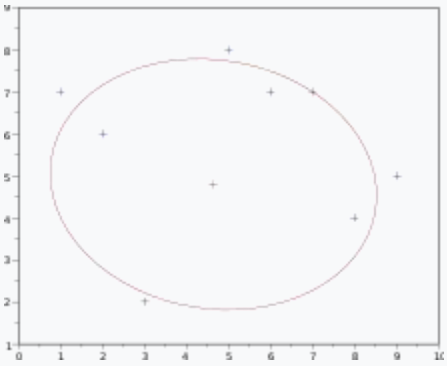
Fitting a circle by geometric fit



Circle fitting with the Coope method, the points describing a circle arc, centre (1 ; 1), radius 4.



different models of ellipse fitting



Ellipse fitting minimizing the algebraic distance (Fitzgibbon method).

Cooper[22] approaches the problem of trying to find the best visual fit of circle to a set of 2D data points. The method elegantly transforms the

ordinarily non-linear problem into a linear problem that can be solved without using iterative numerical methods, and is hence an order of magnitude faster than previous techniques.

Fitting an ellipse by geometric fit

The above technique is extended to general ellipses[23] by adding a non-linear step, resulting in a method that is fast, yet finds visually pleasing ellipses of arbitrary orientation and displacement.

Application to surfaces

Further information: Computer representation of surfaces

Note that while this discussion was in terms of 2D curves, much of this logic also extends to 3D surfaces, each patch of which is defined by a net of curves in two parametric directions, typically called **u** and **v**. A surface may be composed of one or more surface patches in each direction.

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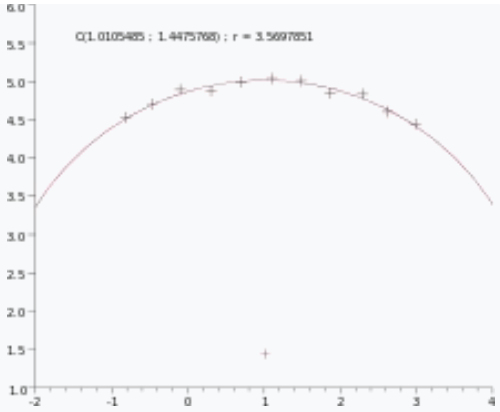

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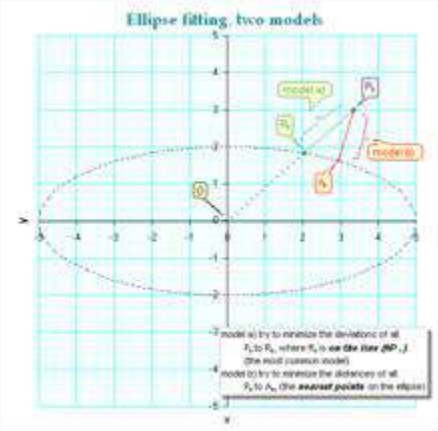

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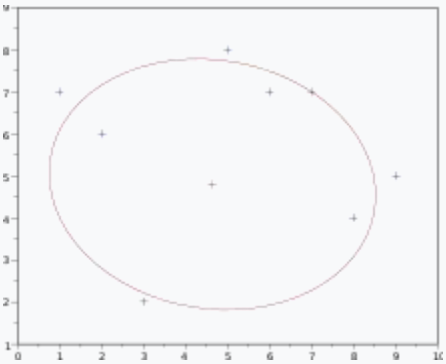

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Fitting functions to data points Edit

Most commonly, one fits a function of the form y=f(x).

Fitting lines and polynomial functions to data points Edit

Main article: Polynomial regression

See also: Polynomial interpolation

Polynomial curves fitting a sine function

Polynomial curves fitting points generated with a sine function. The black dotted line is the "true" data, the red line is a first degree polynomial, the green line is second degree, the orange line is third degree and the blue line is fourth degree.

The first degree polynomial equation

{\displaystyle y=ax+b\;}{\displaystyle y=ax+b\;}

is a line with slope a. A line will connect any two points, so a first degree polynomial equation is an exact fit through any two points with distinct x coordinates.

If the order of the equation is increased to a second degree polynomial, the following results:

{\displaystyle y=ax^{2}+bx+c\;.}y=ax^{2}+bx+c\;.

This will exactly fit a simple curve to three points.

If the order of the equation is increased to a third degree polynomial, the following is obtained:

{\displaystyle y=ax^{3}+bx^{2}+cx+d\;.}y=ax^{3}+bx^{2}+cx+d\;.

This will exactly fit four points.

A more general statement would be to say it will exactly fit four constraints. Each constraint can be a point, angle, or curvature (which is the reciprocal of the radius of an osculating circle). Angle and curvature constraints are most often added to the ends of a curve, and in such cases are called end conditions. Identical end conditions are frequently used to ensure a smooth transition between polynomial curves contained within a single spline. Higher-order constraints, such as "the change in the rate of curvature", could also be added. This, for example, would be useful in highway cloverleaf design to understand the rate of change of the forces applied to a

car (see jerk), as it follows the cloverleaf, and to set reasonable speed limits, accordingly.

The first degree polynomial equation could also be an exact fit for a single point and an angle while the third degree polynomial equation could also be an exact fit for two points, an angle constraint, and a curvature constraint. Many other combinations of constraints are possible for these and for higher order polynomial equations.

If there are more than n + 1 constraints (n being the degree of the polynomial), the polynomial curve can still be run through those constraints. An exact fit to all constraints is not certain (but might happen, for example, in the case of a first degree polynomial exactly fitting three collinear points). In general, however, some method is then needed to evaluate each approximation. The least squares method is one way to compare the deviations.

There are several reasons given to get an approximate fit when it is possible to simply increase the degree of the polynomial equation and get an exact match.:

Even if an exact match exists, it does not necessarily follow that it can be readily discovered. Depending on the algorithm used there may be a divergent case, where the exact fit cannot be calculated, or it might take too much computer time to find the solution. This situation might require an approximate solution.

The effect of averaging out questionable data points in a sample, rather than distorting the curve to fit them exactly, may be desirable.

Runge's phenomenon: high order polynomials can be highly oscillatory. If a curve runs through two points A and B, it would be expected that the curve would run somewhat near the midpoint of A and B, as well. This may not happen with high-order polynomial curves; they may even have values that are very large in positive or negative magnitude. With low-order polynomials, the curve is more likely to fall near the midpoint (it's even

guaranteed to exactly run through the midpoint on a first degree polynomial).

Low-order polynomials tend to be smooth and high order polynomial curves tend to be "lumpy". To define this more precisely, the maximum number of inflection points possible in a polynomial curve is n-2, where n is the order of the polynomial equation. An inflection point is a location on the curve where it switches from a positive radius to negative. We can also say this is where it transitions from "holding water" to "shedding water". Note that it is only "possible" that high order polynomials will be lumpy; they could also be smooth, but there is no guarantee of this, unlike with low order polynomial curves. A fifteenth degree polynomial could have, at most, thirteen inflection points, but could also have eleven, or nine or any odd number down to one. (Polynomials with even numbered degree could have any even number of inflection points from n - 2 down to zero.)

The degree of the polynomial curve being higher than needed for an exact fit is undesirable for all the reasons listed previously for high order polynomials, but also

leads to a case where there are an infinite number of solutions. For example, a first degree polynomial (a line) constrained by only a single point, instead of the usual two, would give an infinite number of solutions. This brings up the problem of how to compare and choose just one solution, which can be a problem for software and for humans, as well. For this reason, it is usually best to choose as low a degree as possible for an exact match on all constraints, and perhaps an even lower degree, if an approximate fit is acceptable.

Relation between wheat yield and soil salinity[18]

Fitting other functions to data points

Other types of curves, such as trigonometric functions (such as sine and cosine), may also be used, in certain cases.

In spectroscopy, data may be fitted with Gaussian, Lorentzian, Voigt and related functions.

In agriculture the inverted logistic sigmoid function (S-curve) is used to describe the relation between crop yield and growth factors. The blue figure was made by a sigmoid regression of data measured in farm lands. It can be seen that initially, i.e. at low soil salinity, the crop yield reduces slowly at increasing soil salinity, while thereafter the decrease progresses faster.

Algebraic fit versus geometric fit for curves

For algebraic analysis of data, "fitting" usually means trying to find the curve that minimizes the vertical (y-axis) displacement of a point from the curve (e.g., ordinary least squares). However, for graphical and image applications geometric fitting seeks to provide the best visual fit; which usually means trying to minimize the orthogonal distance to the curve (e.g., total least squares), or to otherwise include both axes of displacement of a point from the curve. Geometric fits are not popular because they usually require non-linear and/or iterative calculations, although they have the

advantage of a more aesthetic and geometrically accurate result.[19][20][21]

Fitting plane curves to data points

If a function of the form {\displaystyle y=f(x)}y=f(x) cannot be postulated, one can still try to fit a plane curve.

Other types of curves, such as conic sections (circular, elliptical, parabolic, and hyperbolic arcs) or trigonometric functions (such as sine and cosine), may also be used, in certain cases. For example, trajectories of objects under the influence of gravity follow a parabolic path, when air resistance is ignored. Hence, matching trajectory data points to a parabolic curve would make sense. Tides follow sinusoidal patterns, hence tidal data points should be matched to a sine wave, or the sum of two sine waves of different periods, if the effects of the Moon and Sun are both considered.

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Fitting surfaces

Further information: Computer representation of surfaces

See also: Multivariate interpolation and Smoothing

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A STUDY ON

APPLICATIONS OF NUMERICAL ANALYSIS

A Group Project submitted to KRISHNA UNIVERSITY

KRISHNA UNIVERSITY

In partial fulfilment of the requirement For the award of the degree of BACHELOR OF SCIENCE

Submitted by

B. Leela sankar *Register No. 18304203*

UNDER THE GUIDENCE OF

Sk. Parveen

LECTURER IN MATHEMATICS.

S.R.R &C.V.R GOVT.DEGREE COLLEGE

MACHAVARAM, VIJAYAWADA-520004

S.R.R & C.V.R GOVT.DEGREE COLLEGE

CERTIFICATE

This is to certificate that the project entitled ("APPLICATIONS OF NUMERICAL ANALYSIS") bonafide work done by B. Leela sankar (18304203) belongs to department of bachelor science studies,S.R.R & C.V.R GOVT. DEGREE COLLEGE,VIJAYAWADA under my guidance in partial fulfilment of the award for the degree in bachelor of science.

EXTERNAL EXAMINAR K.V. NAGALAKSHMI *sk.Parveen*

H. O. D of Dept of mathematics (PROJECT GUIDE)

DECLARATION

I am B. Leela sankar (18304203) here by declare that the project entitled “APPLICATIONS OF NUMERICAL ANALYSIS “has been prepared by in a partial fulfilment of the requirement for the award of the degree of bachelor of science submitted at S.R.R & C.V.R GOVT DEGREE COLLEGE affiliated to KRISHNA UNIVERSITY is an authentic project record of our own work .

The matter presented as not be submitted to any University or Institute for the award of bachelor of science Degree .

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ACKNOWLEDGEMENT

We would like to convey my deep felt regards to all of my well wishers, Who have contributed their share of cooperation and help me to complete this project work .

Firstly we would like to thank. Dr.I.RAVI principal and Sk. PARVEEN and K.V.NAGALAKSHMI H.O.D OF MATHEMATICS S.R.R &C.V.R GOVT.DEGREE COLLEGE VIJAYAWADA. for their Encouragement and guidance throughout my project work

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1.Interpolation With Equal Intervals

1.1 INTRODUCTION :

Interpolation means insertion or filling up intermediate terms of the series. It is the technique of getting the value of a function for any intermediate value of the argument from the given set of values of function is called interpolation, while the process of computing the value of the function outside of the given range is called interpolation. The study of interpolation is based on the calculus of finite differences which

The following assumptions are to be kept in mind for interpolation.

(1) The values of the function should be either increasing or decreasing

(2) The rise or fall in the values should be uniform plays a vital role in numerical analysis manner.

1.2 INTERPOLATION METHODS :

(i) Graphical method

(i) Method of curve fitting

(iii) Applications of the calculus of finite differences formula.

1.2.1 Graphical Method :

Let y =f(x), then plot a graph for different values of x corresponding to the value of y. From the graph, we can find the value of y for given value of x.

For example

Year(x)	1891	1901	1911	1921	1931
Population (Y)(in 1000's)	46	66	81	93	101

Interpolate population for the year 1919.

The following steps are to be maintained

1. Take a suitable scale for x and y plot the different points on the graph for given data.

2. Join the point by hand curve.

3. Find the point on the curve corresponding to x = 1919 and find corresponding value of y

Remarks : This is simply an approximate method of evaluating the value of y and is not reliable in number of cases.

1.2.2 Method of Curve Fitting :

This method is applicable in some cases in which the form of the function is known. In such cases, by the method of least squares, we can fit the curve of known form to given set of values

and afterwards we can find the unknown value with the help of fitted curve.

Remarks :

(1) This is not exact method.

(2) In case of number of values are sufficiently large this method is complicated

(3) The form of the function in this method for given set of observations is assumed to be known.

1.2.3 Applications of the Calculus of Finite Differences :

(1) Newton-Gregory formula for interpolation. If n is a integer and h is the interval differencing, then $f(a+ nh) = f(a)+{}^{n}c_1\,\Delta f(a)+ {}^{n}c_2\Delta^2 f(a)+....+ {}^{n}c_n\,\Delta^{n}{}_{f(a)}$

We prove the theorem by using the method of induction.

we know that $\Delta f(a) = f(a + h) - f(a)$

$f(a+h)= f(a) +\Delta f(a)$

The theorem is true for n = 1.

Assume that the theorem is true for n=k.

$f(a+kh)=f(a)+{}^{k}c_1\,\Delta^2 f(a)+....+{}^{k}c_n\,\Delta^{k} f(a)$

We have $\Delta f(a+kh) = f(a+(k+ 1)h)-f(a+kh)$

$f(a + (k + 1)h) = f(a+kh) + h + \Delta f(a + kh)$

$= [f(a)+{}^{k}c_1\Delta f(a)+ {}^{k}c_2\Delta^{r} f(a)+....+ \Delta^{k} f(a)]$

$[\Delta^{r} f(a)+{}^{k}c_1\Delta^{r} f(a)+ {}^{k}c_1\Delta^{3} f(a)+.....+ \Delta^{k+1} f(a)]$

$= f(a)+({}^{k}c_1+1)\,\Delta f(a)+({}^{k}c_1+ {}^{k}c_2)\,\Delta^2 f(a)+.....+\Delta^{k+1} f(a)$

$= f(a)+{}^{k+1}c_1+\Delta f(a)+{}^{k+1}c_2\,\Delta^2 f(a)+.....+\Delta^{k+1} f(a)$

The theorem is true for n=k+1

By the method of induction, the theorem is true for any integer.

$f(a+ nh) = f(a)+{}^{n}c_1\,\Delta f(a)+ {}^{n}c_2\Delta^2 f(a)+....+ {}^{n}c_n\,\Delta^{n}{}_{f(a)}$

Note :

1. $f(a+nh) = f(a)+{}^{n}c_1\,\Delta f(a)+ {}^{n}c_2\Delta^2 f(a)+....+ {}^{n}c_n\,\Delta^{n}{}_{f(a)}$

2. Put a=0, h=1

$f(h)=f(0)+{}^{n}c_1\,\Delta f(0)+{}^{n}c_2+ {}^{n}c_2\,\Delta^2 f(0)+.....$

$f(x)=f(0)+\frac{x}{1!}\Delta f(0)+ \frac{x(x-1)}{2!}\Delta^2 f(0)+.....$

This formula is used only when the first term a =0 and h =1.

Alter : Let y=f(x) represent a function which assumes the values f(a),f(a+h)....f(a+nh) for (n +1) equidistant values a, a+ h, a +2h, a+nh of the independent variable x and let $P_n(x)$ be a polynomial in x of degree n.

This polynomial can be expressed in the form.

$P_n(x) = Bo +B (x-a) + B2(x-a)(x-a-h)+.....+ B_n(x-a)(x-a-h)(x-a-2h)....(x-a-(n-1)h)$ ....(1)

where B_o. B_1, B_n are constants and are choosen such that

$P_n(a) = f(a)$, $P_n(a + h) = f(a + h)$, . . . $P_n(a + nh) = f(a + nh)$

By substituting successively the values a, a+ h, a +2h . a+nh in equation(1) and writing the values of $P_n(a)$, $P_n(a+h)$... P(a + nh), we get $f(a)= B_0$ B1=f(a + h)-f(a)

$f(a) = B_0$, $f(a + h) = B_0+ hB_1$

$B_0 =f(a)$ or $B_1 = \frac{f(a+h)-f(a)}{h} = \frac{\Delta f(a)}{h}$

$f(a + 2h) = B_0 + 2hB_1 + h^rB_2$ (or)

A2 = f(a + 2h)-2{f(a+ h)-f(a)}-f(a)/$2h^2$

$= \frac{1}{2!h^r}\Delta^2 f(a)$

Similarly, we get $B_3 = \frac{1}{3!h^3}\Delta^3 f(a)$..... $B_n = \frac{1}{n!h^n}\Delta^n f(a)$

Substituting the values of B_0, B_1,....., B_n, we get

$P_n(x) = f(a) +\frac{\Delta f(a)}{h}$ (x-a)+ $\frac{\Delta^2 \mathbf{f(a)}}{\mathbf{2!}\boldsymbol{h^r}}$ **(**x-a)(x-a-h)+....+ $\frac{\Delta^n \mathbf{f(a)}}{\boldsymbol{n!h^n}}$ (x-a)(x-a-h)...(x-a-(n-1)h)

This is Newton-Gregory formula for forward interpolation putting $\frac{x-a}{h}$ = u or x=a+hu then(1) can be expressed as

$P_n(x)=P_n(a+nh) = f(a)+u\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+....+\frac{u(u-1).....(u-n+1)}{n!}\Delta^n f(a)$

Newton-Gregory Formula for Backward Interpolation

In this case , we write

$P_n(x) = C_0 +C_1$ (x-a-nh)+ C_2**(**x-a-nh)+....+C_n(x-a-nh)(x-a-nh+h).....(x-a-h) .....(1)

where Co, C_1 ,$C_{2...}$ C_n are constants and are choosen such that $f_{n(}a+ nh) = f(a+ nh)$,.., $P_n(a) = f(a)$

Substituting in (1), x=a + nh, a + nh-h, we have

$C_0 = f(a + nh)$, $f(a + nh-h) = C_o + C_1 (-h)$

$C_1 = \frac{f(a + nh) - f(a + nh - h)}{h} = \frac{\Delta f(a + nh)}{h}$

Similarly $C_2 = \frac{1}{2!h^r} \nabla^2 f(a+nh)$

$C_3 = \frac{1}{3!h^3} \nabla^3 f(a+nh)$

……

$C_n = \frac{1}{n!h^n} \nabla^n f(a+nh)$

Substituting these values of $C_o, C_1, \ldots, C_n$ in (1), we obtain

$P_n(x) = f(a+nh) + \frac{\nabla f(a+nh)}{1!h}(x-a+nh) + \frac{\nabla^2 f(a+nh))}{2!h}(x-a+nh)\ (x-(a+nh-h)) + \ldots + \frac{(\nabla^n f(a+nh)}{n!h^n}(x-a+nh)\ (x-(a+nh-h))\ldots\ldots(x-(a+h))$ ……(2)

This is Newton-Gregory formula for backward interpolation. Substituting $n = \frac{x-(a+ nh)}{h}$ or

x= a + nh + hl in eq. (2), then we obtain $P_n(x) = P_n(a + nh + h) = f(a + nh) + u\nabla f(a+nh) + \frac{u(u + 1)}{2!} \nabla^2 f(a+nh) + \ldots\ldots\ldots + \frac{u(u+1)\ldots\ldots(u+n-1)}{n!} \nabla^n f(a+nh)$ Where $P_n(x) = f(x)$.

1.3 SOLVED PROBLEMS

1. Given log x for x = 40, 45, 50, 55, 60, 65 for the following data :

x	40	45	50	55	60	65
logx	1.60206	1.65321	1.69877	1.74036	1.7785	1.81291

Estimate the value of log (58.75) by using Newton backward interpolation formula.

Solution.

X	10^5xlogx=f(x)	10^5x∇f(x)	10^5x∇^2f(x)	10^5x∇^3f(x)	10^5x∇^4f(x)
40	160206				
		5115			
45	165321		-539		
		4516		102	
50	169892		-437		-25
		4319		77	

55	174036		-360		-20
		3729		57	
60	177815		-330		
		3476			
65	181291				

(a + nh) + uh = 58.75

65 +5u = 58.75

u = 1.25

Newton backward interpolation formula

$$f(a+nh)+nh) = f(a+nh) + \frac{h}{1!}\Delta f(a+nh) + \frac{u(u+1)}{2!}\Delta^r f(a+nh) + \frac{u(u+1)(u+2)}{3!}\Delta^r f(a+nh)$$

$10^5 f(58.75) = 181291 - 1.25x(3476) + \frac{(-1.25)(-1.25u)x(-30.5)}{2!} + \frac{(-1.25)(-1.25+1)x(-1.25+2)}{6} x(5)$

= 181291-4345-47.3438+2.2267 = 176900.89

F(58.75) = 1.7690089

2. Find the cubic polynomial for the following data hence find f(u)

X	0	1	2	3
F(x)	-2	1	4	6

Solution :

x	**F(x)**	Δf(x)	Δ^2**f(x)**	Δ^3**f(x)**
0	-2			
		3		
1	1		0	-1
		3		
2	4		-1	
		2		
3	6			

Newton's Gregory's formula is

$$f(x) = f(0) + \frac{x}{1!}\Delta f(0) + \frac{x(x-1)}{2!}\Delta^2 f(0) + \frac{x(x-1)(x-2)}{3!}\Delta^3 f(0)$$

$$= -2 + \frac{x}{1!}(3) + \frac{x(x-1)}{2!}(0) + \frac{x(x-1)(x-2)}{3!}(-1)$$

$= -2+3x-\frac{(x^3+3x^2+2x)}{6} = \frac{-12-18x-x^3+3x^2-2x}{6}$

$= \frac{x^3+3x^2+16x-12}{6} = \frac{-(4)^3+3(4)^2+16(4)-12}{6}$

$= \frac{-64+48+64-12}{6} = \frac{36}{6} = 6$

f(4) = 6

3. Find f(1.5) by using Newton's advancing difference formula given that f(o)= +2, f(1)=4, f(2)=7, f(3)=8

x	F(x)	$\Delta f(x)$	Δ^2**f(x)**	Δ^3**f(x)**
0	**2**			
		2		
1	4		1	
		3		**-3**
2	7		-2	
		1		
3	8			

Solution. Newton's advancing formula is

$f(x) = f(0)+\frac{x}{1!}\Delta f(0)+\frac{x(x-1)}{2!}\Delta^2 f(0)+\frac{x(x-1)(x-2)}{3!}\Delta^3 f(0)$

$= 2+\frac{x}{1!}(2)+\frac{x(x-1)}{2!}(1)+\frac{x(x-1)(x-2)}{3!}(-3)$

$= 2+2x+\frac{x^2-x}{2!}+\frac{x^3-3x^2+2x}{2!}(-1)$

$= \frac{4+4x+x^2-x-x^3+3x^2-2x}{2}$

$F(x) = \frac{-(1.5)^3+4(1.5)^2+(1.5)+4}{2}$

$= \frac{-3.375+9+1.5+4}{2} = \frac{11-125}{2} = 5.5625$

f(1.5) = 5.5625

4. Find the polynomial and hence find f(5) from the following data by using Newton's Gregory's formula.

x	**0**	**1**	**2**	**3**	**4**

F(x)	0	7	26	63	124

Solution.

X	F(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	0				
		7			
1	7		12		
		19		6	
2	26		18		0
		37		6	
3	63		24		
		61			
4	124				

F(x) = f(0)+$\frac{x}{1!}\Delta f(0) + \frac{x(x-1)}{2!}\Delta^2 f(0) + \frac{x(x-1)(x-2)}{3!}\Delta^3 f(0)$

= 0+$\frac{x}{1!}(7) + \frac{x(x-1)}{2!}(12) + \frac{x(x-1)(x-2)}{3!}(6)$

= 7x+$6x^2$-6x+$x^3 - 3x^2 + 2x = x^3 + 3x^2 + 7x - 4x$

F(x) = $x^3 - 3x^2 + 2x$

F(5) = 5^3+$3(5)^2$+3(5) = 125+75+15 = 215

F(5) = 215

5. Find f from the given data by using Newton's Gregory's formula

X	10	20	30	40
F(x)	9	30	35	42

Solution .

X	F(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
10	**9**			
		21		
20	30		-16	
		5		18
30	35		2	
		7		
40	42			

a=10 , h= 10

Let a+uh=1.5

10+10u = 1.5

$u = \frac{5}{10} = \frac{1}{2}$

Newton's Gregory's formula is

$$F(a+uh) = f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)$$

$$F(52) = 0.7071+\frac{7}{5}(0.0589)+\frac{\frac{7}{5}(\frac{7}{5}-1)}{2!}(-0.0057)+\frac{\frac{7}{5}(\frac{7}{5}-1)(\frac{7}{5}-2)}{3!}(-0.0007)$$

= 7071+824.6-15.96+0.392 = 7880

10^4 f(52) = 7880

$F(52) = \frac{7880}{10^4} = 0.788$ $\sin 52^0 = 0.7880$

7. Using Newton's formula for interpolation estimate population for the year 1905 from the table.

Year (x)	Population(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1891	98752				
		33533			
1901	132285		2258		
		35791		-10435	
1911	168076		-8177		41358
		27614		30923	
1921	195690		22746		
		50360			
1931	246050				

Solution.

a = 1891, h = 10

Let a+ uh = 1905

u(10) = 1905-1891 = 14

$u = \frac{14}{10} = \frac{7}{6}$

Newton's Gregory's formula is

$$f(a+uh) = f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)+\frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(a)$$

f(1905) = 98752+$\frac{7}{5}(33533)+\frac{\frac{7}{5}(\frac{7}{5}-1)}{2}(2258)+\frac{\frac{7}{5}(\frac{7}{5}-1)(\frac{7}{5}-2)}{6}$(-10435)+$\frac{\frac{7}{5}(\frac{7}{5}-1)(\frac{7}{5}-2)(\frac{7}{5}-3)}{24}$(41358)

= 98,752 + 48946·2+ 632.24 + 584.36 + 926.4192

= 147841.2192

The population for the year 1905 is 14,784 approximately

8 . From the following table find the no. of students who obtained less than 45 marks.

x	30-40	40-50	50-60	60-70	70-80
F(x)	31	42	51	35	31

Solution .

Marks less than x	No.of students	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
40	31				
		42			
50	73		9		
		51		-25	
60	124		-16		37
		35		12	
70	159		-4		
		31			
80	190				

a=40,h=10

let a +uh = 45

u(10) = 45-40

u = $\frac{5}{10} = \frac{1}{2}$

Newton's advancing formula is

F(a+uh) = f(a)

$+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)+\frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(a)$

= $31+\frac{1}{2}(42)+\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(9)+\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(-25)+\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)(\frac{1}{2}-3)}{4!}(37)$

$= 31+21-\frac{9}{8}+(-\frac{75}{48})-\frac{15}{16}(\frac{37}{21})$

$= 52-1.125-1.5625-1.4453125$

$= 47.8671875 = 47$

: The no. of students getting less than 45 marks is 47 and the no. of students who getting more than 45 is 190-47-143

9. Find the no. of men getting the wages less than is rupees from the following :

0-10	10-20	20-30	30-40
9	30	35	42

Solution .

Wages x	No. of Men	$\Delta\mathbf{f(x)}$	$\Delta^2\mathbf{f(x)}$	$\Delta^3\mathbf{f(x)}$
0-10	9			
		21		
10-20	30		-16	
		5		18
20-30	35		2	
		7		
30-40	42			

Newton's advancing formula is

$f(a+uh) = f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)$

$f(15) = 9+\frac{1}{2}(21)+\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(-16)+\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(18)$

$= 9+\frac{21}{2}+2+\frac{3}{8}(\frac{18}{16}) = 11+\frac{21}{2}+\frac{9}{8}$

$= 11+10.5+1.125- 22.625 = 23$

11. Find the second degree polynomial from the data is then find f(5)

0	1	2	3	4
3	6	11	17	27

Solution .

X	F(x)	$\Delta\mathbf{f(x)}$	$\Delta^2\mathbf{f(x)}$	$\Delta^3\mathbf{f(x)}$
0	3			
		3		
1	6		2	

2	11	5	2	0
3	18	7	2	0
4	27	9		

Newton's Gregory's formula is

$f(x) = f(0)+\frac{x}{1!}\Delta f(0)+\frac{x(x-1)}{2!}\Delta^2 f(0)$

a=0 , h=1

a+uh = 5 u =5

$f(x) = 3+x(3)+\frac{x(x-1)}{2}(2)$

$= 3+3x + x^2-x = x^2 + 2x + 3$

$f(5) = (5)^2 + 2(5) + 3=25+10+ 3 = 38.$

12. Second degree polynomial passes through the points (o, 1), (1, 3), (2, 7),(3, 13) then find the polynomial or find f(x). Also find the value of f(1.5).

Solution.

X	Y = f(x)	$\Delta\mathbf{f}(\mathbf{x})$	$\Delta^{\mathbf{2}}\mathbf{f}(\mathbf{x})$	$\Delta^{\mathbf{3}}\mathbf{f}(\mathbf{x})$
0	1	2		
1	3	4	2	0
2	7	6	2	
3	13			

Newton's Gregory's formula is

$f(x) = f(0)+\frac{x}{1!}\Delta f(0)+\frac{x(x-1)}{2!}\Delta^2 f(0)$

$= 1+x(2)+\frac{x(x-1)}{2}(2)$

$f(x) = 1 + 2x + x^2-x$

$= x^2 + x + 1$

f(1.5) = $(1.5)^2+(1.5)+1$

= 2.25+1.5+1 = 4.75

f(1.5) = 4.75

13. Find the polynomial fits the data in the following table then also find f(13).

X	3	5	7	9	11
Y	6	24	58	109	174

solution. a=3, h = 2

Let a+uh = x = 13 3+2u = x

$$u = \frac{x-3}{2}$$

X	Y =f(x)	$\Delta\mathbf{f(x)}$	$\Delta^{\mathbf{2}}\mathbf{f(x)}$	$\Delta^{\mathbf{3}}\mathbf{f(x)}$	$\Delta^{\mathbf{4}}\mathbf{f(x)}$
3	6				
		18			
5	24		16		
		34		0	
7	58		16		0
		50		0	
9	109		16		
		66			
11	174				

Newton's Gregory's formula is

$$f(a+uh) = f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)$$

$$= 6+\frac{x-3}{1!}(18)+\frac{\left(\frac{x-3}{2}\right)\left(\frac{x-3}{2}-1\right)}{2!}(16)$$

f(a+uh) = $6+9x-27+2(x-3)(x-5)=9x-21+2x^2-16x+30$

f(x) = $2x^2-7x+9$

f(13) = $2(13)^2-7(13)+9=2(169)-91+9=338+9-91$

f(13) = 2.56

14. Find the value of f(35) from the following table :

X	Y=f(x)	$\Delta\mathbf{f(x)}$	$\Delta^{\mathbf{2}}\mathbf{f(x)}$	$\Delta^{\mathbf{3}}\mathbf{f(x)}$

20	512			
30	439	-73	-20	
40	346	-93	-10	10
50	243	-103		

Solution .

Newton's advancing formula is

F(a+uh) =

$f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)+\frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(a)$

a=20 , h=10 => a+uh=35a=30, h=10

20+10h=35 (or) 30+10u=35

h= $\frac{3}{2}$,u= $\frac{1}{2}$

$f(35)=439+\frac{1}{2}(-93)+\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(-10)+\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(-10)$

$=439-\frac{93}{2}+\frac{10}{8}+\frac{30}{48}$

=439-46.5+1,25+6.625=440.875-46.5=394.375

f(35)=394.

Central Difference Interpolation Formulae

1.3.1 INTRODUCTION :

In the previous chapter, we derived and discussed Newton forward and backward interpolation formula, which are applicable for interpolation, near the beginning and end respectively of tabulated data. We shall now discuss in the present chapter, about the central difference formula, which are suitable for interpolation near the

middle of a tabulated data.

The central difference operator (δ) is defined as $\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$

The first central difference operator δf(x) defined by

$$\delta f(x) = f(x+\frac{h}{2}) - f(x-\frac{h}{2})$$

$$\delta f(x) = (E^{\frac{1}{2}} - E^{-\frac{1}{2}}) f(x)$$

We have $E = 1 + \Delta$

$$\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}} = E^{-\frac{1}{2}}(E-1) = E^{-\frac{1}{2}}\Delta \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Again by the first backward difference of f(x)

We have $\nabla f(x) = f(x)-f(x-h) = f(x)-f(x-h) = E(x)- E^{-1}f(x)$

$$\nabla f(x) = (1- E^{-1})f(x)$$

$$\nabla = (1- E^{-1})$$

$$\nabla E^{\frac{1}{2}} = E^{\frac{1}{2}} - E^{-\frac{1}{2}} = \delta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From(1) and (2) $\delta = \Delta E^{-\frac{1}{2}} = \nabla E^{\frac{1}{2}}$ $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$

The average operator u is defined as

$$U = \frac{1}{2}[E^{\frac{1}{2}} + E^{-\frac{1}{2}}]$$

Results :

(1) $U^2 = 1+\frac{1}{4}\delta^2 u^2 f(x) = \frac{1}{2}(E^{\frac{1}{2}} + E^{-\frac{1}{2}})^2$

$= \frac{1}{4}(E^{\frac{1}{2}} + E^{-\frac{1}{2}})^2 = \frac{1}{4}(E+E^{-1}+2) = \frac{1}{4}(E+E^{-1}-2+4)$

$= \frac{(u+k-1)^{(2k+1)}}{(2k+1)!} = -\frac{(k-v)^{(2k+1)}}{(2k+1)!}$

$= \frac{v(v^2-1)(v^2-4)\ldots\ldots((v^2-k^2)}{(2k+1)!}$

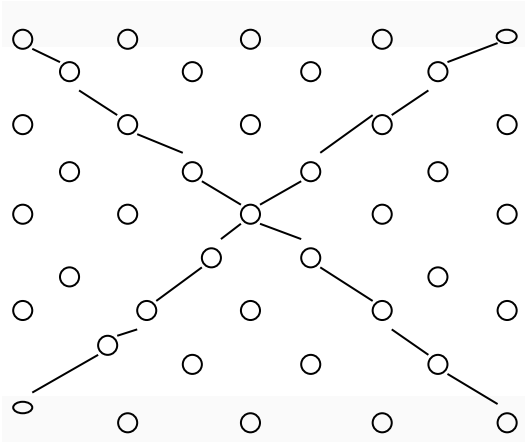
From (3), we obtain

$y_u = vy_o + \frac{v(v^2-1)}{3!}\Delta^2 y_{-1} + \frac{v(v^2-1)(v^2-4)}{5!}\Delta^4 y_{-2} + \ldots\ldots uy_1 + \frac{u(u^2-1)}{3!}\Delta^2 y_0 + \frac{u(u^2-1)(u^2-4)}{5!}\Delta^4 y_{-2} + \ldots\ldots\ldots + \frac{u(u^2-1)(u^2-4)(u^2-k^2)}{(2k+1)!}\Delta^{2k} y_{1-k} + \ldots\ldots\ldots\ldots$ ……..(5)

this can be written as

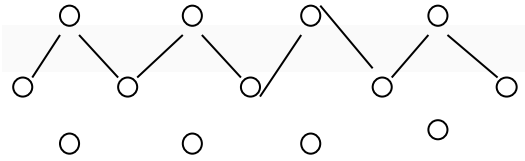
$f(u) = vf(0) + \frac{v(v^2-1)}{3!}\delta^2 f(0) + \frac{v(v^2-1)(v^2-4)}{5!}\delta^2 f(0) + \ldots\ldots + uf(1) + \frac{u(u^2-1)}{3!}\delta^2 f(1) + \frac{u(u^2-1)(u^2-4)}{5!}\delta^2 f(1) + \ldots$

this is known as Laplace-Everette's formula

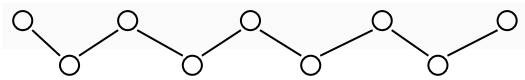


Newtons backward formula

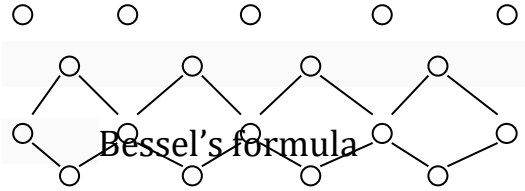
Newtons backward formula



Gauss backward formula

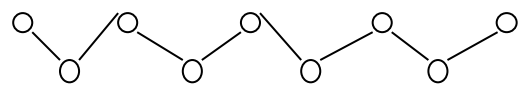


Gauss forward formula



Stirling's formula

Bessel's formula



Note 1: This formula is useful especially when using the tables in which only even order difference are tabulated.

All the difference formula can be explained with the help of following diagram.

Results : (1) $\delta = \Delta E^{-\frac{1}{2}} = \nabla E^{\frac{1}{2}}$

(2) $\Delta\nabla = \nabla\Delta = \Delta - \nabla = \delta^2$

(3) u= $\delta = \frac{1}{2}(\nabla + \Delta) = \frac{1}{2}\Delta E^{-1} + \frac{1}{2}\Delta$ (4) $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$)

Proof :

(1) $\delta = \Delta E^{-\frac{1}{2}} = \nabla E^{\frac{1}{2}}$

$\delta f(x) = f(x+\frac{h}{2}) - f(x-\frac{h}{2}) = \nabla f(x+\frac{h}{2}) = \nabla E^{\frac{1}{2}} f(x)$

$\delta = \nabla E^{\frac{1}{2}}$

(2) (a) $\Delta\nabla f(x) = \Delta\Delta E^{-1}f(x) = \Delta^2 E^{-1}f(x)$

(b) $\Delta\nabla f(x) = \Delta E^{-1}f(x) = \Delta^2 E^{-1}f(x)$

(c) $(\Delta - \nabla)f(x) = (\Delta - \nabla E^{-1})f(x) = \Delta(1-\frac{1}{E})f(x)$

$= \frac{\Delta(E-1)}{E}f(x) = \Delta.\Delta E^{-1}f(x) = \Delta^2 E^{-1}f(x)$

(d) $\delta^2 f(x) = (\Delta E^{-\frac{1}{2}})^2 f(x) = \Delta^2 E^{-1}f(x)$

$\Delta\nabla = \nabla\Delta = \Delta - \nabla = \delta^2$

(3) $u\delta f(x) = \frac{(E^{\frac{1}{2}} + E^{-\frac{1}{2}})}{2}(E^{\frac{1}{2}} - E^{-\frac{1}{2}})f(x)$

$= \frac{1}{2}(e - e^{-1})f(x) = \frac{1}{2}(e - \frac{1}{E})f(x) = \frac{1}{2}(E^2 - \frac{1}{E})f(x)$

$= \frac{(E-1)(E+1)}{2E}f(x) = \frac{\Delta}{2}(1+\frac{1}{E})f(x)$

We know that $\nabla = \Delta E^{-1} = \frac{1}{2}(\Delta + \nabla)f(x) = \frac{1}{2}(\Delta + \nabla E^{-1})f(x)$

$= \frac{1}{2}\nabla E^{-1} + \frac{1}{2}\Delta$

$$U\delta = \frac{1}{2}(\Delta + \nabla E^{-1})$$

(4) $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$

$(\frac{\Delta}{\nabla} - \frac{\nabla}{\Delta})f(x) = (\frac{\Delta}{\Delta E^{-1}} - \frac{\Delta E^{-1}}{\Delta})f(x) = (E - \frac{1}{E})f(x) = (\frac{E^2 1}{E})f(x)$

$= (E-1)\frac{(E+1)}{E} f(x) = \Delta(1+E^{-1})f(x) =$

$(\Delta + \nabla E^{-1})f(x)$

$= (\Delta + \nabla)f(x)$

$(\frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}) = \Delta + \nabla$

1.4 SOLVED PROBLEMS

1. Using Gauss forward formula find the value of y when x = 0.5 from the following table.

X	**-2**	**-1**	**0**	**1**	**2**
F(x)	**22**	**30**	**82**	**106**	**216**

Solution .

X	**F(x)**	$\delta f(x)$	$\delta^2 f(x)$	$\delta^3 f(x)$	$\delta^4 f(x)$
-2	22				
		8			
-1	30		44		
		52		-72	
0	82		-28		186
		24		114	
1	106		86		
		110			
2	216				

Gauss forward interpolation formula is

$f(x) = f(0) + \frac{x}{1!}\delta f(1/2) + \frac{x(x-1)}{2!}\delta^2 f(0) + \frac{x(x-1)(x+1)}{3!}\delta^3 f(1/2) + \frac{x(x-1)(x+1)(x-2)}{4!}\delta^4 f(0)$

$f(x) = 82 + \frac{x}{1!}(24) + \frac{x(x-1)}{2!}(-28) + \frac{x(x-1)(x+1)}{6}(114) + \frac{x(x-1)(x+1)(x-2)}{24}(186)$

$f(0.5) = 82 + (0.5)(24) + \frac{(0.5)(0.5-1)}{2}(-28)$

$$+\frac{(0.5)(0.5-1)(0.5+1)}{6}(114) + \frac{(0.5)(0.5-1)(0.5+1)(0.5-2)}{24}(186)$$

$$= 82 + 12+\frac{7}{2} - \frac{57}{24}+4.359 = 94+4.359+3.5-7.125$$

f(1/2) = 94.734

2. Using Gauss backward interpretation formula find the value at y when x=0.5[0,-1,+1,-2,+2]

Solution.

X	**F(x)**	$\delta f(x)$	$\delta^2 f(x)$	$\delta^3 f(x)$	$\delta^4 f(x)$
-2	22				
		8			
-1	30		44		
		52		-72	
0	82		-28		186
		24		114	
1	106		86		
		110			
2	216				

Gauss backward interpolation formula is

$$F(x) = f(0)+ \frac{x}{1!}\delta f(-1/2)+ \frac{x(x+1)}{2!}\delta^2 f(0) +\frac{x(x-1)(x+1)}{3!}\delta^2 f(-1/2)+ \frac{x(x-1)(x+1)(x+2)}{4!}(186)$$

$$f(0.5) = f(\tfrac{1}{2}) = 82+(0.5)(52) +\frac{(0.5)(0.5+1)}{2}(28) + \frac{(0.5)(0.5+1)(0.5-1)}{6}(-72)+ \frac{(0.5)(0.5+1)(0.5-1)(0.5+2)}{24}(186)$$

$$= 82 + 26-10.5+ 4.5-7.2656 = 112.5-17.7656$$

f(0.5) = 94.7344

a. Using Gauss backward interpolation formula and the value of f(-1/2) from the following data

X	**F(x)**	$\delta f(x)$	$\delta^2 f(x)$	$\delta^3 f(x)$	$\delta^4 f(x)$
-2	22				

-1		8	14		
	30			18	
0		22	32		6
	52			24	
1		54	56		
	106				
2		110			
	216				

Solution. Gauss backward interpolation formula is

$F(x) = f(0)+\frac{x}{1!}\delta f(-1/2)+\frac{x(x+1)}{2!}\delta^2 f(0)+\frac{x(x-1)(x+1)}{3!}\delta^2 f(-1/2)+\frac{x(x-1)(x+1)(x+2)}{4!}\delta^4 f(0)$

$= 52+\frac{x}{1!}(22)+\frac{x(x+1)}{2}(32)+\frac{x(x^2-1)}{6}(18)+\frac{x(x^2-1)(x+2)}{4!}(16)$

$= 52+11+\frac{\frac{1}{2}(\frac{1}{2}+1)}{2}(32)+\frac{\frac{1}{2}(\frac{1}{4}-1)}{6}(18)+\ )+\frac{\frac{1}{2}(\frac{1}{4}-1)(\frac{1}{2}+2)}{24}(16)$

$f(1/2) = 63+ 12-1.125-0.234375 = 75-1.359375 = 73.64$

4. Apply Gauss forward formula to obtain f(33) given that

X	**25**	**30**	**35**	**40**
F(x)	**0.2707**	**0.3027**	**0.3386**	**0.3794**

Solution .

X	10^4 **F(x)**	$10^4\delta f(x)$	$10^4\delta^2 f(x)$	$10^4\ \delta^3 f(x)$
-1	2707			
		320		
0	3027		39	
		359		10
1	3386		49	
		408		
2	3794			

Gauss forward interpolation formula is

$f(x) = f(0)+\frac{x}{1!}\delta f(1/2)+\frac{x(x-1)}{2!}\delta^2 f(0)+\frac{x(x-1)(x+1)}{3!}\delta^3 f(1/2)$

let us take a = 30 as origin h= 5

put a+uh = 32 $\quad 30+5u = 32 \Rightarrow u = \frac{2}{5}$

$$10^4 f(2/5) = 3027 + \frac{2}{5}(359) + \frac{\frac{2}{5}(\frac{2}{5}-1)}{2}(39) + \frac{\frac{2}{5}(\frac{2}{5}-1)(\frac{2}{5}+1)}{6}(10)$$

$$10^4 f(2/5) = 3027+143.6-4.68-0.045 = 3170.6-4.105 = 3165.36$$

$$f(\tfrac{2}{5}) = 0.3165$$

f(x) = 0.3165 when x =32

5. Use Gauss forward formula to find the value of y when y=3.75 from the following table.

X	2.5	3.0	3.5	4.0	4.5	5.0
Y	24.145	22.043	20.225	18.644	17.262	16.047

Solution. Taking 3.2 as the origin and 0.5 as the unit, the value of y required will be value for

$$u = \frac{3.75-3.5}{0.5} = 0.5$$

U	$\mathbf{Y_u}$	$\boldsymbol{\delta Y_u}$	$\boldsymbol{\delta^2 Y_u}$	$\boldsymbol{\delta^3 Y_u}$	$\boldsymbol{\delta^4 Y_u}$	$\boldsymbol{\delta^5 Y_u}$
-2	24.145					
		-2.102				
-1	22.043		0.284			
		-1.818		-0.047		
0	20.225		0.237		0.009	
		-1.581		-0.038		-0.003
1	18.644		0.99		0.006	
		-1.382		-0.032		
2	17.262		0.167			
		-1.215				
3	16.047					

Using the Gauss forward formula we have

$$F(u) = f(0) + u\delta f(0) + \frac{u(u-1)}{2!}\delta f(1) + \frac{u(u-1)(u+1)}{6}\delta^2 f(-1/2) + \frac{u(u-1)(u+1)(u-2)}{24}\delta^4 f(1/2)$$

$$Y_5 = 20.225+5(-1.5881)+\frac{0.5(0.5-1)}{2}(0.237)+\frac{0.5(0.5-1)(0.5+1)}{6}(-0.38)+\frac{0.5(0.5-1)(0.5+1)(0.5-2)}{24}(0.009)$$

$$+\frac{0.5(0.5-1)(0.5+1)(0.5-2)(0.5+2)}{120}(-0.003)$$

= 20.225-0.7905-0.029625+ 0.00238 + 0.9023750 + 0.0002106

= 19.40 approx

Thus value of y for x = 3.75 is 19.40 approximately.

6. Use Gauss forward formula to find y_{30} given that y_{21} = 18.4708, y_{25} = 17.8144, y_{29} = 17.1070, y_{33} = 16.3432, y_{37} = 15.5154.

X	21	25	29	33	37
Y	18.4708	17.8144	17.1070	16.3432	15.5154

Solution. Arrange the above data in the following form :

Take the origin x = 29 and h = u as the unit; then the value of y for u = $\frac{30-29}{4}$ = 0.25 is to be found. Construct the following difference table.

X	$u = \frac{x-29}{4}$	Y_u	δY_u	$\delta^2 Y_u$	$\delta^3 Y_u$	$\delta^4 Y_u$
21	-2	18.4708				
			-0.6564			
25	-1	17.8144		-0.0510		
			-0.7074		-0.0054	
29	0	17.1070		-0.0564		-
			-0.7638		-00076	0.0022
33	1	16.3432		-0.640		
			-0.8278			
37	2	15.5154`				

Now applying the gauss forward interpolation formula.

We get

f(0.4) = 17.1070+ (0.25) (-0.7638)+$\frac{(0.25)(-0.75)}{2}(-0.0564)+\frac{(1.25)(0.25)(-0.75)}{6}(-0.0076)+$

$\frac{(1.25)(0.25)(-0.75)(-1.75)}{24}(-0.0022)$

= 17.1070-0.19095+0.0052875+0.00002968-0.00000375

= 16.9216 approx.

7. Given that $\sqrt{12500}$ = 111.803399; $\sqrt{12510}$ = 111.848111; $\sqrt{12520}$ = 111.892806;

$\sqrt{12530}$ = 111.937483 Show by Gauss's backward formula that $\sqrt{12516}$ = 111.874930

Solution. Taking 12520 as the origin h 10 as the unit, the value of y required is for

$$u = \frac{12516-12520}{10} = -0.4$$

U	Y_u	δY_u	$\delta^2 Y_u$	$\delta^3 Y_u$
-2	111.803399			
		0.044712		
-1	111.848111		-0.00017	
		0.044695		-0.000001
0	111.892806		-0.000018	
		0.044677		
1	111.937483			

Now by using Gauss backward formula, we have

$y_{-0.4}$ = 111.8928+ (-0.4)(0.044695)-+$\frac{(0.6)(-0.4)}{2!}$(-0.000018) +

$\frac{(0.6)(-0.4)(-1.4)}{3!}$(-0.000001)

= 111.8928-0.017878 + negligible quantities = 111.874930

i.e., $\sqrt{12516}$ = 111.874930.

8. Apply Gauss forward formula to find the value of f(x) at x = 3.75 from the following table.

X	2.5	3.0	3.5	4.0	4.5
F(x)	24.145	22.043	20.225	18.644	17.262

Solution. Let us take a = 3.5 as origin, h = 0.5

Put a + uh = 3.75 => 3.5+ 0.5u = 3.75

0.5u = 0.25

u = 0.5 = ½

u	10^3 f(u)	$10^3\ \delta f(u)$	$10^3\ \delta^2$ f(u)	$10^3\ \delta^3$f(u)	$10^3\ \delta^4$ f(u)
-2	24145	-2102			
			284		
-1	22043	-1818		-47	

			237		9
0	20225	-1581		-38	
			199		6
1	18644	-1382		-32	
			167		
2	17262	-1215			

$10^3 f(\frac{1}{2}) = 20225+\frac{1}{2}(-1581)+\frac{\frac{1}{2}(\frac{1}{2}-1)}{2}(237)+\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}+1)}{6}(-38)+$

$$\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}+1)((\frac{1}{2}-2)}{24}(9)$$

$= 20225-790.5-29.625+2.375+0.2709 = 19407.46$

$f(\frac{1}{2}) = 19.408$

$f(x) = 19.408$ when x= 375.

9**. Find the value of cos 51^0. 42' by Gauss backward formula given that**

X	**50^0**	**51^0**	**52^0**	**53^0**	**54^0**
Cosx	**0.6428**	**0.6293**	**0.6157**	**0.6018**	**0.5878**

Solution : Let us take a =50^0 as origin h =1^0

Put a+uh = 51^042^1,52^0 +u = 51^042^I => u= $-18^I = \frac{-18}{60}$ = 0.3, u = -0.3

u	**10^4 f(u)**	**$10^4\ \delta f(u)$**	**$10^4\ \delta^2$ f(u)**	**$10^4\ \delta$f(u)**	**$10^4\ \delta^4$ f(u)**
-2	6428	-135			
-1	6293	-136	-1	-2	
					4
0	6157	-139	-3	2	
1	6018	-140	-1		
2	5878				

Gauss backward interpolation formula is

f(u) = f(0)+$\frac{u}{1!}$ $\boldsymbol{\delta}$ f($\frac{1}{2}$) +$\frac{u(u+1)}{2}$ $\boldsymbol{\delta^2}$ f(0)+ $\frac{u(u+1)(u-1)}{3!}$ δ^3(-$\frac{1}{2}$)+

$\frac{u(u+1)(u-1)(u+2)}{4!}$ δ^4f(0)

10^4f(-0.3) = 6157 + (-0.3)(-136)+$\frac{(-0.3)(0.7)}{2}$(-3) + $\frac{(-0.3)(0.7)(-1.3)}{6}$(-2) +

$\frac{(-0.3)(0.7)(-1.3)(1.7)}{24}$(2)

= 6157 + 40.8 + 0.315 -0,091 + 0.07735 = 6198.1013

f(-0.3) = 0.61981013

cos $57^0 42^{|}$ = 0.6198.

10. Find by Gauss backward formula the sales of the conceived by the year 1936.

Given that

Year	**1901**	**1911**	**1921**	**1931**	**1941**	**1951**
Sales	**12**	**15**	**20**	**27**	**29**	**52**

(in thousand of rupees)

Solution : let us take a = 1931 as origin

H = 10

a + uh = 1936 => 1931 + 10u = 1936 => 10u = 5

=> U=$\frac{1}{2}$[0,-1,1,-2,2,-3]

U	**f(u)**	$\boldsymbol{\delta f(u)}$	$\boldsymbol{\delta^2 f(u)}$	$\boldsymbol{\delta^3 f(u)}$	$\boldsymbol{\delta^4 f(u)}$
-3	12				
		3			
-2	15		2		
		5		0	
-1	20		2		3
		7		3	
0	27		5		-7
		12		-4	
1	39		1		
		13			

2	52				

Gauss backward interpolation formula is

$f(u = f(0) + \frac{u}{1!}\delta f(\frac{1}{2}) + \frac{u(u+1)}{2}\delta^{2}f(0) + \frac{u(u+1)(u-1)}{3!}\delta^{3}f(\frac{1}{2}) +$

$\frac{u(u+1)(u-1)(u+2)}{4!}\delta^{4}f(0$

$f(\frac{1}{2}) = 27 + (0.5)7 + \frac{(0.5)(0.5+1)}{2}(5) + \frac{(-0.5)(0.5+1)(0.5-1)}{6}(3) +$

$\frac{(-0.5)(0.5+1)(0.5-1)(0.5-1)}{24}(-7)$

= 27 + 3.5 + 1.875 -0.1875 + 0.2734 = 32.46 thousand 32.7

In the year 1936 = 32.7 thousands. Difference table:

X	$U = \frac{x-30}{5}$	Y_u	δy_u	$\delta^2 y_u$	$\delta^3 y_u$	$\delta^4 y_u$
20	-2	49225				
			-909			
25	-1	48316		-171		
			-1080		-59	
30	0	47236		-230		-21
			-1310		-80	
35	1	45926		-310		
			-1620			
40	2	44306				

Using Stirlings formula

Putting u = -0.4 we get difference

$y_{-0.4} = 47236 + (-0.4)\frac{-1310-1080}{2} + \frac{0.16}{2}(-230) + \frac{(-0.4)(0.16-1)}{6}(\frac{80-59}{2})$

$+ \frac{(0.16)(0.16-1)}{24}(-21)$

= 47326+ 478 -18.4 – 3.8920 + 1176

= 47692

Y_{28} = 47692

12. Apply Bessel's formula to obtain y_{25}, given y_{20} = 2854, y_{24} = 3162, y_{32} = 3992

Solution. take 24 as origin and 4 $u = \frac{25-24}{4} = \frac{1}{4} = 0.25$

Difference table is

X	$u = \frac{x-24}{4}$	Y_u	δy_u	$\delta^2 y_u$	$\delta^3 y_u$
20	-1	2854			
			308		
24	0	3162		74	
			382		-8
28	1	3544		64	
			448		
32	2	3992			

Using bessel's formula, we get

$Y_{1/4} = \frac{1}{2}(3162+3544)+(\frac{1}{4}-\frac{1}{2})382+\frac{\frac{1}{4}(\frac{1}{4}-\frac{1}{2})}{2!}(\frac{14+66}{2})+\frac{(\frac{1}{4}-\frac{1}{2})(\frac{1}{4})(\frac{1}{4}-1)}{3!}(-8)$

$= 3353 - 95.5 - 6.5625 - 0.0625 = 3250.875.$

13.Given y_{20} =24 , y_{24} =32, y_{28} =35, y_{32} =40 find y_{25} by Bessel's formula.

Solution : Taking origin at x =24 and u as the unit.

Find y_u for $u = \frac{25-24}{4} = \frac{1}{4} = 0.25$

The difference table is

X	$U = \frac{x-24}{4}$	Y_u	δy_u	$\delta^2 y_u$	$\delta^3 y_u$
20	-1	24			
			8		
24	0	32		-5	
			3		7
28	1	35		2	
			5		
32	2	40			

By using Bessel's formula we have

$$y_{\frac{1}{4}} = \frac{32+35}{2} + (\frac{1}{4} - \frac{1}{2})3 + \frac{\frac{1}{4}\left(\frac{1}{4}-1\right)\left(\frac{-5+2}{2}\right)}{2} + \frac{(\frac{1}{4}-\frac{1}{2})\frac{1}{4}\left(\frac{1}{4}-1\right)}{6}(7)$$

= 33.5-0.75+ 0.1406+0.54687 = 32.945287.

14.Given that

θ	0^0	5^0	10^0	15^0	20^0	25^0	30^0
$\tan\theta$	**0.0000**	**0.0875**	**0.1763**	**0.2679**	**0.3640**	**0.4663**	**0.5774**

Show that tan 16^0 = 0.2867 (use Stirilling's formula)

Solution : Taking 15^0 as origin and 5^0 as the unit , the value required will be for

$$U = \frac{16^0-15^0}{15^0} = 0.2^0$$

θ	U	Y_u	δy_u	$\delta^2 y_u$	$\delta^3 y_u$	$\delta^4 y_u$	$\delta^5 y_u$	$\delta^6 y_u$
0^0	-3	0.0000						
5^0	-2	0.0875	0.0872					
10^0	-1	0.1763	0.0888	0.0013	0.0015			
15^0	0	0.2679	0.916	0.0028	0.0017	0.0002	-0.0002	
20^0	1	0.3640	0.961	0.0045	0.0017	0.0000	0.0009	0.0011
25^0	2	0.4663	0.1023	0.0062	0.0026	0.0009		
30^0	3	0.5774	0.1111	0.0088				

By using Stirlings formula, we have

Tan 16^0 = $y_{0.2}$ = 0.2679+ $\frac{0.2}{2}$ (0.0961+0.0916){+ $\frac{0.04}{2}$ (0.0045)+ $\frac{0.2(0.4-1)}{2}$ $\frac{1}{2}$(0.0017+0.0017) +

$\frac{0.4(0.4-1)}{24}$ (0.0000) + $\frac{0.2(0.4-1)(0.04-1)}{120}$ $\frac{1}{2}$ (-0.0002+0.0009) + $\frac{0.2(0.04-1)(0.04-4)}{720}$(0.0011)

= 0.2679 + 0.01877 + 0.00009 + negligible

= 0.2867.

1.5 Applications in engineering :

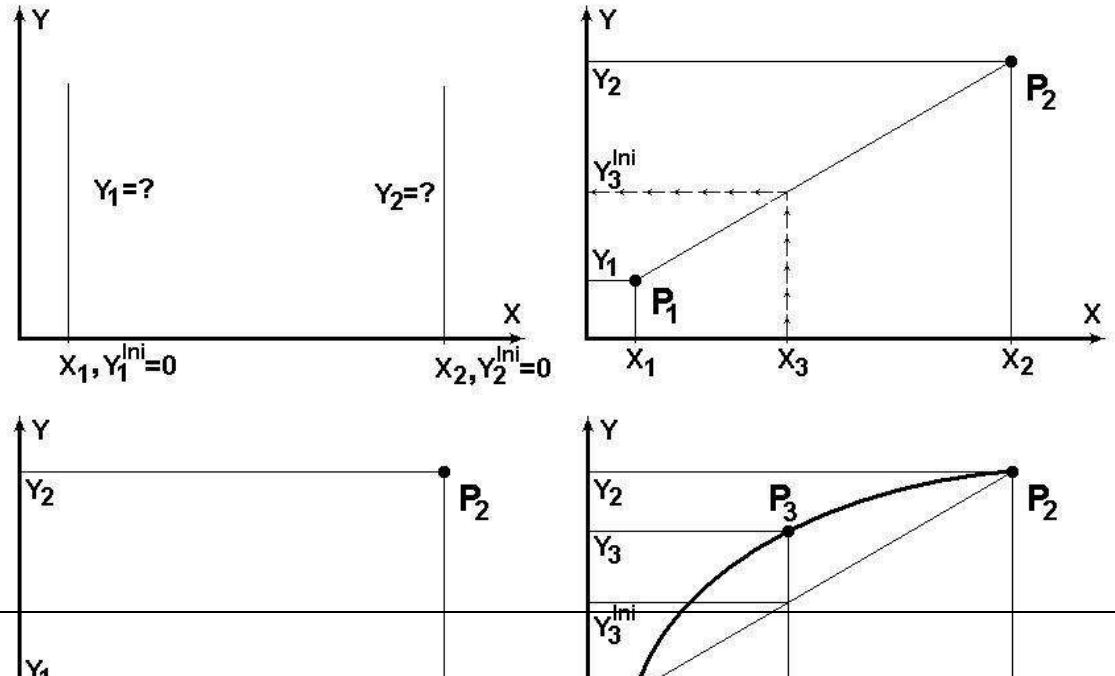
General Ideas Regarding the Use of the Interpolation in the Numerical Analysis

There are certain cases when approximation solutions are useful within the numerical methods field itself.

Some of these cases are presented in the next sections.

Approximation of the Initial Solution for Iterative Methods

Let us consider a theoretical problem where the formulation of the problem depends on some parameters and an iterative solution is possible. It is well known that the final solution depends on the initial approximation in terms of accuracy and of computer runtime.



2. NUMERICAL INTEGRATION

Numerical integration is the approximate computation of an integral using numerical techniques. The numerical computation of an integral is sometimes called quadrature. Ueberhuber (1997, p. 71) uses the word "quadrature" to mean numerical computation of a univariate integral, and "cubature" to mean numerical computation of a multiple integral.

There are a wide range of methods available for numerical integration. A good source for such techniques is Press et al. (1992). Numerical integration is implemented in the Wolfram Language as NIntegrate[f, {x, xmin, xmax}].

The most straightforward numerical integration technique uses the Newton-Cotes formulas (also called quadrature formulas), which approximate a function tabulated at a sequence of regularly spaced intervals by various degree polynomials. If the endpoints are tabulated, then the 2- and 3-point formulas are called the trapezoidal rule

and Simpson's rule, respectively. The 5-point formula is called Boole's rule. A generalization of the trapezoidal rule is Romberg integration, which can yield accurate results for many fewer function evaluations.

If the functions are known analytically instead of being tabulated at equally spaced intervals, the best numerical method of integration is called Gaussian quadrature. By picking the abscissas at which to evaluate the function, Gaussian quadrature produces the most accurate approximations possible. However, given the speed of modern computers, the additional complication of the Gaussian

quadrature formalism often makes it less desirable than simply brute-force calculating twice as many points on a regular grid (which also permits the already computed values of the function to be re-used). An excellent reference for Gaussian quadrature is Hildebrand (1956).

Modern numerical integrations methods based on information theory have been developed to simulate information systems such as computer controlled systems, communication systems, and control systems since in these cases, the classical methods (which are based on

approximation theory) are not as efficient (Smith 1974).

2.1 INTRODUCTION :

The numerical integration is the process of evaluating the approximate Value of the integration $\int_a^b f(x)dx$ from a set of numerical values of f(x). If the problem is to be applied to the integration of function of a single variable, then the process is called as quadrature.

The problem of numerical integration is evolved by first approximating the f(x) by a polynomial with the help of interpolation formula and then integrating this expression between the limits.

$\therefore$ To find the value of $\int_a^b f(x)\,dx$, first express the function f(x) by interpolation formula, say p(x) and then integrate between the limits.

$$\therefore \int_a^b f(x)dx = \int_a^b p(x)dx$$

The error is given by $\int_a^b f(x)dx - \int_a^b p(x)dx$

2.2 A GENERAL QUADRATURE FORMULA FOR NUMERICAL INTEGRATION

Consider the integral $\int_a^b f(x)dx$. Let f(x) be given for some equally spaced values of the orguments say ,x_0 , x_0+h ,x_0+2h,…. Let the range (b-a) be divided into n equal parts, each of which is of width h.

$$b-a = nh$$

$$h = \frac{b-a}{n}$$

let $x_0 = a, x_1 = x_0+h = a+h$, $x_2 = a+2h$ ….. $x_n = a+nh = b$

$$I = \int_a^{a+nh} f(x)dx$$

Putting $x = a+uh$

dx = hdu

x -> a -> u ->0

x ->a+uh -> u->n

I = $\int_a^n f(a+uh)du$ = h $\int_a^n f(a+uh)du$

No write as f(a+uh) on the Newton's interpolation formula as

= h $\int_0^n [f(a) + \frac{u}{1!}\Delta f(a) + \frac{u(u-1)}{2!}\Delta^2 f(a) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(a) + \cdots +$ $\frac{u(u-1)\ldots.(u-n+1)}{n!}\Delta^n f(a)\ ]\ du$

= h [f(a).u + $\frac{u^2}{2}\Delta f(a)$+($\frac{u^3}{3} - \frac{u^2}{2}$) $\Delta^2 f(a)$+ $\frac{(\frac{u^4}{4} - \frac{3u^3}{3} + \frac{2u^2}{2})}{6}\Delta^3 f(a) + \cdots +]$

= h[nf(a) + $\frac{n^2}{2}\Delta f(a)$ + ($\frac{n^2}{6} - \frac{n^2}{4}$) $\Delta^2 f(a)$ +($\frac{n^4}{24} - \frac{n^3}{6} + \frac{n^2}{6}$)$\Delta^3 f(a)$ +]

$\int_a^b f(x)dx$ = h[ny_0 + $\frac{n^2}{2}$(y_1-y_0)+ ($\frac{n^3}{6} - \frac{n^2}{4}$)($y_2-2y_1$+$y_0$)+$\left(\frac{n^4}{24} - \frac{n^3}{6} + \frac{n^2}{6}\right)$

(y_3-3y_2+3y_1-y_0)+……..] ……..(1)

$\int_a^b f(x)dx$ = $\int_a^{a+nh} f(x)dx$

This is known as general quadrature formula we deduce the following important quadratic formulas by taking n= 1,2,3….

2.3 TRAPEZOIDAL RULE :

Put n=1 eq(1) and neglect the second and higher order differences when n=1 , the argument are a, a+h and its corresponding functional values are y_0 and y_1 thus we have

$\int_a^{a+nh} f(x)dx$ = h[y_0 +$\frac{1}{2}$(y_1-y_0)] = $\frac{h}{2}$ (y_0+y_1)

I_1 = $\int_a^{a+nh} f(x)dx$ = $\frac{h}{2}$ (y_0+y_1)

I_2 = $\int_a^{a+nh} f(x)dx$ = $\frac{h}{2}$ (y_0+y_1)

I_3 = $\int_{a+2h}^{a+3h} f(x)dx$ = $\frac{h}{2}$ (y_2+y_3)

……..

……..

$I_n = \int_{a+(n-1)h}^{a+nh} f(x)dx = \frac{h}{2}(y_{n-1}+y_n)$

Adding these n integrals, we have

$\int_a^{a+nh=b} f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

This rule is called the trapezoidal rule.

2.4 SIMPSON'S $\frac{1}{3}$ RULE :

Put n=2 in the equation (1) and neglect the third and higher order difference. In this case the arguments are a, a+h, a+2h the corresponding entries are y_0,y_1 and y_2, thus we have

$\int_a^{a+2h} f(x)dx = h[2y_0 + 2(y_1-y_0)+(\frac{y}{3}-1)(y_2-2y_1+y_0)]$

$= \frac{h}{3}[y_0+4y_1+y_2]$

Similarly $I_2 = \int_a^{a+2h} f(x)dx = \frac{h}{3}(y_0+4y_1+y_2)$

$I_3 = \int_{a+4h}^{a+6h} f(x)dx = \frac{h}{3}(y_4+4y_5+y_6)$

……..

……..

$I_n = \int_{a+(n-2)h}^{a+6h} f(x)dx = \frac{h}{3}(y_{n-2}+4y_{n-1}+y_n)$

Adding these integrals, we have $I = \int_a^b f(x)dx$

$= \frac{h}{3}[(y_0+y_n)+(y_1+y_3+y_5+\ldots+y_{n-1})+2(y_2+y_4+\ldots.+y_{n-2})]$

This is known as SIMPSON'S $\frac{1}{3}$ RULE

2.5 SIMPSON'S $\frac{3}{8}$ RULE :

Put n=3 in equation(1) and neglect fourth and higher order differences. In this case, the arguats are a, a+h, a+2h and a+3h and the corresponding fractional values are y_0, y_1, y_2 and y_3, thus we have

$$\int_a^{a+3h} f(x)dx = n[3y_0+\frac{9}{2}(y_1-y_0)+(\frac{9}{2}-\frac{9}{4})(y_2-2y_1+y_0) + (\frac{27}{8}-\frac{27}{6}-\frac{9}{6})(y_3-3y_2+3y_1-y_0)]$$

$$= h[3y_0+\frac{9}{2}(y_1-y_0)+(\frac{9}{2}-\frac{9}{4})(y_2-2y_1+y_0) + (\frac{27}{8}-\frac{27}{6}-\frac{9}{6})(y_3-3y_2+3y_1-y_0)]$$

$$= h[3y_0+\frac{9}{2}(y_1-y_0)+(\frac{9}{4})(y_2-2y_1+y_0) + (\frac{3}{8})(y_3-3y_2+3y_1-y_0)]$$

$$I_1 = \int_a^{a+3h} f(x)dx = \frac{3h}{8}(y_0+3y_1+3y_2+y_3)$$

$$I_2 = \int_{a+4h}^{a+6h} f(x)dx = \frac{3h}{8}(y_3+3y_4+3y_5+y_6)$$

……..

……..

$$I_n = \int_{a+(n-2)h}^{a+6h} f(x)dx = \frac{3h}{8}(y_{n-3}+3y_{n-2}+3y_{n-1}+y_n)$$

Adding all these integrals we get

$$\int_a^{a+nh} f(x)dx = \frac{3h}{8}[(y_0+y_n)+3(y_1+y_2+y_4+\ldots+y_{n-1})+2(y_3+y_6+\ldots.+y_{n-3})]$$

This formulas is called the simpson's rule.

Note : Applying this formula, the number of sub-internals should be taken as multiple of three

2.6 WEDDLE'S RULE :

put n=6 in equation (1) and neglect the seventh and higher order differences. in this case the arguments are from a to a+6h and there will be only seven functional values, thus we get

$$\int_a^{a+6h} f(x)dx = h[6y_0+18\Delta y_0+27\Delta^2 y_0+24\Delta^3 y_0+\frac{123}{10}\Delta^4 y_0+\frac{33}{10}\Delta^5 y_0+\frac{41}{140}\Delta^6 y_0]$$

Replacing $\frac{41}{140}\Delta^6 y_0$ by $\frac{3}{10}\Delta^6 y_0$,

Since the error involved is negligible.

Thus above equation can be expressed as

$$\int_a^{a+6h} f(x)dx = h[6y_0+18\Delta y_0+27\Delta^2 y_0+24\Delta^3 y_0+\frac{123}{10}\Delta^4 y_0+\frac{33}{10}\Delta^5 y_0+\frac{3}{10}\Delta^6 y_0]$$

$= \frac{3h}{10}(y_0+5y_1+y_2+ 6y_3+y_4+5y_5+y_6)$

Similarly $\int_{a+6h}^{a+12h} f(x)dx = \frac{3h}{10}(y_6+5y_7+y_8+ 6y_9+y_{10}+5y_{11}+y_{12})$

…….

…….

$\int_{a+(n-6)h}^{a+nh} f(x)dx = \frac{3h}{10}(y_{n-6}+5y_{n-5}+y_{n-4}+ 6y_{n-3}+y_{n-2}+5y_{n-1}+y_n)$

Adding these integrals, we have

$\int_a^{a+nh} f(x)dx = \frac{3h}{10}(y_0+5y_1+y_2+ 6y_3+y_4+5y_5+2y_6+ 5y_7+y_8 +\ldots\ldots\ldots)$

This formula is known as Weddle's rule.

Note:

1. This rule is more accurate than the Simpson's rule, but it requires atleast seven consecutive values of the function
2. This method is applicable when n is a multiple of 6.

2.7 ERROR IN QUADRATURE FORMULA :

The error terms in the quadrature formula is given by

$$E = \int_a^b y dx - \int_a^b p(x)dx$$

where P(x) is the polynomial representing the function y = f(x) in the interval [a, b].

(i) Error in Trapezoidal Rule : Expanding y = f(x), about $x = x_o$ by Taylor's series, we obtain

$$y = y_0 + (x-x_0)y_0 + \frac{(x-x0)^2}{2!} y_0^{11} + \ldots\ldots\ldots\ldots(1)$$

$$\int_{x_0}^{x_0+h} y\,dx = \int_{x_0}^{x_0+h} [y_0 + (x - x_0)y_0 + \frac{(x-x_0)^2}{2!} y_0 + \cdots]dx$$

$$= y_0 n + \frac{h^2}{2!} y_0 + \frac{h^2}{3!} y_0 + \ldots\ldots\ldots\ldots(2)$$

But area of the first trapezium in the interval $[x_o, x_1] = \frac{1}{2}h(y_0 + y_1)$ ….(3)

Then putting $x = x_o + h$ and $y = y_1$ in (1), we obtain

$$y_1 = y_0 + h\,y_0 + \frac{h^2}{2!}y_0 + \ldots\ldots\ldots\ldots$$

Putting this value of y, in (3), we have

$$B = \frac{1}{2}h[y_0 + y_0 + h\,y_0 + \frac{h^2}{2!}y_0 + \ldots\ldots\ldots] = h\,y_0 + \frac{h^2}{2!}y_0 + \frac{h^3}{2.2!}y_0 + \ldots\ldots \quad \ldots\ldots(4)$$

Hence the error in the interval $[x_o, x_1]$

$$= \int_{x_0}^{x_1} y\,dx - B_1 = [\frac{1}{3!} - \frac{1}{2.2!}]\,h^3 y_0 + \ldots\ldots\ldots\ldots = -\frac{h^3}{12}y_0 + \ldots\ldots$$

Principal part of the error in $[x_o, x_1]$ = $-\frac{h^3}{12}y_0$

And similarly principal part of the error in $[x_1, x_2]$ = $-\frac{h^3}{12}y_1$ etc.,

Thus, the total error = $-\frac{h^3}{12}[y_0 + y_1 + y_{n-1}]$ = E , say

Then assuming that y''(x) is the largest of the n quantities y_o , y_1 ,.. . ., y_{n-1} ,

we can say that

$$E < -\frac{nh^3}{12}y^{11}(x) = -\frac{(b-a)h^2}{12}y^{11}(x) \text{ as } b\text{-}a = nh \quad \ldots\ldots\ldots\ldots\ldots(5)$$

Thus the error in the trapezoidal rule is of the order h^2.

(ii) Error in Simpson's $\frac{1}{3}$ Rule : Expanding y = f(x) about $x = x_o$ by Taylor's series, we have

$$y = y_0 + (x\text{-}x_0)y_0 + \frac{(x-x0)^2}{2!}y_0 + \ldots\ldots\ldots\ldots$$

$$\int_{x_0}^{x^2} y\,dx = \int_{x_0}^{x_0+2h}[y_0 - (x - x_0) + \frac{(x-x_0)^2}{2!}\,y_{0+\ldots}]\,dr$$

$$2hy_0 + \frac{4h^2}{2!}y_0 + \frac{8h^3}{3!}y_0 + \frac{16h^4}{4!}y_0 + \ldots + \frac{32h^5}{5!}y_0 + \ldots\ldots\ldots \quad \ldots\ldots\ldots\ldots(2)$$

and area over the first double strip by Simpson's 1/3 rule

$$= \frac{1}{3}h(y_0 + 4y_1 + y_2) = A_1 \text{ say}$$

Then putting $x = x_0 + h$ and $y = y_1$ in (1), we get

$$y_2 = y_0 + 2hy_0 + \frac{4h^2}{2!}y_0 + \frac{8h^3}{3!}y_0 + \ldots\ldots\ldots\ldots$$

Further putting $x = x_o + 2h$ and y = 1/2 in (1), we get

$y_2 = y_0 + 2hy_0 + \frac{4h^2}{2!}y_0 + \frac{8h^3}{3!}y_0 + \ldots\ldots\ldots$

(3) → $A_1 = \frac{h}{3}[y_0+4(y_0+ hy_0+ \frac{h^2}{2!} y_{0+\ldots\ldots\ldots})+(y_0+2h\, y_0+ \frac{4h^2}{2!} y_0 + \frac{8h^3}{3!} y_0 + \ldots\ldots\ldots)]$

$= 2hy_0 +2h^2y_0 + \frac{4h^3}{3} y_0 + \frac{2h^2}{8} y_0 + \frac{5h^5}{18} y_0 +\ldots\ldots\ldots$ ……(4)

Hence the error in the interval $[x_o, x_2] = \int_{x_0}^{x} y\, dx - A_1$

$= [\frac{4}{15} - \frac{5}{18}] h^5y_0^5 +\ldots\ldots$ …….(5)

and the principal part of the error in $[x_o, x_2]$

$= [\frac{4}{15} - \frac{5}{18}] h^5y_0^5 = - -\frac{h^5}{90}y_0^5$

On the same lines principal part of the error in $[x_2, x_4]$

$= \frac{h^5}{90}y_2^5$, etc.,

Total error = $- -\frac{x^5}{90}[y_0^4 + y_2^4 +\ldots\ldots\ldots.+ y_2^4 (n-1)] = E$, Say ……(6)

The assuming $y_{1v}(x)$ as the largest of y_0^{1V} , y_2^{1V} …… , y_{2n-2}^{1V}

We readily obtain $E<-\frac{n/5}{90} y_0^{1V}(x) = - \frac{(b-a)h^y y^5(x)}{180}$ …….(7) [2nh = b-a]

(iii) Error in Simpson's 3/8 rule : Proceeding as above, we have the principal part of the error in the interval $[x_o, x_3]$ as $= -\frac{3h^5}{80} y^5$

(iv) Error in Weddle's rule: Principal part of the error in the interval$[x_0,x_6] = -(h^7/140)y_0^5$

2.8 SOLVED PROBLEMS ::

1.Evalaute the value of the integral using $\int_2^{1.4}(sinx - logx + e^x)$dx

(a) The Trapoidal rule, (b) Simpson's one-third rule,

(c) Simpson's three eight rule.

Solution. In this case divide the range of integration into twelve equal parts by taking h = -1 and then the values of the function y = sin x -$\log_e x$ +e are computed for each point of subdivision which are as under

x	Sinx	$\text{Log}_e x=\log_{10} x x 2.30258$	e^x	$Y=\sin x-\log_e x+e^x$

0.2	0.19867	-1.60943	1.22140	3.02950
0.3	0.29552	-1.20397	1.34986	2.84935
0.4	0.38942	-0.91629	1.49182	2.79753
0.5	0.47943	-0.69315	1.64872	2.82130
0.6	0.5646	-0.51083	1.82212	2.89759
0.7	0.64422	-0.35667	2.01375	3.01464
0.8	0.71736	-0.22314	2.22554	3.16604
0.9	0.78333	-0.10537	2.45960	3.34830
1.0	0.84147	+0.00000	2.71828	3.55975
1.1	0.89121	0.09530	3.00417	3.80008
1.2	0.93204	0.18232	3.32012	4.06984
1.3	0.96356	0.26236	3.66930	4.37050
1.4	0.98545	0.33647	4.05520	4.70418

(a)By Trapezoidal rule, we have

$\int_2^{1.4}$(sin x -$\log_e{}^x$ + e^x)dx = $\int_2^{2+12x\ 0.1} y\ dx$

Here n = 12; h = 0.1, x_o = 0.2, y = sin x -$\log_e{}^r$+ e^x

Again w.k.t. $\int_{x0}^{xo\ +nh} y\ dx$ = $\frac{h}{2}$[y_0+y_{n+2}(y_1+y_2+…+y_{n-1})]

Substituting these values of, h_1, y_o, y_n…, in the above formula we get

$\int_2^{1.4}$(sin x − logex + e^x) dx = $\frac{1}{20}$[3.02950+ 4.70418+2(2.8493+…+4.37050)]

= $\frac{1}{20}$[7.73368+2(36.69492)]

= 4.05617

(b) By Simpson's rule $\frac{1}{3}$ we have

$\int_{x0}^{xo\ +nh} y\ dx$ = $\frac{h}{3}$[y_0+y_6+4(y_1+y_3+…)+2(y_2+y_4+…)]

Put the values of h, y_o, $y_{1..}$... from the table, we have

= $\int_2^{1.4}$(sin x − logex + e^x) dx

= $\frac{1}{3}$[7.73368+ 4(20.20417)-3(16.49075)]

= $\frac{1}{3}$ [121.53186] = 4.05106

(c) By Simpson's 3/8 rule we have

$\int_{x0}^{xo\ +nh} y\ dx$ = $\frac{3h}{8}$[(y_0+ y_n)+3(y_1+y_2+y_4+…+$y_{n-1)}$+2(y_3+y_6+….+y_{n-3})]

$= \frac{3(0.1)}{8}[7.73368+ (26.90750)+ 2(9.78742)] = 4.05116$

2. A curve is drawn to pass through the points given by the following table.

x	1	1.5	2	2.5	3	3.5	4
y	2	2.4	2.7	2.8	3	2.6	2.1

Estimate the area bounded by the curve, the x-axis and the lines x=1,x=4.

Solution. By Simpson's rule, we have required area

$= \frac{h}{3}[y_0+4y_1+2y_2+4y_3+.....+4y_{n-1}+y_n]$

$= \frac{0.5}{3}$ [2+ 9.6+5.4+11.2+10.4+ 2.1]

$= \frac{50}{30}$x46.7 $= \frac{23.35}{3}$ = 7.783 unit of area

By Weddle's rule, we have required area

$= \frac{3h}{10}[y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6]$

$= \frac{3x0.5}{10}[2+12.0+2.7+16.8+3.13.0+2.1]$ = 7.74 units of area

3. Calculate by Simpson's rule an approximate value of by taking seven equidistant ordinates. Compare it with the exact value and the value obtained by using the Trapezoidal rule.

Solution. Let us divide the range of integration (-3, 3) into six equal parts each of width $\frac{3(-3)}{6}$ = 1 unit. Hence h-1, value of the function for each point of sub division are as given below.

x	$y= x^4$
-3	$(-3)^4 = 81$
-2	$(-2)^4 = 16$
-1	$(-1)^4 = 1$
0	$(0)^4 = 0$
1	$(1)^4 = 1$
2	$(2)^4 = 16$
3	$(3)^4 = 81$

Now by Simpson's rule, we obtain

$\int_{-3}^{3} x^4$ dx = $\frac{h}{3}$[y_0+y_6+4(y_1+y_3+y_5)+2(y_2+y_4)]

= $\frac{1}{3}$[162+4x32+ 2x2] = $\frac{1}{3}$x294 = 98

But the exact value of

$\int_{-3}^{3} x^4$ dx = $\left[\frac{x^5}{5}\right]_{-3}^{3}$ = $\frac{1}{5}$[$(3)^5$ –$(-3)^5$] = $\frac{1}{5}$x486 = 97.2

Also by Trapezoidal rule, we have

$\int_{-3}^{3} x^4$ dx = $\frac{h}{2}$[y_0+y_6+2(y_1+ y_2+y_3+ y_4+y_5)] = $\frac{1}{2}$[162+2x34]

Thus it is obvious that the Trapezoidal rule does not give an accurate result.

4.Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rule. Hence obtain the approximate value of π in each case.

Solution. Divide the range into six equal parts each of with h = $\frac{1}{6}$ and compute

The value of y = $\frac{1}{1+x^2}$ at each point of sub division.

These values are given below.

X	$Y_x = 1/(1+x^2)$
0	1.0000
1/6	36/37 = 0.97297
2/6	36/40 = 0.90000
3/6	36/45 = -0.80000
4/6	36/52 = 0.69231
5/6	36/61 = 0.59016
1	1 / 2 = 0.5000

By Simpson's rule, we have

$\int_{x_0}^{x_0+6h} y\,dx$ = $\frac{h}{3}$[y_0+y_6+4(y_1+y_3+y_5)+2(y_2+y_4)]

= $\frac{1}{18}$ [1.00 + 0.500 + 4(0.97292 + 0.50000 + 0.59016) +

2(0.900+ 0.69231)] ……(1)

= 0.785395

By Simpson's 3/8 rule we have

$\int_{x_0}^{x_0+6h} y\,dx = \frac{3h}{8}[y_0+y_6+3(y_1+y_2+ y_4+y_5)+2(y_3)]$

$= \frac{1}{16}$ [1.5000 + 3 (0.97297 + 0.900 + 0.69231+0.59016 +

2 x (0.800))] = 0.785395 ……(2)

Again, $\int_0^1 \frac{dx}{1+x^2} = [\tan^{-1} x]^1_0 = \pi/4$ …….(3)

From (1) and (2), we get π = 3.14188 and from (2) and (3) we get

π = 3.141580

4.Evaluate $\int_2^{10} \frac{dx}{x}$ by dividing the range into '8' equal parts by using Trapezoidal rule.

Solution. Given integral, $\int_2^{10} \frac{dx}{x}$

Let f(x) = $\frac{1}{4}$

a = 2, b = 10;n = 8

h = $\frac{b-a}{n} = \frac{10-2}{8} = 1$

x	y = f(x) = $\frac{1}{x}$
2	1/2 = 0.5000
3	1/3 = 0.3333
4	1/4 = 0.2500
5	1/5 = 0.2000
6	1/6 = 0.1666
7	1/7 = 0.1428
8	1/8 = 0.1250
9	1/9 = 0.1000
10	1/10 = 0.1000

Trapezoidal rule is $\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

$\int_2^{10} \frac{dx}{x} = \frac{h}{2}[(y_0+y_8)+2(y_1+y_2+y_3+ y_{4+} y_{5+} y_6+y_7)]$

$= \frac{1}{2}[(0.5+ 0.1)+ 2(0.333+ 0.25)+ 0.2+ 0.125+ 0.166 + 0.1428 +0.1111]$

$= \frac{1}{2}[(0.6)+2(1.3288)]$

$= 0.3+1.3288$

$= 1.62880$

5.. **Evaluate $\int_4^{5.2} f(x)dx$ by using Trapezoidal rule from the following.**

X	4	4.2	4.4	4.6	4.8	5.0	5.2
Y	1.4	1.4	1.5	1.5	1.6	1.6	1.6

Where h = 0.2

Solution. Trapezoidal rule is, $\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

$\int_4^{5.2} f(x)dx = \frac{0.2}{2}[(1.4+ 1.6)+ 2(1.4+1.5+ 1.5+1.6+ 1.6)]$

$= 0.1[3+2(7.6)]$

$= 0.1[18.2] = 1.82$

6. Evaluate $\log_e 7$ from the integral $\int_1^7 \frac{dx}{x}$

Solution. Given $f(x) = \frac{1}{x}$ a = 1, b = 7, n = 6, $h = \frac{b-a}{n} = \frac{7-1}{6} = \frac{6}{6} = 1$

x	y= f(x)
1	1.0000
2	0.5000
3	0.333
4	0.2500
5	0.2000
6	0.1666

7	0.1428

Trapezoidal rule is, $\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

$= \frac{h}{2}[(1+ 0.1428)+ 20.5+ 0.333+0.25+ 0.2+ 0.1666)]$

$= \frac{1}{2}[1.1428+ 2(1.4499)]$

$= \frac{1}{2}[4.0426]$ and $\int_1^7 \frac{dx}{x} = [\log_e x]_1^7$

$= \log_e 7 = \log_e 1 = \log_e 7$

$\log_e 7 = 2.0113$

7. A curve is drawn to passes through the point given by the following table.

X	1	1.5	2	2.5	3	3.5	4
Y	2	2.4	2.7	2.8	3	2.6	2

Solution. Estimate the area bounded by the curve, the axis-x and the lines x=1,x=4by using Trapezoidal rule.

Trapezoidal rule is h= $\frac{4-1}{6}$=0.5

$\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+y_4+y_5)]$

$= \frac{0.5}{2}[2+2.1+2+ (2.4 +2.7+2.8+2.6+3)]$

$\frac{0.5}{2}(4.1+27.0) = \frac{15.35}{2} = 7.775$ units of area

8.**. calculate e $\int_2^{10} \frac{dx}{1+x}$ by dividing the arrange into 8 equal parts by using Simpson's 1/3 rule**.

Solution.: Simpson's 1/3 rule is

$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_5+\ldots..)+2(y_2+ y_4+\ldots....)]$

$\int_2^{10} \frac{dx}{1+x} = \frac{h}{3}[(O.333+ 0.0909)+ 4(0.25+ 0.1666+ 0.125+0.1)$

$+ 200.2+ 0.142+ 0.111)]$

$= \frac{1}{3}[0.4242 + 4(0.6416) + 2(0.4539)) = \frac{1}{3}[3.8984] = 1.2994666$

9.**calculate $\int_a^b f(x)dx$ by using Simpson's $\frac{1}{3}$ rule**

X	4	4.2	4.4	4.6	4.8	5.0	5.2
Y	1.4	1.4	1.5	1.5	1.6	1.6	1.6

where h = 0.2

Solution: Simpson's $\frac{1}{3}$ rule is

$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_5+.....)+2(y_2+ y_4+.....)]$

$\int_4^{5.2} f(x)dx = \frac{0.2}{3}[(1.4+1.6)+ 4(1.4+1.5+ 1.6)+ 2(1.5+ 1.6)]$

$= \frac{0.2}{3}[3+4(4.5)+2(3.1)] = \frac{0.2}{3}[27.2]$

$= 1.813333.....$

10.Evaluate $\int_2^{10} \frac{dx}{1+x}$ by dividing the range iato 8 equal parts by using Simpson's $\frac{1}{3}$ rule.

Solution:. Let f(x)= $\frac{1}{x}$

a = 2, b = 10, n =number of equal parts = 8

h = $\frac{b-a}{n} = \frac{8}{8} = 1$

X	Y = f(x) = $\frac{1}{x}$
2	0.5000
3	0.3333
4	0.2500
5	0.2000
6	0.1666
7	0.1428
8	0.1250

9	0.1111
10	0.1000

Simpson's $\frac{1}{3}$ rule is

$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_5+.....+y_{n-1})+2(y_2+ y_4+y_6.....+y_{n-2})]$

$= \frac{1}{3}$ [I0.6)+ 4(0.3333+ 0.2+ 0.1428+ 0.1111)] +2(0.25+0.1666+0.125)]

$=\frac{1}{3}$[0.6+4(0.7872+ 1.0832)]

$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_6+.....)+2(y_2+ y_4+.....)]$

$\int_0^1 \frac{dx}{1+x^2} = \frac{1}{6*3}$ [(1 +0.5)+ 4(0. 9729+0.8+0.3+ 2(09+0.6923)]

$= \frac{1}{18}$[1.5+9.452+3.1846] = $\frac{1}{18}$[14.1366]

= 0.7853666

On the other hand $\int_0^1 \frac{1}{1+x^2}dx = (\tan^{-1})^1_0$

$= \tan^{-1}(1)-\tan^{-1}(0)$

$= \frac{\pi}{4}-0 = \frac{\pi}{4}$

$= \frac{\pi}{4} = 0.7853$

$= \pi = 3142$

11.. **Evaluate $\int_0^4 e^x\,dx$ by using Simpson's rule using the data e = 2°.72, e2= 739, e3=20.09, e4=54.60 and compare it will the actual value.**

Solution.: Divide the whole range (0,4) into four equal parts by taking h=1. By

Simpson's rule 1/3 we have

$\int_0^4 e^x$ dx = $\int_{x_0}^{x_0+4h} y\,dx$ = $\frac{h}{3}$(y_0+y_4+4(y_1+y_3)+242)

= $\frac{1}{3}$[1+54.60+4(2.72+20.09)+2(7.39)]

= $\frac{1}{3}$[55.60+91.24+14.78] = $\frac{1}{3}$[161.62]

= 553.373

The actual value of the integral is

$\int_0^4 e^x$ dx = e^4 - e^0 = 54.60 – 1.0 = 53.60

12.Evaluate $\int_4^{5.2} \log, x$ dx by (a) Simpson's $\frac{1}{3}$ (b) $\frac{3}{8}$ rule . After finding the true value of the integral.

Solution. Divide the whole range of integration ie., (4, 5, 2) into six equal parts taking h 0.2 and then compute the values of the function f(x)= $\log^x_e$ for each of sub division. These computed values are as follows.

X	Y = $\log^x_e$ x	X	Y = $\log^x_e$ x
Xo = 4.0	1.38629436	Xo+ 4h= 4.8	1.56861592
Xo+ h =4.2	1.43508453		
		Xo+5h= 5.0	1.60943791
X_0 + 2h= 4.4	1.48160454		
		Xo+ 6h= 5.2	1.64865863
Xo +3h= 4.6	1.52605630		

(a) By Simpson's $\frac{1}{3}$ rule, we have

$\int_4^{5.2}$ log, x dx = $\frac{h}{3}$ [y_0+y_6+(y_1+y_3++y_5)+2(y_2+y_4)]

= $\frac{2}{8}$ [3.03495299+ 4(4.57056874)+ 2(3.05022046)]

=1.82784726.

(b) By Simpson's 3/8 rule, we have

$\int_4^{5.2} \log_e x \, dx = \frac{3h}{8}[y_0+y_2+3(y_1+y_2++y_4+ y_5)+2 y_3]$

$= \frac{3(0.2)}{8}[03495299+ 3(6.09424290)+ 2 (1.52605630)]$

$= \frac{0.3}{4}[24.371294291]$

= = 1.82784707

Again exact value of the integral is

$\int_4^{5.2} \log_e x \, dx = x(\log_e x-1)\int_{4.0}^{5.4} 1.82784741$

13. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ rule.

Solution : let f(x) = $\frac{1}{1+x^2}$ let n =6 , a= -0 , b = 1 ,h = $\frac{b-a}{n} = \frac{1-0}{6} = \frac{1-0}{6} = \frac{1}{6}$ = 0.1666

X	f(x) = $\frac{1}{1+x^2}$
0	1
1/6	0.9729
2/6	0.9000
3/6	0.8000
4/6	0.6923
5/6	0.5901
$\frac{6}{6}$=1	0.5000

Simpson's $\frac{1}{3}$ rule is

a = 0, b = 6, n= 6

$h = \frac{b-a}{n} = \frac{6-0}{6} = 1$

$y_0 = f(0) -= \frac{1}{1+0^2} = \frac{1}{1} = 1$

$y_1 = f(1) = \frac{1}{1+1^2} = \frac{1}{2} = 0.5000$

$y_2 = f(2) = \frac{1}{1+2^2} = \frac{1}{5} = 0.2000$

$y_3 = f(3) = \frac{1}{1+3^2} = \frac{1}{10} = 0.1000$

$y_4 = f(4) = \frac{1}{1+4^2} = \frac{1}{17} = 0.0588$

$y_5 = f(5) = \frac{1}{1+5^2} = \frac{1}{26} = 0.0385$

$y_6 = f(6) = \frac{1}{1+6^2} = \frac{1}{37} = 0.0270$

Simpson's $\frac{1}{3}$ rule is

$$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_5+.....)+2(y_2+ y_4+.....)]$$

$$= \frac{1}{3}[(1 + 0.027)+ 4(0.5+ 0.1 + 0.0385)+ 2(0.2+ 0.0588)]$$

$$= \frac{1}{3} [3.7104] = 1.2368$$

14. Find the value of $\int_0^1 \frac{x^2}{1+x^2}$ dx by using Simpson's $\frac{1}{3}$ rule and also find the error

Solution. : Dividing the whole range (o, 1) into 4 equal parts nd making use of Simpson's $\frac{1}{3}$ rule we have

$$\int_0^1 \frac{x^2}{1+x^2} dx = \frac{h}{3}[(y_0+y_4)+4(y_1+ y_3+)+2y_2)]$$

$h = \frac{1}{4}$, a =0 , b= 1 ,$n = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4}$

X	F(x)

0	0
0.25	0.0615
0.5	0.2222
0.75	0.3956
1.0	-0.5000

$\int_0^1 \frac{x^2}{1+x^2}$ dx = $\frac{1}{12}$ [(0+ 0.5)+ 4C0.0615+ 0.3956)+ 2(0.222)]

= $\frac{1}{12}$ [0.5+ 1.8284+ 0.444]

=$\frac{2.7728}{12}$ = 0.2310

15. Find $\int_4^{5.2} \log x_e$ dx by using Simpson's $\frac{1}{3}$ rule.

solution.: Given integral is $\int_4^{5.2} \log x_e$ dx

A = 4, b = 52 let n= 6 = $\frac{b-a}{n}$ = $\frac{5.2-2}{6}$ = 0.2

X	F(x)
4	1.3862
4.2	1.4350
4.4	1.4816
4.6	1.5260
4.8	1.5686
5.0	1.6094
5.2	1.6486

$\int_4^{5.2} \log x_e$ = $\frac{h}{3}$[(y_0+y_6)+4(y_1+ y_3+ y_5)+2(y_2+ y_4)]]

= $\frac{0.2}{3}$[(1.3862+1.6481+4(1.435071 +1.5260+1.6094)

+2(1.4816+1.5686)]

= $\frac{0.2}{3}$ [(3.0348)+ 4 (4.5704)+ 2(3.05027)]

= $\frac{0.2}{3}$[3.0348+18.2816+ 6.1004] = $\frac{5.48336}{3}$ = 1.8277866.

16. Evaluate $\int_{-3}^{3} x^4$dx by using Simpson's $\frac{1}{3}$ rule.

Solution. We know the integral is $\int_{-3}^{3} x^4$dx

Where h = $\frac{b-a}{h}$ = $\frac{3-(-3)}{6}$ = $\frac{6}{6}$ = 1

x	F(x)
-3	81
-2	16
-1	1
0	0
1	1
2	16
3	81

Simpson's $\frac{1}{3}$ rule is

$\int_b^a f(x)dx$ = $\frac{h}{3}$ [(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)]

= $\frac{1}{3}$[(81+81)+4(16+0+16)+2(1+1)]

a = 3.b = 3 , n = 6

$$h = \frac{b-a}{h} = \frac{3-(-3)}{6} = 1$$

let $f(x) = x^2$

$y_0 = f(-3) = (-3)^2 = 9$

$y_1 = f(-2) = (-2)^2 = 4$

$y_2 = f(-1) = (-1)^2 = 1$

$y_3 = f(0) = (0)^2 = 0$

$y_4 = f(1) = (1)^2 = 1$

$y_5 = f(2) = (2)^2 = 4$

$y_6 = f(3) = (3)^2 = 9$

Simpson's $\frac{1}{3}$ rule

$$\int_a^b f(x) = \frac{h}{3}[(y_0+y_n)+4(y_1+y_{3+\ldots})+2(y_2+y_{4+\ldots})]$$

$$= \frac{1}{3}[(9+9)+4(4+0+4)+2(1+1)]$$

$$= \frac{54}{3} = 18 = \frac{1}{3}[162+128+4] = \frac{1}{3}[294] = 98$$

17. calculate an approximate value of $\int_a^{\pi/2} sinx\, dx$ by (a) Trapezoidal rule (b) Simpson's rule using 11 ordinates

Solution : Divide the range of integration into a ten equal parts taking h = $\pi/20$ and then compute the values of the function f(x) = sin x for each point of sub-division. The computed values are as follows :

X	Y=sin x

0	0.00000
$\pi/20$	0.15663
$2\pi/20$	0.30902
$3\,\pi/20$	0.45399
$4\pi/20$	0.58778
$5\pi/20$	0.707111
$6\,\pi/20$	0.80900
$7\,\pi/20$	0.89101
$8\,\pi/20$	0.95106
$9\pi/20$	0.98769
$10\,\pi/20$	1.00000

(a) By the trapezoidal rule, we have

$$\int_0^{\pi/2} sinx\ \mathrm{dx} = \frac{h}{3}[(y_0+y_{10})+2(y_1+y_{2+\ldots})]$$
$$= \frac{\pi/20}{2}[1.0000+2(5.85311)]$$
$$= \frac{\pi}{40}(12.70622) = 0.9981$$

(b) By Simpson's $\frac{1}{3}$ rule , we have

$$\int_0^{\pi/2} sinx\ \mathrm{dx} = \frac{h}{3}[(y_0+y_n)+4(y_1+y_{3+\ldots})+2(y_2+y_{4+\ldots})]$$
$$= \frac{\pi}{60}[1.00000+4(3.19623)+2(2.65688)]$$
$$= \frac{\pi}{60}[1.00000+12.78492+5.31376]$$
$$= \frac{\pi}{60}[19.09868] = 1.0006$$

Again the exact value of the integral

$$\int_0^{\pi/2} sinx\ \mathrm{dx} = [-\cos x]_0^{\pi/2} = 1.0000$$

The error (i) Due to Simpson's $\frac{1}{3}$ rule , we have = -0.0006

(ii) Due to trapezoidal rule = 0.0019

18.Evaluate $\int_a^b f(x)dx$,by Simpson's $\frac{3}{8}$ rule from the following

X	4	4.2	4.4	4.6	4.8	5.0	5.2
Y	1.4	1.4	1.5	1.5	1.6	1.6	1.6

Where

Solution : Simpson's $\frac{3}{8}$ rule is

$\int_a^b f(x)dx = \frac{3h}{8}[y_0+3y_1+2y_2+3y_2+2y_3+3y_4+3y_5+y_6]$

$= \frac{3(0.2)}{8}[1.4+3(1.4)+3(1.5)+2(1.5)+3(1.6)+3(1.6)+1.6]$

$= \frac{0.6}{8}[1.4+4.2+4.5+3.0+4.8+4.8+1.6]$

$= \frac{0.6}{8}(24.3) = 1.8225$

19. Evaluate $\int_{-3}^{3} x^4 dx$,by using Simpson's $\frac{3}{8}$ rule

Solution. we know by the above problem.

Let $f(x) = x^4$

X	F(X)
-3	81
-2	16
-1	1
0	0
1	1
2	16
3	81

Where h=1

Simpson's 3/8 rule is

$\int_a^b f(x)dx = \frac{3h}{8}[(y_0+ y_6)+3y_1+2y_3+3y_4+3y_5]$

$= \frac{3}{8}[81+81+3(16+1+1+16)]$

$= \frac{3}{8}[162+102] = \frac{3}{8}(264) = 99$

23. A curve is drawn to passes through the part given by the following .

X	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
Y	**2**	**2.4**	**2.7**	**2.8**	**3**	**2.6**	**2.1**

Estimate the area bounded by the curve the x- axis and the lines x =1, x=4 by using Simpson's $\frac{3}{8}$rule.

Solution. h $= \frac{4-1}{6} = 0.5$

Simpson's 3/8 rule is

$$\int_a^b f(x)dx = \frac{3h}{8}[(y_0+ y_6)+3y_1+2y_3+3y_4+3y_5]$$

$$= \frac{3(0.5)}{8}[2+2.1+3(2.4+2.7+3+2.6)+2(2.8)]$$

$$= \frac{6.35}{8} = 8.04375 \text{ units of area}$$

20. Evaluate $\int_a^b f(x)$ dx by using weddles formula from the following.

x	4	4.2	4.4	4.6	4.8	5.0	5.2
F(x)	1.4	1.4	1.5	1.5	1.6	1.6	1.6

Where h=0.4

Weddle's formula is

$$\int_a^b f(x)dx = \frac{3h}{10}[y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6]$$

$$= \frac{3(0.2)}{10}[(1.4+1.6)+5(1.4+1.6)+(1.5+1.6)+6(1.5)]$$

$$= \frac{0.6}{10}[3+15+3.1+9]$$

$$= \frac{0.6}{10}(30.1)$$

$$= 1.806$$

21. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using weddle's formula.

Solution. a =0 , b= 1 ,h = $\frac{1-0}{6}$ = $\frac{1}{6}$ = 0.1666

X	f(x)
0	1
1/6	0.9729
2/6	0.9000
3/6	0.8000
4/6	0.6923
5/6	0.5901
$\frac{6}{6}$=1	0.5000

Weddle's formula is

$\int_a^b f(x)dx$ = $\frac{3h}{10}$[y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6]

= $\frac{3}{6x10}$[(1+0.5)+5(0.972+0.590)+(0.9+0.6923)+6(0.8)]

= $\frac{1}{20}$[1.5+7.815+1.5923+4.8]

= $\frac{1}{20}$[15.703]

= 0.785365

2.9 Applications of Numerical Methods

Considering the following model equation:

$$(\mu u_y)_y + y^n u_y = C, \qquad (1)$$

defined in a domain $\Omega = [0\ 1]$.

Where $\mu = (1 - exp(-y/\varepsilon) + \delta)/Re$, $\varepsilon << 1$, $\delta << 1$, $Re >> 1$, $n > 0$.

The first term simulates dissipative terms in the Navier-Stokes equations, whereas the second term models the contribution of convective terms and the right hand side represents the pressure gradient term or source in the transport equations. The "viscosity" coefficient μ corresponds to the effective viscosity coefficient. The coefficient is rapidly changed from a relatively small value $\mu_l = \delta /Re$ (laminar viscosity) to a "turbulent" viscosity $\mu_t \approx 1/Re$.

The equation simulates the low-Reynolds-number effects and can be considered as a model equation for the LR model. The left hand side point in the domain Ω will be treated as a "wall". The low-Reynolds-number effects occur nearby this point. If we set $\mu = (1+ \delta)/Re$ in (1), we have the HR approach.

Furthermore, we will consider the following values for the constants: $Re = 10^2$, $\varepsilon = 3*10^{-2}$, $\delta = 10^{-2}$. For simplicity the right-hand side will be considered as the constant $C = -1$

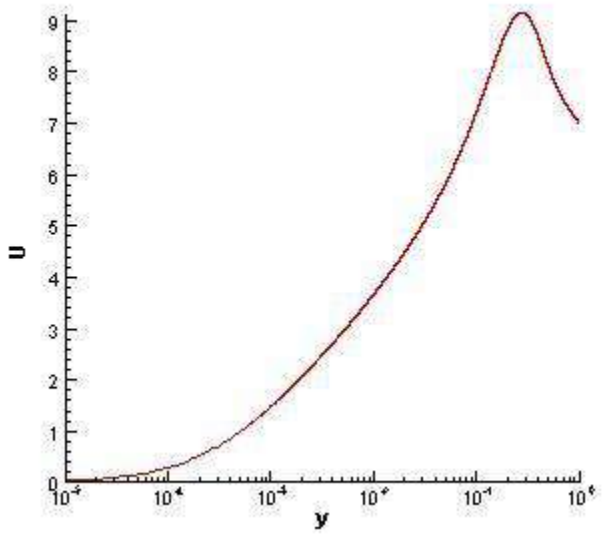
although this assumption is not important for our consideration.

Consider the following boundary-value Dirichlet problem:

$$(\mu u_{y)y} + y^n u_y = C \qquad (2)$$

$$u(0) = u_0 \quad u(1) = u_1$$

In the case of $n = 2$, $u_0 = 0$ and $u_1 = 7$, the solution is shown in Figure 1. The profile includes both the linear near-wall and logarithmic parts. Near the wall, $u = \varepsilon\, ln(1+y/(\varepsilon\delta))$. The thickness of the viscous "sub-layer", where u is a linear function, can be approximated as $y_v \approx \varepsilon\delta$.



In the calculations, the compact scheme [4] is used. The scheme allows us to calculate both the function and its derivative with a fourth-order of approximation. In Table 1, the results of calculations of the "friction" $\tau_w = \mu_w du/dy(0)$ (here $\mu_w = \mu(0)$) are given on different meshes. To exclude questions concerned with a mesh adaptation, a uniform mesh is used

Grid	10^2	$5*10^2$	10^3	$5*10^3$	10^4
τ_w	$3.1*10^{-1}$	$3.2*10^{-1}$	$3.3*10^{-1}$	$3.36*10^{-1}$	$3.36*10^{-1}$

Table 1. Calculation of τ_w on different meshes.o

If we use the HR model with the same boundary conditions, then $\tau_w = 4.9*10^{-1}$, which shows the importance of the sub-layer.

3. Linear System of Equations

Solutions of simultaneous linear equations:

Rate, Distance and Time:

You can calculate the best routes for your running or cycling schedule by creating a mathematical expression that takes into account the distance and your average speed for various parts of the route. You can use the equations to set different goals, such as to maximize time for build endurance, or to maximize speed for performance.

Planes, Trains and Automobiles:

The same formula used to calculate running times can be used to determine speed, distances and time duration when traveling by car, plane or train and you want to know the values for the unknown variables in your travel situations.

The Best Deal:

You want to find out the better deal when renting a car, and you're comparing two rental companies. By putting the variable and fixed costs, such as the per - mile and daily rate, into an algebraic expression, then solving for the total cost, you ca see which company saves you money for different amounts of driving.

The Best Plan:

You can use this same process with a system of equations when trying to decide on the best call phone plan, determining at how many minutes both companies charge the same amount and deciding from there which is the best plan for you and your intended usage.

Deciding on a Loan:

Simultaneous equations can be used to determine the best loan

choice to make when buying a car or a house when you consider the duration of the loan, the interest rate and the monthly payment of the loan. Other variables may be involved as well, with the information at hand, you can calculate which loan is the best choice for you.

Cost and Demand:

Simultaneous equations can be used when considering the relationship between the price of a commodity and the quantities of the commodity people want to buy at a certain price. An equation can be written that describes the relationship between quantity, price and other variables, such as income. These relationship equations can be solved simultaneously to determine the best way to price the commodity and sell it.

In the Air:

An air traffic controller can use Simultaneous equations to ensure two airplanes don't intersect at the same time.

The Best job for the Money:

System of equations can be used when trying to determine if you'll make more money at one job or another, taking multiple variables into account, such as salary, benefits and commissions.

Investing Wisely:

You can use Simultaneous equations to decide on your best investment option, taking into account the duration of the investment, the interest it will accrue, as well as other variables that will affect the end result. If you know the amount you'd like to accrue, you can set the options equal to each other and figure out which option is best for your future.

Mixing it up:

With respect to mixtures Simultaneous equations can be used for achieving a certain consistency in a resultant product, which is dependent on the consistency of the compounds mixed together to produce it.

Rates :

Linear equations can be useful tool for comparing rates of pay. For example, if one company offers to pay you $450 per week and other offers $10 per hour, and both ask you to work 40 hours per week, which company is offering the better rate of pay? A linear equation can help you figure it out! The first company's offer expressed as 450=40x. the second company's offer is expressed as y=10(40). After computing the two offers, the equations tell you that the first company is offering the better rate of pay at $11.25 per hour.

Variable costs:

Imagine that you taking a taxi while on vacation. You know that the taxi service charges $9 to pick your family up form your hotel and another $0.15 per mile for the trip. Without knowing how many miles it will be to each destination, you can set up a linear equation that can be used to find the cost of any taxi trip you take on your trip. By using "x" to represent the number of miles to your destination and "y" to represent the cost of that taxi ride, the linear equation would be: y= 0.15x+9.

3.1 MATRIX INVERSION METHOD:

* Solve x + 2y = 10, 2x - y = 15 by matrix inverse method?

Solution:-

Let $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 10 \\ 15 \end{bmatrix}$

Given equations are equivalent to AX = B

$\Rightarrow X = A^{-1} B$.

$$A^{-1} = \frac{1}{-1-4}\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix} \frac{1}{-1-4} = -\frac{-1}{5}\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix}$$

$$\therefore X = A^{-1} B = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} \frac{1}{5}\begin{bmatrix} 10 \\ 15 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 10 + 30 \\ 20 - 15 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 40 \\ 5 \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

$\therefore$ x = 18, y = 1 is the solution.

-----0-----

* Solve the equations 3x + 2y + 4z = 7; 2x + y + z = 7; x + 3y + 5z = 2. by matrix inversion method ?

Solution:-

Let $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 7 \\ 7 \\ 2 \end{bmatrix}$

Given equations are equivalent to AX = B

det A = 3(5 - 3) - 2(10 - 1) +4(6 - 1) = 6 - 18 + 20 = 8 ≠ 0.

$\therefore$ X = A^{-1} B is the solution.

Now adj A = $\begin{bmatrix} 2 & 2 & -2 \\ -9 & 11 & 5 \\ 5 & -7 & -1 \end{bmatrix}$

=> $A^{-1} = \frac{AdjA}{\det A} = \frac{1}{8}\begin{bmatrix} 2 & 2 & -2 \\ -9 & 11 & 5 \\ 5 & -7 & -1 \end{bmatrix}$

$\therefore$ $X = A^{-1} B = \frac{1}{8}\begin{bmatrix} 2 & 2 & -2 \\ -9 & 11 & 5 \\ 5 & -7 & -1 \end{bmatrix}\begin{bmatrix} 7 \\ 7 \\ 2 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ 24 \\ -16 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ -2 \end{bmatrix}$

$\therefore$ x = 3, y = 3, z = -2 is the solution.

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3.2 GAUSS ELIMINATION METHOD:

* Solve the equations 2x - y + 4z = 12; 8x - 3y =2z = 23; 4x + 11y - z = 33 by Gauss Elimination Method ?

Solution:-

Given equations are $2x - y + 4z = 12$, $8x - 3y + 2z = 23$, $4x + 11y - z = 33$.

Dividing first equation by 2, we get $x - \frac{1}{2}y + 2z = 6$ ----------(1)

Now subtract (1) after multiplying by 8 and 4 respectively from second and third equations we get $y - 14z = -25$----->(2), $13y - 9z = 9$ ------(3)

Subtract (2) after multiplied by 13 from (3) we get $173z = 334$

$\Rightarrow z = \frac{334}{173} = 1.931$

From (3); $13y - 9\left(\frac{334}{173}\right) = 9 \Rightarrow 13y = 9 + \frac{3006}{173} \Rightarrow y = \frac{4563}{2249} = 2.029$

From (1); $x = 6 + \frac{1}{2}\left(\frac{4563}{2249}\right) - 2\left(\frac{334}{173}\right) = 6 + \frac{1}{2}(2.029) - 2(1.931)$

$= 6 + 1.0145 - 3.862 = 3.153.$

$\therefore$ The solution is $x = 3.153$, $y = 2.029$, $z = 1.931$.

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3.3GAUSS JORDAN METHOD:

* Solve the system of equations $2x + y + z = 1$, $x - 2y - 3z = 1$, $3x + 2y + 4z = 5$ by Gauss Jordan Method.

Solution:-

The augmented matrix is $\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & -2 & -3 & 1 \\ 3 & 2 & 4 & 5 \end{bmatrix}$

$2R_2 - R_1, 2R_3 - 3R_1 \sim \begin{bmatrix} 2 & 1 & 1 & 1 \\ 0 & -5 & -7 & 1 \\ 0 & 1 & 5 & 7 \end{bmatrix}$

$5R_1 + R_2, 5R_3 + R_2 \sim \begin{bmatrix} 10 & 0 & -2 & 6 \\ 0 & -5 & -7 & 1 \\ 0 & 0 & 18 & 36 \end{bmatrix}$

$9R_1 + R_3, 18R_2 + 7R_3 \sim \begin{bmatrix} 90 & 0 & 0 & 90 \\ 0 & -90 & 0 & 270 \\ 0 & 0 & 18 & 36 \end{bmatrix}$ $\frac{1}{90}$

$R_1, \frac{1}{90}R_2, \frac{1}{18}R_3 \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

$\therefore$ **By** Gauss Jordan method, $x = 1$, $y = -3$, $z = 2$.

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* Solve the equations by Gauss Jordan Method . $5x + 2y + z = 12$, $x + 4y + 2z = 15$, $x + 2y + 5z = 20$.

Solution:-

The augmented matrix is $\begin{bmatrix} 5 & 2 & 1 & 12 \\ 1 & 4 & 2 & 11 \\ 1 & 2 & 5 & 20 \end{bmatrix}$

$5R_2 - R_1, 5R_3 - R_1 \sim \begin{bmatrix} 5 & 2 & 1 & 12 \\ 0 & 18 & 9 & 63 \\ 0 & 8 & 24 & 88 \end{bmatrix}$

$9R_1 - R_2, 9R_3 - 4R_1 \sim \begin{bmatrix} 45 & 0 & 0 & 45 \\ 0 & 18 & 9 & 63 \\ 0 & 0 & 180 & 540 \end{bmatrix}$

$180R_3 \sim \begin{bmatrix} 45 & 0 & 0 & 45 \\ 0 & 18 & 9 & 63 \\ 0 & 0 & 1 & 3 \end{bmatrix}$ $R_2 - 9R_3 \sim \begin{bmatrix} 45 & 0 & 0 & 45 \\ 0 & 18 & 0 & 36 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

$\frac{1}{45}R_1, \frac{1}{18}R_2 \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

$\therefore$ **By** Gauss Jordan method, $x = 1$, $y = 2$, $z = 3$.

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3.4 LU - DECOMPOSITION METHOD:

* Solve the system of equations $2x_1 + x_2 + x_3 = 2$, $x_1 + 3x_2 + 2x_3 = 2$, $3x_1 + x_2 + 2x_3 = 2$ by LU Decomposition Method?

Solution:-

Given equations are $2x_1 + x_2 + x_3 = 2$----------(1)

$x_1 + 3x_2 + 2x_3 = 2$---------(2)

$3x_1 + x_2 + 2x_3 = 2$---------(3)

Here the coefficient matrix A is given by $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 3 & 1 & 2 \end{bmatrix}$ and

$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$

$\therefore AX = B$

Now $A = LU \Rightarrow \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$

$$= \begin{bmatrix} u_{11} & u_{12} & u_{12} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{bmatrix}$$

On comparing two matrices, we get $U_{11} = 2$, $U_{12} = 1$, $U_{13} = 1$;

$L_{21}U_{11} = 1 \Rightarrow L_{21} = \frac{1}{U_{11}} = \frac{1}{2}$; $L_{31}U_{11} = 3 \Rightarrow L_{31} = \frac{3}{U_{11}} = \frac{3}{2}$;

$L_{21}U_{12}+U_{22} = 3 \Rightarrow 3 - L_{21}U_{12} = 3 - \frac{1}{2}(1) = \frac{5}{2}$

$L_{21}U_{13}+U_{2} = 3 \Rightarrow 2 - L_{21}U_{12} = 2 - \frac{1}{2}(1) = \frac{3}{2}$

$L_{31}U_{12}+L_{32}U_{22} = 1 \Rightarrow L_{32} = \frac{1-L_{31}U_{12}}{U_{22}} = \frac{1-\frac{3}{2}}{\frac{5}{2}}(1) = \frac{-1}{5}$

$L_{31}U_{13}+L_{32}U_{23}+U_{33} = 2$

$\Rightarrow U_{33} = 2 - L_{31}U_{13} - L_{32}U_{23} = 2 - \frac{3}{2} + \frac{1}{5}\left(\frac{3}{2}\right) = 2 - \frac{3}{2} + \frac{3}{10} = \frac{8}{10} = \frac{4}{5}$

Thus $L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{3}{2} & \frac{-1}{5} & 1 \end{bmatrix}$ and

$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 0 & \frac{5}{2} & \frac{3}{2} \\ 0 & 0 & \frac{4}{5} \end{bmatrix}$

LY = B => $\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{3}{2} & \frac{-1}{5} & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$

From this we get $y_1 = 1$, $\frac{1}{2}y_1 + y_2 = 2$, $\frac{3}{2}y_1 - \frac{1}{5}y_2 + y_3 = 2$.

On solving, we get $y_1 = 2$, $y_2 = 1$, $y_3 = \frac{-4}{5}$.

And since, we have UX =Y => $\begin{bmatrix} 2 & 1 & 1 \\ 0 & \frac{5}{2} & \frac{3}{2} \\ 0 & 0 & \frac{4}{5} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ \frac{-4}{5} \end{bmatrix}$

$2x_1 + x_2 + x_3 = 2$, $\frac{5}{2}x_2 + \frac{3}{2}x_3 = 1$, $\frac{4}{5}x_3 = \frac{-4}{5}$

By back substitution, we get $x_1 = 1$, $x_2 = 1$, $x_3 = -1$.

$\therefore$ Hence, solution is $x_1 = 1$, $x_2 = 1$, $x_3 = -1$

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* Solve the equations 2x + 3y + z = 9, x + 2y + 3z = 6, 3x + y 2z = 8 by LU Decomposition method.

Solution:-

Given equations can be written as AX= B where A = $\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$, X= $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, B = $\begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$

Let A = LU

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{12} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{bmatrix}$$

Comparing, we get U_{11}= 2, U_{12}= 3, U_{13}= 1 and $L_{21}U_{11}$= 1 => L_{21}= $\frac{1}{2}$;

$L_{21}U_{12}+U_{22}$= 2 => U_{22}= 2 - $L_{21}U_{12}$ = 2 - $\frac{3}{2}$ = $\frac{1}{2}$;

$L_{21}U_{13}+U_{23}$= 3 => U_{23}= 3 - $L_{21}U_{13}$ = 3 - $\frac{1}{2}$ = $\frac{5}{2}$;

$L_{31}U_{11}$ = 3 => L_{31}= $\frac{3}{U_{11}}$ = $\frac{3}{2}$ and

$L_{31}U_{12}$ +$L_{32}U_{22}$ = 1 => L_{32}= $\frac{1-L_{31}U_{12}}{U_{22}}$ = $\frac{1-\frac{9}{2}}{\frac{1}{2}}$ = -7

$L_{31}U_{13} + L_{32}U_{23} = 2 \Rightarrow U_{33} = 1 - L_{31}U_{13} - L_{32}U_{23} = 2 - \frac{3}{2} - (-7)\left(\frac{5}{2}\right) = 2 - \frac{3}{2} + \frac{35}{2} = \frac{36}{2} = 18.$

Thus L and U are given by $L = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -7 & 1 \end{bmatrix}$, $U = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1/2 & 5/2 \\ 0 & 0 & 18 \end{bmatrix}$

$LY = B \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -7 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$

From this we get $y_1 = 9$, $\frac{1}{2}y_1 + y_2 = 6$, $\frac{3}{2}y_1 - 7y_2 + y_3 = 8$.

On solving, we get $y_1 = 9$, $y_2 = \frac{3}{2}$, $y_3 = 5$.

Further since $UX = Y \Rightarrow \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1/2 & 5/2 \\ 0 & 0 & 18 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 3/2 \\ 8 \end{bmatrix}$

$2x + 3y + z = 9$, $1/2y + 5/2z = 3/2$, $18z = 5$.

By back substitution, we get $z = 5/18$ and $1/2y + 5/2(5/18) = 3/2$

$\Rightarrow y = 3 - 25/18 = 29/18$

And $2x + 3(29/18) + (5/18) = 9 \Rightarrow 2x = 9 - 29/6 - 5/18 = 9 - 92/18$

$\Rightarrow 2x = 70/18 \Rightarrow x = 35/18$

Hence, the solution is $x = 35/18$, $y = 29/18$, $z = 5/18$.

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3.5 ILL CONDITIONED OR WELL CONDITIONED:

* Establish whether the following matrix is singular, ill conditioned or well conditioned: $\begin{bmatrix} 3 & -1 & 3 \\ 1 & 0 & 4 \\ 2 & 2 & 1 \end{bmatrix}$?

Solution:-

Let A = $\begin{bmatrix} 3 & -1 & 3 \\ 1 & 0 & 4 \\ 2 & 2 & 1 \end{bmatrix}$

det A = 3(0 - 8) + 1(1 - 8) + 3(2 - 0) = -24 - 7 + 6 = -25

$\therefore$ A is not singular.

$\begin{vmatrix} 3.01 & -1 & 3 \\ 1 & 0 & 4 \\ 2 & 2 & 1 \end{vmatrix}$ = (3.01)(0 - 8) + 1(1 - 8) + 3(2 - 0)= -24.08-7+6 =-25.08.

$\therefore$ The matrix is well conditioned.

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* Show that A = $\begin{bmatrix} -73 & 78 & 24 \\ 92 & 66 & 25 \\ -80 & 37 & 10 \end{bmatrix}$ is ill- conditioned?

Solution :-

The ,matrix A = $\begin{bmatrix} -73 & 78 & 24 \\ 92 & 66 & 25 \\ -80 & 37 & 10 \end{bmatrix}$ is well conditioned, since $|A|$ = 1 whereas

$\begin{vmatrix} -73 & 78 & 24 \\ 92.01 & 66 & 25 \\ -80 & 37 & 10 \end{vmatrix}$ = 2.08, $\begin{vmatrix} -73 & 78.01 & 24 \\ 92 & 66 & 25 \\ -80 & 37 & 10 \end{vmatrix}$ = -28.20

and $\begin{vmatrix} -73 & 78 & 24 \\ 92 & 66 & 25 \\ -80 & 37 & 10.01 \end{vmatrix}$ = -118.94

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3.6 GAUSS - JACOBI METHOD:

* Solve the following equations by Jacobi method 20x + y -2z = 17, 3x + 20y - z = -18, 2x - 3y + 20z = 25?

Solution:-

Given equations can be written as $x = \frac{1}{20}(17 - y + 2z)$---------(1)

$y = \frac{1}{20}(-18 - 3x + z)$--------(2)

$z = \frac{1}{20}(25- 2x + 3y)$--------(3)

Let us take first approximation $x^{(1)} = 0$, $y^{(1)} = 0$, $z^{(1)} = 0$.

Then the second approximation using (1), we get

$x^{(2)} = \frac{1}{20}(17 - y^{(1)} + 2z^{(1)}) = \frac{17}{20} = 0.85$

$y^{(2)} = \frac{1}{20}(-18 - 3x^{(1)} + z^{(1)}) = \frac{1}{20}(-18 - 0 + 0) = -\frac{18}{20} = -0.9.$

$z^{(2)} = \frac{1}{20}(25- 2x^{(1)} + 3y^{(1)}) = \frac{1}{20}(25 - 0+ 0) = \frac{25}{20} = 1.25.$

Substituting these values in R.H.S of (1), we get third approximation

$x^{(3)} = \frac{1}{20}(17 - y^{(2)} + 2z^{(2)}) = \frac{1}{20}(17 + 0.9 + 2(1.25)) = \frac{20.4}{20} = 1.02.$

$y^{(3)} = \frac{1}{20}(-18 - 3x^{(2)} + z^{(2)}) = \frac{1}{20}(-18 - 3(0.85) + 1.25) = \frac{-19.3}{20} = -0.965.$

$z^{(3)} = \frac{1}{20}(25 - 2x^{(2)} + 3y^{(2)}) = \frac{1}{20}(25 - 2(0.85) + 3(-0.9)) = \frac{20.6}{20} = 1.03.$

Now Substitute these values in (1), we get fourth approximation

$x^{(4)} = \frac{1}{20}(17 - y^{(3)} + 2z^{(3)}) = \frac{1}{20}(17 + 0.965 + 2(1.03)) = \frac{20.025}{20} = 1.0013.$

$y^{(4)} = \frac{1}{20}(-18 - 3x^{(3)} + z^{(3)}) = \frac{1}{20}(-18 - 3(1.02) + 1.03) = -\frac{20.03}{20} = -1.0015.$

$z^{(4)} = \frac{1}{20}(25 - 2x^{(3)} + 3y^{(3)}) = \frac{1}{20}(25 - 2(1.02) + 3(-0.965)) = \frac{20.065}{20} = 1.0032.$

Substitute these values in (1), we get fifth approximation

$x^{(5)} = \frac{1}{20}(17 - y^{(4)} + 2z^{(4)}) = \frac{1}{20}(17 + 1.0015 + 2(1.0032)) = \frac{20.0079}{20} =$

1.0004.

$y^{(5)} = \frac{1}{20}(-18 - 3x^{(4)} + z^{(4)}) = \frac{1}{20}(-18 - 3(1.0013) + 1.0032) =$

$- = -\frac{20.0007}{20} = -1.00003.$

$z^{(5)} = \frac{1}{20}(25 - 2x^{(4)} + 3y^{(4)}) = \frac{1}{20}(25 - 2(1.0013) + 3(1.0015)) = \frac{19.9929}{20} =$

1.9996.

Substitute these values in (1), we get sixth approximation

$x^{(6)} = \frac{1}{20}(17 - y^{(5)} + 2z^{(5)}) = \frac{1}{20}(17 + 1.00003 + 2(0.9996)) = \frac{19.99923}{20} =$

0.99996.

$y^{(6)} = \frac{1}{20}(-18 - 3x^{(5)} + z^{(3)}) = \frac{1}{20}(-18 - 3(1.0004) + 0.9996) = \frac{-20.0016}{20} =$

$-1.00008.$

$z^{(6)} = \frac{1}{20}(25 - 2x^{(5)} + 3y^{(5)}) = \frac{1}{20}(25 - 2(1.0004) + 3(-1.00003)) = \frac{19.99911}{20} =$

$0.99995.$

The values in the fifth and sixth approximation are nearly same.

$\therefore$ Hence the solution is $x = 1, y = -1, z = 1$.

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* Solve the equations by Jacobi Method $83x + 11y - 4z = 95$; $7x + 52y + 13z = 104$; $3x + 8y + 29z = 71$?

Solution:- Given equation can be written as

$x = \frac{1}{83}(95 - 11y + 4z)$, $y = \frac{1}{52}(104 - 7x - 13z)$, $z = \frac{1}{83}(71 - 3x - 8y)$------(1).

Let us take the first approximation to be $x^{(1)} = 0, y^{(1)} = 0, z^{(1)} = 0$.

Then the second approximation using (1), we get

$x^{(2)} = \frac{1}{83}(95 - 11y^{(1)} + 4z^{(1)}) = \frac{1}{83}(95 - 0 + 0) = 1.14$

$y^{(2)} = \frac{1}{52}(104 - 7x^{(1)} - 13z^{(1)}) = \frac{1}{52}(104 - 0 - 0) = \frac{104}{52} = 2.$

$z^{(2)} = \frac{1}{29}(71 - 3x^{(1)} - 8y^{(1)}) = \frac{1}{29}(71 - 0 - 0) = \frac{71}{23} = 2.4.$

Substituting these values in R.H.S of (1), we get third approximation

$x^{(3)} = \frac{1}{83}(95 - 11y^{(2)} + 4z^{(2)}) = \frac{1}{83}(95 - 11(2) + 4(2.4)) = \frac{82.6}{83} = 0.995 \sim 1.$

$y^{(3)} = \frac{1}{52}(104 - 7x^{(2)} - 13z^{(2)}) = \frac{1}{52}(104 - 7(1.14) - 13(2.4)) = \frac{64.82}{52} = 1.25.$

$z^{(3)} = \frac{1}{29}(71- 3x^{(2)} - 8y^{(2)}) = \frac{1}{29}(71 - 3(1.14) - 8(2)) = \frac{51.58}{29} = 1.78.$

Now Substitute these values in (1), we get fourth approximation

$x^{(4)} = \frac{1}{83}(95 - 11y^{(3)} + 4z^{(3)}) = \frac{1}{83}(95 -11(1.25)+ 4(1.78)) = \frac{88.37}{83} = 1.1.$

$y^{(4)} = \frac{1}{52}(104 - 7x^{(3)} - 13z^{(3)}) = \frac{1}{52}(104 - 7(1) - 13(1.78)) = \frac{73.86}{52} = 1.42.$

$z^{(4)} = \frac{1}{29}(71- 3x^{(3)} - 8y^{(3)}) = \frac{1}{29}(71 - 3(1) - 8(1.25)) = \frac{58}{29} = 2.$

Substitute these values in (1), we get fifth approximation

$x^{(5)} = \frac{1}{83}(95 - 11y^{(4)} + 4z^{(4)}) = \frac{1}{83}(95 -11(1.42)+ 4(2)) = \frac{-87.38}{83} = 1.1.$

$y^{(5)} = \frac{1}{52}(104 - 7x^{(4)} - 13z^{(4)}) = \frac{1}{52}(104 - 7(1.1) - 13(2)) = \frac{70.3}{52} = 1.4.$

$z^{(5)} = \frac{1}{29}(71- 3x^{(4)} - 8y^{(4)}) = \frac{1}{29}(71 - 3(1.1) - 8(1.42)) = \frac{51.7}{29} = 1.78.$

$\therefore$ Hence the solution is x = 1.1, y = 1.4, z = 1.78.

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3.7 GAUSS SEIDEL METHOD

* Solve by Gauss Seidel method $10x_1 + x_2 + x_3 = 12$, $2x_1 + 10x_2 + x_3 = 13$, $2x_1 + 2x_2 + 10x_3 = 14$.

Solution:-

From the given equations we have

$$X_1 = \frac{1}{10}[12 - x_1 - x_2]\text{---------(1)}$$

$$X_2 = \frac{1}{10}[13 - 2x_1 - x_3]\text{---------(2)}$$

$$X_3 = \frac{1}{10}[14 - 2x_1 - 2x_2]\text{---------(3)}$$

Putting x = 0 in right side of the equation (1), we get $x_1^{(1)} = \frac{12}{10} = 1.2$

Putting $x_1 = x_1^{(1)} = 1.2$, $x_3 = 0$ in (2), we get $x_2^{(1)} = \frac{1}{10}(13 - 2.4 - 0) = 10.6/10 = 1.06$

Putting $x_1 = x_1^{(1)} = 1.2$, and $x_2 = x_2^{(1)} = 1.06$ in (3), we get

$x_3^{(2)} = \frac{1}{10}[14 - 2.4 - 2.12] = 0.948.$

For the second iteration we have $x_1^{(2)} = \frac{1}{10}[12 - 1.06 - 0.948] = 0.9992.$

$x_2^{(2)} = \frac{1}{10}[13 - 2(0.9992) - 0.948] = 1.00536.$

$x_3^{(2)} = \frac{1}{10}[14 - 2(0.9992) - 2(1.00536)] = 0.999098.$

Thus the iteration process is continued

The results are tabulated as follows correcting to four decimal places.

i	$x_1^{(i)}$	$x_2^{(i)}$	$x_3^{(i)}$
0	1.2000	0.0000	0.0000
1	1.2000	1.0600	0.9480

2	0.9992	1.0054	0.9991
3	0.9996	1.0000	1.0000
4	1.0000	1.0000	1.0000
5	1.0000	1.0000	1.0000

$\therefore$ The exact value of the roots are $x_1 = 1$, $x_2 = 1$, $x_3 = 1$.

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Network Analysis in linear system of Equations:

Set up a system of linear equations to represent the network shown in the figure. Then solve the system.

Solution:-

Each of the network's five junctions give rise to a linear equations as follows:

$X_1 + X_2 = 20$ Junction 1

$X_3 - X_4 = -20$ Junction

$X_2 + X_3 = 20$ Junction 3

$X_1 - X_5 = -10$ Junction 4

$-X_4 + X_5 = -10$ Junction 5

The augmented matrix for this system is

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 20 \\ 0 & 0 & 1 & -1 & 0 & -20 \\ 0 & 1 & 1 & 0 & 0 & 20 \\ 1 & 0 & 0 & 0 & -1 & -10 \\ 0 & 0 & 0 & -1 & 1 & -10 \end{bmatrix}$$

Gauss Jordan elimination produces the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & -10 \\ 0 & 1 & 0 & 0 & 1 & 30 \\ 0 & 0 & 1 & 0 & -1 & -10 \\ 0 & 0 & 0 & 1 & -1 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

From the matrix above, you can see that

$X_1 - X_5 = -10$, $X_2 - X_5 = 30$,

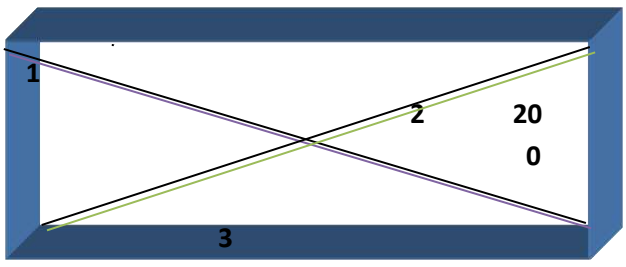
$X_3 - X_5 = -10$ and $X_4 - X_5 = -10$

Letting $t = X_5$, we have

$X_1 = t - 10$, $X_2 = -t + 30$, $X_3 = t + 30$, $X_4 = t + 10$, $X_5 = t$

Where t is the real number, so this system has infinitely many solutions.

1



2

4

5

Figure

In example 5, suppose you could control the mount of flow along the branch labeled X_5. using the solution of example 5 you could then control the flow represented by each of the other variables. For instance, letting t =10 would reduces the flow of X_1 and X_3 to zero, as shown In the figure.

4. LAPLACE TRANSFORM

4.1 Definition and properties of Laplace Transform, piecewise continuous functions, the Laplace Transform method of solving initial value problems

The method of Laplace transforms is a system that relies on algebra (rather than calculus-based methods) to solve linear differential equations. While it might seem to be a somewhat cumbersome method at times, it is a very powerful tool that enables us to readily deal with linear differential equations with discontinuous forcing functions.

***Definition*:** Let $f(t)$ be defined for $t \geq 0$. The Laplace transform of $f(t)$, denoted by $F(s)$ or $\mathcal{L}\{f(t)\}$, is an *integral transform* given by the *Laplace integral*:

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\, dt .$$

Provided that this (improper) integral exists, i.e. that the integral is convergent.

The Laplace transform is an operation that transforms a function of t (i.e., a function of *time domain*), defined on [0, ∞), to a function of s (i.e., of *frequency domain*)*. $F(s)$ is the *Laplace transform*, or simply *transform*, of $f(t)$. Together the two functions $f(t)$ and $F(s)$ are called a *Laplace transform pair*.

For functions of t continuous on [0, ∞), the above transformation to the frequency domain is *one-to-one*. That is, different continuous functions will have different transforms.

The integral is divergent whenever $s \le 0$. However, when $s > 0$, it converges to

$$\frac{1}{}(0 - e^{0}) = \frac{-1}{}(-1) = \frac{1}{} = F(s)$$

Example: Let $f(t) = t$, then $F(s) = \dfrac{1}{s^2}, \quad s > 0.$

[This is left to you as an exercise.]

The integral is divergent whenever $s \le a$. However, when $s > a$, it converges to

$$\frac{1}{}(0 - e^{0}) = \frac{1}{}(-1) = \frac{1}{} = F(s).$$

$$a - s \qquad a - s \qquad s - a$$

Definition: A function $f(t)$ is called *piecewise continuous* if it only has finitely many (or none whatsoever – a continuous function is considered to be "piecewise continuous"!) discontinuities on any interval $[a, b]$, and that both one-sided limits exist as t approaches each of those discontinuity from within the interval. The last part of the definition means that f could have removable and/or jump discontinuities only; it cannot have any infinity discontinuity.

Theorem: Suppose that

1. f is piecewise continuous on the interval $0 \le t \le A$ for any $A > 0$.

2. $|f(t)| \le K e^{at}$ when $t \ge M$, for any real constant a, and some positive constants K and M. (This means that f is "of *exponential order*", i.e. its rate of growth is no faster than that of exponential functions.)

3.Then the Laplace transform, $F(s) = \mathcal{L}\{f(t)\}$, exists for $s > a$.

***Note*:** The above theorem gives a sufficient condition for the existence of Laplace transforms. It is not a necessary condition. A function does not need to satisfy the two conditions in order to have a Laplace transform.

Examples of such functions that nevertheless have Laplace transforms are logarithmic functions and the unit impulse function.

Some properties of the Laplace Transform:

1. $\mathcal{L}\{0\} = 0$
2. $\mathcal{L}\{f(t) \pm g(t)\} = \mathcal{L}\{f(t)\} \pm \mathcal{L}\{g(t)\}$
3. $\mathcal{L}\{c\, f(t)\} = c\, \mathcal{L}\{f(t)\}$, for any constant c.

Properties 2 and 3 together means that the Laplace transform is *linear*.

[The derivative of Laplace transforms]

$\mathcal{L}\{(-t)f(t)\} = F'(s)$ or, equivalently $\mathcal{L}\{t\, f(t)\} = -F'(s)$

Example: $\mathcal{L}\{t^2\} = -(\mathcal{L}\{t\})' = -\frac{d}{ds}\left(\frac{1}{s}\right)^2 = -\left(-\frac{2}{s^3}\right) = \frac{2}{s^3}$

In general, the derivatives of Laplace transforms satisfy

$\mathcal{L}\{(-t)^n f(t)\} = F^{(n)}(s)$ or, equivalently $\mathcal{L}\{t^n f(t)\} = (-1)^n F^{(n)}(s)$

Warning: The Laplace transform, while a linear operation, is **not** *multiplicative*. That is, in general $\mathcal{L}\{f(t)\, g(t)\} \neq \mathcal{L}\{f(t)\}\, \mathcal{L}\{g(t)\}$.

Exercise: (a) Use property 4 above, and the fact that $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$,

to deduce that $\mathcal{L}\{t\, e^{at}\} = \frac{1}{(s-a)^2}$. (b) What will $\mathcal{L}\{t^2 e^{at}\}$ be?

Exercises C-1.1:

1 – 5 Use the (integral transformation) definition of the Laplace transform to find the Laplace transform of each function below.

1. t^2 2. te^{6t}

3. $\cos 3t$ 4. $e^{-t}\sin 2t$

5.* $e^{i\alpha t}$, where i and α are constants, $i = \sqrt{-1}$.

6 – 8 Each function $F(s)$ below is defined by a definite integral. Without integrating, find an explicit expression for each $F(s)$.

[Hint: each expression is the Laplace transform of a certain function. Use your knowledge of Laplace Transformation, or with the help of a table of common Laplace transforms to find the answer.]

6. $\int_{+0}^{\infty} e^{-(s+7)t}\,dt$

7. $\int_{+0}^{\infty} t^2 e^{-(s-3)t}\,dt$

8. $\int_{+0}^{\infty} 4e^{-st}\sin 6t\,dt$

9. Let $f(t)$ be a continuous function and $F(s)$ be its Laplace transform. Suppose $f(t) \geq 0$ for $t \geq 0$. (a) Show that $F(0)$ is equal to the area of the region under the curve $y = f(t)$, $0 \leq t \leq \infty$. (b) Verify this fact by comparing the value of $F(0)$ with the area under the curve obtained the usual way (i.e. by integration) for each of the functions e^{-t}, te^{-2t}, and t^2e^{-4t}.

Answers C-1.1:

1. $\dfrac{2}{s^3}$ 2. $\dfrac{1}{(s-6)^2}$

3. $\dfrac{s}{s^2 + 9}$ 4. $\dfrac{2}{s^2 + 2s + 5}$

5. $\dfrac{s}{s^2 + \alpha^2} + i\dfrac{\alpha}{s^2 + \alpha^2}$

Note: Since the Euler's formula says that $e^{i\alpha t} = \cos \alpha t + i \sin \alpha t$, therefore, $\mathcal{L}\{e^{i\alpha t}\} = \mathcal{L}\{\cos \alpha t + i \sin \alpha t\}$. That is, the real part of its Laplace transform corresponds to that of $\cos \alpha t$, the imaginary part corresponds to that of $\sin \alpha t$. (Check it for yourself!)

6. $\dfrac{1}{s + 7}$ 7. $\dfrac{2}{(s - 3)^3}$ 8. $\dfrac{24}{s^2 + 36}$

9. Set $s = 0$ in the definition $F(s) = \int_0^\infty e^{-st} f(t)\, dt$ to show that

$F(0) = \int_0^\infty f(t) dt = A$.

Solution of Initial Value Problems :

We now shall meet "the new System": how the Laplace transforms can be used to solve linear differential equations algebraically.

Theorem: [**Laplace transform of derivatives**] Suppose f is of exponential order, and that f is continuous and f' is piecewise continuous on any interval $0 \le t \le A$. Then $\mathcal{L}\{f'(t)\} = s\,\mathcal{L}\{f(t)\} - f(0)$

Applying the theorem multiple times yields:

$$\mathcal{L}\{f''(t)\} = s^2\,\mathcal{L}\{f(t)\} - s f(0) - f'(0),$$

$$\mathcal{L}\{f'''(t)\} = s^3\,\mathcal{L}\{f(t)\} - s^2 f(0) - s f'(0) - f''(0),$$

$$\vdots \qquad \mathcal{L}\{f^{(n)}(t)\} = s^n\,\mathcal{L}\{f(t)\} - s^{n-1} - s^2 f^{(n-3)}(0) - s f^{(n-2)}(0) - f$$

$$\vdots$$

This is an extremely useful aspect of the Laplace transform: that it changes differentiation with respect to *t* into multiplication by *s* (and, as seen a little earlier, differentiation with respect to *s* into multiplication by −*t*, on the other hand). Equally importantly, it says that the Laplace transform, when applied to a differential equation, would change derivatives into algebraic expressions in terms of *s* and (the transform of) the dependent variable itself. Thus, it can transform a differential equation into an algebraic equation.

We are now ready to see how the Laplace transform can be used to solve differentiation equations.

Solving initial value problems using the method of Laplace transforms

To solve a linear differential equation using Laplace transforms, there are only 3 basic steps:

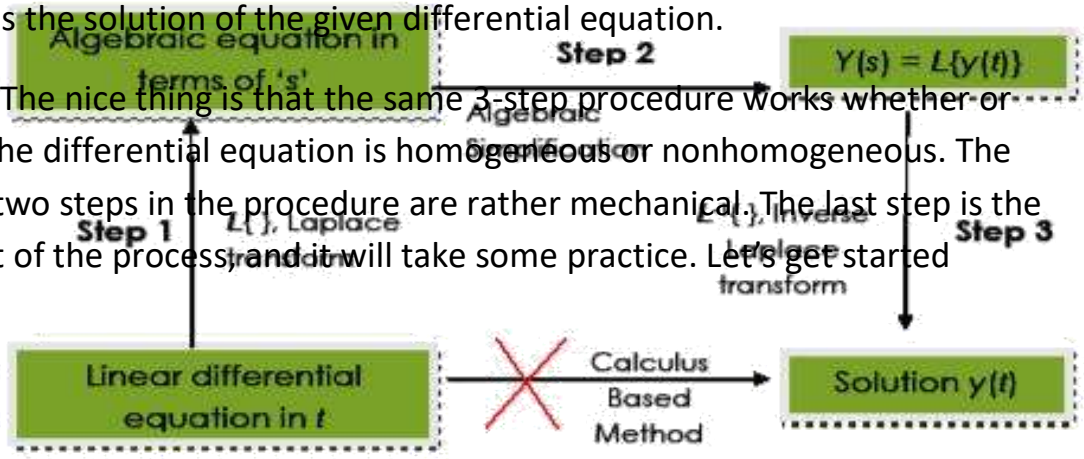
1.Take the Laplace transforms of both sides of an equation.

2.Simplify algebraically the result to solve for $\mathcal{L}\{y\} = Y(s)$ in terms of *s*.

3.Find the inverse transform of *Y*(*s*). (Or, rather, find a function *y*(*t*) whose Laplace transform matches the expression of *Y*(*s*).) This inverse transform, *y*(*t*), is the solution of the given differential equation.

The Laplace Transform Method of Solving an Initial Value Problem

Flow Chart



The nice thing is that the same 3-step procedure works whether or not the differential equation is homogeneous or nonhomogeneous. The first two steps in the procedure are rather mechanical. The last step is the heart of the process, and it will take some practice. Let's get started

***Example*:** $y'' - 6y' + 5y = 0, \qquad y(0) = 1, y'(0) = -3$

[Step 1] Transform both sides

$\mathcal{L}\{y'' - 6y' + 5y\} = \mathcal{L}\{0\}$

$(s^2\,\mathcal{L}\{y\} - s\,y(0) - y'(0)) - 6(s\mathcal{L}\{y\} - y(0)) + 5\mathcal{L}\{y\} = 0$

[Step 2] Simplify to find $Y(s) = \mathcal{L}\{y\}$

$(s^2\,\mathcal{L}\{y\} - s - (-3)) - 6(s\,\mathcal{L}\{y\} - 1) + 5\mathcal{L}\{y\} = 0\ (s^2 - 6\,s + 5)$

$\mathcal{L}\{y\} - s + 9 = 0(s^2 - 6\,s + 5)\ \mathcal{L}\{y\} = s - 9$

[Step 3] Find the inverse transform $y(t)$

Use partial fractions to simplify,

$$\mathcal{L}\{y\} = \frac{s-9}{s^2-6s+5} = \frac{a}{s-1} + \frac{b}{s-5}$$

$$\frac{s-9}{s^2-6s+5} = \frac{a(s-5)}{(s-1)(s-5)} + \frac{b(s-1)}{(s-5)(s-1)}$$

$s - 9 = a(s - 5) + b(s - 1) = (a + b)s + (-5a - b)$

Equating the corresponding coefficients:

$1 = a + b \qquad a = 2$

$-9 = -5a - b \qquad b = -1$

Hence,

$$\mathcal{L}\{y\} = \frac{s-9}{s^2-6s+5} = \frac{2}{s-1} - \frac{1}{s-5}\ .$$

The last expression corresponds to the Laplace transform of $2e^{t} - e^{5t}$. Therefore, it must be that, $y(t) = 2e^{t} - e^{5t}$.

Many of the observant students no doubt have noticed an interesting aspect (out of many) of the method of Laplace transform: that it finds the particular solution of an initial value problem directly, without solving for the general solution first. Indeed, it usually takes more effort to find the general solution of an equation than it takes to find a particular solution!

The Laplace Transform method can be used to solve linear differential equations of any order, rather than just second order equations as in the previous example. The method will also solve a nonhomogeneous linear differential equation directly, using the exact same three basic steps, without having to separately solve for the complementary and particular solutions. These points are illustrated in the next two examples.

Example: $y' + 2y = 4t\,e^{-2t}$, $y(0) = -3$.

[Step 1] Transform both sides

$$\mathcal{L}\{y' + 2y\} = \mathcal{L}\{4t\,e^{-2t}\}$$

$$(s\mathcal{L}\{y\} - y(0)) + 2\mathcal{L}\{y\} = \mathcal{L}\{4t\,e^{-2t}\} = \frac{4}{(s+2)^2}$$

$$(s\mathcal{L}\{y\} - (-3)) + 2\mathcal{L}\{y\} = \frac{4}{(s+2)^2}$$

$$(s+2)\,\mathcal{L}\{y\} + 3 = \frac{4}{(s+2)^2}$$

$$(s+2)\,\mathcal{L}\{y\} = \frac{4}{(s+2)^2} - 3$$

$$(s+2)^2$$

$$\mathcal{L}\{y\} = \frac{4}{3} - \frac{3}{s+2} = \frac{4-3(s+2)}{3} \quad 2 = \frac{-3s^2-12s-8}{3}$$
$$(s+2) \qquad (s+2) \qquad (s+2) \qquad (s+2)$$

[Step 3] Find the inverse transform $y(t)$

By partial fractions,

$$\mathcal{L}\{y\} = \frac{-3s^2 \quad -12s-8}{3} = \frac{a}{3} + \frac{b}{2} + \frac{c}{s+2}.$$
$$(s+2) \qquad (s+2) \qquad (s+2) \qquad s+2$$

$$\frac{-3s^{\,2}-12s-8}{(s+2)^3} = \frac{a}{(s+\ 2)^3} + \frac{b(s+2)}{(s+2)^3} + \frac{c(s+2)^2}{(s+2)^3}$$

$$= \frac{a+bs+2b+cs^2 \quad +4cs+4c}{(s+2)^3} = \frac{cs^2+(b+4c)s+(a+2b+4c)}{(s+2)^3}$$

$-3 = c$ $\qquad$ $a = 4$

$-12 = b + 4c$ $\qquad$ $b = 0$

$-8 = a + 2b + 4c$ $\qquad$ $c = -3$

$$\mathcal{L}\{y\} = \frac{-3s^2 - 12s - 8}{(s+2)^3} = \frac{4}{(s+2)^3} - \frac{3}{s+2}.$$

This expression corresponds to the Laplace transform of $2t^2 e^{-2t} - 3e^{-2t}$. Therefore,

$$y(t) = 2t^2 e^{-2t} - 3e^{-2t}.$$

Note: $\mathcal{L}\{t^n e^{at}\} = \frac{}{(s-a)^{n+1}}$

Example: $y'' - 3y' + 2y = e^{3t}$, $y(0) = 1, y'(0) = 0$

[Step 1] Transform both sides

$(s^2 \mathcal{L}\{y\} - s\,y(0) - y'(0)) - 3(s\mathcal{L}\{y\} - y(0)) + 2\mathcal{L}\{y\} = \mathcal{L}\{e^{3t}\}$

[Step 2] Simplify to find $Y(s) = \mathcal{L}\{y\}$

$(s^2 \mathcal{L}\{y\} - s - 0) - 3(s\mathcal{L}\{y\} - 1) + 2\mathcal{L}\{y\} = 1 / (s - 3)$

$(s^2 - 3s + 2)\,\mathcal{L}\{y\} - s + 3 = 1 / (s - 3)$

$(s^2 - 3$

s

$$+ 2)\,\mathcal{L}\{y\} = s - 3 + \frac{1}{s-3} = \frac{(s-3)^2 + 1}{s-3}$$

$$\mathcal{L}\{y\} = \frac{s^2 - 6s + 10}{(s^2 - 3s + 2)(s-3)} = \frac{s^2 - 6s + 10}{(s-1)(s-2)(s-3)}$$

[Step3] Find the inverse transform $y(t)$

By partial fractions,

$$\mathcal{L}\{y\} = \frac{s^2 - 6s + 10}{(s-1)(s-2)(s-3)} = \frac{5}{2}\cdot\frac{1}{s-1} - 2\cdot\frac{1}{s-2} + \frac{1}{2}\cdot\frac{1}{s-3}.$$

Therefore, $y(t) = \frac{5}{2}e^{t} - 2e^{2t} + \frac{1}{2}e^{3t}$.

For the next example, we will need the following Laplace transforms:

$$\mathcal{L}\{\cos bt\} = \frac{s}{s^2 + b^2}, \quad s > 0$$

$$\mathcal{L}\{\sin bt\} = \frac{b}{s^2 + b^2}, \quad s > 0$$

$$\mathcal{L}\{e^{at}\cos bt\} = \frac{s-a}{(s-a)^2 + b^2}, \quad s > a$$

$$\mathcal{L}\{e^{at}\sin bt\} = \frac{b}{(s-a)^2 + b^2}, \quad s > a$$

$$\frac{}{(s-a)^2+b^2}$$

***Note*:** The values of a and b in the last two expressions' denominators can be determined without using the method of completing the squares. Any irreducible quadratic polynomial $s^2 + Bs + C$ can always be written in the

required from of $(s-a)^2 + b^2$ by using the quadratic formula to find (necessarily complex-valued roots) s . The value a is the real part of s, and the value b is just the absolute value of the imaginary part of s. That is, if $s = \lambda \pm \mu\, i$, then $a = \lambda$ and $b = \mu$.

Example: $y'' - 2y' + 2y = \cos(t)$, $y(0) = 1, y'(0) = 0$

[Step 1] Transform both sides

$(s^2\, \mathcal{L}\{y\} - s\, y(0) - y'(0)) - 2(s\mathcal{L}\{y\} - y(0)) + 2\mathcal{L}\{y\} = \mathcal{L}\{\cos(t)\}$

[Step 2] Simplify to find $Y(s) = \mathcal{L}\{y\}$

$(s^2\, \mathcal{L}\{y\} - s - 0) - 2(s\mathcal{L}\{y\} - 1) + 2\mathcal{L}\{y\} = s\,/\,(s^2 + 1)$

$(s^2 - 2\,s + 2)\, \mathcal{L}\{y\} - s + 2 = s\,/\,(s^2 + 1)$

$$(s^2 - 2\,s + 2)\, \mathcal{L}\{y\} = s - 2 + \frac{s}{s^2+1} = \frac{(s-2)(s^2+1) + s}{s^2+1}$$

$$\mathcal{L}\{y\} = \frac{s^3 - 2s^2 + s - 2 + s}{(s^2+1)(s^2 - 2s + 2)} = \frac{s^3 - 2s^2 + 2s - 2}{(s^2+1)(s^2 - 2s + 2)}$$

[Step 3] Find the inverse transform $y(t)$

By partial fractions,

$$\mathcal{L}\{y\} = \frac{s^3 - 2s^2 + 2s - 2}{} = \frac{1\,s - 2}{} + \frac{4s - 6}{}$$

$$\frac{}{(s^2+1)(s^2-2s+2)} = \frac{}{5(s^2+1)} + \frac{}{5(s^2-2s+2)}$$

$$= \frac{1}{5}\left[\frac{s}{s^2+1} - \frac{2}{s^2+1} + \frac{4(s-1)}{(s-1)^2+1} - \frac{2}{(s-1)^2+1}\right]$$

which corresponds to

$$y(t) = \frac{1}{5}\left[\cos(t) - 2\sin(t) + 4e^t\cos(t) - 2e^t\sin(t)\right]$$

Examples: Find the inverse Laplace transform of each

(i) $F(s) = \dfrac{2s-5}{s^2+4s+8}$

Rewrite $F(s)$ as:

$$F(s) = \frac{2s-5}{(s+2)^2+2^2} = \frac{2(s+2)}{(s+2)^2+2^2} - \frac{9}{2}\cdot\frac{2}{(s+2)^2+2^2}$$

Answer: $f(t) = 2e^{-2t}\cos(2t) - \frac{9}{2}e^{-2t}\sin(2t)$

(ii) $F(s) = \frac{s+4}{(s-2)^3}$

Use partial fractions to rewrite $F(s)$ as:

$$F(s) = \frac{1}{(s-2)^2} + \frac{6}{(s-2)^3}$$

Answer: $f(t) = te^{2t} + 3t^2e^{2t}$

Appendix A

4.2 Some Additional Properties of Laplace Transforms

I. Suppose $f(t)$ is discontinuous at $t = 0$, then the Laplace transform of its derivative becomes

$$\mathcal{L}\{f'(t)\} = s\,\mathcal{L}\{f(t)\} - \lim_{t \to 0-} f(t).$$

Suppose $f(t)$ is a periodic function of period T, that is, $f(t + T) = f(t)$, for all t in its domain, then

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt.$$

***Comment*:** This property is convenient when finding the Laplace transform of a discontinuous periodic function, whose discontinuities (necessarily there are infinitely many, due to the periodic nature of the function) would make the usual approach of integrating from 0 to ∞ unwieldy.

Let c > 0 be a constant, the *time-scaling* property of Laplace transform states that

$$\mathcal{L}\{f(ct)\} = \frac{1}{c} F\left(\frac{s}{c}\right).$$

IV. We have known that $\mathcal{L}\{-t\,f(t)\} = F'(s)$. Taking the inverse transforms on both sides yields $-t\,f(t) = \mathcal{L}^{-1}\{F'(s)\}$. Therefore, $f(t) = -\frac{1}{t}\mathcal{L}^{-1}\{F'(s)\}$. That is, if we know how to invert the function $F'(s)$, then we also know how to find the inverse of its anti-derivative $F(s)$. Formally,

$$\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} F(u)\,du.$$

V. Similarly, dividing $F(s)$ by s corresponding an integral with respect to t.

$$\mathcal{L}\left\{\int_0^t f(u)\,du\right\} = \frac{F(s)}{s}$$

Comment: Therefore, dividing a function by its independent variable has the effect of anti-differentiation with respect to the other independent variable for Laplace transforms. These two properties (IV and V) are the f

$$\mathcal{L}\left\{\int_0^t f(\tau)g(t-\tau)d\tau\right\} = \mathcal{L}\{f(t)\}\mathcal{L}\{g(t)\}.$$

In addition, observe that, by taking the inverse Laplace transform of both sides the property VI, we have

$$\int_0^{\tau} f(\tau)g(t-\tau)d\tau = \mathcal{L}^{-1}\{\mathcal{L}\{f(t)\}\mathcal{L}\{g(t)\}\}.$$

In other words, we can obtain the inverse Laplace transform of a (simple) function of s that is itself a product of Laplace transforms of two known functions of t by taking the convolution of the two functions of t.

Example: Find the inverse Laplace transform of

$$F(s) = \frac{1}{s(s^2 + 4)} .$$

First, note that

$$F(s) = \frac{1}{s} \times \frac{1}{s^2 + 4} = \frac{1}{2} \quad \mathcal{L}\,\mathcal{L} \quad \{1\}\,\{\sin 2t\}$$

Therefore, its inverse can be found by the convolution integral (letting $f(t) = 1$ and $g(t) = \sin 2t$):

4.3 Applications of Laplace transform

This section describes the applications of laplace transform in the domain of engineering. Laplace transform has several applications in almost all Engineering disciplines such as System Modelling, Analysis of Electrical and Electronic Circuits, Digital Signal Processing and Process Controls. Laplace transforms are critical for process controls. It helps analyze the variables, which when altered, produces

desired manipulations in the result. For example, while study heat experiments, Laplace transform is used to find out to what extent the given input can be altered by changing temperature, hence one can alter temperature to get desired output for a while. This is an efficient and easier way to control processes that are guided by differential equations

Electric Circuit theory

Example 1 :: The switching transient phenomenon in the RL , RC or RLC circuits can be solved by laplace transform. Let us consider a series RLC circuit as shown figure 1 to which a dc voltage V_0 is suddenly applied.

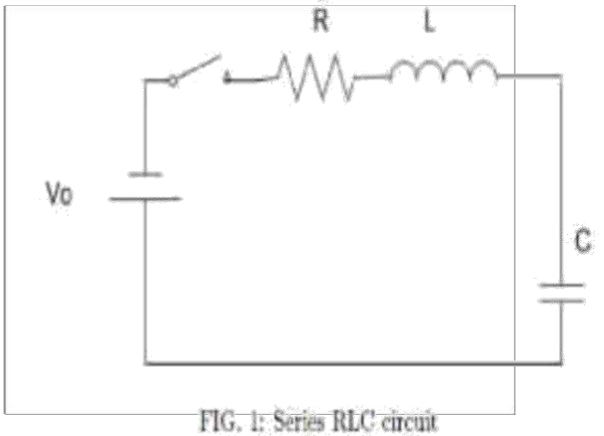


FIG. 1: Series RLC circuit

Now applying kirchoff's voltage law (KVL) to the circuit , we have

Differentiating bothsides, $R_i + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = V_0$

$$L\frac{d^2i}{dt^2} + \frac{1}{Ci} + R\frac{di}{dt} = 0 \text{ or}$$

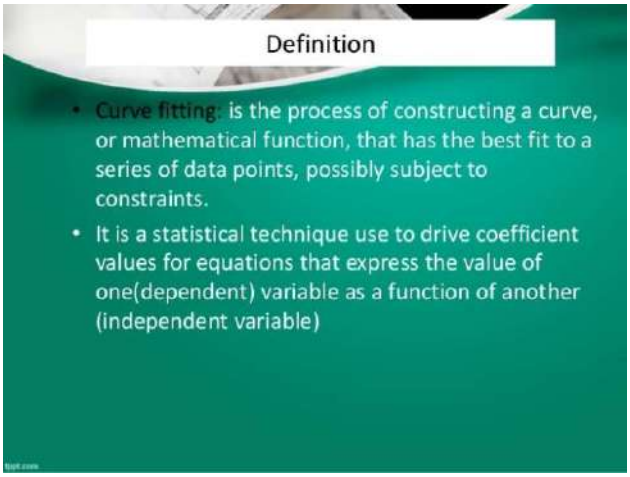
$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0 \quad \rightarrow (1)$$

Applying laplace transform to this equation, let us assume that the solution of this equation is

$$i(t) = K e^{sT}$$

$$L K s^2 e^{sT} + R K e^{sT} + \frac{1}{C K} e^{sT} = 0$$

5.CURVE FITTING



5.1 INTRODUCTION :

Curve and Surface Fitting

Curve fitting is one of the most powerful and most widely used analysis tools in Origin. Curve fitting examines the relationship between one or more predictors (independent variables) and a response variable (dependent variable), with the goal of defining a "best fit" model of the relationship.

Origin provides tools for linear, polynomial, and nonlinear curve fitting along with validation and goodness-of-fit tests. You can summarize and present your results with customized fitting reports. There are many time-saving options such as a copy-and-paste-operation feature which allows you to "paste" a just-completed fitting operation to another curve or data column. Curve fitting operations can also be part of an Analysis Template™, allowing you to perform batch fitting operations on any number of data files or data columns.

In this chapter we are concerned with the problem of fitting

an equation or a curve to data involving period values. An approximate non-mathematical relationship between the two variables , can be established by a diagram called **scatter diagram .** The exact mathematical relationship between the

two variables is given by simple algebra expression called **curve fitting .** Though there are infinite variety of curve in mathematics , the curve used for the purpose of curve fitting are relatively limited in type. The straight line is the simple and one of the most important curve used.Application of Curve Fitting in Indian Structure

The straight line :

The equation $y = a + bx$…. (1) Is an equation of the degree in x and y .It represents a straight line .

The difference $y_i - (a + bx_i)$….. (2)

Is zero if and only if the point (x_i, y_i) lies on the line given by (1)

Fitting a straight line:

Usually fitting a straight line means finding the value of the parameter a and b of the straight line given by (1) , as well as actually constructing the line itself .The Graphic method and the method of least square are two useful method for fitting a straight line .

The graphical method:

The method can be used whenever the given formula can be plotted as a straight line either directly or after suitable transformation. The straight line drawn after a careful visual estimate of its position has been made with the aid of a ruler. The coordinated of any two points on the measures and substitute in equation (1). The resulting equations in a and b are then solved for their *parameters.* Graphical method , whatever its theoretical attractions , suffers from the disadvantage that it is difficult apply in practice except for the straight line .This method will give fairly good results when finely divided coordinate paper is used , but in general it is recommended .

An alternative method, which id in almost universal use at present time, is known as the of **Least Squares** and we proceed to discuss it at length.

The method of lest squares:

This method of curve fitting was suggested early in the nineteenth century by the French mathematician Adrian Legendre. The method of least squares assumes that the best fitting line in the curve for which the sum of the squares of the vertical distances of the points (x_r , y_r) from the line is minimum. This method is more accurate that the graphical method.

Let $y = a + bx$ Refer (1)

Be the equation of the line. The ordinate of any point Q_r on the line vertically above or below a given point P_r , can be found by substituting the abscissa x_r , in the right-hand side of (1) . The two coordinates of Q_r will be $(x_r , a + bx_r)$ (see in the above figure). The vertical distances e_r from the line of any point p_r with coordinates (x_r , y_r) will therefore be given by the equation

$$e_r = y_r - (a + x_r) \ldots\ldots \quad (3)$$

We may say that e_r represents the difference between the actual ordinate y_r , of a pint and its theoretical ordinate $a + bx_r$.

Let $q = \sum_{r=1}^{n} e_r^2 = \sum_{r=1}^{n} (y_r - (a + bx_r))^2$ ……. (4)

The best fitting line is that line for which the sum of the square, $q = \sum e_r^2$ is a minimum. We find the value of a and b which make q minimum as follows :Differentiating q partially with respect to a and b equally these partially to zero , we obtain

$$\therefore \frac{\partial_q}{\partial_a} = \sum_{r=1}^{n} (-2)[y_r - (a + bx_r)] = 0 \ ,$$

And $$\frac{\partial_q}{\partial_o} = \sum_{r=1}^{n} (-2) x_r [y_r - (a + bx_r)] = 0 \ ,$$

Which yield the so – called **System of normal equations,**

$$\sum_{r=1}^{n} y_r = an + b \sum_{r=1}^{n} x_r \quad ,\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

And $\sum_{r=1}^{n} x_r \, y_r = a \sum_{r=1}^{n} x_r + b \sum_{r=1}^{n} x_r^2$ ……..(6)

Solving this system of equations we get

$$a = x = \frac{\sum_1^n x_r^2 \ \sum_1^n y_r - \sum_1^n x_r \sum x_r y_r}{n\sum_1^n x_r^2 - [\sum_1^n x_r} \quad \text{..... (7)}$$

$$b = \frac{n[\sum_{r=1}^n x_r y_r] - [\sum_{r=1}^n x_r][\sum_{r=1}^n x_r^2]}{n[\sum_{r=1}^n x_r^2] - [\sum_{r=1}^n x_r]^2} \quad \text{,.....(8)}$$

Where n is the number of points (x_r , y_r).

To simplify the for formulae, we get

$$t_r = \frac{x_r - \bar{x}}{h} \ , \qquad (9)$$

Where h is the interval between successive value of x assumed to be equally spaced. Shifting the origin to the mean $\bar{x}$ and compressing the intervals to unity we get transformed equation of (2) referred to the new (t, y) axes as

$$Y = \bar{a} + \bar{b}t \ , \qquad (10)$$

Where $\bar{a}\ and\ \bar{b}$ are the parameter in the new equation . In terms of the new coordinate formula (7) and (8) becomes

$$\bar{a} = \frac{\sum_1^n t_r^2 \sum_1^n y_r - \sum_1^n t_r \sum t_r y_r}{n\sum t_r^2 - (\sum t_r)^2} \ , \qquad (11)$$

$$\bar{b} = \frac{n\sum_1^n t_r y_1 - \sum_1^n t_r \sum t_r y_r}{n\sum_1^n t_r^2 - [\sum_1^n t_r]^2} \ , \qquad \text{.....} \qquad (12)$$

Transformation (9)

replace the $x_r\ \grave{s}$ by unit deviations t_r from the mean $\bar{x}$.

Since

$$\sum_{1}^{n} t_r = 0$$

The formula (11)and (12) reduce to

$$\bar{a} = \frac{\sum_1^n t_r^2 \sum_1^n y_r - 0}{n \sum t_r^2 - 0}$$

And

$$\bar{b} = \frac{n \sum_1^n t_r y_r - 0}{n \sum t_r^2 - 0}$$

Which further simplify to the form

5.2 Fitting a parabola:

When a set of point exhibits a parabolic trend , the fitting of a quadratic functions

$$y = a + bx + cx^2 ,$$

To the data may be carried out by the method of least square , Which leads to the three equations

$$na + b \sum_1^n x_r + c \sum x_r^2 = \sum_1^n y_r$$

$$a \sum_{1}^{n} x_r + b \sum_{1}^{n} x_r^2 + c \sum_{1}^{n} x_r^3 = \sum_{1}^{n} x_r y_r .$$

$$a \sum_{1}^{n} x_r^2 + b \sum_{1}^{n} x_r^3 + c \sum_{1}^{n} x_r^4 = \sum_{1}^{n} x_r^2 y_r .$$

The above equations can be solve for a , b, c.

5.3 EXPONETIAL FUNCTION Y= Ae^{bx}

Transforming the exponential equation $y = ae^{bx}$ by taking logarithms on both sides we get

$$\log y = \log a + bx \log e .$$

If we replace $\log y$ by y and the constant $\log a$ and b $\log e$ by $\grave{a}$ and $\acute{b}$

Respectively , we obtain

$$y = \acute{a} + b\acute{x} ,$$

Which defines a straight line .

Example:

1.Find the lest square line fir the data points (-1,10) ,(0,9),(1,7),(,4,5), (,2,5),(3,4),(4,3),(5,0) and (6,-1).

Solution: Here

x_r	y_r	x_r^2	y_r^2
-1	10	1	-10
0	9	0	0
1	7	1	7
2	5	4	10
3	4	9	12
4	3	9	12
5	0	25	0
6	-1	36	-6

20	37	92	25

From the table we have

$$n = 8\,, \sum x_r = 20\,, \sum y_r = 37, \sum x_r^2 = 92\,, and\ \sum x_r y_r = 25\,.$$

The normal equation are

$$na + b\sum x_r = \sum y_r\ ,$$

$$a\sum x_r + \sum x_r^2 = \sum x_r y_r\ .$$

Putting the value in normal equations

$$8a + 20b = 37\ ,$$

And

$$20a + 97b + 25\quad ,$$

And solving these equations we get

$$a = 8.6428571\ ,$$

$$b = -1.6071429\ \ .$$

Therefore the least square line is

$$y = 8.6428571 + (-1)1.6071429x\ ,$$

i.e.,

$$y = -1.6071429x + 8.6428571\,.$$

Example 2: **Find the least square line $y = a + bx$ for the data**

X	**-2**	**-1**	**0**	**1**	**2**
Y	**1**	**2**	**3**	**3**	**4**

Solution:

x_r	y_r	x_r^2	$x_r y_r$
-2	1	4	-2
-1	2	1	-2
0	3	0	0
1	3	1	3
2	4	4	8
0	13	10	7

$\therefore$ *In this case* $n = 5 , \sum x_r = 0 , \sum y_r = 13 , \sum x_{r2} = 10 , \sum x_r y_r = 7.$

The normal equation are

$$na + b \sum x_r = \sum y_r \ ,$$

$$a \sum x_r + b \sum x_r^2 = \sum x_r y_r \ .$$

Putting the value of $n , \sum x_r , \sum y_r , \sum x_r^2 \ and \ \sum x_r y_r$ in the above equation we get

$$5a = 13 , 10b = 7 \ .$$

Solving

$$a = 2.6 , b = 0.7 \ .$$

$\therefore$The required line of fit is

$$y = 2.6 + b = (0.7)x \ .$$

Example 3:

Find a formula for the line of the form $y = a + bx + cx^2$ Which will fit the following data

X	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Y	3.1950	3.2299	3.2532	3.2516	3.2516	3.2282	3.1807	3.1266	3.0594	2.9759

Solution :

The normal equation are

$$na = b\sum x_r + c\sum x_r^2 = \sum y_r^2 \ ,$$

$$a\sum x_r + b\sum x_r^2 + c\sum x_r^3 = \sum x_r y_r$$

$$a\sum x_r^2 + b\sum x_r^3 + c\sum x_r^4 = \sum x_r^2 y_r \ .$$

Substituting the value of $\sum x_r \ , \sum x_r^2 \ , \sum x_r^3 , \sum x_r y_r , \sum x_r^2 \ y_r , \sum y_r ,$ *and* n, *we get*

$$10a + 4.5b + 2.85c = 31.7616 \ ,$$

$$4.5a + 2.85b + 2.025c = 14.0896 \ ,$$

$$2.85a + 2.025b + 1.5333 = 8.82881 \ ,$$

And solving these equations we obtain

$$a = 3.1951 \ , b = 0.44254 \ , c = -0.76531.$$

$\therefore$ *The requied equation is*

$$y = 3.1951 + 0.44254x - 0.76531x^2.$$

Example 4:

Fit second degree parabola to the following data

X	0	1	2	3	4

y	0	1.8	1.3	2.5	6.3

Solution : The values of x are 0,1,2,3,4

The number of value is odd. Shifting the origin to the middle value 2 of x, and making the substituting

$$u = x - 2, v = y ,$$

The curve of fit as

$$v = a + bu + cu^2 \ ,$$

We obtain

x	v	u	v	uv	u^2	u^2v	u^3	u^4
0	1	-1	1	-2	4	4	-8	16
1	1.8	-2	1.8	1.6	1	1.8	-1	1
2	1.2	0	1.3	0	0	0	0	0
3	2.5	1	2.5	2.5	1	2.5	1	1
4	6.3	2	6.3	12.6	4	25.2	8	16
		0	12.9	11.3	10	33.5	0	34

The normal equation are

$$\sum u = na + b\sum u + c\sum u_r^2 \ ,$$

$$\sum uv = a\sum u + bu^2 + c\sum u^3 \ ,$$

$$\sum u^2 v = a\sum u^2 + b\sum u^3 + c\sum u^4 \ .$$

Putting the values of n ,$\sum u$, $\sum v$, $\sum uv$, … … … . , etc,

12.9=5a+40c,

$$11.3 = 10b \ ,$$

33.5=10a+34c,

Solving these normal equations we get

$$a = 1.48, b = 1.13, c = 0.550.$$

∴The required equation is

$$u = 1.48 + 1.13u + 1.55u^2 \quad ,$$

Substituting $u = x - 2$, $v = y$ in the above equation we get,

$$y = 1.42 + 1.13(x - 2) + (0.55)(x - 2)^2 \quad .$$

$$\therefore The\ requird\ curve\ of\ fit\ is\ the\ parabola$$

$$y = 1.42 - 1.07x + 0.55x^2.$$

5.4 Solved problems:

1)Find the lest square line $y = a + bx$ for the data

X	-4	-2	0	2	4
Y	1.2	2.8	6.2	7.8	13.2

A)	X	y	x^2	xy
	-4	1.2	16	-4.8
	-2	2.8	4	-5.6
	0	6.2	0	0
	2	7.8	4	15.6
	4	13.2	16	52.8
	0	31.2	4.	58

The normal equations are

$$\sum y = na + b\sum xi$$

$$\sum xy = a\sum x_i + b\sum x_i^2$$

31.2=5a

$$a = \frac{31.2}{5}$$

= 6.24

$$58 = b(40)$$

$$\therefore y = 6.24 + 1.45 \ .$$

2) **Fit a straight line to the following data regarding x as the independent variable**

X	0	1	2	3	4
Y	1	1.8	3.3	4.5	6.3

A)	x	y	x^2	xy
	0	1	0	0
	1	1.8	1	1.8
	2	3.3	4	6.6
	3	4.5	4	13.5
	4	6.3	16	25.2
	10	16.9	30	47.1

$$na + b\sum xi = \sum y$$

$$5a + b(10) = 16.9 \qquad (1)$$

$$a\sum x + b\sum x^2 = \sum xy$$

$$a(10) + b(30) = 47.1$$

$$1*2\rightarrow\ 10a + 20b = 33.8$$

$$\underline{2\rightarrow\ 10a + 30b = 47.1}$$

$$-10b = -11.3$$

$$b = \frac{13.3}{10}$$

=1.33

Substituting b in (1)

$$5a + (1.33)10 = 16.9$$

$$5a = 16.9 - 13.3$$

$$a = \frac{3.6}{5}$$

=0.72

$$\therefore y = 0.72 + 1.33x$$

3) Find the least square line $y = a + bx$

	X	-2	0	2	4	6
	Y	1	3	6	8	13

A)	X	y	x^2	xy
	-2	1	4	-2

0	3	0	-2
2	6	4	12
4	8	16	12
6	13	36	78
10	31	60	78

$$na + b\sum x_i = \sum xy$$

$$5a + 10b = 31 \quad (1)$$

$$a\sum x + b\sum x^2 = \sum xy$$

$$10a + 60b = 102 \quad (2)$$

$$1*2 \rightarrow 10a + 20b = 62$$

10a+60b=120

$$-40b = -58$$

$$b = 1.45$$

Substituting b in (1)

$$5a + (1.45)10 = 31$$

$$5a = 31 - 14.5$$

$$a = \frac{16.5}{5}$$

=3.3

$$a = 3.3 \ , b = 1.45$$

$$\therefore y = 3.3 + 1.45x$$

4) Find the least squares parabolic lift $y = a + bx + cx^2$

A)	x	y	x^2	x^3	x^4	xy	x^2y

-3	15	9	-27	81	-45	135
-1	5	1	-1	1	-5	5
1	1	1	1	1	1	1
3	5	9	27	81	15	45
0	26	20	0	164	-34	186

$$\sum y = na + b\sum x + c\sum x^2$$

$$26 = 4a + 0 + 20c$$

$$4a + 20c = 26 \qquad (1)$$

$$\sum xy = a\sum ax + b\sum x^2 + c\sum x^3$$

$$-34 = 0 + b(20) + c(0)$$

$$20b = -34$$

$$b = -\frac{34}{20}$$

$$= -1.7$$

$$\sum x^2y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

$$186 = 20a + 164c \qquad (2)$$

$$1*5 \quad \rightarrow \quad 20a + 100c = 130$$

$$\underline{2 \quad \rightarrow \quad 20a + 164c = 186}$$

$$-64c = -56$$

$$c = 0.875$$

$$4a + 20(0.875) = 26$$

$$4a = 36 - 17.5$$

$$a = \frac{8.5}{4}$$

$$= 2.125$$

$$y = a + bx + cx^2$$

$$\therefore y = 2.215 - 1.70x + 0.875x^2$$

5.5 APPLICATIONS OF CURVE FITTING

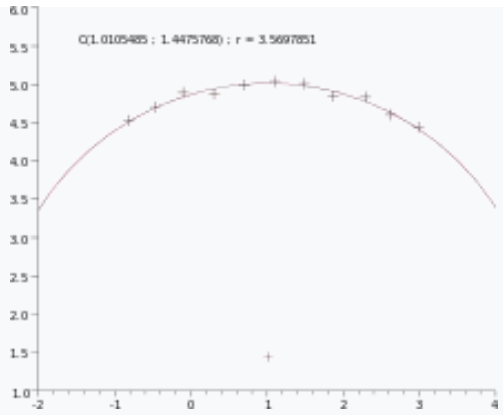
Fitting plane curves to data points :

If a function of the form cannot be postulated, one can still try to fit a plane curve.

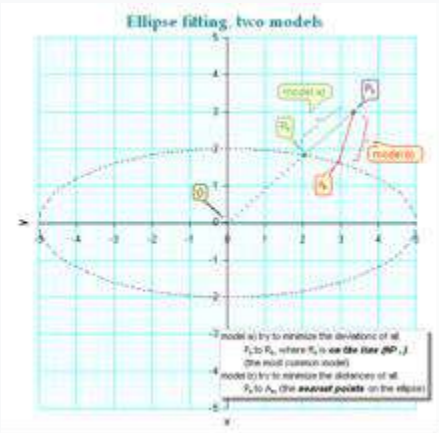
Other types of curves, such as conic sections (circular, elliptical, parabolic, and hyperbolic arcs) or trigonometric functions (such as sine and cosine), may also be used, in certain cases. For example, trajectories of objects under the influence of gravity follow a parabolic path, when air resistance is ignored. Hence, matching trajectory data points to a parabolic curve would make sense. Tides follow sinusoidal patterns, hence tidal data points should be matched to a sine wave, or the sum of two sine waves of different periods, if the effects of the Moon and Sun are both considered.

For a parametric curve, it is effective to fit each of its coordinates as a separate function of arc length; assuming that data points can be ordered, the chord distance may be used.

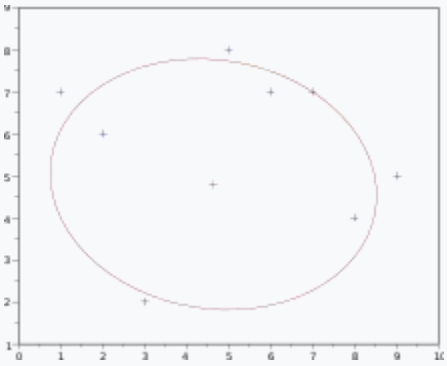
Fitting a circle by geometric fit



Circle fitting with the Coope method, the points describing a circle arc, centre (1 ; 1), radius 4.



different models of ellipse fitting



Ellipse fitting minimizing the algebraic distance (Fitzgibbon method).

Cooper[22] approaches the problem of trying to find the best visual fit of circle to a set of 2D data points. The method elegantly transforms the

ordinarily non-linear problem into a linear problem that can be solved without using iterative numerical methods, and is hence an order of magnitude faster than previous techniques.

Fitting an ellipse by geometric fit

The above technique is extended to general ellipses[23] by adding a non-linear step, resulting in a method that is fast, yet finds visually pleasing ellipses of arbitrary orientation and displacement.

Application to surfaces

Further information: Computer representation of surfaces

Note that while this discussion was in terms of 2D curves, much of this logic also extends to 3D surfaces, each patch of which is defined by a net of curves in two parametric directions, typically called **u** and **v**. A surface may be composed of one or more surface patches in each direction.

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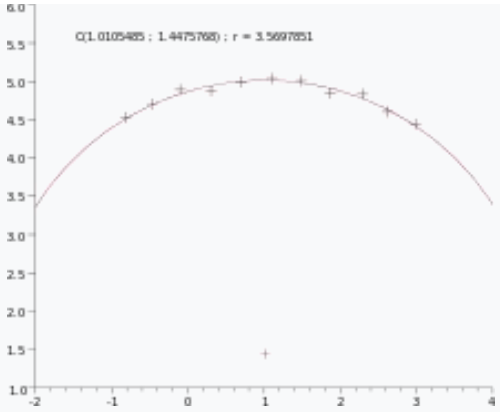

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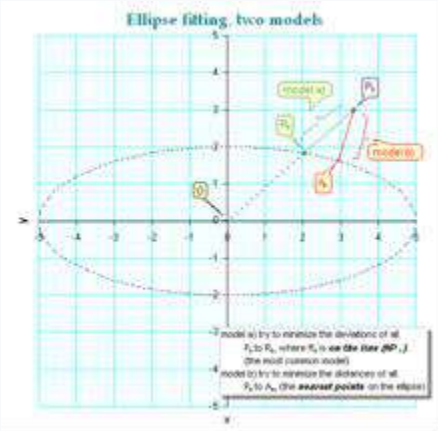

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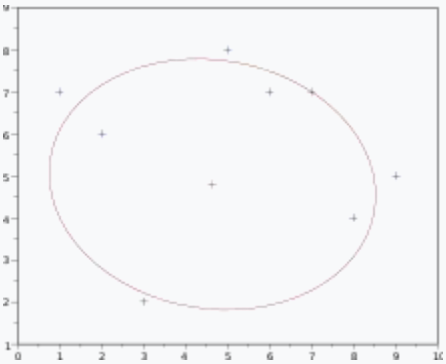

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Fitting functions to data points Edit

Most commonly, one fits a function of the form y=f(x).

Fitting lines and polynomial functions to data points Edit

Main article: Polynomial regression

See also: Polynomial interpolation

Polynomial curves fitting a sine function

Polynomial curves fitting points generated with a sine function. The black dotted line is the "true" data, the red line is a first degree polynomial, the green line is second degree, the orange line is third degree and the blue line is fourth degree.

The first degree polynomial equation

{\displaystyle y=ax+b\;}{\displaystyle y=ax+b\;}

is a line with slope a. A line will connect any two points, so a first degree polynomial equation is an exact fit through any two points with distinct x coordinates.

If the order of the equation is increased to a second degree polynomial, the following results:

{\displaystyle y=ax^{2}+bx+c\;.}y=ax^{2}+bx+c\;.

This will exactly fit a simple curve to three points.

If the order of the equation is increased to a third degree polynomial, the following is obtained:

{\displaystyle y=ax^{3}+bx^{2}+cx+d\;.}y=ax^{3}+bx^{2}+cx+d\;.

This will exactly fit four points.

A more general statement would be to say it will exactly fit four constraints. Each constraint can be a point, angle, or curvature (which is the reciprocal of the radius of an osculating circle). Angle and curvature constraints are most often added to the ends of a curve, and in such cases are called end conditions. Identical end conditions are frequently used to ensure a smooth transition between polynomial curves contained within a single spline. Higher-order constraints, such as "the change in the rate of curvature", could also be added. This, for example, would be useful in highway cloverleaf design to understand the rate of change of the forces applied to a

car (see jerk), as it follows the cloverleaf, and to set reasonable speed limits, accordingly.

The first degree polynomial equation could also be an exact fit for a single point and an angle while the third degree polynomial equation could also be an exact fit for two points, an angle constraint, and a curvature constraint. Many other combinations of constraints are possible for these and for higher order polynomial equations.

If there are more than n + 1 constraints (n being the degree of the polynomial), the polynomial curve can still be run through those constraints. An exact fit to all constraints is not certain (but might happen, for example, in the case of a first degree polynomial exactly fitting three collinear points). In general, however, some method is then needed to evaluate each approximation. The least squares method is one way to compare the deviations.

There are several reasons given to get an approximate fit when it is possible to simply increase the degree of the polynomial equation and get an exact match.:

Even if an exact match exists, it does not necessarily follow that it can be readily discovered. Depending on the algorithm used there may be a divergent case, where the exact fit cannot be calculated, or it might take too much computer time to find the solution. This situation might require an approximate solution.

The effect of averaging out questionable data points in a sample, rather than distorting the curve to fit them exactly, may be desirable.

Runge's phenomenon: high order polynomials can be highly oscillatory. If a curve runs through two points A and B, it would be expected that the curve would run somewhat near the midpoint of A and B, as well. This may not happen with high-order polynomial curves; they may even have values that are very large in positive or negative magnitude. With low-order polynomials, the curve is more likely to fall near the midpoint (it's even

guaranteed to exactly run through the midpoint on a first degree polynomial).

Low-order polynomials tend to be smooth and high order polynomial curves tend to be "lumpy". To define this more precisely, the maximum number of inflection points possible in a polynomial curve is n-2, where n is the order of the polynomial equation. An inflection point is a location on the curve where it switches from a positive radius to negative. We can also say this is where it transitions from "holding water" to "shedding water". Note that it is only "possible" that high order polynomials will be lumpy; they could also be smooth, but there is no guarantee of this, unlike with low order polynomial curves. A fifteenth degree polynomial could have, at most, thirteen inflection points, but could also have eleven, or nine or any odd number down to one. (Polynomials with even numbered degree could have any even number of inflection points from n - 2 down to zero.)

The degree of the polynomial curve being higher than needed for an exact fit is undesirable for all the reasons listed previously for high order polynomials, but also

leads to a case where there are an infinite number of solutions. For example, a first degree polynomial (a line) constrained by only a single point, instead of the usual two, would give an infinite number of solutions. This brings up the problem of how to compare and choose just one solution, which can be a problem for software and for humans, as well. For this reason, it is usually best to choose as low a degree as possible for an exact match on all constraints, and perhaps an even lower degree, if an approximate fit is acceptable.

Relation between wheat yield and soil salinity[18]

Fitting other functions to data points

Other types of curves, such as trigonometric functions (such as sine and cosine), may also be used, in certain cases.

In spectroscopy, data may be fitted with Gaussian, Lorentzian, Voigt and related functions.

In agriculture the inverted logistic sigmoid function (S-curve) is used to describe the relation between crop yield and growth factors. The blue figure was made by a sigmoid regression of data measured in farm lands. It can be seen that initially, i.e. at low soil salinity, the crop yield reduces slowly at increasing soil salinity, while thereafter the decrease progresses faster.

Algebraic fit versus geometric fit for curves

For algebraic analysis of data, "fitting" usually means trying to find the curve that minimizes the vertical (y-axis) displacement of a point from the curve (e.g., ordinary least squares). However, for graphical and image applications geometric fitting seeks to provide the best visual fit; which usually means trying to minimize the orthogonal distance to the curve (e.g., total least squares), or to otherwise include both axes of displacement of a point from the curve. Geometric fits are not popular because they usually require non-linear and/or iterative calculations, although they have the

advantage of a more aesthetic and geometrically accurate result.[19][20][21]

Fitting plane curves to data points

If a function of the form {\displaystyle y=f(x)}y=f(x) cannot be postulated, one can still try to fit a plane curve.

Other types of curves, such as conic sections (circular, elliptical, parabolic, and hyperbolic arcs) or trigonometric functions (such as sine and cosine), may also be used, in certain cases. For example, trajectories of objects under the influence of gravity follow a parabolic path, when air resistance is ignored. Hence, matching trajectory data points to a parabolic curve would make sense. Tides follow sinusoidal patterns, hence tidal data points should be matched to a sine wave, or the sum of two sine waves of different periods, if the effects of the Moon and Sun are both considered.

For a parametric curve, it is effective to fit each of its coordinates as a separate function of arc length; assuming that data points can be ordered, the chord distance may be used.[22]

Fitting a circle by geometric fit

Circle fitting with the Coope method, the points describing a circle arc, centre (1 ; 1), radius 4.

different models of ellipse fitting

Ellipse fitting minimising the algebraic distance (Fitzgibbon method).

Coope[23] approaches the problem of trying to find the best visual fit of circle to a set of 2D data points. The method elegantly transforms the ordinarily non-linear problem into a linear problem that can be solved without using iterative numerical methods, and is hence much faster than previous techniques.

Fitting an **ellipse by geometric fit**

The above technique is extended to general ellipses[24] by adding a non-linear step, resulting in a method that is fast, yet finds visually pleasing ellipses of arbitrary orientation and displaceme

Fitting surfaces

Further information: Computer representation of surfaces

See also: Multivariate interpolation and Smoothing

Note that while this discussion was in terms of 2D curves, much of this logic also extends to 3D surfaces, each patch of which is defined by a net of curves in two parametric directions, typically called u and v. A surface may be composed of one or more surface patches in each direction.

A STUDY ON

APPLICATIONS OF NUMERICAL ANALYSIS

A Group Project submitted to KRISHNA UNIVERSITY

In partial fulfilment of the requirement for the award of the degree of

BACHELOR OF SCIENCE

Submitted by

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CERTIFICATE

This is to certificate that the project entitled ***"APPLICATIONS OF NUMERICAL ANALYSIS"*** bonafide work done by ***E. BHAVANI (18302228), P. LAKSHMINARAYANA (18302229), B. VASUDEVA RAO (18302234), D. CHIRANJEEVI (18302235), P. NITHIN BABU (18302238), N. APPANNA (18302241), J. VAMSI (18302258)*** belongs to department of bachelor of science studies, ***S.R.R & C.V.R GOVT. DEGREE COLLEGE, VIJAYAWADA*** under my guidance in partial fulfilment of the award for the degree in bachelor of science.

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DECLARATION

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The matter presented as not been submitted to any University or Institute for the award of Bachelor of Science Degree.

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1. INTERPOLATION WITH EQUAL INTERVALS

1.1 INTRODUCTION :

Interpolation means insertion or filling up intermediate terms of the series. It is the technique of getting the value of a function for any intermediate value of the argument from the given set of values of function is called interpolation, while the process of computing the value of the function outside of the given range is called interpolation. The study of interpolation is based on the calculus of finite differences which

The following assumptions are to be kept in mind for interpolation.

(1) The values of the function should be either increasing or decreasing

(2) The rise or fall in the values should be uniform plays a vital role in numerical analysis manner.

1.2 INTERPOLATION METHODS :

(i) Graphical method

(i) Method of curve fitting

(iii) Applications of the calculus of finite differences formula.

1.2.1 Graphical Method :

Let y =f(x), then plot a graph for different values of x corresponding to the value of y. From the graph, we can find the value of y for given value of x.

For example

Year(x)	1891	1901	1911	1921	1931
Population (Y)(in 1000's)	46	66	81	93	101

Interpolate population for the year 1919.

The following steps are to be maintained

1. Take a suitable scale for x and y plot the different points on the graph for given data.

2. Join the point by hand curve.

3. Find the point on the curve corresponding to x = 1919 and find corresponding value of y

Remarks : This is simply an approximate method of evaluating the value of y and is not reliable in number of cases.

1.2.2 Method of Curve Fitting :

This method is applicable in some cases in which the form of the function is known. In such cases, by the method of least squares, we can fit the curve of known form to given set of values

and afterwards we can find the unknown value with the help of fitted curve.

Remarks :

(1) This is not exact method.

(2) In case of number of values are sufficiently large this method is complicated

(3) The form of the function in this method for given set of observations is assumed to be known.

1.2.3 Applications of the Calculus of Finite Differences :

(1) Newton-Gregory formula for interpolation. If n is a integer and h is the interval differencing, then $f(a+ nh) = f(a)+{}^{n}c_1 \Delta f(a)+ {}^{n}c_2\Delta^2 f(a)+....+ {}^{n}c_n \Delta^{n}{}_{f(a)}$

We prove the theorem by using the method of induction.

we know that $\Delta f(a) = f(a + h) - f(a)$

$f(a+h)= f(a) +\Delta f(a)$

The theorem is true for n = 1.

Assume that the theorem is true for n=k.

$f(a+kh)=f(a)+{}^{k}c_1 \Delta^2 f(a)+....+{}^{k}c_n \Delta^k f(a)$

We have $\Delta f(a+kh) = f(a+(k+ 1)h)-f(a+kh)$

$f(a + (k + 1)h) = f(a+kh) + h + \Delta f(a + kh)$

$= [f(a)+{}^{k}c_1\Delta f(a)+ {}^{k}c_2\Delta^r f(a)+....+ \Delta^k f(a)]$

$[\Delta^r f(a)+{}^{k}c_1\Delta^r f(a)+ {}^{k}c_1\Delta^3 f(a)+.....+ \Delta^{k+1} f(a)]$

$= f(a)+({}^{k}c_1+1) \Delta f(a)+({}^{k}c_1+ {}^{k}c_2) \Delta^2 f(a)+.....+ \Delta^{k+1} f(a)$

$= f(a)+{}^{k+1}c_1+ \Delta f(a)+{}^{k+1}c_2 \Delta^2 f(a)+.....+ \Delta^{k+1} f(a)$

The theorem is true for n=k+1

By the method of induction, the theorem is true for any integer.

$f(a+ nh) = f(a)+{}^{n}c_1 \Delta f(a)+ {}^{n}c_2\Delta^2 f(a)+....+ {}^{n}c_n \Delta^{n}{}_{f(a)}$

Note :

1. $f(a+nh) = f(a)+{}^{n}c_1\,\Delta f(a)+ {}^{n}c_2\Delta^2 f(a)+....+ {}^{n}c_n\,\Delta^{n}{}_{f(a)}$

2. Put a=0, h=1

$f(h)=f(0)+{}^{n}c_1\,\Delta f(0)+{}^{n}c_2+ {}^{n}c_2\,\Delta^2 f(0)+\ldots\ldots$

$f(x)=f(0)+\frac{x}{1!}\,\Delta f(0)+ \frac{x(x-1)}{2!}\,\Delta^2 f(0)+\ldots\ldots.$

This formula is used only when the first term a =0 and h =1.

Alter : Let y=f(x) represent a function which assumes the values f(a),f(a+h)....f(a+nh) for (n +1) equidistant values a, a+ h, a +2h, a+nh of the independent variable x and let $P_n(x)$ be a polynomial in x of degree n.

This polynomial can be expressed in the form.

$P_n(x) = Bo +B\,(x-a) + B2(x-a)(x-a-h)+\ldots\ldots\ldots+ B_n(x-a)(x-a-h)(x-a-2h)....(x-a-(n-1)h)$ ....(1)

where B_o. B_1, B_n are constants and are choosen such that

$P_n(a) = f(a)$, $P_n(a + h) = f(a + h)$, . . . $P_n(a + nh) = f(a + nh)$

By substituting successively the values a, a+ h, a +2h . a+nh in equation(1) and writing the values of $P_n(a)$, $P_n(a+h)$... P(a + nh), we get $f(a)= B_0$ $B1=f(a + h)-f(a)$

$f(a) = B_0$, $f(a + h) = B_0+ hB_1$

$B_0 =f(a)$ or $B_1 = \frac{f(a+h)-f(a)}{h} = \frac{\Delta f(a)}{h}$

$f(a + 2h) = B_0 + 2hB_1 + h^rB_2$ (or)

$A2 = f(a + 2h)-2\{f(a+ h)-f(a)\}-f(a)/2h^2$

$= \frac{1}{2!h^r}\,\Delta^2 f(a)$

Similarly, we get $B_3 = \frac{1}{3!h^3}\,\Delta^3 f(a)\ldots\ldots$ $B_n = \frac{1}{n!h^n}\,\Delta^n f(a)$

Substituting the values of B_0, B_1,....., B_n, we get

$P_n(x) = f(a) +\frac{\Delta f(a)}{h}\,(x-a)+ \frac{\Delta^2\mathbf{f(a)}}{\mathbf{2!}\boldsymbol{h^r}}\,(x-a)(x-a-h)+....+ \frac{\Delta^n\mathbf{f(a)}}{\boldsymbol{n!h^n}}\,(x-a)(x-a-h)\ldots(x-a-(n-1)h)$

This is Newton-Gregory formula for forward interpolation putting $\frac{x-a}{h}$ = u or x=a+hu then(1) can be expressed as

$P_n(x)=P_n(a+nh) = f(a)+u\Delta f(a)+\frac{u(u-1)}{2!}\,\Delta^2 f(a)+....+\frac{u(u-1)\ldots\ldots(u-n+1)}{n!}\,\Delta^n f(a)$

Newton-Gregory Formula for Backward Interpolation

In this case , we write

$P_n(x) = C_0 +C_1\,(x-a-nh)+ C_2(x-a-nh)+....+C_n(x-a-nh)(\,x-a-nh+h).....(x-a-h)$ .....(1)

where Co, C_1 ,C_2... C_n are constants and are choosen such that $f_n(a+ nh) = f(a+ nh)$,.., $P_n(a) = f(a)$

Substituting in (1), x=a + nh, a + nh-h, we have

$C_0 = f(a + nh)$, $f(a + nh-h) = C_o + C_1(-h)$

$C_1 = \frac{f(a + nh) - f(a + nh - h)}{h} = \frac{\Delta f(a + nh)}{h}$

Similarly $C_2 = \frac{1}{2!h^r} \nabla^2 f(a+nh)$

$C_3 = \frac{1}{3!h^3} \nabla^3 f(a+nh)$

……

$C_n = \frac{1}{n!h^n} \nabla^n f(a+nh)$

Substituting these values of $C_o, C_1, \ldots, C_n$ in (1), we obtain

$P_n(x) = f(a+nh) + \frac{\nabla f(a+nh)}{1!h}(x-a+nh) + \frac{\nabla^2 f(a+nh))}{2!h}(x-a+nh)\,(x-(a+nh-h))$

$+\ldots.. + \frac{(\nabla^n f(a+nh)}{n!h^n}(x-a+nh)\,(x-(a+nh-h))\ldots\ldots(x-(a+h))\ \ldots\ldots\ldots(2)$

This is Newton-Gregory formula for backward interpolation. Substituting $n = \frac{x-(a+nh)}{h}$ or

x= a + nh + hl in eq. (2), then we obtain $P_n(x) = P_n(a + nh + h) = f(a + nh) + u\nabla f(a+nh)$

$+ \frac{u(u+1)}{2!} \nabla^2 f(a+nh) + \ldots\ldots\ldots + \frac{u(u+1)\ldots\ldots(u+n-1)}{n!} \nabla^n f(a+nh)$ Where $P_n(x) = f(x)$.

1.3 SOLVED PROBLEMS

1. Given log x for x = 40, 45, 50, 55, 60, 65 for the following data :

x	40	45	50	55	60	65
logx	1.60206	1.65321	1.69877	1.74036	1.7785	1.81291

Estimate the value of log (58.75) by using Newton backward interpolation formula.

Solution.

X	10^5xlogx=f(x)	$10^5 x\nabla f(x)$	$10^5 x\nabla^2 f(x)$	$10^5 x\nabla^3 f(x)$	$10^5 x\nabla^4 f(x)$
40	160206				
		5115			
45	165321		-539		
		4516		102	

50	169892		-437		-25
		4319		77	
55	174036		-360		-20
		3729		57	
60	177815		-330		
		3476			
	181291				
65					

(a + nh) + uh = 58.75

65 +5u = 58.75

u = 1.25

Newton backward interpolation formula

$f(a+nh)+nh) = f(a+nh) + \frac{h}{1!}\Delta f(a+nh) + \frac{u(u+1)}{2!}\Delta^r f(a+nh) + \frac{u(u+1)(u+2)}{3!}\Delta^r f(a+nh)$

$10^5 f(58.75) = 181291 - 1.25x(3476) + \frac{(-1.25)(-1.25u)x(-30.5)}{2!} + \frac{(-1.25)(-1.25+1)x(-1.25+2)}{6}x(5)$

= 181291-4345-47.3438+2.2267 = 176900.89

F(58.75) = 1.7690089

2. Find the cubic polynomial for the following data hence find f(u)

X	0	1	2	3
F(x)	-2	1	4	6

Solution :

x	**F(x)**	Δf(x)	Δ^2**f(x)**	Δ^3**f(x)**
0	-2			
		3		
1	1		0	-1

		3		
2	4		-1	
		2		
3	6			

Newton's Gregory's formula is

$$f(x) = f(0)+\frac{x}{1!}\Delta f(0)+\frac{x(x-1)}{2!}\Delta^2 f(0)+\frac{x(x-1)(x-2)}{3!}\Delta^3 f(0)$$

$$= -2+\frac{x}{1!}(3)+\frac{x(x-1)}{2!}(0)+\frac{x(x-1)(x-2)}{3!}(-1)$$

$$= -2+3x-\frac{(x^3+3x^2+2x)}{6} = \frac{-12-18x-x^3+3x^2-2x}{6}$$

$$= \frac{x^3+3x^2+16x-12}{6} = \frac{-(4)^3+3(4)^2+16(4)-12}{6}$$

$$= \frac{-64+48+64-12}{6} = \frac{36}{6} = 6$$

f(4) = 6

3. Find f(1.5) by using Newton's advancing difference formula given that f(o)= +2, f(1)=4, f(2)=7, f(3)=8

x	F(x)	$\Delta f(x)$	Δ^2**f(x)**	Δ^3**f(x)**
0	**2**			
		2		
1	4		1	
		3		**-3**
2	7		-2	
		1		
3	8			

Solution. Newton's advancing formula is

$$f(x) = f(0)+\frac{x}{1!}\Delta f(0)+\frac{x(x-1)}{2!}\Delta^2\mathbf{f(0)}+\frac{x(x-1)(x-2)}{3!}\Delta^3\mathbf{f(0)}$$

$$= \mathbf{2}+\frac{x}{1!}(2)+\frac{x(x-1)}{2!}(1)+\frac{x(x-1)(x-2)}{3!}(-3)$$

$= 2+2x+\frac{x^2-x}{2!}+\frac{x^3-3x^2+2x}{2!}(-1)$

$= \frac{4+4x+x^2-x-x^3+3x^2-2x}{2}$

$F(x) = \frac{-(1.5)^3+4(1.5)^2+(1.5)+4}{2}$

$= \frac{-3.375+9+1.5+4}{2} = \frac{11-125}{2} = 5.5625$

$f(1.5) = 5.5625$

4. Find the polynomial and hence find f(5) from the following data by using Newton's Gregory's formula.

x	0	1	2	3	4
F(x)	0	7	26	63	124

Solution.

X	F(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	0				
		7			
1	7		12		
		19		6	
2	26		18		0
		37		6	
3	63		24		
		61			
4	124				

$F(x) = f(0)+\frac{x}{1!}\Delta f(0) + \frac{x(x-1)}{2!}\Delta^2 f(0) + \frac{x(x-1)(x-2)}{3!}\Delta^3 f(0)$

$= 0+\frac{x}{1!}(7) + \frac{x(x-1)}{2!}(12) + \frac{x(x-1)(x-2)}{3!}(6)$

$= 7x+6x^2-6x+x^3 - 3x^2 + 2x = x^3 + 3x^2 + 7x - 4x$

$F(x) = x^3 - 3x^2 + 2x$

F(5) = 5^3+3$(5)^2$+3(5) = 125+75+15 = 215

F(5) = 215

5. Find f from the given data by using Newton's Gregory's formula

X	**10**	**20**	**30**	**40**
F(x)	**9**	**30**	**35**	**42**

Solution .

X	F(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
10	**9**			
		21		
20	30		-16	
		5		18
30	35		2	
		7		
40	42			

a=10 , h= 10

Let a+uh=1.5

10+10u = 1.5

u = $\frac{5}{10}$ = $\frac{1}{2}$

Newton's Gregory's formula is

F(a+uh)= f(a)+$\frac{u}{1!}$ $\Delta f(a)$+ $\frac{u(u-1)}{2!}$ $\Delta^2 f(a)$+ $\frac{u(u-1)(u-2)}{3!}$ $\Delta^3 f(a)$

F(52) = 0.7071+$\frac{7}{5}$(0.0589)+$\frac{\frac{7}{5}(\frac{7}{5}-1)}{2!}$(-0.0057)+ $\frac{\frac{7}{5}(\frac{7}{5}-1)(\frac{7}{5}-2)}{3!}$(-0.0007)

= 7071+824.6-15.96+0.392 = 7880

10^4 f(52) = 7880

F(52) = $\frac{7880}{10^4}$ = 0.788 sin 52^0 = 0.7880

7. Using Newton's formula for interpolation estimate population for the year 1905 from the table.

Year (x)	Population(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1891	98752				
		33533			
1901	132285		2258		
				-10435	
1911	168076	35791	-8177		41358
				30923	
1921	195690	27614	22746		
		50360			
1931	246050				

Solution.

a = 1891, h = 10

Let a+ uh = 1905

u(10) = 1905-1891 = 14

$u = \frac{14}{10} = \frac{7}{6}$

Newton's Gregory's formula is

f(a+uh) =

$f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)+\frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(a)$

$f(1905) = 98752+\frac{7}{5}(33533)+\frac{\frac{7}{5}(\frac{7}{5}-1)}{2}(2258)+\frac{\frac{7}{5}(\frac{7}{5}-1)(\frac{7}{5}-2)}{6}(-10435)+\frac{\frac{7}{5}(\frac{7}{5}-1)(\frac{7}{5}-2)(\frac{7}{5}-3)}{24}(41358)$

= 98,752 + 48946·2+ 632.24 + 584.36 + 926.4192

= 147841.2192

The population for the year 1905 is 14,784 approximately

8 . From the following table find the no. of students who obtained less than 45 marks.

x	30-40	40-50	50-60	60-70	70-80
F(x)	31	42	51	35	31

Solution .

Marks less than x	No.of students	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
40	31				
		42			
50	73		9		
		51		-25	
60	124		-16		37
		35		12	
70	159		-4		
		31			
80	190				

a=40,h=10

let a +uh = 45

u(10) = 45-40

$$u = \frac{5}{10} = \frac{1}{2}$$

Newton's advancing formula is

F(a+uh) = f(a)

$$+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)+\frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(a)$$

$$= 31+\frac{1}{2}(42)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(9)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(-25)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\left(\frac{1}{2}-3\right)}{4!}(37)$$

$$= 31+21-\frac{9}{8}+\left(-\frac{75}{48}\right)-\frac{15}{16}\left(\frac{37}{21}\right)$$

$$= 52-1.125-1.5625-1.4453125$$

$$= 47.8671875 = 47$$

: The no. of students getting less than 45 marks is 47 and the no. of students who getting more than 45 is 190-47-143

9. Find the no. of men getting the wages less than is rupees from the following :

0-10	10-20	20-30	30-40
9	30	35	42

Solution .

Wages x	No. of Men	$\Delta \mathbf{f(x)}$	$\Delta^2 \mathbf{f(x)}$	$\Delta^3 \mathbf{f(x)}$
0-10	9			
		21		
10-20	30		-16	
		5		18
20-30	35		2	
		7		
30-40	42			

Newton's advancing formula is

$$f(a+uh) = f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)$$

$$f(15) = 9+\frac{1}{2}(21)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(-16)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(18)$$

$$= 9+\frac{21}{2}+2+\frac{3}{8}\left(\frac{18}{16}\right) = 11+\frac{21}{2}+\frac{9}{8}$$

$$= 11+10.5+1.125-22.625 = 23$$

11. Find the second degree polynomial from the data is then find f(5)

0	1	2	3	4
3	6	11	17	27

Solution .

X	F(x)	$\Delta \mathbf{f(x)}$	$\Delta^2 \mathbf{f(x)}$	$\Delta^3 \mathbf{f(x)}$

0	3			
		3		
1	6		2	
		5		0
2	11		2	
		7		0
3	18		2	
		9		
4	27			

Newton's Gregory's formula is

f(x) = f(0)+$\frac{x}{1!}\Delta f(0)$+ $\frac{x(x-1)}{2!}\Delta^2 f(0)$

a=0 , h=1

a+uh = 5 u =5

f(x) = 3+x(3)+$\frac{x(x-1)}{2}(2)$

= 3+3x + x^2-x = x^2 + 2x + 3

f(5) = $(5)^2$ + 2(5) + 3=25+10+ 3 = 38.

12. Second degree polynomial passes through the points (o, 1), (1, 3), (2, 7),(3, 13) then find the polynomial or find f(x). Also find the value of f(1.5).

Solution.

X	Y = f(x)	$\Delta\mathbf{f(x)}$	$\Delta^{\mathbf{2}}\mathbf{f(x)}$	$\Delta^{\mathbf{3}}\mathbf{f(x)}$
0	1			
		2		
1	3		2	
		4		0
2	7		2	
		6		
3	13			

Newton's Gregory's formula is

$$f(x) = f(0)+\frac{x}{1!}\Delta f(0)+\frac{x(x-1)}{2!}\Delta^2 f(0)$$

$$= 1+x(2)+\frac{x(x-1)}{2}(2)$$

$$f(x) = 1 + 2x + x^2-x$$

$$= x^2 + x + 1$$

$$f(1.5) = (1.5)^2+(1.5)+1$$

$$= 2.25+1.5+1 = 4.75$$

$$f(1.5) = 4.75$$

13. Find the polynomial fits the data in the following table then also find f(13).

X	3	5	7	9	11
Y	6	24	58	109	174

solution. a=3, h = 2

Let $a+uh = x = 13$ $3+2u = x$

$$u = \frac{x-3}{2}$$

X	Y =f(x)	$\Delta\mathbf{f(x)}$	$\Delta^2\mathbf{f(x)}$	$\Delta^3\mathbf{f(x)}$	$\Delta^4\mathbf{f(x)}$
3	6				
		18			
5	24		16		
		34		0	
7	58		16		0
		50		0	
9	109		16		
		66			
11	174				

Newton's Gregory's formula is

$$f(a+uh) = f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)$$

$$= 6+\frac{x-3}{1!}(18)+\frac{\left(\frac{x-3}{2}\right)\left(\frac{x-3}{2}-1\right)}{2!}(16)$$

$f(a+uh) = 6+9x-27+2(x-3)(x-5)=9x-21+2x^2-16x+30$

$f(x) = 2x^2-7x+9$

$f(13) = 2(13)^2-7(13)+9=2(169)-91+9=338+9-91$

$f(13) = 2.56$

14. Find the value of f(35) from the following table :

X	Y=f(x)	$\Delta\mathbf{f(x)}$	$\Delta^{\mathbf{2}}\mathbf{f(x)}$	$\Delta^{\mathbf{3}}\mathbf{f(x)}$
20	512			
30	439	-73		
			-20	
40	346	-93		10
			-10	
50	243	-103		

Solution .

Newton's advancing formula is

F(a+uh) =

$$f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)+\frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(a)$$

a=20 , h=10 => a+uh=35a=30, h=10

20+10h=35 (or) 30+10u=35

h= $\frac{3}{2}$,u= $\frac{1}{2}$

$f(35)=439+\frac{1}{2}(-93)+\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(-10)+\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(-10)$

$=439-\frac{93}{2}+\frac{10}{8}+\frac{30}{48}$

=439-46.5+1,25+6.625=440.875-46.5=394.375

f(35)=394.

Central Difference Interpolation Formulae

1.3.1 INTRODUCTION :

In the previous chapter, we derived and discussed Newton forward and backward interpolation formula, which are applicable for interpolation, near the beginning and end respectively of tabulated data. We shall now discuss in the present chapter, about the central difference formula, which are suitable for interpolation near the middle of a tabulated data.

The central difference operator (δ) is defined as $\delta = E^{\frac{1}{2}}-E^{-\frac{1}{2}}$

The first central difference operator δf(x) defined by

δ f(x) =f(x+$\frac{h}{2}$) - f(x-$\frac{h}{2}$)

δ f(x) =$(E^{\frac{1}{2}} - E^{-\frac{1}{2}})$ f(x)

We have E = 1 + Δ

$\delta = E^{\frac{1}{2}}-E^{-\frac{1}{2}} = E^{-\frac{1}{2}}(E-1) = E^{-\frac{1}{2}}\Delta$ ..... (1)

Again by the first backward difference of f(x)

We have $\nabla f(x) = f(x)-f(x-h) = f(x)-f(x-h) = E(x)- E^{-1}f(x)$

∇ f(x) = (1- E^{-1})f(x)

$\nabla = (1- E^{-1})$

$\nabla E^{\frac{1}{2}} = E^{\frac{1}{2}}-E^{-\frac{1}{2}} = \delta$ .....(2)

From(1) and (2) $\delta = \Delta E^{-\frac{1}{2}} = \nabla E^{\frac{1}{2}}$ .....(3)

The average operator u is defined as

$$U = \frac{1}{2}[E^{\frac{1}{2}}+E^{-\frac{1}{2}}]$$

Results :

(1) $U^2 = 1+\frac{1}{4}\delta^2 u^2 f(x) = \frac{1}{2}(E^{\frac{1}{2}}+E^{-\frac{1}{2}})^2$

$= \frac{1}{4}(E^{\frac{1}{2}}+E^{-\frac{1}{2}})^2 = \frac{1}{4}(E+E^{-1}+2) = \frac{1}{4}(E+E^{-1}-2+4)$

$= \frac{(u+k-1)^{(2k+1)}}{(2k+1)!} = -\frac{(k-v)^{(2k+1)}}{(2k+1)!}$

$= \frac{v(v^2-1)(v^2-4)\ldots\ldots((v^2-k^2)}{(2k+1)!}$

From (3), we obtain

$y_u = vy_o + \frac{v(v^2-1)}{3!}\Delta^2 y_{-1} + \frac{v(v^2-1)(v^2-4)}{5!}\Delta^4 y_{-2} + \ldots\ldots uy_1 + \frac{u(u^2-1)}{3!}\Delta^2 y_0 + \frac{u(u^2-1)(u^2-4)}{5!}$

$\Delta^4 y_{-2} + \ldots\ldots\ldots + \frac{u(u^2-1)(u^2-4)(u^2-k^2)}{(2k+1)!}\Delta^{2k} y_{1-k} + \ldots\ldots\ldots\ldots$ .....(5)

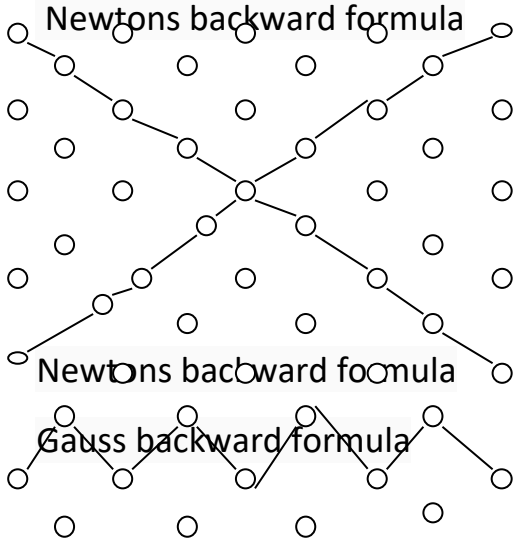
this can be written as

$f(u) = vf(0) + \frac{v(v^2-1)}{3!}\delta^2 f(0) + \frac{v(v^2-1)(v^2-4)}{5!}\delta^2 f(0) + \ldots\ldots + uf(1) + \frac{u(u^2-1)}{3!}$

$\delta^2 f(1) + \frac{u(u^2-1)(u^2-4)}{5!}\delta^2 f(1) + \ldots$

this is known as Laplace-Everette's formula

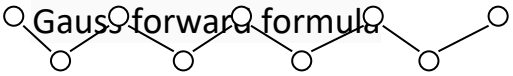
Newtons backward formula



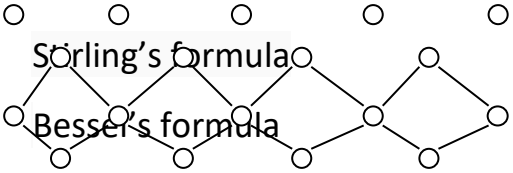
Newtons backward formula

Gauss backward formula

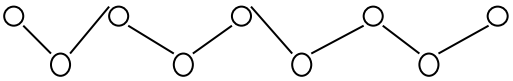
Gauss forward formula



Sörling's formula



Besser's formula



Note 1: This formula is useful especially when using the tables in which only even order difference are tabulated.

All the difference formula can be explained with the help of following diagram.

Results : (1) $\delta = \Delta E^{-\frac{1}{2}} = \nabla E^{\frac{1}{2}}$

(2) $\Delta\nabla = \nabla\Delta = \Delta - \nabla = \delta^2$

(3) u= $\delta = \frac{1}{2}(\nabla + \Delta) = \frac{1}{2}\Delta E^{-1} + \frac{1}{2}\Delta$ (4) $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$)

Proof :

(1) $\delta = \Delta E^{-\frac{1}{2}} = \nabla E^{\frac{1}{2}}$

$\delta f(x) = f(x+\frac{h}{2}) - f(x-\frac{h}{2}) = \nabla f(x+\frac{h}{2}) = \nabla E^{\frac{1}{2}} f(x)$

$\delta = \nabla E^{\frac{1}{2}}$

(2) (a) $\Delta\nabla f(x) = \Delta\Delta E^{-1} f(x) = \Delta^2 E^{-1} f(x)$

(b) $\Delta\nabla f(x) = \Delta E^{-1} f(x) = \Delta^2 E^{-1} f(x)$

(c) $(\Delta - \nabla) f(x) = (\Delta - \nabla E^{-1}) f(x) = \Delta(1-\frac{1}{E}) f(x)$

$= \frac{\Delta(E-1)}{E} f(x) = \Delta.\Delta E^{-1} f(x) = \Delta^2 E^{-1} f(x)$

(d) $\delta^2 f(x) = (\Delta E^{-\frac{1}{2}})^2 f(x) = \Delta^2 E^{-1} f(x)$

$\Delta\nabla = \nabla\Delta = \Delta - \nabla = \delta^2$

(3) $u\delta f(x) = \frac{(E^{\frac{1}{2}}+E^{-\frac{1}{2})}}{2}(E^{\frac{1}{2}}-E^{-\frac{1}{2}})f(x)$

$= \frac{1}{2}(e-e^{-1})f(x) = \frac{1}{2}(e-\frac{1}{E})f(x) = \frac{1}{2}(E^2-\frac{1}{E})f(x)$

$= \frac{(E-1)(E+1)}{2E}f(x) = \frac{\Delta}{2}(1+\frac{1}{E})f(x)$

We know that $\nabla = \Delta E^{-1} = \frac{1}{2}(\Delta+\nabla)f(x) = \frac{1}{2}(\Delta+\nabla E^{-1})f(x)$

$= \frac{1}{2}\nabla E^{-1}+\frac{1}{2}\Delta$

$U\delta = \frac{1}{2}(\Delta+\nabla E^{-1})$

(4) $\Delta+\nabla = \frac{\Delta}{\nabla}-\frac{\nabla}{\Delta}$

$(\frac{\Delta}{\nabla}-\frac{\nabla}{\Delta})f(x) = (\frac{\Delta}{\Delta E^{-1}}-\frac{\Delta E^{-1}}{\Delta})f(x) = (E-\frac{1}{E})f(x) = (\frac{E^2 1}{E})f(x)$

$= (E-1)\frac{(E+1)}{E}f(x) = \Delta(1+E^{-1})f(x) =$

$(\Delta+\nabla E^{-1})f(x)$

$= (\Delta+\nabla)f(x)$

$(\frac{\Delta}{\nabla}-\frac{\nabla}{\Delta}) = \Delta+\nabla$

1.4 SOLVED PROBLEMS

1. Using Gauss forward formula find the value of y when x = 0.5 from the following table.

X	-2	-1	0	1	2
F(x)	22	30	82	106	216

Solution .

X	F(x)	$\delta f(x)$	$\delta^2 f(x)$	$\delta^3 f(x)$	$\delta^4 f(x)$
-2	22				
		8			
-1	30		44		

0	82	52 24	-28	-72 114	186
1	106	110	86		
2	216				

Gauss forward interpolation formula is

$f(x) = f(0) + \frac{x}{1!}\delta f(1/2) + \frac{x(x-1)}{2!}\delta^2 f(0) + \frac{x(x-1)(x+1)}{3!}\delta^3 f(1/2) + \frac{x(x-1)(x+1)(x-2)}{4!}\delta^4 f(0)$

$f(x) = 82 + \frac{x}{1!}(24) + \frac{x(x-1)}{2!}(-28) + \frac{x(x-1)(x+1)}{6}(114) + \frac{x(x-1)(x+1)(x-2)}{24}(186)$

$f(0.5) = 82 + (0.5)(24) + \frac{(0.5)(0.5-1)}{2}(-28)$

$+ \frac{(0.5)(0.5-1)(0.5+1)}{6}(114) + \frac{(0.5)(0.5-1)(0.5+1)(0.5-2)}{24}(186)$

$= 82 + 12 + \frac{7}{2} - \frac{57}{24} + 4.359 = 94 + 4.359 + 3.5 - 7.125$

$f(1/2) = 94.734$

2. Using Gauss backward interpretation formula find the value at y when x=0.5[0,-1,+1,-2,+2]

Solution.

X	**F(x)**	$\delta f(x)$	$\delta^2 f(x)$	$\delta^3 f(x)$	$\delta^4 f(x)$
-2	22	8			
-1	30	52	44	-72	
0	82	24	-28	114	186
1	106	110	86		
	216				

2					

Gauss backward interpolation formula is

$F(x) = f(0)+\frac{x}{1!}\delta f(-1/2)+\frac{x(x+1)}{2!}\delta^2 f(0)+\frac{x(x-1)(x+1)}{3!}\delta^2 f(-1/2)+\frac{x(x-1)(x+1)(x+2)}{4!}$ (186)

$f(0.5) = f(\frac{1}{2}) = 82+(0.5)(52)+\frac{(0.5)(0.5+1)}{2}(28)+\frac{(0.5)(0.5+1)(0.5-1)}{6}(-72)+\frac{(0.5)(0.5+1)(0.5-1)(0.5+2)}{24}$ (186)

$= 82 + 26-10.5+ 4.5-7.2656 = 112.5-17.7656$

$f(0.5) = 94.7344$

a. Using Gauss backward interpolation formula and the value of f(-1/2) from the following data

X	**F(x)**	$\delta f(x)$	$\delta^2 f(x)$	$\delta^3 f(x)$	$\delta^4 f(x)$
-2	22				
		8			
-1	30		14		
		22		18	
0	52		32		6
		54		24	
1	106		56		
		110			
2	216				

Solution. Gauss backward interpolation formula is

$F(x) = f(0)+\frac{x}{1!}\delta f(-1/2)+\frac{x(x+1)}{2!}\delta^2 f(0)+\frac{x(x-1)(x+1)}{3!}\delta^2 f(-1/2)+\frac{x(x-1)(x+1)(x+2)}{4!}$ $\delta^4 f(0)$

$$= 52+\frac{x}{1!}(22)+\frac{x(x+1)}{2}(32)+\frac{x(x^2-1)}{6}(18)+\frac{x(x^2-1)(x+2)}{4!}(16)$$

$$= 52+11+\frac{\frac{1}{2}(\frac{1}{2}+1)}{2}(32)+\frac{\frac{1}{2}(\frac{1}{4}-1)}{6}(18)+\ )+\frac{\frac{1}{2}(\frac{1}{4}-1)(\frac{1}{2}+2)}{24}(16)$$

f(1/2) = 63+ 12-1.125-0.234375 = 75-1.359375 = 73.64

4. Apply Gauss forward formula to obtain f(33) given that

X	25	30	35	40
F(x)	0.2707	0.3027	0.3386	0.3794

Solution .

X	10^4 F(x)	$10^4\delta f(x)$	$10^4\delta^2 f(x)$	10^4 $\delta^3 f(x)$
-1	2707			
		320		
0	3027		39	
		359		10
1	3386		49	
		408		
2	3794			

Gauss forward interpolation formula is

$$f(x) = f(0)+\frac{x}{1!}\delta f(1/2)+\frac{x(x-1)}{2!}\delta^2 f(0)+\frac{x(x-1)(x+1)}{3!}\delta^3 f(1/2)$$

let us take a = 30 as origin h= 5

put a+uh = 32 30+5u = 32 => u = $\frac{2}{5}$

$$10^4 f(2/5) = 3027+\frac{2}{5}(359)+\frac{\frac{2}{5}(\frac{2}{5}-1)}{2}(39)+\frac{\frac{2}{5}(\frac{2}{5}-1)(\frac{2}{5}+1)}{6}(10)$$

$$10^4 f(2/5) = 3027+143.6-4.68-0.045 = 3170.6-4.105 = 3165.36$$

$f(\frac{2}{5}) = 0.3165$

f(x) = 0.3165 when x =32

5. Use Gauss forward formula to find the value of y when y=3.75 from the following table.

X	2.5	3.0	3.5	4.0	4.5	5.0
Y	24.145	22.043	20.225	18.644	17.262	16.047

Solution. Taking 3.2 as the origin and 0.5 as the unit, the value of y required will be value for

$$u = \frac{3.75-3.5}{0.5} = 0.5$$

U	Y_u	δY_u	$\delta^2 Y_u$	$\delta^3 Y_u$	$\delta^4 Y_u$	$\delta^5 Y_u$
-2	24.145					
		-2.102				
-1			0.284			
	22.043			-0.047		
0		-1.818			0.009	
			0.237			-0.003
1	20.225			-0.038		
		-1.581				
			0.99		0.006	
2	18.644			-0.032		
		-1.382				
3			0.167			
	17.262					
		-1.215				
	16.047					

Using the Gauss forward formula we have

F(u) = f(0)+ u$\delta f(0)$+ $\frac{u(u-1)}{2!}$ δf(1) +$\frac{u(u-1)(u+1)}{6}$ δ^2f(-1/2)+ $\frac{u(u-1)(u+1)(u-2)}{24}$ δ^4f(1/2)

Y_5 = 20.225+5(-1.5881)+$\frac{0.5(0.5-1)}{2}$(0.237)+ $\frac{0.5(0.5-1)(0.5+1)}{6}$(-0.38)+ $\frac{0.5(0.5-1)(0.5+1)(0.5-2)}{24}$(0.009)

+ $\frac{0.5(0.5-1)(0.5+1)(0.5-2)(0.5+2)}{120}$(-0.003)

= 20.225-0.7905-0.029625+ 0.00238 + 0.9023750 + 0.0002106

= 19.40 approx

Thus value of y for x = 3.75 is 19.40 approximately.

6. Use Gauss forward formula to find y_{30} given that y_{21} = 18.4708, y_{25} = 17.8144, y_{29} = 17.1070, y_{33} = 16.3432, y_{37} = 15.5154.

X	21	25	29	33	37
Y	18.4708	17.8144	17.1070	16.3432	15.5154

Solution. Arrange the above data in the following form :

Take the origin x = 29 and h = u as the unit; then the value of y for u = $\frac{30-29}{4}$ = 0.25 is to be found. Construct the following difference table.

X	u = $\frac{x-29}{4}$	Y_u	δY_u	$\delta^2 Y_u$	$\delta^3 Y_u$	$\delta^4 Y_u$
21	-2	18.4708	-0.6564			
				-0.0510		
25	-1	17.8144	-0.7074		-0.0054	
				-0.0564		-0.0022
29	0	17.1070	-0.7638		-00076	
				-0.640		
	1					
	2					

3 3 3 7		16.343 2 15.515 4`	- 0.8278			

Now applying the gauss forward interpolation formula.

We get

f(0.4) = 17.1070+ (0.25) (-0.7638)+$\frac{(0.25)(-0.75)}{2}(-0.0564)+\frac{(1.25)(0.25)(-0.75)}{6}(-0.0076)+$

$\frac{(1.25)(0.25)(-0.75)(-1.75)}{24}(-0.0022)$

= 17.1070-0.19095+0.0052875+0.00002968-0.00000375

= 16.9216 approx.

7. Given that $\sqrt{12500}$ = 111.803399; $\sqrt{12510}$ = 111.848111; $\sqrt{12520}$ = 111.892806;

$\sqrt{12530}$ = 111.937483 Show by Gauss's backward formula that $\sqrt{12516}$ = 111.874930

Solution. Taking 12520 as the origin h 10 as the unit, the value of y required is for

u = $\frac{12516-12520}{10}$ = -0.4

U	Y_u	δY_u	$\delta^2 Y_u$	$\delta^3 Y_u$
-2	111.803399			
		0.044712		
-1	111.848111		-0.00017	
				-0.000001
0	111.892806	0.044695	-0.000018	
1	111.937483	0.044677		

Now by using Gauss backward formula, we have

$y_{-0.4}$ = 111.8928+ (-0.4)(0.044695)-+$\frac{(0.6)(-0.4)}{2!}$(-0.000018) +

$\frac{(0.6)(-0.4)(-1.4)}{3!}$(-0.000001)

= 111.8928-0.017878 + negligible quantities = 111.874930

i.e., $\sqrt{12516}$ = 111.874930.

8. Apply Gauss forward formula to find the value of f(x) at x = 3.75 from the following table.

X	**2.5**	**3.0**	**3.5**	**4.0**	**4.5**
F(x)	**24.145**	**22.043**	**20.225**	**18.644**	**17.262**

Solution. Let us take a = 3.5 as origin, h = 0.5

Put a + uh = 3.75 => 3.5+ 0.5u = 3.75

0.5u = 0.25

u = 0.5 = ½

u	**10^3 f(u)**	**10^3 $\delta f(u)$**	**10^3 δ^2 f(u)**	**10^3 δ^3f(u)**	**10^3 δ^4 f(u)**
-2	24145	-2102			
			284		
-1	22043	-1818		-47	
			237		9
0	20225	-1581		-38	
			199		6
1	18644	-1382		-32	
			167		
2	17262	-1215			

$10^3 f(\frac{1}{2}) = 20225+\frac{1}{2}(-1581)+\frac{\frac{1}{2}(\frac{1}{2}-1)}{2}(237)+\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}+1)}{6}(-38)+\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}+1)((\frac{1}{2}-2)}{24}(9)$

$= 20225-790.5-29.625+2.375+0.2709 = 19407.46$

$f(\frac{1}{2}) = 19.408$

f(x) = 19.408 when x= 375.

9**. Find the value of cos 51^0. 42' by Gauss backward formula given that**

X	50^0	51^0	52^0	53^0	54^0
Cosx	0.6428	0.6293	0.6157	0.6018	0.5878

Solution : Let us take a =50^0 as origin h =1^0

Put a+uh = $51^0 42^1$,52^0 +u = $51^0 42^|$ => u= -$18^|$ = $\frac{-18}{60}$ = 0.3, u = -0.3

u	10^4 f(u)	10^4 $\delta f(u)$	10^4 δ^2 f(u)	10^4 δf(u)	10^4 δ^4 f(u)
-2	6428	-135			
-1	6293	-136	-1	-2	
					4
0	6157	-139	-3	2	
1	6018	-140	-1		
2	5878				

Gauss backward interpolation formula is

f(u) = f(0)+$\frac{u}{1!}$ $\boldsymbol{\delta}$ f($\frac{1}{2}$) +$\frac{u(u+1)}{2}$ $\boldsymbol{\delta^2}$ f(0)+ $\frac{u(u+1)(u-1)}{3!}$ δ^3(-$\frac{1}{2}$)+

$\frac{u(u+1)(u-1)(u+2)}{4!}$ δ^4f(0)

10^4f(-0.3) = 6157 + (-0.3)(-136)+$\frac{(-0.3)(0.7)}{2}$(-3) + $\frac{(-0.3)(0.7)(-1.3)}{6}$(-2) +

$\frac{(-0.3)(0.7)(-1.3)(1.7)}{24}$(2)

= 6157 + 40.8 + 0.315 -0,091 + 0.07735 = 6198.1013

f(-0.3) = 0.61981013

cos $57^0 42^{|}$ = 0.6198.

10. Find by Gauss backward formula the sales of the conceived by the year 1936.

Given that

Year	**1901**	**1911**	**1921**	**1931**	**1941**	**1951**
Sales	**12**	**15**	**20**	**27**	**29**	**52**

(in thousand of rupees)

Solution : let us take a = 1931 as origin

H = 10

a + uh = 1936 => 1931 + 10u = 1936 => 10u = 5

=> U=$\frac{1}{2}$[0,-1,1,-2,2,-3]

U	**f(u)**	$\boldsymbol{\delta f(u)}$	$\boldsymbol{\delta^2 f(u)}$	$\boldsymbol{\delta^3 f(u)}$	$\boldsymbol{\delta^4 f(u)}$
-3	12				
		3			
-2	15		2		
		5		0	
-1	20		2		3

		7		3	
0	27		5		-7
				-4	
1	39	12	1		
2	52	13			

Gauss backward interpolation formula is

$$f(u = f(0) + \frac{u}{1!}\delta f(\frac{1}{2}) + \frac{u(u+1)}{2}\delta^{2}f(0) + \frac{u(u+1)(u-1)}{3!}\delta^{3}f(\frac{1}{2}) +$$

$$\frac{u(u+1)(u-1)(u+2)}{4!}\delta^{4}f(0$$

$$f(\frac{1}{2}) = 27 + (0.5)7 + \frac{(0.5)(0.5+1)}{2}(5) + \frac{(-0.5)(0.5+1)(0.5-1)}{6}(3) +$$

$$\frac{(-0.5)(0.5+1)(0.5-1)(0.5-1)}{24}(-7)$$

$$= 27 + 3.5 + 1.875 - 0.1875 + 0.2734 = 32.46 \text{ thousand } 32.7$$

In the year 1936 = 32.7 thousands. Difference table:

X	$U = \frac{x-30}{5}$	Y_u	δy_u	$\delta^2 y_u$	$\delta^3 y_u$	$\delta^4 y_u$
20	-2	49225				
			-909			
	-1	48316		-171		
25			-1080		-59	
	0	47236		-230		-21
			-1310		-80	
30	1	45926		-310		

35 40	2	44306	-1620			

Using Stirlings formula

Putting u = -0.4 we get difference

$y_{-0.4} = 47236+(-0.4)\frac{-1310-1080}{2}+\frac{0.16}{2}(-230)+\frac{(-0.4)(0.16-1)}{6}(\frac{80-59}{2})$

$+\frac{(0.16)(0.16-1)}{24}(-21)$

$= 47326+ 478 -18.4 – 3.8920 + 1176$

$= 47692$

$Y_{28} = 47692$

12. Apply Bessel's formula to obtain y_{25}, given $y_{20} = 2854$, $y_{24} = 3162$, $y_{32} = 3992$

Solution. take 24 as origin and 4 $u = \frac{25-24}{4} = \frac{1}{4} = 0.25$

Difference table is

X	**$u = \frac{x-24}{4}$**	**Y_u**	**δy_u**	**$\delta^2 y_u$**	**$\delta^3 y_u$**
20	-1	2854			
			308		
24	0	3162		74	
			382		-8
28	1	3544		64	
			448		
32	2	3992			

Using bessel's formula, we get

$Y_{1/4} = \frac{1}{2}(3162+3544)+(\frac{1}{4}-\frac{1}{2})382+\frac{\frac{1}{4}(\frac{1}{4}-\frac{1}{2})}{2!}(\frac{14+66}{2})+\frac{(\frac{1}{4}-\frac{1}{2})(\frac{1}{4})(\frac{1}{4}-1)}{3!}(-8)$

$= 3353 – 95.5 – 6.5625 – 0.0625 = 3250.875.$

13.Given y_{20} =24 , y_{24} =32, y_{28} =35, y_{32} =40 find y_{25} by Bessel's formula.

Solution : Taking origin at x =24 and u as the unit.

Find y_u for u = $\frac{25-24}{4} = \frac{1}{4}$ = 0.25

The difference table is

X	U =$\frac{x-24}{4}$	Y_u	δy_u	$\delta^2 y_u$	$\delta^3 y_u$
20	-1	24			
			8		
	0	32		-	
24			3	5	7
	1	35			
			5	2	
28					
	2	40			
32					

By using Bessel's formula we have

$y_{\frac{1}{4}} = \frac{32+35}{2} + (\frac{1}{4} - \frac{1}{2})3 + \frac{\frac{1}{4}(\frac{1}{4}-1)(\frac{-5+2}{2})}{2} + \frac{(\frac{1}{4}-\frac{1}{2})\frac{1}{4}(\frac{1}{4}-1)}{6}(7)$

= 33.5-0.75+ 0.1406+0.54687 = 32.945287.

14.Given that

0	0^0	5^0	10^0	15^0	20^0	25^0	30^0
$\tan\theta$	**0.0000**	**0.0875**	**0.1763**	**0.2679**	**0.3640**	**0.4663**	**0.5774**

Show that $\tan 16^0$ = 0.2867 (use Stirilling's formula)

Solution : Taking 15^0 as origin and 5^0 as the unit , the value required will be for

$$U = \frac{16^0 - 15^0}{15^0} = 0.2^0$$

θ	U	Y_u	δy_u	$\delta^2 y_u$	$\delta^3 y_u$	$\delta^4 y_u$	$\delta^5 y_u$	$\delta^6 y_u$
0^0	-3	0.0000						
5^0	-2	0.0875	0.0872	0.0013	0.0015	0.0002	-0.0002	0.0011
10^0	-1	0.1763	0.0888	0.0028	0.0017	0.0000	0.0009	
15^0	0	0.2679	0.0916	0.0045	0.0017	0.0009		
20^0	1	0.3640	0.0961	0.0062	0.0026			
25^0	2	0.4663	0.1023	0.0088				
30^0	3	0.5774	0.1111					

By using Stirlings formula, we have

Tan 16^0 = $y_{0.2}$ = 0.2679+ $\frac{0.2}{2}$ (0.0961+0.0916){+ $\frac{0.04}{2}$ (0.0045)+ $\frac{0.2(0.4-1)}{2}$ $\frac{1}{2}$(0.0017+0.0017) +

$\frac{0.4(0.4-1)}{24}$ (0.0000) + $\frac{0.2(0.4-1)(0.04-1)}{120}$ $\frac{1}{2}$ (-0.0002+0.0009) + $\frac{0.2(0.04-1)(0.04-4)}{720}$(0.0011)

= 0.2679 + 0.01877 + 0.00009 + negligible

= 0.2867.

1.5 Applications in engineering :

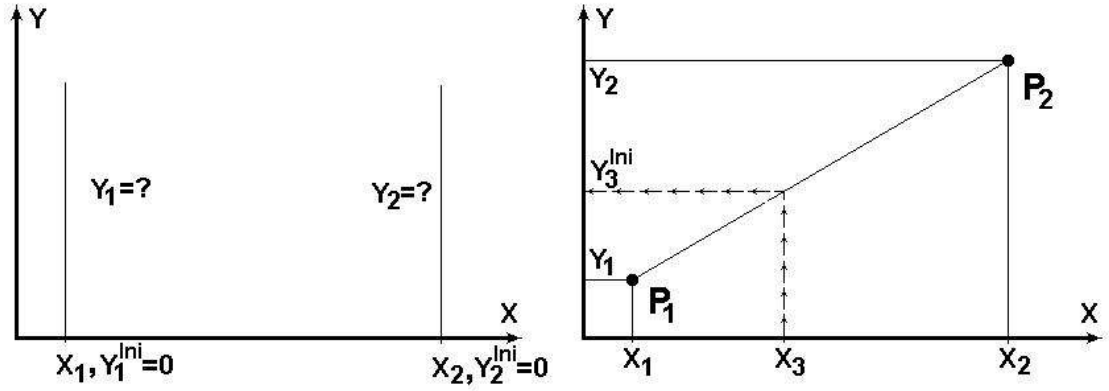
General Ideas Regarding the Use of the Interpolation in the Numerical Analysis

There are certain cases when approximation solutions are useful within the numerical methods field itself.

Some of these cases are presented in the next sections.

Approximation of the Initial Solution for Iterative Methods

Let us consider a theoretical problem where the formulation of the problem depends on some parameters and an iterative solution is possible. It is well known that the final solution depends on the initial approximation in terms of accuracy and of computer runtime.



Calculus of the Integrals :

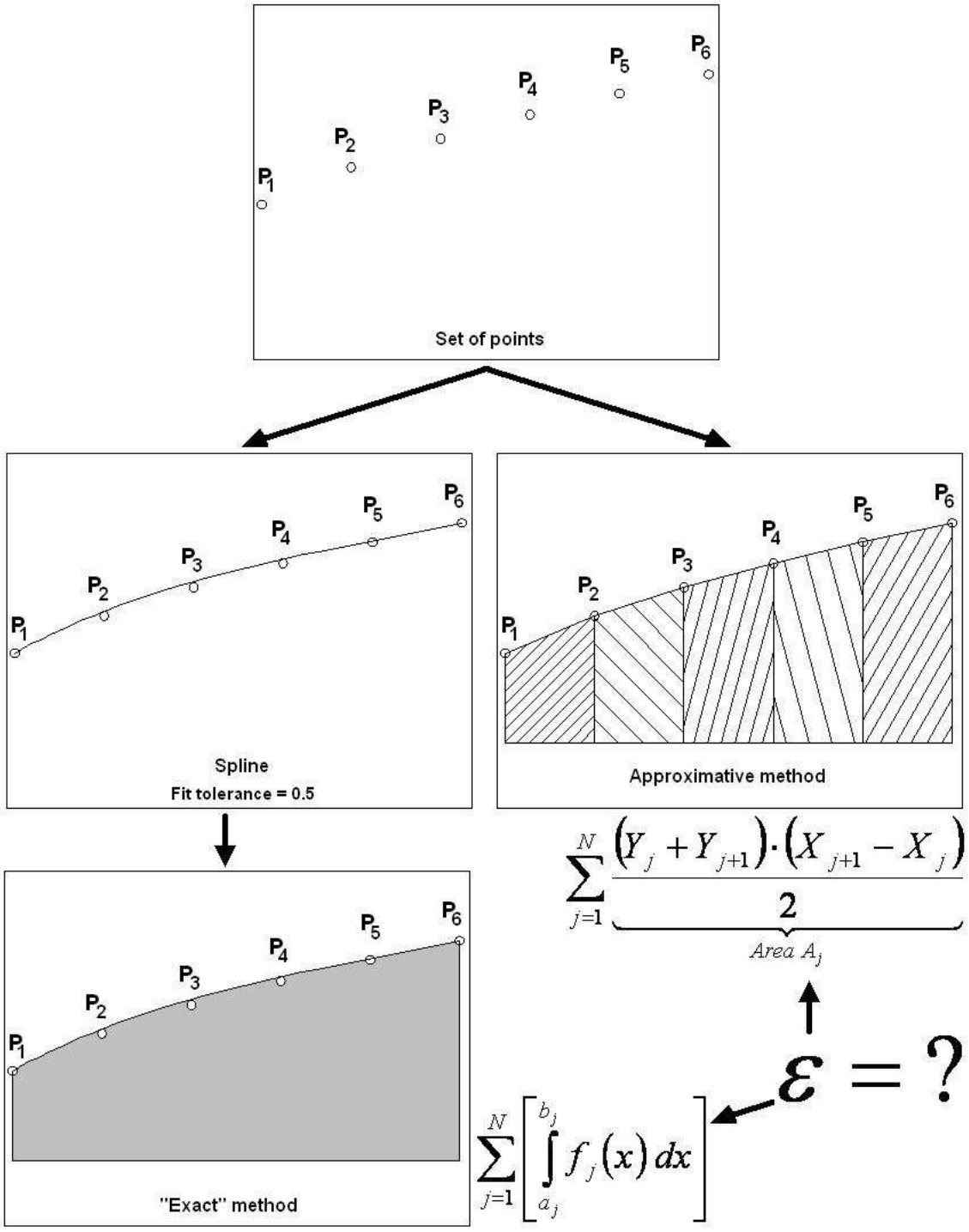
Let us consider that a set of points are known, that is $P_j\, x_j , y_j , j\ 1, N$. Usually these points result from experimental studies. There must be computed the integral of the function approximated by this set of points.

A first method to compute the integral is to use the summation of the areas of the trapezoids defined by two consecutive points. This is a coarse approximation, but if the points are close one to the other the accuracy may be considered satisfactory.

The second method has two stages. The first one is to approximate the set of points using one of the previously mentioned interpolation methods, and it results a set of equations like (6). The second stage is to analytically integrate the functions and to calculate the result. In his case the source of errors is in the interpolation stage.

A similar approach can be used to compute the derivative of the functions, either using the initial points, or using the appropriate interpolation functions.

The error for each of the methods may be evaluated for a certain dataset. Basically, it is necessary to use as many methods exist, in order to have several results which, ideally, must be close one to the other.


P1
P2
P3
P4
P5
P6
Set of points
Spline
Fit tolerance = 0.5
Approximative method
"Exact" method
$\sum_{j=1}^{N} \underbrace{\frac{(Y_j + Y_{j+1}) \cdot (X_{j+1} - X_j)}{2}}_{Area\ A_j}$
$\varepsilon = ?$
$\sum_{j=1}^{N} \left[\int_{a_j}^{b_j} f_j(x)\, dx \right]$

2. NUMERICAL INTEGRATION

2.1 INTRODUCTION :

The numerical integration is the process of evaluating the approximate

Value of the integration $\int_a^b f(x)dx$ from a set of numerical values of f(x). If the problem is to be applied to the integration of function of a single variable, then the process is called as quadrature.

The problem of numerical integration is evolved by first approximating the f(x) by a polynomial with the help of interpolation formula and then integrating this expression between the limits.

∴ To find the value of $\int_a^b f(x)\,dx$, first express the function f(x) by interpolation formula, say p(x) and then integrate between the limits.

$$\therefore \int_a^b f(x)dx = \int_a^b p(x)dx$$

The error is given by $\int_a^b f(x)dx - \int_a^b p(x)dx$

2.2 A GENERAL QUADRATURE FORMULA FOR NUMERICAL INTEGRATION

Consider the integral $\int_a^b f(x)dx$. Let f(x) be given for some equally spaced values of the orguments say ,x_0 , x_0+h ,x_0+2h,…. Let the range (b-a) be divided into n equal parts, each of which is of width h.

$$b-a = nh$$

$$h = \frac{b-a}{n}$$

let $x_0 = a, x_1 = x_0+h = a+h$, $x_2 = a+2h$ ….. $x_n = a+nh = b$

$$I = \int_a^{a+nh} f(x)dx$$

Putting $x = a+uh$

$$dx = hdu$$

x -> a -> u ->0

x ->a+uh -> u->n

I = $\int_a^n f(a+uh)du$ = h $\int_a^n f(a+uh)du$

No write as f(a+uh) on the Newton's interpolation formula as

= h $\int_0^n [f(a) + \frac{u}{1!}\Delta f(a) + \frac{u(u-1)}{2!}\Delta^2 f(a) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(a) + \cdots + \frac{u(u-1)\ldots.(u-n+1)}{n!}\Delta^n f(a)\,]\,du$

= h [f(a).u + $\frac{u^2}{2}\Delta f(a)$+($\frac{u^3}{3} - \frac{u^2}{2}$) $\Delta^2 f(a)$+ $\frac{(\frac{u^4}{4} - \frac{3u^3}{3} + \frac{2u^2}{2})}{6}\Delta^3 f(a) + \cdots +]$

= h[nf(a) + $\frac{n^2}{2}\Delta f(a)$ + ($\frac{n^2}{6} - \frac{n^2}{4}$) $\Delta^2 f(a)$ +($\frac{n^4}{24} - \frac{n^3}{6} + \frac{n^2}{6}$)$\Delta^3 f(a) +]$

$\int_a^b f(x)dx$ = h[ny_0 + $\frac{n^2}{2}$(y_1-y_0)+ ($\frac{n^3}{6} - \frac{n^2}{4}$)($y_2-2y_1$+$y_0$)+$\left(\frac{n^4}{24} - \frac{n^3}{6} + \frac{n^2}{6}\right)$

(y_3-3y_2+3y_1-y_0)+……..] …….(1)

$\int_a^b f(x)dx$ = $\int_a^{a+nh} f(x)dx$

This is known as general quadrature formula we deduce the following important quadratic formulas by taking n= 1,2,3….

2.3 TRAPEZOIDAL RULE :

Put n=1 eq(1) and neglect the second and higher order differences when n=1 , the argument are a, a+h and its corresponding functional values are y_0 and y_1 thus we have

$\int_a^{a+nh} f(x)dx$ = h[y_0 +$\frac{1}{2}$(y_1-y_0)] = $\frac{h}{2}$ (y_0+y_1)

I_1 = $\int_a^{a+nh} f(x)dx$ = $\frac{h}{2}$ (y_0+y_1)

I_2 = $\int_a^{a+nh} f(x)dx$ = $\frac{h}{2}$ (y_0+y_1)

I_3 = $\int_{a+2h}^{a+3h} f(x)dx$ = $\frac{h}{2}$ (y_2+y_3)

……..

…….

$$I_n = \int_{a+(n-1)h}^{a+nh} f(x)dx = \frac{h}{2}(y_{n-1}+y_n)$$

Adding these n integrals, we have

$$\int_a^{a+nh=b} f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$$

This rule is called the trapezoidal rule.

2.4 SIMPSON'S $\frac{1}{3}$ RULE :

Put n=2 in the equation (1) and neglect the third and higher order difference. In this case the arguments are a, a+h, a+2h the corresponding entries are y_0,y_1 and y_2, thus we have

$$\int_a^{a+2h} f(x)dx = h[2y_0 + 2(y_1-y_0)+(\frac{y}{3}-1)(y_2-2y_1+y_0)]$$

$$= \frac{h}{3}[y_0+4y_1+y_2]$$

Similarly $I_2 = \int_a^{a+2h} f(x)dx = \frac{h}{3}(y_0+4y_1+y_2)$

$$I_3 = \int_{a+4h}^{a+6h} f(x)dx = \frac{h}{3}(y_4+4y_5+y_6)$$

……..

……..

$$I_n = \int_{a+(n-2)h}^{a+6h} f(x)dx = \frac{h}{3}(y_{n-2}+4y_{n-1}+y_n)$$

Adding these integrals, we have $I = \int_a^b f(x)dx$

$$= \frac{h}{3}[(y_0+y_n)+(y_1+y_3+y_5+\ldots+y_{n-1})+2(y_2+y_4+\ldots.+y_{n-2})]$$

This is known as SIMPSON'S $\frac{1}{3}$ RULE

2.5 SIMPSON'S $\frac{3}{8}$ RULE :

Put n=3 in equation(1) and neglect fourth and higher order differences. In this case, the arguats are a, a+h, a+2h and a+3h and the corresponding fractional values are y_0,y_1,y_2 and y_3, thus we have

$$\int_a^{a+3h} f(x)dx = n[3y_0+\frac{9}{2}(y_1-y_0)+(\frac{9}{2}-\frac{9}{4})(y_2-2y_1+y_0)+(\frac{27}{8}-\frac{27}{6}-\frac{9}{6})(y_3-3y_2+3y_1-y_0)]$$

$= h[3y_0+\frac{9}{2}(y_1-y_0)+(\frac{9}{2}-\frac{9}{4})(y_2-2y_1+y_0)+(\frac{27}{8}-\frac{27}{6}-\frac{9}{6})(y_3-3y_2+3y_1-y_0)]$

$= h[3y_0+\frac{9}{2}(y_1-y_0)+(\frac{9}{4})(y_2-2y_1+y_0)+(\frac{3}{8})(y_3-3y_2+3y_1-y_0)]$

$I_1 = \int_a^{a+3h} f(x)dx = \frac{3h}{8}(y_0+3y_1+3y_2+y_3)$

$I_2 = \int_{a+4h}^{a+6h} f(x)dx = \frac{3h}{8}(y_3+3y_4+3y_5+y_6)$

……

……

$I_n = \int_{a+(n-2)h}^{a+6h} f(x)dx = \frac{3h}{8}(y_{n-3}+3y_{n-2}+3y_{n-1}+y_n)$

Adding all these integrals we get

$\int_a^{a+nh} f(x)dx = \frac{3h}{8}[(y_0+y_n)+3(y_1+y_2+y_4+\ldots+y_{n-1})+2(y_3+y_6+\ldots.+y_{n-3})]$

This formulas is called the simpson's rule.

Note : Applying this formula, the number of sub-internals should be taken as multiple of three

2.6 WEDDLE'S RULE :

put n=6 in equation (1) and neglect the seventh and higher order differences. in this case the arguments are from a to a+6h and there will be only seven functional values, thus we get

$\int_a^{a+6h} f(x)dx = h[6y_0+18\Delta y_0+27\Delta^2 y_0+24\Delta^3 y_0+\frac{123}{10}\Delta^4 y_0+\frac{33}{10}\Delta^5 y_0+\frac{41}{140}\Delta^6 y_0]$

Replacing $\frac{41}{140}\Delta^6 y_0$ by $\frac{3}{10}\Delta^6 y_0$,

Since the error involved is negligible.

Thus above equation can be expressed as

$\int_a^{a+6h} f(x)dx = h[6y_0+18\Delta y_0+27\Delta^2 y_0+24\Delta^3 y_0+\frac{123}{10}\Delta^4 y_0+\frac{33}{10}\Delta^5 y_0+\frac{3}{10}\Delta^6 y_0]$

$= \frac{3h}{10}(y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6)$

Similarly $\int_{a+6h}^{a+12h} f(x)dx = \frac{3h}{10}(y_6+5y_7+y_8+6y_9+y_{10}+5y_{11}+y_{12})$

……

……

$$\int_{a+(n-6)h}^{a+nh} f(x)dx = \frac{3h}{10}(y_{n-6}+5y_{n-5}+y_{n-4}+ 6y_{n-3}+y_{n-2}+5y_{n-1}+y_n)$$

Adding these integrals, we have

$$\int_a^{a+nh} f(x)dx = \frac{3h}{10}(y_0+5y_1+y_2+ 6y_3+y_4+5y_5+2y_6 + 5y_7+y_8 +\ldots\ldots\ldots\ldots)$$

This formula is known as Weddle's rule.

Note:

1. This rule is more accurate than the Simpson's rule, but it requires atleast seven consecutive values of the function
2. This method is applicable when n is a multiple of 6.

2.7 ERROR IN QUADRATURE FORMULA :

The error terms in the quadrature formula is given by

$$E = \int_a^b y dx - \int_a^b p(x)dx$$

where P(x) is the polynomial representing the function y = f(x) in the interval [a, b].

(i) Error in Trapezoidal Rule : Expanding y = f(x), about $x = x_o$ by Taylor's series, we obtain

$$y = y_0 +(x-x_0)y_0 + \frac{(x-x0)^2}{2!} y_0^{11} +\ldots\ldots\ldots\ldots(1)$$

$$\int_{x_0}^{x_0+h} y\,dx = \int_{x_0}^{x_0+h}[y_0 + (x - x_0)y_0 + \frac{(x-x_0)^2}{2!}y_0 + \cdots]dx$$

$$= y_0 n + \frac{h^2}{2!}y_0+\frac{h^2}{3!}y_0+\ldots\ldots\ldots\ldots\ldots(2)$$

But area of the first trapezium in the interval $[x_o, x_1] = \frac{1}{2}h(y_0 + y_1)$ ....(3)

Then putting $x = x_o + h$ and $y = y_1$ in (1), we obtain

$$y_1 = y_0 + h\, y_0 + \frac{h^2}{2!}y_0 +\ldots\ldots\ldots\ldots$$

Putting this value of y, in (3), we have

$$B = \frac{1}{2}h[y_0 + y_0 +h\, y_0+ \frac{h^2}{2!}y_0+\ldots\ldots\ldots] = h\, y_0 + \frac{h^2}{2!}y_0 + \frac{h^3}{2.2!}y_0+\ldots\ldots \ldots\ldots(4)$$

Hence the error in the interval $[x_o, x_1]$

$$= \int_{x_0}^{x_1} y\,dx - B_1 = [\frac{1}{3!} - \frac{1}{2.2!}]\, h^3y_0+\ldots\ldots\ldots\ldots = -\frac{h^3}{12}y_0+\ldots\ldots\ldots$$

Principal part of the error in $[x_o,x_1]$ = $-\frac{h^3}{12} y_0$

And similarly principal part of the error in $[x_1, x_2]$ = $-\frac{h^3}{12} y_1$ etc.,

Thus, the total error = $-\frac{h^3}{12} [y_0 + y_1 + y_{n-1}]$ = E , say

Then assuming that y''(x) is the largest of the n quantities y_o , y_1 ,.. . ., y_{n-1} , we can say that

$$E < -\frac{nh^3}{12} y^{11}(x) = -\frac{(b-a)h^2}{12} y^{11}(x) \text{ as } b-a = nh \quad \ldots\ldots\ldots\ldots(5)$$

Thus the error in the trapezoidal rule is of the order h^2.

(ii) Error in Simpson's $\frac{1}{3}$ Rule : Expanding y = f(x) about $x = x_o$ by Taylor's series, we have

$$y = y_0 + (x-x_0)y_0 + \frac{(x-x0)^2}{2!} y_0 + \ldots\ldots\ldots$$

$$\int_{x_0}^{x^2} y\, dx = \int_{x_0}^{x_0+2h} [y_0 - (x - x_0) + \frac{(x-x_0)^2}{2!} y_{0+\ldots}]\, dr$$

$$2hy_0 + \frac{4h^2}{2!}y_0 + \frac{8h^3}{3!}y_0 + \frac{16h^4}{4!}y_0 + \ldots + \frac{32h^5}{5!}y_0 + \ldots\ldots\ldots \quad \ldots\ldots\ldots(2)$$

and area over the first double strip by Simpson's 1/3 rule

$$= \frac{1}{3} h(y_0 + 4y_1 + y_2) = A_1 \text{ say}$$

Then putting $x = x_0 + h$ and $y = y_1$ in (1), we get

$$y_2 = y_0 + 2hy_0 + \frac{4h^2}{2!}y_0 + \frac{8h^3}{3!}y_0 + \ldots\ldots\ldots$$

Further putting $x = x_o + 2h$ and y = 1/2 in (1), we get

$$y_2 = y_0 + 2hy_0 + \frac{4h^2}{2!}y_0 + \frac{8h^3}{3!}y_0 + \ldots\ldots\ldots$$

$$(3) \rightarrow A_1 = \frac{h}{3} [y_0 + 4(y_0 + hy_0 + \frac{h^2}{2!} y_{0+\ldots\ldots\ldots}) + (y_0 + 2h\, y_0 + \frac{4h^2}{2!} y_0 + \frac{8h^3}{3!} y_0 + \ldots\ldots\ldots)]$$

$$= 2hy_0 + 2h^2y_0 + \frac{4h^3}{3} y_0 + \frac{2h^2}{8} y_0 + \frac{5h^5}{18} y_0 + \ldots\ldots\ldots \quad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Hence the error in the interval $[x_o, x_2] = \int_{x_0}^{x} y\, dx - A_1$

$$= [\frac{4}{15} - \frac{5}{18}] h^5 y_0^5 + \ldots\ldots \quad \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

and the principal part of the error in $[x_o, x_2]$

$= [\frac{4}{15} - \frac{5}{18}] h^5 y_0^5 = - -\frac{h^5}{90} y_0^5$

On the same lines principal part of the error in $[x_2, x_4]$

$= \frac{h^5}{90} y_2^5$, etc.,

Total error = $- -\frac{x^5}{90} [y_0^4 + y_2^4 + \ldots\ldots\ldots + y_2^4 (n-1)] = E$, Say ……(6)

The assuming $y_{1v}(x)$ as the largest of $y_0^{1V}, y_2^{1V} \ldots\ldots, y_{2n-2}^{1V}$

We readily obtain $E < -\frac{n/5}{90} y_0^{1V}(x) = - \frac{(b-a)h^y y^5(x)}{180}$ ……(7) [2nh = b-a]

(iii) Error in Simpson's 3/8 rule : Proceeding as above, we have the principal part of the error in the interval $[x_o, x_3]$ as $= -\frac{3h^5}{80} y^5$

(iv) Error in Weddle's rule: Principal part of the error in the interval$[x_0, x_6] = -(h^7/140) y_0^5$

2.8 SOLVED PROBLEMS ::

1.Evalaute the value of the integral using $\int_2^{1.4} (sinx - logx + e^x)$dx

(a) The Trapoidal rule, (b) Simpson's one-third rule,

(c) Simpson's three eight rule.

Solution. In this case divide the range of integration into twelve equal parts by taking h = -1 and then the values of the function y = sin x -$\log_e x$ +e are computed for each point of subdivision which are as under

x	Sinx	$\log_e x = \log_{10} x \times 2.30258$	e^x	$Y = \sin x - \log_e x + e^x$
	0.19867	-1.60943	1.22140	3.02950
0.2	0.29552	-1.20397	1.34986	2.84935
		-0.91629	1.49182	2.79753
0.3	0.38942	-0.69315	1.64872	2.82130
	0.47943	-0.51083	1.82212	2.89759
0.4	0.5646	-0.35667	2.01375	3.01464
		-0.22314	2.22554	3.16604
	0.64422	-0.10537	2.45960	3.34830
0.5	0.71736	+0.00000	2.71828	3.55975
	0.78333	0.09530	3.00417	3.80008
	0.84147	0.18232	3.32012	4.06984

0.6	0.89121 0.93204	0.26236 0.33647	3.66930 4.05520	4.37050 4.70418
0.7	0.96356 0.98545			
0.8				
0.9				
1.0				
1.1				
1.2				
1.3				
1.4				

(a)By Trapezoidal rule, we have

$\int_2^{1.4}$(sin x -$\log_e x$ + e^x)dx = $\int_2^{2+12x\,0.1} y\,dx$

Here n = 12; h = 0.1, x_o = 0.2, y = sin x -$\log_e r$+ e^x

Again w.k.t. $\int_{x0}^{xo+nh} y\,dx = \frac{h}{2}[y_0+y_{n+2}(y_1+y_2+\ldots+y_{n-1})]$

Substituting these values of, h_1, y_o, y_n…, in the above formula we get

$\int_2^{1.4}(\sin x - \log ex + e^x)\,dx = \frac{1}{20}$[3.02950+ 4.70418+2(2.8493+…+4.37050)]

$= \frac{1}{20}$[7.73368+2(36.69492)]

= 4.05617

(b) By Simpson's rule $\frac{1}{3}$ we have

$\int_{x0}^{xo+nh} y\,dx = \frac{h}{3}[y_0+y_6+4(y_1+y_3+\ldots)+2(y_2+y_4+\ldots)]$

Put the values of h, y_o, $y_{1..}$.. from the table, we have

$= \int_2^{1.4} (\sin x - \log ex + e^x)\, dx$

$= \frac{1}{3}[7.73368 + 4(20.20417) - 3(16.49075)]$

$= \frac{1}{3}[121.53186] = 4.05106$

(c) By Simpson's 3/8 rule we have

$\int_{x0}^{xo+nh} y\, dx = \frac{3h}{8}[(y_0 + y_n) + 3(y_1+y_2+y_4+\ldots+y_{n-1}) + 2(y_3+y_6+\ldots.+y_{n-3})]$

$= \frac{3(0.1)}{8}[7.73368 + (26.90750) + 2(9.78742)] = 4.05116$

2. A curve is drawn to pass through the points given by the following table.

x	1	1.5	2	2.5	3	3.5	4
y	2	2.4	2.7	2.8	3	2.6	2.1

Estimate the area bounded by the curve, the x-axis and the lines x=1,x=4.

Solution. By Simpson's rule, we have required area

$= \frac{h}{3}[y_0 + 4y_1 + 2y_2 + 4y_3 + \ldots\ldots + 4y_{n-1} + y_n]$

$= \frac{0.5}{3}[2 + 9.6 + 5.4 + 11.2 + 10.4 + 2.1]$

$= \frac{50}{30} \times 46.7 = \frac{23.35}{3} = 7.783$ unit of area

By Weddle's rule, we have required area

$= \frac{3h}{10}[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6]$

$= \frac{3x0.5}{10}[2 + 12.0 + 2.7 + 16.8 + 3.13.0 + 2.1] = 7.74$ units of area

3. Calculate by Simpson's rule an approximate value of by taking seven equidistant ordinates. Compare it with the exact value and the value obtained by using the Trapezoidal rule.

Solution. Let us divide the range of integration (-3, 3) into six equal parts each of width $\frac{3(-3)}{6} = 1$ unit. Hence h-1, value of the function for each point of sub division are as given below.

x	$y = x^4$

-3	$(-3)^4 = 81$
-2	$(-2)^4 = 16$
-1	$(-1)^4 = 1$
0	$(0)^4 = 0$
1	$(1)^4 = 1$
2	$(2)^4 = 16$
3	$(3)^4 = 81$

Now by Simpson's rule, we obtain

$$\int_{-3}^{3} x^4 \, dx = \frac{h}{3}[y_0+y_6+4(y_1+y_3+y_5)+2(y_2+y_4)]$$

$$= \frac{1}{3}[162+4x32+ 2x2] = \frac{1}{3}x294 = 98$$

But the exact value of

$$\int_{-3}^{3} x^4 \, dx = [\frac{x^5}{5}]_{-3}^{3} = \frac{1}{5}[(3)^5 - (-3)^5] = \frac{1}{5}x486 = 97.2$$

Also by Trapezoidal rule, we have

$$\int_{-3}^{3} x^4 \, dx = \frac{h}{2}[y_0+y_6+2(y_1+ y_2+y_3+ y_4+y_5)] = \frac{1}{2}[162+2x34]$$

Thus it is obvious that the Trapezoidal rule does not give an accurate result.

4.Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rule. Hence obtain the approximate value of π in each case.

Solution. Divide the range into six equal parts each of with h = $\frac{1}{6}$ and compute

The value of y = $\frac{1}{1+x^2}$ at each point of sub division.

These values are given below.

X	$Y_x = 1/(1+x^2)$
0	1.0000
1/6	36/37 = 0.97297
2/6	36/40 = 0.90000
3/6	36/45 = -0.80000
	36/52 = 0.69231

4/6 5/6 1	36/61 = 0.59016 1 / 2 = 0.5000

By Simpson's rule, we have

$\int_{x_0}^{x_0+6h} y\,dx = \frac{h}{3}[y_0+y_6+4(y_1+y_3+y_5)+2(y_2+y_4)]$

$= \frac{1}{18}$ [1.00 + 0.500 + 4(0.97292 + 0.50000 + 0.59016) +

2(0.900+ 0.69231)] ……(1)

= 0.785395

By Simpson's 3/8 rule we have

$\int_{x_0}^{x_0+6h} y\,dx = \frac{3h}{8}[y_0+y_6+3(y_1+y_2+ y_4+y_5)+2(y_3)]$

$= \frac{1}{16}$ [1.5000 + 3 (0.97297 + 0.900 + 0.69231+0.59016 +

2 x (0.800))] = 0.785395 ……(2)

Again, $\int_0^1 \frac{dx}{1+x^2} = [\tan^{-1} x]^1_0 = \pi/4$ …….(3)

From (1) and (2), we get π = 3.14188 and from (2) and (3) we get

π = 3.141580

5. Evaluate $\int_2^{10} \frac{dx}{x}$ by dividing the range into '8' equal parts by using Trapezoidal rule.

Solution. Given integral, $\int_2^{10} \frac{dx}{x}$

Let f(x) = $\frac{1}{4}$

a = 2, b = 10;n = 8

h = $\frac{b-a}{n} = \frac{10-2}{8} = 1$

x	y = f(x) = $\frac{1}{x}$

2	1/2 = 0.5000
3	1/3 = 0.3333
4	1/4 = 0.2500
5	1/5 = 0.2000
6	1/6 = 0.1666
7	1/7 = 0.1428
8	1/8 = 0.1250
9	1/9 = 0.1000
10	1/10 = 0.1000

Trapezoidal rule is $\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

$\int_2^{10} \frac{dx}{x} = \frac{h}{2}[(y_0+y_8)+2(y_1+y_2+y_3+ y_{4+} y_{5+} y_6+y_7)]$

$= \frac{1}{2}[(0.5+ 0.1)+ 2(0.333+ 0.25)+ 0.2+ 0.125+ 0.166 + 0.1428 +0.1111]$

$= \frac{1}{2}[(0.6)+2(1.3288)]$

$= 0.3+1.3288$

$= 1.62880$

6. Evaluate $\int_2^{10} \frac{dx}{1+x}$ by dividing the range into '8' equal parts by using Trapezoidal rule.

Solution. Let $f(x) = \frac{1}{1+x}$

a = 2, b = 10, n = number of equal parts = 8

$h = \frac{b-a}{n} = \frac{10-2}{8} = 1$

h = 1

x	$f(x) = \frac{1}{1+x}$

2	1/3 = 0.333
3	1/4 = 0.2500
4	1/5 = 0.2000
5	1/6 = 0.1666
6	1/7 = 0.1428
7	1/8 = 0.1250
8	1/9 = 0.1111
9	1/10 = 0.1000
10	1/11 = 0.0909

Trapezoidal rule is $\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

$\int_2^{10} \frac{dx}{1+x} = \frac{h}{2}[(y_0+y_8)+2(y_1+y_2+y_3+ y_{4+} y_{5+} y_6+y_7)]$

$= \frac{1}{2}[(0.333+ 0.090)+ 2(0.25+ 0.2+ 0.1661 + 0.1425+ 0.125+ 0.111+ 0.1)]$

$= \frac{1}{2}[0.4242+2(1.0955)] = 0.2121 + 1.0955 = 1.3076$

8. Evaluate $\int_4^{5.2} f(x)dx$ by using Trapezoidal rule from the following.

X	**4**	**4.2**	**4.4**	**4.6**	**4.8**	**5.0**	**5.2**
Y	**1.4**	**1.4**	**1.5**	**1.5**	**1.6**	**1.6**	**1.6**

Where h = 0.2

Solution. Trapezoidal rule is, $\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

$\int_4^{5.2} f(x)dx = \frac{0.2}{2}[(1.4+ 1.6)+ 2(1.4+1.5+ 1.5+1.6+ 1.6)]$

$= 0.1[3+2(7.6)]$

$= 0.1[18.2] = 1.82$

9. Evaluate $\log_e 7$ from the integral $\int_1^7 \frac{dx}{x}$

Solution. Given f(x) = $\frac{1}{x}$ a = 1, b = 7, n = 6, h = $\frac{b-a}{n}$ = $\frac{7-1}{6}$ = $\frac{6}{6}$ = 1

x	y= f(x)
1	1.0000
2	0.5000
3	0.333
4	0.2500
5	0.2000
6	0.1666
7	0.1428

Trapezoidal rule is, $\int_a^b f(x)dx$ = $\frac{h}{2}$ [$(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})$]

= $\frac{h}{2}$[(1+ 0.1428)+ 20.5+ 0.333+0.25+ 0.2+ 0.1666)]

= $\frac{1}{2}$ [1.1428+ 2(1.4499)]

= $\frac{1}{2}$ [4.0426] and $\int_1^7 \frac{dx}{x}$ = $[\log_e x]_1^7$

= $\log_e 7$ = $\log_e 1$ = $\log_e 7$

$\log_e 7$ = 2.0113

10. A curve is drawn to passes through the point given by the following table.

X	1	1.5	2	2.5	3	3.5	4
Y	2	2.4	2.7	2.8	3	2.6	2

Solution. Estimate the area bounded by the curve, the axis-x and the lines x=1,x=4by using Trapezoidal rule.

Trapezoidal rule is h= $\frac{4-1}{6}$=0.5

$\int_a^b f(x)dx$ = $\frac{h}{2}$[$(y_0+y_n)+2(y_1+y_2+y_3+y_4+y_5)$]

$= \frac{0.5}{2}[2+2.1+2+ (2.4 +2.7+2.8+2.6+3)]$

$\frac{0.5}{2}(4.1+27.0) = \frac{15.35}{2}$ = 7.775 units of area

11. calculate e $\int_2^{10} \frac{dx}{1+x}$ by dividing the arrange into 8 equal parts by using Simpson's 1/3 rule.

Solution.: Simpson's 1/3 rule is

$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_5+.....)+2(y_2+ y_4+.....)]$

$\int_2^{10} \frac{dx}{1+x} = \frac{h}{3}[(O.333+ 0.0909)+ 4(0.25+ 0.1666+ 0.125+0.1)$

$+ 200.2+ 0.142+ 0.111)]$

$= \frac{1}{3}[0.4242+ 4(0.6416)+ 2(0.4539)) = \frac{1}{3}[3.8984] = 1.2994666$

12. calculate $\int_a^b f(x)dx$ by using Simpson's $\frac{1}{3}$ rule

X	4	4.2	4.4	4.6	4.8	5.0	5.2
Y	1.4	1.4	1.5	1.5	1.6	1.6	1.6

where h = 0.2

Solution: Simpson's $\frac{1}{3}$ rule is

$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_5+.....)+2(y_2+ y_4+.....)]$

$\int_4^{5.2} f(x)dx = \frac{0.2}{3}[(1.4+1.6)+ 4(1.4+1.5+ 1.6)+ 2(1.5+ 1.6)]$

$= \frac{0.2}{3}[3+4(4.5)+2(3.1)] = \frac{0.2}{3}[27.2]$

= 1.813333…..

13. Evaluate $\int_2^{10} \frac{dx}{1+x}$ by dividing the range iato 8 equal parts by using Simpson's $\frac{1}{3}$ rule.

Solution:. Let f(x)= $\frac{1}{x}$

a = 2, b = 10, n =number of equal parts = 8

h = $\frac{b-a}{n} = \frac{8}{8}$ = 1

X	Y = f(x) = $\frac{1}{x}$
2	0.5000
3	0.3333
4	0.2500
5	0.2000
6	0.1666
7	0.1428
8	0.1250
9	0.1111
10	0.1000

Simpson's $\frac{1}{3}$ rule is

$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_5+.....+y_{n-1})+2(y_2+ y_4+y_6.....+y_{n-2})]$

$= \frac{1}{3}$ [I0.6)+ 4(0.3333+ 0.2+ 0.1428+ 0.1111)] +2(0.25+0.1666+0.125)]

$=\frac{1}{3}$[0.6+4(0.7872+ 1.0832)]

$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_6+.....)+2(y_2+ y_4+.....)]$

$\int_0^1 \frac{dx}{1+x^2} = \frac{1}{6*3}$ [(1 +0.5)+ 4(0. 9729+0.8+0.3+ 2(09+0.6923)]

$= \frac{1}{18}[1.5+9.452+3.1846] = \frac{1}{18}[14.1366]$

$= 0.7853666$

On the other hand $\int_0^1 \frac{1}{1+x^2} dx = (\tan^{-1})^1_0$

$= \tan^{-1}(1)-\tan^{-1}(0)$

$= \frac{\pi}{4}-0 = \frac{\pi}{4}$

$= \frac{\pi}{4} = 0.7853$

$= \pi = 3142$

14. Evaluate $\int_0^4 e^x\, dx$ by using Simpson's rule using the data e = 2°.72, e2= 739, e3=20.09, e4=54.60 and compare it will the actual value.

Solution.: Divide the whole range (0,4) into four equal parts by taking h=1. By Simpson's rule 1/3 we have

$\int_0^4 e^x\, dx = \int_{x_0}^{x_0+4h} y\, dx = \frac{h}{3}(y_0+y_4+4(y_1+y_3)+242)$

$= \frac{1}{3}[1+54.60+4(2.72+20.09)+2(7.39)]$

$= \frac{1}{3}[55.60+91.24+14.78] = \frac{1}{3}[161.62]$

$= 553.373$

The actual value of the integral is

$\int_0^4 e^x\, dx = e^4 - e^0 = 54.60 – 1.0 = 53.60$

15. Evaluate $\int_4^{5.2} \log, x\ dx$ by (a) Simpson's $\frac{1}{3}$ (b) $\frac{3}{8}$ rule . After finding the true value of the integral.

Solution. Divide the whole range of integration ie., (4, 5, 2) into six equal parts taking h 0.2 and then compute the values of the function f(x)= $\log^x_e$ for each of sub division. These computed values are as follows.

X	Y = $\log^x_e$ x	X	Y = $\log^x_e$ x
Xo = 4.0	1.38629436	Xo+ 4h= 4.8	1.56861592
Xo+ h =4.2	1.43508453		
	1.48160454	Xo+5h= 5.0	1.60943791
X_0 + 2h= 4.4	1.52605630		
		Xo+ 6h= 5.2	1.64865863
Xo +3h= 4.6			

(a) By Simpson's $\frac{1}{3}$ rule, we have

$\int_4^{5.2} \log, x \ dx = \frac{h}{3}$ [y_0+y_6+(y_1+y_3++y_5)+2(y_2+y_4)]

$= \frac{2}{8}$ [3.03495299+ 4(4.57056874)+ 2(3.05022046)]

=1.82784726.

(b) By Simpson's 3/8 rule, we have

$\int_4^{5.2} \log e, x \ dx = \frac{3h}{8}$ [y_0+y_2+3(y_1+y_2++y_4+ y_5)+2 y_3]

$= \frac{3(0.2)}{8}$[03495299+ 3(6.09424290)+ 2 (1.52605630)]

$= \frac{0.3}{4}$[24.371294291]

= = 1.82784707

Again exact value of the integral is

$\int_4^{5.2} \log_e x \, dx = x(\log_e x - 1) \int_{4.0}^{5.4} 1.82784741$

16. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ rule.

Solution : let f(x) = $\frac{1}{1+x^2}$ let n =6 , a= -0 , b = 1 ,h = $\frac{b-a}{n} = \frac{1-0}{6} = \frac{1-0}{6} = \frac{1}{6} = 0.1666$

X	f(x) = $\frac{1}{1+x^2}$
0	1
1/6	0.9729
2/6	0.9000
3/6	0.8000
4/6	0.6923
5/6	0.5901
$\frac{6}{6}$=1	0.5000

Simpson's $\frac{1}{3}$ rule is

a = 0, b = 6, n= 6

h = $\frac{b-a}{n} = \frac{6-0}{6} = 1$

y_0 = f(0) -= $\frac{1}{1+0^2} = \frac{1}{1} = 1$

y_1 = f(1) = $\frac{1}{1+1^2} = \frac{1}{2} = 0.5000$

$y_2 = f(2) = \frac{1}{1+2^2} = \frac{1}{5} = 0.2000$

$y_3 = f(3) = \frac{1}{1+3^2} = \frac{1}{10} = 0.1000$

$y_4 = f(4) = \frac{1}{1+4^2} = \frac{1}{17} = 0.0588$

$y_5 = f(5) = \frac{1}{1+5^2} = \frac{1}{26} = 0.0385$

$y_6 = f(6) = \frac{1}{1+6^2} = \frac{1}{37} = 0.0270$

Simpson's $\frac{1}{3}$ rule is

$$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_5+.....)+2(y_2+ y_4+.....)]$$

$$= \frac{1}{3}[(1 + 0.027)+ 4(0.5+ 0.1 + 0.0385)+ 2(0.2+ 0.0588)]$$

$$= \frac{1}{3} [3.7104] = 1.2368$$

17. Find the value of $\int_0^1 \frac{x^2}{1+x^2} dx$ by using Simpson's $\frac{1}{3}$ rule and also find the error

Solution. : Dividing the whole range (o, 1) into 4 equal parts nd making use of Simpson's $\frac{1}{3}$ rule we have

$$\int_0^1 \frac{x^2}{1+x^2} dx = \frac{h}{3}[(y_0+y_4)+4(y_1+ y_3+)+2y_2)]$$

$h = \frac{1}{4}$, a =0 , b= 1 ,n = $\frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4}$

X	F(x)
0	0
0.25	0.0615
0.5	0.2222

0.75	0.3956
1.0	- 0.5000

$\int_0^1 \frac{x^2}{1+x^2}$ dx = $\frac{1}{12}$ [(0+ 0.5)+ 4C0.0615+ 0.3956)+ 2(0.222)]

= $\frac{1}{12}$ [0.5+ 1.8284+ 0.444]

=$\frac{2.7728}{12}$ = 0.2310

18. Find $\int_4^{5.2} \log x_e$ dx by using Simpson's $\frac{1}{3}$ rule.

solution.: Given integral is $\int_4^{5.2} \log x_e$ dx

A = 4, b = 52 let n= 6 = $\frac{b-a}{n}$ = $\frac{5.2-2}{6}$ = 0.2

X	F(x)
4	1.3862
4.2	1.4350
4.4	1.4816
4.6	1.5260
4.8	1.5686
5.0	1.6094

5.2	1.6486

$\int_4^{5.2} \log x_e$ = $\frac{h}{3}$[(y_0+y_6)+4(y_1+ y_3+ y_5)+2(y_2+ y_4)]]

= $\frac{0.2}{3}$[(1.3862+1.6481+4(1.435071 +1.5260+1.6094)

+2(1.4816+1.5686)]

= $\frac{0.2}{3}$ [(3.0348)+ 4 (4.5704)+ 2(3.05027)]

= $\frac{0.2}{3}$[3.0348+18.2816+ 6.1004] = $\frac{5.48336}{3}$ = 1.8277866.

19. Evaluate $\int_{-3}^{3} x^4 dx$ by using Simpson's $\frac{1}{3}$ rule.

Solution. We know the integral is $\int_{-3}^{3} x^4 dx$

Where h = $\frac{b-a}{h}$ = $\frac{3-(-3)}{6}$ = $\frac{6}{6}$ = 1

x	F(x)
-3	81
-2	16
-1	1
0	0
1	1
2	16
3	81

Simpson's $\frac{1}{3}$ rule is

$\int_b^a f(x)dx$ = $\frac{h}{3}$ [(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)]

= $\frac{1}{3}$[(81+81)+4(16+0+16)+2(1+1)]

a = 3.b = 3 , n = 6

$$h = \frac{b-a}{h} = \frac{3-(-3)}{6} = 1$$

let $f(x) = x^2$

$y_0 = f(-3) = (-3)^2 = 9$

$y_1 = f(-2) = (-2)^2 = 4$

$y_2 = f(-1) = (-1)^2 = 1$

$y_3 = f(0) = (0)^2 = 0$

$y_4 = f(1) = (1)^2 = 1$

$y_5 = f(2) = (2)^2 = 4$

$y_6 = f(3) = (3)^2 = 9$

Simpson's $\frac{1}{3}$ rule

$$\int_a^b f(x) = \frac{h}{3}[(y_0+y_n)+4(y_1+y_{3+\ldots})+2(y_2+y_{4+\ldots})]$$

$$= \frac{1}{3}[(9+9)+4(4+0+4)+2(1+1)]$$

$$= \frac{54}{3} = 18 = \frac{1}{3}[162+128+4] = \frac{1}{3}[294] = 98$$

20 .calculate an approximate value of $\int_a^{\pi/2} sinx\ dx$ by (a) Trapezoidal rule (b) Simpson's rule using 11 ordinates

Solution : Divide the range of integration into a ten equal parts taking h = $\pi/20$ and then compute the values of the function f(x) = sin x for each point of sub-division. The computed values are as follows :

X	Y=sin x

0	0.00000
$\pi/20$	0.15663
$2\pi/20$	0.30902
$3\,\pi/20$	0.45399
$4\pi/20$	0.58778
$5\pi/20$	0.707111
$6\,\pi/20$	0.80900
$7\,\pi/20$	0.89101
$8\,\pi/20$	0.95106
$9\pi/20$	0.98769
$10\,\pi/20$	1.00000

(a) By the trapezoidal rule, we have

$$\int_0^{\pi/2} sinx \text{ dx} = \frac{h}{3}[(y_0+y_{10})+2(y_1+y_{2+....})]$$

$$= \frac{\pi/20}{2}[1.0000+2(5.85311)]$$

$$= \frac{\pi}{40}(12.70622) = 0.9981$$

(b) By Simpson's $\frac{1}{3}$ rule , we have

$$\int_0^{\pi/2} sinx \text{ dx} = \frac{h}{3}[(y_0+y_n)+4(y_1+y_{3+....})+2(y_2+y_{4+...})]$$

$$= \frac{\pi}{60}[1.00000+4(3.19623)+2(2.65688)]$$

$$= \frac{\pi}{60}[1.00000+12.78492+5.31376]$$

$$= \frac{\pi}{60}[19.09868] = 1.0006$$

Again the exact value of the integral

$$\int_0^{\pi/2} sinx \text{ dx} = [-\cos x]_0^{\pi/2} = 1.0000$$

The error (i) Due to Simpson's $\frac{1}{3}$ rule , we have = -0.0006

(ii) Due to trapezoidal rule = 0.0019

21.Evaluate $\int_a^b f(x)dx$,by Simpson's $\frac{3}{8}$ rule from the following

X	4	4.2	4.4	4.6	4.8	5.0	5.2
Y	1.4	1.4	1.5	1.5	1.6	1.6	1.6

Where

Solution : Simpson's $\frac{3}{8}$ rule is

$\int_a^b f(x)dx$ = $\frac{3h}{8}$[y_0+3y_1+2y_2+3y_2+2y_3+3y_4+3y_5+y_6]

= $\frac{3(0.2)}{8}$[1.4+3(1.4)+3(1.5)+2(1.5)+3(1.6)+3(1.6)+1.6]

= $\frac{0.6}{8}$[1.4+4.2+4.5+3.0+4.8+4.8+1.6]

= $\frac{0.6}{8}$(24.3) = 1.8225

22. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's $\frac{3}{8}$ rule.

Solution : By the above problem we have y_0 =1 ,y_1 = 0.5000,y_2 = 0.1000,

y_4 = 0.0588 , y_5 = 0.0385, y_6 = 0.0270 and h = 1

Simpson's $3/8\ rule\ is$

$\int_a^b f(x)dx$ = $\frac{3h}{8}$[(y_0+ y_6)+3y_1+2y_3+3y_4+3y_5]

$\int_0^6 \frac{dx}{1+x^2}$ = $\frac{3}{8}$[(1+0.027)+3(0.5+0.2+0.0588+0.0385+2(0.1))]

= $\frac{3}{8}$[1.027+2.3919+0.2] = 13.09437/8 = 1.6770

23.Evaluate $\int_{-3}^{3} x^2 dx$, using by Simpson's $\frac{3}{8}$ rule we know by the above problem

Solution. y_0=9 ,y_1= 4,y_2= 1 , y_3 = 0 ,y_4 = 1 , y_5 = 4 ,y_6 = 9

Simpson's $3/8\ rule\ is$

$\int_a^b f(x)dx$ = $\frac{3h}{8}$[(y_0+ y_6)+3y_1+2y_3+3y_4+3y_5]

= $\frac{3h}{8}$[(9+9)+3(4+1+4+1)+0]

$= \frac{3}{8}[48] = 18$

24. Evaluate $\int_{-3}^{3} x^4 dx$,by using Simpson's $\frac{3}{8}$ rule

Solution. we know by the above problem.

Let f(x) = x^4

X	F(X)
-3	81
-2	16
-1	1
0	0
1	1
2	16
3	81

Where h=1

Simpson's 3/8 rule is

$$\int_a^b f(x)dx = \frac{3h}{8}[(y_0+ y_6)+3y_1+2y_3+3y_4+3y_5]$$

$$= \frac{3}{8}[81+81+3(16+1+1+16)]$$

$$= \frac{3}{8}[162+102] = \frac{3}{8}(264) = 99$$

25. A curve is drawn to passes through the part given by the following .

X	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
Y	**2**	**2.4**	**2.7**	**2.8**	**3**	**2.6**	**2.1**

Estimate the area bounded by the curve the x- axis and the lines x =1, x=4 by using Simpson's $\frac{3}{8}$rule.

Solution. h = $\frac{4-1}{6}$ = 0.5

Simpson's 3/8 rule is

$$\int_a^b f(x)dx = \frac{3h}{8}[(y_0+ y_6)+3y_1+2y_3+3y_4+3y_5]$$

$$= \frac{3(0.5)}{8}[2+2.1+3(2.4+2.7+3+2.6)+2(2.8)]$$

$$= \frac{6.35}{8} = 8.04375 \text{ units of area}$$

26. Evaluate $\int_a^b f(x)$ dx by using weddles formula from the following.

x	4	4.2	4.4	4.6	4.8	5.0	5.2
F(x)	1.4	1.4	1.5	1.5	1.6	1.6	1.6

Where h=0.4

Weddle's formula is

$$\int_a^b f(x)dx = \frac{3h}{10}[y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6]$$

$$= \frac{3(0.2)}{10}[(1.4+1.6)+5(1.4+1.6)+(1.5+1.6)+6(1.5)]$$

$$= \frac{0.6}{10}[3+15+3.1+9]$$

$$= \frac{0.6}{10}(30.1)$$

$$= 1.806$$

27. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using weddle's formula.

Solution. a =0 , b= 1 ,h = $\frac{1-0}{6} = \frac{1}{6} = 0.1666$

X	f(x)
0	1
1/6	0.9729
2/6	0.9000
3/6	0.8000

4/6	0.6923
5/6	0.5901
$\frac{6}{6}$=1	0.5000

Weddle's formula is

$$\int_a^b f(x)dx = \frac{3h}{10}[y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6]$$

$$= \frac{3}{6x10}[(1+0.5)+5(0.972+0.590)+(0.9+0.6923)+6(0.8)]$$

$$= \frac{1}{20}[1.5+7.815+1.5923+4.8]$$

$$= \frac{1}{20}[15.703]$$

$$= 0.785365$$

2.9 Applications of Numerical Methods

Considering the following model equation:

$$(\mu u_y)_y + y^n u_y = C, \qquad (1)$$

defined in a domain Ω = [0 1].

Where $\mu = (1 - exp(-y/\varepsilon) + \delta)/Re$, $\varepsilon << 1$, $\delta << 1$, $Re >> 1$, $n > 0$.

The first term simulates dissipative terms in the Navier-Stokes equations, whereas the second term models the contribution of convective terms and the right hand side represents the pressure gradient term or source in the transport equations. The "viscosity" coefficient μ corresponds to the effective viscosity coefficient. The

coefficient is rapidly changed from a relatively small value $\mu_l = \delta /Re$ (laminar viscosity) to a "turbulent" viscosity $\mu_t \approx 1/Re$.

The equation simulates the low-Reynolds-number effects and can be considered as a model equation for the LR model. The left hand side point in the domain Ω will be treated as a "wall". The low-Reynolds-number effects occur nearby this point. If we set $\mu = (1+ \delta)/Re$ in (1), we have the HR approach.

Furthermore, we will consider the following values for the constants: $Re = 10^2$, $\varepsilon = 3*10^{-2}$, $\delta = 10^{-2}$. For simplicity the right-hand side will be considered as the constant $C = -1$

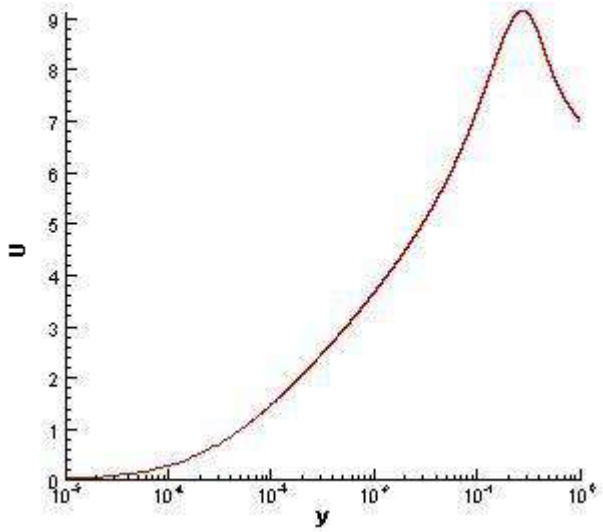
although this assumption is not important for our consideration.

Consider the following boundary-value Dirichlet problem:

$$(\mu u_{y})_y + y^n u_y = C \qquad (2)$$

$$u(0) = u_0 \quad u(1) = u_1$$

In the case of $n = 2$, $u_0 = 0$ and $u_1 = 7$, the solution is shown in Figure 1. The profile includes both the linear near-wall and logarithmic parts. Near the wall, $u = \varepsilon \ln(1+y/(\varepsilon\delta))$. The thickness of the viscous "sub-layer", where u is a linear function, can be approximated as $y_v \approx \varepsilon\delta$.



In the calculations, the compact scheme [4] is used. The scheme allows us to calculate both the function and its derivative with a fourth-order of approximation. In Table 1, the results of calculations of the "friction" $\tau_w = \mu_w du/dy(0)$ (here $\mu_w = \mu(0)$) are given on different meshes. To exclude questions concerned with a mesh adaptation, a uniform mesh is used

Grid	10^2	$5*10^2$	10^3	$5*10^3$	10^4
τ_w	$3.1*10^{-1}$	$3.2*10^{-1}$	$3.3*10^{-1}$	$3.36*10^{-1}$	$3.36*10^{-1}$

Table 1. Calculation of τ_w on different meshes.

If we use the HR model with the same boundary conditions, then $\tau_w = 4.9*10^{-1}$, which shows the importance of the sub-layer.

3. LINEAR SYSTEM OF EQUATIONS

Linear Equations:

Linear equations are nothing but yet another subset of "equations". Any linear calculations requiring more than one variable can be done with the help of linear equations. The standard form of a linear equation in one variable is of the form ax + b = 0. Here, x is a variable, and a and b are constants. While the standard form of a linear equation in two variables is of the form ax + by = c. Here, x and y are variables, and a, b and c are constants.

A mathematical statement that has an equal to the "=" symbol is called an equation. Linear equations are equations of degree 1.

Linear Equation Definition:

A linear equation is an equation that is written for two different variables. This equation will be a linear combination of these two variables, and a constant can be present. Surprisingly, when any linear equation is plotted on a graph, it will necessarily produce a straight line - hence the name: Linear equation

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Linear Equation Examples:

A linear equation can be written in different ways. Any simple equation in x and y can be termed as a linear equation if it follows a certain set of rules. For example, the highest (and the only) degree of both - x and y - variables in the equation should be 1. :Other than that, constants (zero degree variables) can be there. In order to have a better understanding of which equations can be termed as linear or not, have a look at the following equations.

Equations	Linear or Non-Linear
$y = 8x - 9.$	Linear
$y = x - 7$	Non-Linear
$\sqrt{y} + x = 6$	Non-Linear

$y + 3x - 1 = 0$ Linear

$y - x = 9$ Non-Linear

how it works.

Linear Equation Formula:

Linear equation formula is upon case to case and based on the number of variables and the variables used themselves. Firstly, the variables should be independent of each other. Suppose you have x as a variable, then you can't keep x2 as another variable. Secondly, the highest (and the only) degree of all variables in the equation should be 1. Other than that, constants (zero degree variables) can be there. Let us have a look at the standard form of a linear equation with variables x and y:

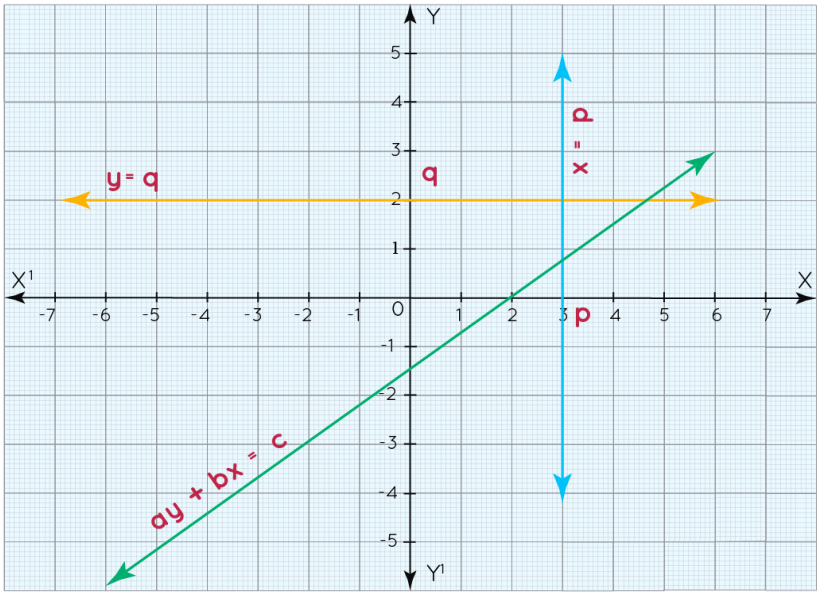
The standard form of a linear equation is $Ax+By=C$.

Linear Equation Graph:

The graph of a linear equation in one variable x forms a vertical line parallel to the y-axis and vice-versa, whereas the graph of a linear equation in two variables x and y forms a straight line. The reason an equation of degree one is called a linear equation is that its geometrical representation is a straight line. Given above are a

few examples of how we plot linear eq.

Graphical Representation of Linear Equations



The graphical representation of linear equations with one and two variables.

Systems of Linear Equations:

Definition An equation in the unknowns x,y,z,... is called linear if both sides of the equation are a sum of (constant) multiples of x,y,z,..., plus an optional constant.

For instance,

3x+4y=2

−x−z=100 are linear equations, but

3x+yz=3

sin(x)−cos(y)=2 are not.

We will usually move the unknowns to the left side of the equation, and move the constants to the right.

A system of linear equations is a collection of several linear equations, like

x+2y+3z=62

x−3y+2z=143

x+y−z=−2

Two or more equations that share variables.

Example:two equations that share the variables x and y:

x + y = 6

−3x + y = 2

Those two equations are shown in this graph:

When we have as many equations as variables we may be able to solve them. The solution for this example is where the lines cross at (1,5)

Application

For each of the systems of equations, determine and give the geometric meaning (if any) of the solution.

1. $$\begin{cases} x + y - z = 5 \\ y - 5z = 8 \\ y - 5z = -1 \end{cases}$$

2. $$\begin{cases} x + y - z = 5 \\ y - 5z = 8 \\ z = -1 \end{cases}$$

3. $\begin{cases} x + y - z = 5 \\ y - 5z = 8 \\ 2x + 2y - 2z = 10 \end{cases}$

3.1 MATRIX INVERSION METHOD:

This method can be applied only when the coefficient matrix is a square matrix and non-singular.

Consider the matrix equation

AX = B , … (1)

where A is a square matrix and non-singular. Since A is non-singular, A^{-1}exists and A^{-1} A = AA^{-1} = I. Pre-multiplying both sides of (1) by A^{-1}, we get A^{-1} (AX) = A^{-1}B. That is, (A^{-1}A) X = A^{-1}B. Hence, we get X = A^{-1}B.

Example 1.22

Solve the following system of linear equations, using matrix inversion method, 5x + 2 y = 3, 3x + 2 y = 5 .

Solution:

The matrix form of the system is AX = B , where

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}.$$

We find |A| = $\begin{vmatrix} 5 & 2 \\ 3 & 2 \end{vmatrix}$ = 10 - 6= 4 ≠ 0. So, A^{-1} exists and A^{-1}= $\frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$.

Then, applying the formula X = A^{-1}B , we get

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 6-10 \\ -9+25 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -4 \\ 16 \end{bmatrix} = \begin{bmatrix} \frac{-4}{4} \\ \frac{16}{4} \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}.$$

So the solution is ($x = -1$, $y = 4$).

Example 1.23

Solve the following system of equations, using matrix inversion method, $2x_1 + 3x_2 + 3x_3 = 5$, $x_1 - 2x_2 + x_3 = 4$, $3x_1 - x_2 - 2x_3 = 3$

Solution:

The matrix form of the system is AX = B,where

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}.$$

$$\text{We find } |A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix} = 2(4+1) - 3(-2-3) + 3(-1+6) = 10+15+15 = 40 \neq 0.$$

So, A^{-1} exists and

$$A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{40}\begin{bmatrix} +(4+1) & -(-2-3) & +(-1+6) \\ -(-6+3) & +(-4-9) & -(-2-9) \\ +(3+6) & -(2-3) & +(-4-3) \end{bmatrix}^T = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

Then, applying $X = A^{-1}B$, we get

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 25-12+27 \\ 25+52+3 \\ 25-44-21 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}.$$

So, the solution is ($x_1 = 1$, $x_2 = 2$, $x_3 = -1$) .

Example 1.24

If , $A=\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$ find the products AB and BA and hence solve the system of equations

x − y + z = 4, x – 2y – 2z = 9, 2x + y +3z =1.

Solution:

$$\text{We find } AB=\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}=\begin{bmatrix}-4+4+8 & 4-8+4 & -4-8+12\\ -7+1+6 & 7-2+3 & -7-2+9\\ 5-3-2 & -5+6-1 & 5+6-3\end{bmatrix}=\begin{bmatrix}8 & 0 & 0\\ 0 & 8 & 0\\ 0 & 0 & 8\end{bmatrix}=8I_3$$

$$\text{and } BA=\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}=\begin{bmatrix}-4+7+5 & 4-1-3 & 4-3-1\\ -4+14-10 & 4-2+6 & 4-6+2\\ -8-7+15 & 8+1-9 & 8+3-3\end{bmatrix}=\begin{bmatrix}8 & 0 & 0\\ 0 & 8 & 0\\ 0 & 0 & 8\end{bmatrix}=8I_3.$$

$$\text{So, we get } AB=BA=8I_3. \text{ That is, } \left(\frac{1}{8}A\right)B=B\left(\frac{1}{8}A\right)=I_3. \text{ Hence, } B^{-1}=\frac{1}{8}A.$$

Writing the given system of equations in matrix form, we get

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}. \text{ That is, } B\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}.$$

$$\text{So, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = B^{-1}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \left(\frac{1}{8}A\right)\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -16+36+4 \\ -28+9+3 \\ 20-27-1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

Hence, the solution is (x = 3, y = - 2, z = −1).

3.2 GAUSS ELIMINATION METHOD:

Gauss elimination method is used to solve a system of linear equations. Let's recall the definition of these systems of equations. A system of linear equations is a group of linear equations with various unknown factors. As we know, unknown factors exist in multiple equations. Solving a system involves finding the value for the unknown factors to verify all the equations that make up the system.

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Gauss Elimination Method with Example:

Let's have a look at the gauss elimination method example with a solution.

Question:

Solve the following system of equations:

x + y + z = 2

x + 2y + 3z = 5

2x + 3y + 4z = 11

Solution:

Given system of equations are:

$x + y + z = 2$

$x + 2y + 3z = 5$

$2x + 3y + 4z = 11$

Let us write these equations in matrix form.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 2 & 3 & 5 \\ 2 & 3 & 4 & 11 \end{array}\right]$$

Subtracting $R_1$from R_2 to get the new elements of R_1,

i.e. $R_2 \rightarrow R_2 - R_1$

From this we get,

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 2 & 3 & 4 & 11 \end{array}\right]$$

Let us make another operation as $R_3 \rightarrow R_3 - 2R_1$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 7 \end{array}\right]$$

Subtract R_2 from R_1 to get the new elements of R_1,

i.e. $R_1 \rightarrow R_1 - R_1$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 7 \end{array}\right]$$

Now, subtract R_2 from R_3 to get the new elements of R_3,

i.e. $R_3 \rightarrow R_3 - R_2$.

$$= \left[\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 4 \end{array}\right]$$

Here,

$x - z = -1$

$y + 2z = 3$

$0 = 4$

That means, there is no solution for the given system of equations.

3.3GAUSS JORDAN METHOD:

Let A be a non-singular square matrix of order n . Let B be the inverse of A.

Then, we have $AB = BA = I_n$. By the property of I_n, we have $A = I_n A = AI_n$.

Consider the equation $A=I_nA$ ……..(1)

Since A is non-singular, pre-multiplying by a sequence of elementary matrices (row operations) on both sides of (1), A on the left-hand-side of (1) is transformed to the identity matrix In and the same sequence of elementary matrices (row operations) transforms In of the right-hand-side of (1) to a matrix B. So, equation (1) transforms to I_n= BA. Hence, the inverse of A is B. That is, A^{-1}= B.

Note

If E_1 , E_2 ,L, $E_k$are elementary matrices (row operations) such that $(E_k \, L \, E_2 \, E_1) A = I_n$, then $A^{-1} = E_k \, L \, E_2 \, E_1$.

Transforming a non-singular matrix A to the form In by applying elementary row operations, is called Gauss-Jordan method. The steps in finding A^{-1} by Gauss-Jordan method are given below:

Step 1:

Augment the identity matrix In on the right-side A to get the matrix $[A \mid I_n]$.

Step 2:

Obtain elementary matrices (row operations) E_1 , E_2 ,L, E_k such that $(E_k \, L \, E_2 \, E_1) A = I_n$

Apply E_1 , E_2 ,L, E_k on $[A \mid I_n]$. Then $[(E_k \ldots\ldots E_2E_1) A \mid (E_k \ldots.. E_2 \, E_1) I_n]$. That is, $[I_n \mid A^{-1}]$.

Example 1.20

Find the inverse of the non-singular matrix A = , $\begin{bmatrix} 0 & 5 \\ -1 & 6 \end{bmatrix}$ by Gauss-Jordan method.

Solution

Applying Gauss-Jordan method, we get

$$[A \mid I_2] = \left[\begin{array}{cc|cc} 0 & 5 & 1 & 0 \\ -1 & 6 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cc|cc} -1 & 6 & 0 & 1 \\ 0 & 5 & 1 & 0 \end{array}\right] \xrightarrow{R_1 \to (-1)R_1} \left[\begin{array}{cc|cc} 1 & -6 & 0 & -1 \\ 0 & 5 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \to \frac{1}{5}R_2} \left[\begin{array}{cc|cc} 1 & -6 & 0 & -1 \\ 0 & 1 & (1/5) & 0 \end{array}\right] \xrightarrow{R_1 \to R_1 + 6R_2} \left[\begin{array}{cc|cc} 1 & 0 & (6/5) & -1 \\ 0 & 1 & (1/5) & 0 \end{array}\right].$$

So, we get $A^{-1} = \begin{bmatrix} (6/5) & -1 \\ (1/5) & 0 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 6 & -5 \\ 1 & 0 \end{bmatrix}$.

Example 1.21

Find the inverse of A = $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss-Jordan method.

Solution

Applying Gauss-Jordan method, we get

$$[A|I_3]=\left[\begin{array}{ccc|ccc}2&1&1&1&0&0\\3&2&1&0&1&0\\2&1&2&0&0&1\end{array}\right]\xrightarrow{R_1\to\frac{1}{2}R_1}\left[\begin{array}{ccc|ccc}1&(1/2)&(1/2)&(1/2)&0&0\\3&2&1&0&1&0\\2&1&2&0&0&1\end{array}\right]$$

$$\xrightarrow[R_3\to R_3-2R_1]{R_2\to R_2-3R_1}\left[\begin{array}{ccc|ccc}1&(1/2)&(1/2)&(1/2)&0&0\\0&(1/2)&-(1/2)&-(3/2)&1&0\\0&0&1&-1&0&1\end{array}\right]\xrightarrow{R_2\to 2R_2}\left[\begin{array}{ccc|ccc}1&(1/2)&(1/2)&(1/2)&0&0\\0&1&-1&-3&2&0\\0&0&1&-1&0&1\end{array}\right]$$

$$\xrightarrow{R_1\to R_1-\frac{1}{2}R_2}\left[\begin{array}{ccc|ccc}1&0&1&2&-1&0\\0&1&-1&-3&2&0\\0&0&1&-1&0&1\end{array}\right]\xrightarrow[R_3\to R_2+R_3]{R_1\to R_1-R_3}\left[\begin{array}{ccc|ccc}1&0&0&3&-1&-1\\0&1&0&-4&2&1\\0&0&1&-1&0&1\end{array}\right].$$

$$\text{So, } A^{-1}=\begin{bmatrix}3&-1&-1\\-4&2&1\\-1&0&1\end{bmatrix}.$$

3.4 LU - DECOMPOSITION METHOD:

Method of LU Decomposition:

Step 1: The system AX =B can be written as LUX = B

Step 2: Define the matrix: n × 1 matrix y by UX =Y.

Step 3: The system LUX = B can be written as LY =B by using UX =Y .

Step 4: Solve the system of equations for y.

Step 5: Substitute y in UX =y.

Step 6: Solve for x.

General representation of LU decomposition of a 4×4 matrix:

$$\begin{bmatrix} l_{11} & 0 & 0 & 0 \\ l_{21} & l_{22} & 0 & 0 \\ l_{31} & l_{32} & l_{33} & 0 \\ l_{41} & l_{42} & l_{43} & l_{44} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ 0 & u_{22} & u_{23} & u_{24} \\ 0 & 0 & u_{33} & u_{34} \\ 0 & 0 & 0 & u_{44} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

Example:

Consider the matrix $A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$

The matrix A is of the form A=LU. That is,

$$A = LU$$

$$= \begin{bmatrix} 1 & 0 \\ l_{21} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{bmatrix}$$

$$= \begin{bmatrix} u_{11} & u_{12} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} \end{bmatrix}$$

Therefore, $\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} \end{bmatrix}$

Compare the entries of the matrix on both the sides.

$$\left\{\begin{array}{c} u_{11}=2 \\ u_{12}=1 \\ l_{21}u_{11}=-1 \\ l_{21}u_{12}+u_{22}=3 \end{array}\right\}$$

Solve the equations and obtain the results as .

$$u_{11}=2,\ u_{12}=1,\ l_{21}=-\frac{1}{2}\ \text{and}\ u_{22}=\frac{7}{2}$$

Thus, the decomposition of the matrix A is, .

$$\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & \frac{7}{2} \end{bmatrix}$$

3.5 ILL CONDITIONED OR WELL CONDITIONED:

* Establish whether the following matrix is singular, ill conditioned or well conditioned: $\begin{bmatrix} 3 & -1 & 3 \\ 1 & 0 & 4 \\ 2 & 2 & 1 \end{bmatrix}$?

Solution:-

Let A = $\begin{bmatrix} 3 & -1 & 3 \\ 1 & 0 & 4 \\ 2 & 2 & 1 \end{bmatrix}$

det A = 3(0 - 8) + 1(1 - 8) + 3(2 - 0) = -24 - 7 + 6 = -25

A is not singular.

$$\begin{vmatrix} 3.01 & -1 & 3 \\ 1 & 0 & 4 \\ 2 & 2 & 1 \end{vmatrix}$$ = (3.01)(0 - 8) + 1(1 - 8) + 3(2 - 0)= -24.08-7+6=-25.08.

The matrix is well conditioned.

* Show that A = $\begin{bmatrix} -73 & 78 & 24 \\ 92 & 66 & 25 \\ -80 & 37 & 10 \end{bmatrix}$ is ill- conditioned?

Solution :-

The ,matrix A = is well conditioned, since = 1 whereas

$$\begin{vmatrix} -73 & 78 & 24 \\ 92.01 & 66 & 25 \\ -80 & 37 & 10 \end{vmatrix} = 2.08, \quad \begin{vmatrix} -73 & 78.01 & 24 \\ 92 & 66 & 25 \\ -80 & 37 & 10 \end{vmatrix} = -28.20$$

and $$\begin{vmatrix} -73 & 78 & 24 \\ 92 & 66 & 25 \\ -80 & 37 & 10.01 \end{vmatrix} = -118.94$$

3.6 GAUSS - JACOBI METHOD:

Given an exact approximation $x^{(k)}= (x_1^{(k)}, x_2^{(k)}, x_3^{(k)}..., x_n^{(k)}$ for x, the procedure of Jacobian's method helps to use the first equation and the present values of $x_2^{(k)}$, $x_3^{(k)}$, …, $x_n^{(k)}$ to calculate a new value $x_1^{(k+1)}$.

Likewise, to evaluate a new value $x_i^{(k)}$ using the ith equation and the old values of the other variables. That is, given current values $x^{(k)} = (x_1^{(k)}, x_2^{(k)}, ..., x_n^{(k)})$, determine new values by solving for $x^{(k+1)}= (x_1^{(k+1)}, x_2^{(k+2)}, ..., x_n^{(k+n)})$ in the below expression of linear equations.

$$a_{11}x_1^{(k+1)} + a_{12}x_2^{(k)} + \cdots + a_{1n}x_n^{(k)} = b_1$$

$$a_{21}x_1^{(k)} + a_{22}x_2^{(k+1)} + \cdots + a_{2n}x_n^{(k)} = b_2$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$a_{n1}x_1^{(k)} + a_{n2}x_2^{(k)} + \cdots + a_{nn}x_n^{(k+1)} = b_n$$

The above system of equations can also be written as below.

$$\begin{bmatrix} a_{11} & 0 & .. & 0 \\ 0 & a_{22} & .. & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & .. & a_{nn} \end{bmatrix} \begin{bmatrix} x_1^{k+1} \\ x_2^{k+1} \\ \vdots \\ x_n^{k+1} \end{bmatrix} + \begin{bmatrix} 0 & a_{12} & .. & a_{1n} \\ a_{21} & 0 & .. & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & .. & 0 \end{bmatrix} \begin{bmatrix} x_1^{(k)} \\ x_2^{(k)} \\ \vdots \\ x_n^{(k)} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

Example;

A system of linear equation of the form Ax = b with an initial estimate $x^{(o)}$ is given below.

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 7 \end{bmatrix}, b = \begin{bmatrix} 13 \\ 11 \end{bmatrix}, x^0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Solve the above using the Jacobian method.

Solution:

Given that

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 7 \end{bmatrix}, b = \begin{bmatrix} 13 \\ 11 \end{bmatrix}, x^0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

We know that $x^{(k+1)} = D^{-1}(b - Rx^{(K)})$ is used to estimate x.

Let us rewrite the above expression in a more convenient form, i.e. $D^{-1}(b - Rx^{(k)}) = Tx^{(k)} + C$

Here,

$T = -D^{-1}R$

$C = D^{-1}b$

$R = L + U$

Let us split the matrix A as diagonal matrix and remainder.

$$D = \begin{bmatrix} 2 & 0 \\ 0 & 7 \end{bmatrix} \Rightarrow D^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{7} \end{bmatrix}$$

Upper and lower parts of reminder of A:

$$R = \begin{bmatrix} 0 & 1 \\ 5 & 0 \end{bmatrix}, L = \begin{bmatrix} 0 & 0 \\ 5 & 0 \end{bmatrix}, U = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Here,

R = Remainder matrix

L = Lower part of R

U = Upper part of R

$T = -D^{-1}(L + U) = D^{-1}[-L + (-U)]$

$$T = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{7} \end{bmatrix} \left\{ \begin{bmatrix} 0 & 0 \\ -5 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \right\} = \begin{bmatrix} 0 & -\frac{1}{2} \\ -\frac{5}{7} & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{7} \end{bmatrix} \begin{bmatrix} 13 \\ 11 \end{bmatrix} = \begin{bmatrix} \frac{13}{2} \\ \frac{11}{7} \end{bmatrix}$$

$$x^1 = \begin{bmatrix} 0 & -\frac{1}{2} \\ -\frac{5}{7} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} \frac{13}{2} \\ \frac{11}{7} \end{bmatrix} = \begin{bmatrix} \frac{12}{2} \\ \frac{6}{7} \end{bmatrix} \approx \begin{bmatrix} 6 \\ 0.857 \end{bmatrix}$$

$$x^2 = \begin{bmatrix} 0 & -\frac{1}{2} \\ -\frac{5}{7} & 0 \end{bmatrix} \begin{bmatrix} 6 \\ 6/7 \end{bmatrix} + \begin{bmatrix} \frac{13}{2} \\ \frac{11}{7} \end{bmatrix} = \begin{bmatrix} 85/14 \\ -19/7 \end{bmatrix} \approx \begin{bmatrix} 6.071 \\ -2.714 \end{bmatrix}$$

Repeat the above process until it converges, i.e. until the value of $||Ax^n - b||$ is small.

3.7 GAUSS SEIDEL METHOD:

Solve Equations 2x+5y=16,3x+y=11 using Gauss Seidel method

Solution:

Total Equations are 2

2x+5y=16

3x+y=11

The coefficient matrix of the given system is not diagonally dominant.

Hence, we re-arrange the equations as follows, such that the elements in the coefficient matrix are diagonally dominant.

3x+y=11

2x+5y=16

From the above equations

x=1/3(11-y)

y=1/5(16-2x)

Solution steps are

1st Approximation

x_1=1/3 [11-(0)]=1/3 [11]=3.666667

y_1=1/5 [16-2(3.666667)]=1/5 [8.666667]=1.733333

2nd Approximation

x_2=1/3 [11-(1.733333)]=1/3 [9.266667]=3.088889

y_2=1/5 [16-2(3.088889)]=15 [9.822222]=1.964444

3rd Approximation

x_3=1/3 [11-(1.964444)]=1/3[9.035556]=3.011852

y_3=1/5 [16-2(3.011852)]=1/5 [9.976296]=1.995259

4th Approximation

x_4=1/3[11-(1.995259)]=1/3 [9.004741]=3.00158

y_4=1/5 [16-2(3.00158)]=1/5 [9.99684]=1.999368

5th Approximation

x_5=1/3 [11-(1.999368)]=1/3 [9.000632]=3.000211

y_5=1/5 [16-2(3.000211)]=1/5 [9.999579]=1.999916

6th Approximation

x_6=1/3[11-(1.999916)]=1/3 [9.000084]=3.000028

y_6=1/5 [16-2(3.000028)]=1/5[9.999944]=1.999989

Solution By Gauss Seidel Method.

x=3.000028≅3

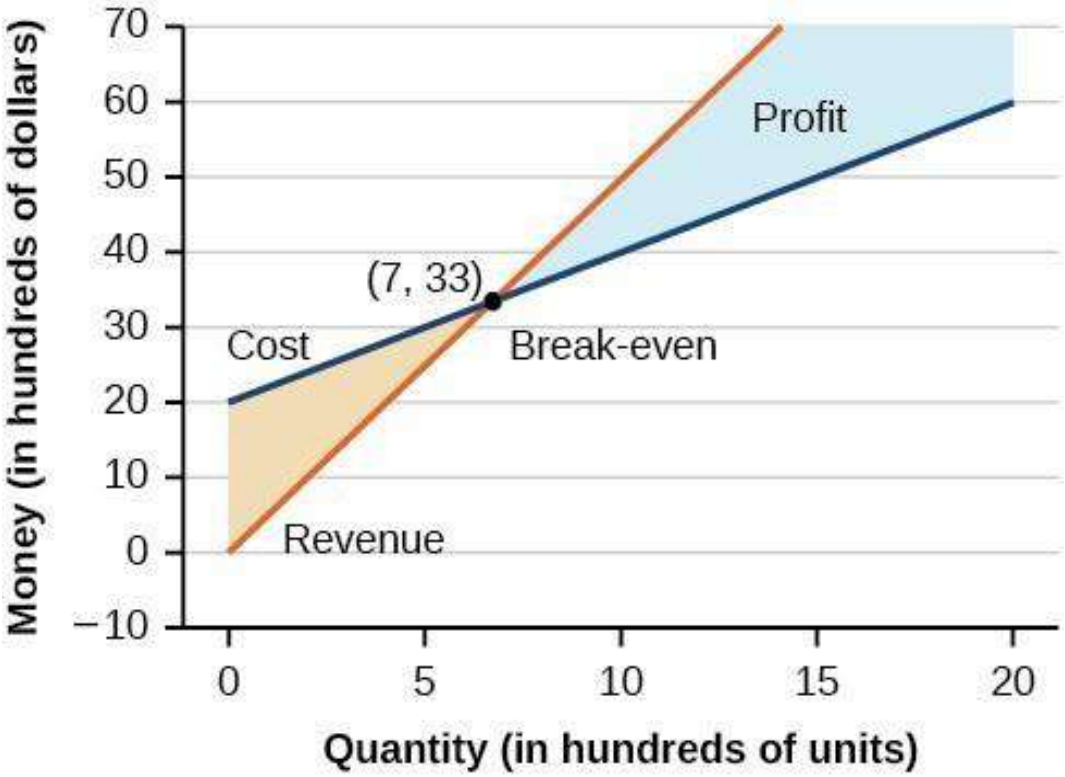
y=1.999989≅2

Applications of Systems of Linear Equations:

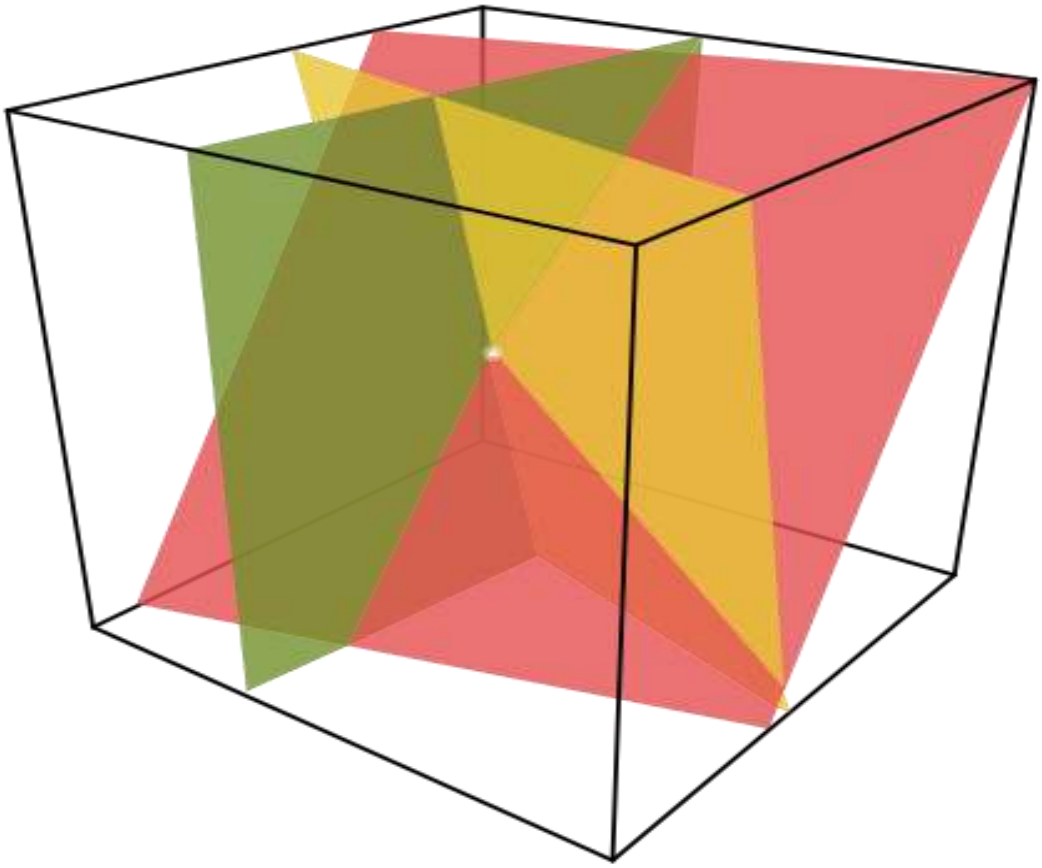
Solve Cost And Revenue Problems:-

Using what we have learned about systems of equations, we can return to the skateboard manufacturing problem at the beginning of the section. The skateboard manufacturer's revenue function is the function used to calculate the amount of money that comes into the business. It can be represented by the equation R=xp, where x= quantity and p= price. The revenue function is shown in orange in the graph below.

The cost function is the function used to calculate the costs of doing business. It includes fixed costs, such as rent and salaries, and variable costs, such as utilities. The cost function is shown in blue in the graph below. The x -axis represents quantity in hundreds of units. The y-axis represents either cost or revenue in hundreds of dollars.



In mathematics, a system of linear equations (or linear system) is a collection of one or more linear equations involving the same set of variables. For example,



A linear system in three variables determines a collection of planes. The intersection point is the solution.

is a system of three equations in the three variables x, y, z. A

$3x+2y-z=1$

$2x-2y+4z=-2$

$-x+y-z=0$

solution to a linear system is an assignment of values to the variables such that all the equations are simultaneously satisfied. A solution to the system above is given by

$X=1$

$Y=-2$

Z=-2

since it makes all three equations valid. The word "system" indicates that the equations are to be considered collectively, rather than individually.

4. LAPLACE TRANSFORM

4.1 Definition and properties of Laplace Transform

The method of Laplace transforms is a system that relies on algebra (rather than calculus-based methods) to solve linear differential equations. While it might seem to be a somewhat cumbersome method at times, it is a very powerful tool that enables us to readily deal with linear differential equations with discontinuous forcing functions.

***Definition*:** Let $f(t)$ be defined for $t \geq 0$. The Laplace transform of $f(t)$, denoted by $F(s)$ or $\mathcal{L}\{f(t)\}$, is an *integral transform* given by the *Laplace integral*:

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^{\infty} e^{-st} f(t)\, dt .$$

Provided that this (improper) integral exists, i.e. that the integral is convergent.

The Laplace transform is an operation that transforms a function of t (i.e., a function of *time domain*), defined on $[0, \infty)$, to a function of s (i.e., of *frequency domain*)*. $F(s)$ is the *Laplace transform*, or simply *transform*, of $f(t)$. Together the two functions $f(t)$ and $F(s)$ are called a *Laplace transform pair*.

For functions of t continuous on $[0, \infty)$, the above transformation to the frequency domain is *one-to-one*. That is, different continuous functions will have different transforms.

The integral is divergent whenever $s \leq 0$. However, when $s > 0$, it converges to

$$\frac{1}{}[0 - e^{0}] = \frac{-1}{}(-1) = \frac{1}{} = \frac{1}{F(s)}$$

***Example*:** Let $f(t) = t$,

then $$F(s) = \frac{2}{s}, \qquad s > 0.$$

[This is left to you as an exercise.]

The integral is divergent whenever s ≤ a. However, when s > a, it converges to

$$\frac{1}{a-s}\left[0-e^{0}\right]=\frac{1}{a-s}(-1)=\frac{1}{s-a}=F(s).$$

***Definition*:** A function $f(t)$ is called *piecewise continuous* if it only has finitely many (or none whatsoever – a continuous function is considered to be "piecewise continuous"!) discontinuities on any interval $[a, b]$, and that both one-sided limits exist as t approaches each of those discontinuity from <u>within</u> the interval. The last part of the definition means that f could have removable and/or jump discontinuities only; it cannot have any infinity discontinuity.

Theorem: Suppose that

1.f is piecewise continuous on the interval $0 \le t \le A$ for any $A > 0$.

2.$|f(t)| \le K e^{at}$ when $t \ge M$, for any real constant a, and some positive constants K and M. (This means that f is "of *exponential order*", i.e. its rate of growth is no faster than that of exponential functions.)

3.Then the Laplace transform, $F(s) = \boldsymbol{L}\{f(t)\}$, exists for $s > a$.

***Note*:** The above theorem gives a <u>sufficient</u> condition for the existence of Laplace transforms. It is <u>not</u> a <u>necessary</u> condition. A function does not need to satisfy the two conditions in order to have a Laplace transform.

Examples of such functions that nevertheless have Laplace transforms are logarithmic functions and the unit impulse function.

<u>Some properties of the Laplace Transform</u>:

1. $\boldsymbol{L}\{0\} = 0$
2. $\boldsymbol{L}\{f(t) \pm g(t)\} = \boldsymbol{L}\{f(t)\} \pm \boldsymbol{L}\{g(t)\}$
3. $\boldsymbol{L}\{c\, f(t)\} = c\, \boldsymbol{L}\{f(t)\}$, for any constant c.

Properties 2 and 3 together means that the Laplace transform is *linear*.

[The derivative of Laplace transforms]

$\boldsymbol{L}\{(-t)f(t)\} = F'(s)$ or, equivalently $\boldsymbol{L}\{t\,f(t)\} = -F'(s)$

Example: $\boldsymbol{L}\{t^2\} = -(\boldsymbol{L}\{t\})' = -\frac{d}{ds}\left(\frac{1}{s^2}\right) = -\left(-\frac{2}{s^3}\right) = \frac{2}{s^3}$

In general, the derivatives of Laplace transforms satisfy

$\boldsymbol{L}\{(-t)^n f(t)\} = F^{(n)}(s)$ or, equivalently $\boldsymbol{L}\{t^n f(t)\} = (-1)^n F^{(n)}(s)$

Warning: The Laplace transform, while a linear operation, is **not** *multiplicative*. That is, in general $\boldsymbol{L}\{f(t)\,g(t)\} \neq \boldsymbol{L}\{f(t)\}\,\boldsymbol{L}\{g(t)\}$.

Exercise: (a) Use property 4 above, and the fact that $\boldsymbol{L}\{e^{at}\} = \frac{1}{s-a}$,

to deduce that $\boldsymbol{L}\{t\,e^{at}\} = \frac{1}{(s-a)^2}$. (b) What will $\boldsymbol{L}\{t^2\,e^{at}\}$ be?

Exercises C-1.1:

1 – 5 Use the (integral transformation) definition of the Laplace transform to find the Laplace transform of each function below.

t^2 2. $t\,e^{6t}$

$\cos 3t$ 4. $e^{-t}\sin 2t$

5.* $e^{i\alpha t}$, where i and α are constants, $i = \sqrt{-1}$.

6 – 8 Each function $F(s)$ below is defined by a definite integral. Without integrating, find an explicit expression for each $F(s)$.

[Hint: each expression is the Laplace transform of a certain function. Use your knowledge of Laplace Transformation, or with the help of a table of common Laplace transforms to find the answer.]

6. $\int_0^\infty e^{-(s+7)t}\,dt$

7. $\int_0^\infty t^2 e^{-(s-3)t}\,dt$

8. $\int_0^\infty 4e^{-st}\sin 6t\,dt$

9. Let $f(t)$ be a continuous function and $F(s)$ be its Laplace transform. Suppose $f(t) \geq 0$ for $t \geq 0$. (a) Show that $F(0)$ is equal to the area of the region under the curve $y = f(t)$, $0 \leq t \leq \infty$. (b) Verify this fact by comparing the value of $F(0)$ with the area under the curve obtained the usual way (i.e. by integration) for each of the functions e^{-t}, te^{-2t}, and t^2e^{-4t}.

Answers C-1.1:

1. $\frac{}{s^3}$

2. $\frac{1}{(s-6)^2}$

3. $\frac{s}{s^2+9}$

4. $\frac{2}{^2+2s+5}$

5. $\frac{s}{s^2+\alpha^2} + i\frac{\alpha}{s^2+\alpha^2}$

Note: Since the Euler's formula says that $e^{i\alpha t} = \cos \alpha t + i \sin \alpha t$, therefore, $\boldsymbol{L}\{e^{i\alpha t}\} = \boldsymbol{L}\{\cos \alpha t + i \sin \alpha t\}$. That is, the real part of its Laplace transform corresponds to that of $\cos \alpha t$, the imaginary part corresponds to that of $\sin \alpha t$. (Check it for yourself!)

6. $\frac{1}{}$

7. $\frac{2}{}$

8. $\frac{24}{}$

$$\frac{s}{+7} \qquad \frac{}{(s-3)^3} \qquad \frac{^2}{+36}$$

9. Set $s = 0$ in the definition $F(s) = \int_0^\infty e^{-st} f(t)\, dt$ to show that

$F(0) = \int_0^\infty f(t)dt = A$.

Solution of Initial Value Problems :

We now shall meet "the new System": how the Laplace transforms can be used to solve linear differential equations algebraically.

Theorem: [**Laplace transform of derivatives**] Suppose f is of exponential order, and that f is continuous and f' is piecewise continuous on any interval $0 \le t \le A$. Then $\boldsymbol{L}\{f'(t)\} = s\,\boldsymbol{L}\{f(t)\} - f(0)$

Applying the theorem multiple times yields:

$$\boldsymbol{L}\{f''(t)\} = s^2\,\boldsymbol{L}\{f(t)\} - s f(0) - f'(0),$$

$$\boldsymbol{L}\{f'''(t)\} = s^3\,\boldsymbol{L}\{f(t)\} - s^2 f(0) - s f'(0) - f''(0),$$

$$\vdots$$

$$\vdots$$

$$\boldsymbol{L}\{f^{(n)}(t)\} = s^n\,\boldsymbol{L}\{f(t)\} - s^{n-1} - s^2 f^{(n-3)}(0) - s f^{(n-2)}(0) - f$$

This is an extremely useful aspect of the Laplace transform: that it changes differentiation with respect to t into multiplication by s (and, as seen a little earlier, differentiation with respect to s into multiplication by $-t$, on the other hand). Equally importantly, it says that the Laplace transform, when applied to a differential equation, would change derivatives into algebraic expressions in terms of s and (the transform of) the dependent variable itself. Thus, it can transform a differential equation into an algebraic equation.

We are now ready to see how the Laplace transform can be used to solve differentiation equations.

Solving initial value problems using the method of Laplace transforms

To solve a linear differential equation using Laplace transforms, there are only 3 basic steps:

1.Take the Laplace transforms of both sides of an equation.

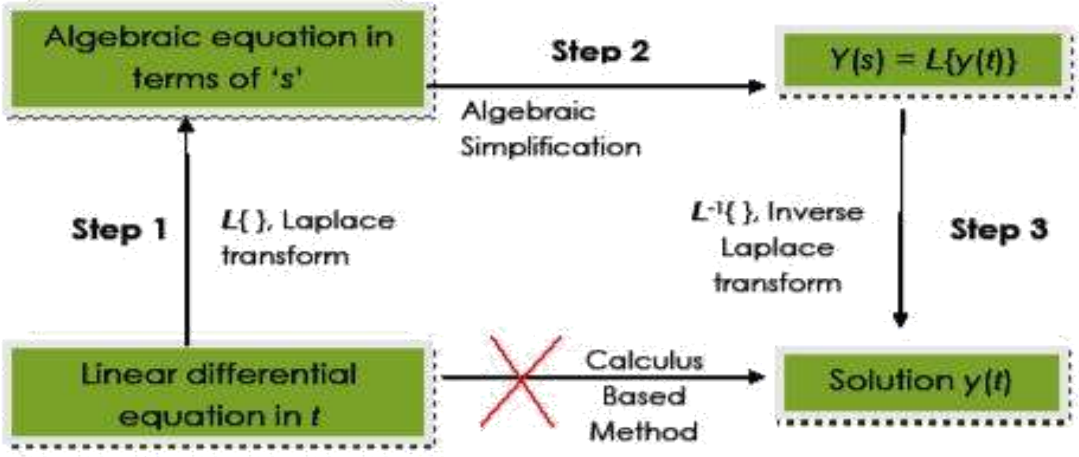
2.Simplify algebraically the result to solve for $\boldsymbol{L}\{y\} = Y(s)$ in terms of s.

3.Find the inverse transform of $Y(s)$. (Or, rather, find a function $y(t)$ whose Laplace transform matches the expression of $Y(s)$.) This inverse transform, $y(t)$, is the solution of the given differential equation.

The nice thing is that the same 3-step procedure works whether or not the differential equation is homogeneous or nonhomogeneous. The first two steps in the procedure are rather mechanical. The last step is the heart of the process, and it will take some practice. Let's get started

The Laplace Transform Method of Solving an Initial Value Problem

Flow Chart



***Example*:** $y'' - 6y' + 5y = 0, \qquad y(0) = 1, y'(0) = -3$

[Step 1] Transform both sides

$\boldsymbol{L}\{y'' - 6y' + 5y\} = \boldsymbol{L}\{0\}$

$(s^2\,\boldsymbol{L}\{y\} - s\,y(0) - y'(0)) - 6(s\boldsymbol{L}\{y\} - y(0)) + 5\boldsymbol{L}\{y\} = 0$

[Step 2] Simplify to find $Y(s) = \boldsymbol{L}\{y\}$

$(s^2\,\boldsymbol{L}\{y\} - s\,-(-3)) - 6(s\,\boldsymbol{L}\{y\} - 1) + 5\boldsymbol{L}\{y\} = 0\ (s^2 - 6\,s + 5)$

$\boldsymbol{L}\{y\} - s + 9 = 0(s^2 - 6\,s + 5)\ \boldsymbol{L}\{y\} = s - 9$

[Step 3] Find the inverse transform $y(t)$

Use partial fractions to simplify,

$$\boldsymbol{L}\{y\} = \frac{s-9}{s^2-6s+5} = \frac{a}{s-1} \quad \frac{b}{s-5}$$

$$\frac{s-9}{s^2-6s+5} = \frac{a(s-5)}{(s-1)(s-5)} \quad \frac{b(s-1)}{(s-5)(s-1)}$$

$s - 9 = a(s - 5) + b(s - 1) = (a + b)s + (-5a - b)$

Equating the corresponding coefficients:

$1 = a + b \qquad a = 2$

$-9 = -5a - b \qquad b = -1$

Hence,

$$\boldsymbol{L}\{y\} = \frac{s-9}{s^2-6s+5} \quad \frac{2}{s-1} \quad \frac{1}{s-5}.$$

The last expression corresponds to the Laplace transform of $2e^{t} - e^{5t}$. Therefore, it must be that, $y(t) = 2e^{t} - e^{5t}$.

Many of the observant students no doubt have noticed an interesting aspect (out of many) of the method of Laplace transform: that it finds the particular solution of an initial value problem directly, without solving for the general solution first. Indeed, it usually takes more effort to find the general solution of an equation than it takes to find a particular solution!

The Laplace Transform method can be used to solve linear differential equations of any order, rather than just second order equations as in the previous example. The method will also solve a nonhomogeneous linear differential equation directly, using the exact same three basic steps, without having to separately solve for the complementary and particular solutions. These points are illustrated in the next two examples.

***Example*:** $y' + 2y = 4t\,e^{-2t}$, $y(0) = -3$.

[Step 1] Transform both sides

$$\boldsymbol{L}\{y' + 2y\} = \boldsymbol{L}\{4t\,e^{-2t}\}$$

$$(s\boldsymbol{L}\{y\} - y(0)) + 2\boldsymbol{L}\{y\} = \boldsymbol{L}\{4t\,e^{-2t}\} = \frac{4}{(s+2)^2}$$

$$(s\boldsymbol{L}\{y\} - (-3)) + 2\boldsymbol{L}\{y\} = \frac{4}{(s+2)^2}$$

$$(s + 2)\,\boldsymbol{L}\{y\} + 3 = \frac{4}{}$$

$$(s \square 2)^2$$

$$(s+2)\,\boldsymbol{L}\{y\} = \frac{}{(s \square 2)^2} - 3$$

$$\boldsymbol{L}\{y\} = \frac{4}{(s+2)} - \frac{3}{s+2} \frac{4-3(s+2)}{(s+} = \frac{-3s^2-12s-8}{3} \quad (s+2)$$

[Step 3] Find the inverse transform $y(t)$

By partial fractions,

$$\boldsymbol{L}\{y\} = \frac{-3s^2-12s-8}{3} = \frac{a}{(s+2)} + \frac{b}{(s+2)} - \frac{c}{s+2}$$

$$\frac{-3s^2-12s-8}{(s+2)^3} = \frac{a}{(s+)^3} - \frac{b(s+2)}{(s+2)^3} - \frac{c(s+2)^2}{(s+2)^3}$$

$$\frac{a+bs+2b+cs^2+4cs+4c}{(s+2)^3} \qquad \frac{c}{s^2+(b+4c)s+(a+2b+4c)}{(s+2)^3}$$

$-3 = c$ $\qquad\qquad$ $a = 4$

$-12 = b + 4c$ $\qquad\qquad$ $b = 0$

$-8 = a + 2b + 4c$ $\qquad\qquad$ $c = -3$

$$\boldsymbol{L}\{y\} = \frac{-3s^2 - 12s - 8}{(s+2)} - \frac{4}{(s+2)} - \frac{3}{s+}$$

This expression corresponds to the Laplace transform of $2t^2 e^{-2t} - 3e^{-2t}$. Therefore,

$$y(t) = 2t^2 e^{-2t} - 3e^{-2t}.$$

Note: $\boldsymbol{L}\{t^n e^{at}\} = \frac{}{(s-a)^{n+1}}$

Example: $y'' - 3y' + 2y = e^{3t}$, $\qquad$ $y(0) = 1, y'(0) = 0$

[Step 1] Transform both sides

$(s^2 \boldsymbol{L}\{y\} - s\,y(0) - y'(0)) - 3(s\boldsymbol{L}\{y\} - y(0)) + 2\boldsymbol{L}\{y\} = \boldsymbol{L}\{e^{3t}\}$

[Step 2] Simplify to find $Y(s) = \boldsymbol{L}\{y\}$

$(s^2 \boldsymbol{L}\{y\} - s - 0) - 3(s\boldsymbol{L}\{y\} - 1) + 2\boldsymbol{L}\{y\} = 1 / (s - 3)$

$(s^2 - 3s + 2)\,\boldsymbol{L}\{y\} - s + 3 = 1 / (s - 3)$

$(s^2 - 3s$

$$+ 2)\,\boldsymbol{L}\{y\} = s - 3 + \frac{1}{s-3} = \frac{(s-3)^2 + 1}{s-3}$$

$$\boldsymbol{L}\{y\} = \frac{s^2 - 6s + 10}{(s^2 - 3s + 2)(s-3)} = \frac{s^2 - 6s + 10}{}$$

(
s
–
1
)
(
s
–
2
)
(
s
–
3
)

[Step3] Find the inverse transform $y(t)$

By partial fractions,

$$\mathcal{L}^{-1}\{y\} = \frac{s^2 - 6s + 10}{(s-1)(s-2)(s-3)} = \frac{5}{2}\cdot\frac{1}{s-1} - 2\cdot\frac{1}{s-2} + \frac{1}{2}\cdot\frac{1}{s-3}$$

Therefore, $y(t) = \frac{1}{2}e^{t} - 2e^{2t} + \frac{1}{2}e^{3t}$.

For the next example, we will need the following Laplace transforms:

$$\mathcal{L}\{\cos bt\} = \frac{s}{s^2 + b^2}, \quad s > 0$$

$$\mathcal{L}\{\sin bt\} = \frac{b}{s^2 + b^2}, \quad s > 0$$

$$\mathcal{L}\{e^{at}\cos bt\} = \frac{s-a}{(s-a)^2 + b^2}, \quad s > a$$

$$\mathcal{L}\{e^{at}\sin bt\} = \frac{b}{(s-a)^2 + b^2}, \quad s > a$$

***Note*:** The values of a and b in the last two expressions' denominators can be determined without using the method of completing the squares. Any irreducible quadratic polynomial $s^2 + Bs + C$ can always be written in the

required from of $(s - a)^2 + b^2$ by using the quadratic formula to find (necessarily complex-valued roots) s . The value a is the real part of s, and the value b is just the absolute value of the imaginary part of s. That is, if $s = \lambda \pm \mu\, i$, then $a = \lambda$ and $b = \mu$.

Example: $y'' - 2y' + 2y = \cos(t), \qquad y(0) = 1, y'(0) = 0$

[Step 1] Transform both sides

$(s^2 \boldsymbol{L}\{y\} - s\, y(0) - y'(0)) - 2(s\boldsymbol{L}\{y\} - y(0)) + 2\boldsymbol{L}\{y\} = \boldsymbol{L}\{\cos(t)\}$

[Step 2] Simplify to find $Y(s) = \boldsymbol{L}\{y\}$

$(s^2 \boldsymbol{L}\{y\} - s - 0) - 2(s\boldsymbol{L}\{y\} - 1) + 2\boldsymbol{L}\{y\} = s / (s^2 + 1)$

$(s^2 - 2\,s + 2)\, \boldsymbol{L}\{y\} - s + 2 = s / (s^2 + 1)$

$$(s^2 - 2\,s + 2)\, \boldsymbol{L}\{y\} = s - 2 + \frac{\frac{1}{s^2+1}}{\frac{(s-2)(s^2+1)+s}{s^2+1}}$$

$$\boldsymbol{L}\{y\} = \frac{s^3 - 2s^2 + s - 2 + s}{(s^2+1)(s^2 - 2s + 2)} \quad \frac{s^3 - 2s^2 + 2s - 2}{(s^2+1)(s^2 - 2s + 2)}$$

[Step 3] Find the inverse transform $y(t)$

By partial fractions,

$$\boldsymbol{L}\{y\} = \frac{s^3 - 2s^2 + 2s - 2}{(s+1)(s^2 - 2s + 2)} = \frac{s-2}{s} - \frac{4s - 6}{s^2 - 2s + 2}$$

$$= \frac{1}{s} - \frac{s+1}{(s+1)^2+1} + \frac{4(s-1)}{(s-1)^2+1} - \frac{2}{(s-1)^2+1}$$

which corresponds to

$$y(t) = \frac{1}{5}\left[\cos(t) - 2\sin(t) + 4e^{t}\cos(t) - 2e^{t}\sin(t)\right]$$

Examples: Find the inverse Laplace transform of each

(i) $F(s) = \dfrac{2s-5}{s^2+4s+8}$

Rewrite $F(s)$ as:

$$F(s) = \frac{2s-5}{(s+2)^2+2^2} = \frac{2(s+2)}{(s+2)^2+2^2} - \frac{9}{2}\frac{2}{(s+2)^2+2^2}$$

Answer: $f(t) = 2e^{-2t}\cos(2t) - \frac{9}{2}e^{-2t}\sin(2t)$

(ii) $F(s) = \dfrac{s+4}{(s-2)^3}$

Use partial fractions to rewrite $F(s)$ as:

$$F(s) = \frac{1}{(s-2)^2} + \frac{6}{(s-2)^3}$$

Answer: $f(t) = te^{2t} + 3t^2e^{2t}$

Appendix A

4.2 Some Additional Properties of Laplace Transforms

I. Suppose $f(t)$ is discontinuous at $t = 0$, then the Laplace transform of its derivative becomes

$$\boldsymbol{L}\{f'(t)\} = s\,\boldsymbol{L}\{f(t)\} - \lim_{t \to 0^-} f(t).$$

Suppose $f(t)$ is a periodic function of period T, that is, $f(t + T) = f(t)$, for all t in its domain, then

$$\boldsymbol{L}\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\, dt.$$

***Comment*:** This property is convenient when finding the Laplace transform of a discontinuous periodic function, whose discontinuities (necessarily there are infinitely many, due to the periodic nature of the function) would make the usual approach of integrating from 0 to ∞ unwieldy.

Let c > 0 be a constant, the *time-scaling* property of Laplace transform states that

$$\boldsymbol{L}\{f(ct)\} = \frac{1}{c} F\left(\frac{s}{c}\right).$$

IV. We have known that $\boldsymbol{L}\{-t\,f(t)\} = F'(s)$. Taking the inverse transforms on both sides yields $-t\,f(t) = \boldsymbol{L}^{-1}\{F'(s)\}$. Therefore, $f(t) = -\frac{1}{t}\boldsymbol{L}^{-1}\{F'(s)\}$. That is, if we know how to invert the function $F'(s)$, then

we also know how to find the inverse of its anti-derivative $F(s)$. Formally,

$$\mathcal{L}\{f_t^{(t)}\} = \int_s^{\infty} F(u)\,du\,.$$

V. Similarly, dividing $F(s)$ by s corresponding an integral with respect to t.

$$\mathcal{L}\left\{\int_0^t f(u)\,du\right\} = \frac{F(s)}{s}$$

Comment: Therefore, dividing a function by its independent variable has the effect of anti-differentiation with respect to the other independent variable for Laplace transforms. These two properties (IV and V) are the

counter parts of the multiplication-corresponds-to-differentiation properties seen earlier.

VI. After learning the fact that $\mathcal{L}\{f(t)\,g(t)\} \neq \mathcal{L}\{f(t)\}\,\mathcal{L}\{g(t)\}$, one might have wondered whether there is an operation of two functions f and g whose result has a Laplace transform equal to the product of the individual transforms of f and g. Such operation does exist:

$$\mathcal{L}\left\{\int_0^t f(\tau)g(t-\tau)\,d\tau\right\} = \mathcal{L}\{f(t)\}\mathcal{L}\{g(t)\}.$$

In addition, observe that, by taking the inverse Laplace transform of both sides the property VI, we have

$$\int_0 f(\tau)g(t-\tau)\,d\tau = \mathcal{L}^{-1}\{\mathcal{L}\{f(t)\}\mathcal{L}\{g(t)\}\}.$$

In other words, we can obtain the inverse Laplace transform of a (simple) function of s that is itself a product of Laplace transforms of two known functions of t by taking the convolution of the two functions of t.

Example: Find the inverse Laplace transform of

$$F(s) = \frac{1}{\quad}\,.$$

$$s(s^2 + 4)$$

First, note that

$$F(s) = \frac{1}{s} \cdot \frac{1}{s^2 + 4} = \mathcal{L}\{1\}\,\mathcal{L}\{\sin 2t\}$$

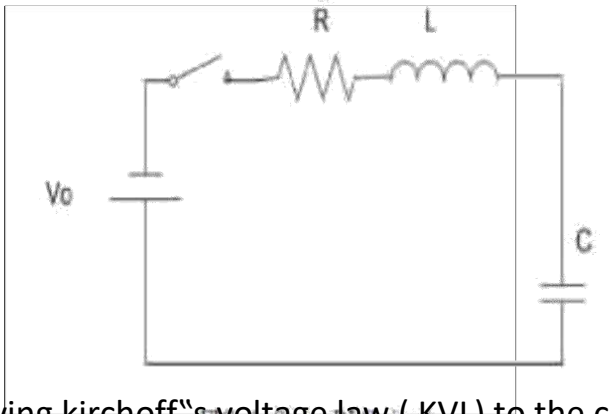
Therefore, its inverse can be found by the convolution integral (letting $f(t) = 1$ and $g(t) = \sin 2t$):

4.3 Applications of Laplace transform

This section describes the applications of laplace transform in the domain of engineering. Laplace transform has several applications in almost all Engineering disciplines such as System Modelling, Analysis of Electrical and Electronic Circuits, Digital Signal Processing and Process Controls. Laplace transforms are critical for process controls. It helps analyze the variables, which when altered, produces desired manipulations in the result. For example, while study heat experiments, Laplace transform is used to find out to what extent the given input can be altered by changing temperature, hence one can alter temperature to get desired output for a while. This is an efficient and easier way to control processes that are guided by differential equations

Electric Circuit theory

Example 1 :: The switching transient phenomenon in the RL , RC or RLC circuits can be solved by laplace transform. Let us consider a series RLC circuit as shown figure 1 to which a dc voltage V_0 is suddenly applied.



Now applying kirchoff"s voltage law (KVL) to the circuit , we have

Differentiating bothsides,

$$R_i + L\frac{d\,i}{d\,t} + \frac{1}{C}\int i\,dt = V_0$$

$$L\frac{d^2i}{dt^2} + \frac{1}{C\,i} + R\frac{d\,i}{d\,t} = 0 \;\; or$$

$$\frac{d^2i}{d\,t^2} + \frac{R}{L}\frac{d\,i}{d\,t} + \frac{1}{L\,C}\,i = 0 \quad \rightarrow (1)$$

Applying laplace transform to this equation, let us assume that the solution of this equation is

$$i\,(t) = K\,e^{s\,T}$$

$$L\,K\,s^2\,e^{s\,T} + R\,K\,e^{s\,T} + \frac{1}{C\,K}\,e^{s\,T} = 0$$

5. CURVE FITTING

5.1 INTRODUCTION :

In this chapter we are concerned with the problem of fitting

an equation or a curve to data involving period values. An approximate non-mathematical relationship between the two variables , can be established by a diagram called **scatter diagram .** The exact mathematical relationship between the two variables is given by simple algebra expression called **curve fitting .** Though there are infinite variety of curve in mathematics , the curve used for the purpose of curve fitting are relatively limited in type. The straight line is the simple and one of the most important curve used.

The straight line :

The equation $y = a + bx$…. (1) Is an equation of the degree in x and y .It represents a straight line .

The difference $y_i - (a + bx_i)$….. (2)

Is zero if and only if the point (x_i, y_i) lies on the line given by (1)

Fitting a straight line:

Usually fitting a straight line means finding the value of the parameter a and b of the straight line given by (1) , as well as actually constructing the line itself .The Graphic method and the method of least square are two useful method for fitting a straight line .

The graphical method:

The method can be used whenever the given formula can be plotted as a straight line either directly or after suitable transformation. The straight line drawn after a careful visual estimate of its position has been made with the aid of a ruler. The coordinated of any two points on the measures and substitute in equation (1). The resulting equations in a and b are then solved for their *parameters.* Graphical method , whatever its theoretical attractions , suffers from the disadvantage that it is difficult apply in practice except for the straight line .This method will give fairly good results when finely divided coordinate paper is used , but in general it is recommended .

An alternative method, which id in almost universal use at present time, is known as the of **Least Squares** and we proceed to discuss it at length.

<u>The method of lest squares:</u>

This method of curve fitting was suggested early in the nineteenth century by the French mathematician Adrian Legendre. The method of least squares assumes that the best fitting line in the curve for which the sum of the squares of the vertical distances of the points (x_r , y_r) from the line is minimum. This method is more accurate that the graphical method.

Let $y = a + bx$ Refer (1)

Be the equation of the line. The ordinate of any point Q_r on the line vertically above or below a given point P_r , can be found by substituting the abscissa x_r , in the right-hand side of (1) . The two coordinates of Q_r will be $(x_r , a + bx_r)$ (see in the above figure). The vertical distances e_r from the line of any point p_r with coordinates (x_r , y_r) will therefore be given by the equation

$$e_r = y_r - (a + x_r) \ldots\ldots \quad (3)$$

We may say that e_r represents the difference between the actual ordinate y_r , of a pint and its theoretical ordinate $a + bx_r$.

Let $q = \sum_{r=1}^{n} e_r^2 = \sum_{r=1}^{n} (y_r - (a + bx_r))^2$ ……. (4)

The best fitting line is that line for which the sum of the square, $q = \sum e_r^2$ is a minimum. We find the value of a and b which make q minimum as follows :Differentiating q partially with respect to a and b equally these partially to zero , we obtain

$$\therefore \frac{\partial_q}{\partial_a} = \sum_{r=1}^{n} (-2)[y_r - (a + bx_r)] = 0 \ ,$$

And $$\frac{\partial_q}{\partial_o} = \sum_{r=1}^{n} (-2) x_r [y_r - (a + bx_r)] = 0 \ ,$$

Which yield the so – called **System of normal equations,**

$\sum_{r=1}^{n} y_r = an + b \sum_{r=1}^{n} x_r$,……(5)

And $\sum_{r=1}^{n} x_r \, y_r = a \sum_{r=1}^{n} x_r + b \sum_{r=1}^{n} x_r^2$ ……..(6)

Solving this system of equations we get

$$a = x = \frac{\sum_1^n x_r^2 \sum_1^n y_r - \sum_1^n x_r \sum x_r y_r}{n\sum_1^n x_r^2 - [\sum_1^n x_r} \quad \text{..... (7)}$$

$$b = \frac{n[\sum_{r=1}^n x_r y_r] - [\sum_{r=1}^n x_r][\sum_{r=1}^n x_r^2]}{n[\sum_{r=1}^n x_r^2] - [\sum_{r=1}^n x_r]^2} \quad \text{,.....(8)}$$

Where n is the number of points (x_r , y_r).

To simplify the for formulae, we get

$$t_r = \frac{x_r - \bar{x}}{h} \quad , \qquad (9)$$

Where h is the interval between successive value of x assumed to be equally spaced. Shifting the origin to the mean $\bar{x}$ and compressing the intervals to unity we get transformed equation of (2) referred to the new (t, y) axes as

$$Y = \bar{a} + \bar{b}t \quad , \qquad (10)$$

Where $\bar{a}\ and\ \bar{b}$ are the parameter in the new equation . In terms of the new coordinate formula (7) and (8) becomes

$$\bar{a} = \frac{\sum_1^n t_r^2 \sum_1^n y_r - \sum_1^n t_r \sum t_r y_r}{n\sum t_r^2 - (\sum t_r)^2} \quad , \qquad (11)$$

$$\bar{b} = \frac{n\sum_1^n t_r y_1 - \sum_1^n t_r \sum t_r y_r}{n\sum_1^n t_r^2 - [\sum_1^n t_r]^2} \quad , \qquad \text{.....} \qquad (12)$$

Transformation (9) replace the $x_r\ \grave{s}$ by unit deviations t_r from the mean $\bar{x}$.

Since

$$\sum_1^n t_r = 0$$

The formula (11)and (12) reduce to

$$\bar{a} = \frac{\sum_1^n t_r^2 \sum_1^n y_r - 0}{n \sum t_r^2 - 0}$$

And

$$\bar{b} = \frac{n \sum_1^n t_r y_r - 0}{n \sum t_r^2 - 0}$$

Which further simplify to the form

5.2 Fitting a parabola:

When a set of point exhibits a parabolic trend , the fitting of a quadratic functions

$$y = a + bx + cx^2 ,$$

To the data may be carried out by the method of least square , Which leads to the three equations

$$na + b \sum_1^n x_r + c \sum x_r^2 = \sum_1^n y_r$$

$$a \sum_1^n x_r + b \sum_1^n x_r^2 + c \sum_1^n x_r^3 = \sum_1^n x_r y_r .$$

$$a \sum_1^n x_r^2 + b \sum_1^n x_r^3 + c \sum_1^n x_r^4 = \sum_1^n x_r^2 y_r .$$

The above equations can be solve for a , b, c.

5.3 EXPONETIAL FUNCTION Y= Ae^{bx}

Transforming the exponential equation $y = ae^{bx}$ by taking logarithms on both sides we get

$$\log y = \log a + bx \log e .$$

If we replace $\log y$ by y and the constant $\log a$ and b $\log e$ by $\grave{a}$ and $\acute{b}$

Respectively , we obtain

$$y = \acute{a} + b\acute{x} \ ,$$

Which defines a straight line .

Example:

1.Find the lest square line fir the data points (-1,10) ,(0,9),(1,7),(,4,5), (,2,5),(3,4),(4,3),(5,0) and (6,-1).

Solution: Here

x_r	y_r	x_r^2	y_r^2
-1	10	1	-10
0	9	0	0
1	7	1	7
2	5	4	10
3	4	9	12
4	3	9	12
5	0	25	0
6	-1	36	-6
20	37	92	25

From the table we have

$$n = 8 \ , \sum x_r \ = 20 \ , \sum y_r = 37, \sum x_r^2 = 92 \ , and \ \sum x_r y_r = 25 \ .$$

The normal equation are

$$na + b \sum x_r \ = \sum y_r \ ,$$

$$\text{a} \sum x_r + \sum x_r^2 = \sum x_r y_r \ .$$

Putting the value in normal equations

$$8a + 20b = 37 \ ,$$

And

$$20a + 97b + 25 \quad ,$$

And solving these equations we get

$$a = 8.6428571 \ ,$$

$$b = -1.6071429 \ .$$

Therefore the least square line is

$$y = 8.6428571 + (-1)1.6071429x \ ,$$

i.e.,

$$y = -1.6071429x + 8.6428571 \ .$$

Example 2: **Find the least square line $y = a + bx$ for the data**

X	**-2**	**-1**	**0**	**1**	**2**
Y	**1**	**2**	**3**	**3**	**4**

Solution:

x_r	y_r	x_r^2	$x_r y_r$
-2	1	4	-2
-1	2	1	-2
0	3	0	0
1	3	1	3
2	4	4	8
0	13	10	7

$\therefore$ *In this case* $n = 5\ , \sum x_r = 0\ , \sum y_r = 13\ , \sum x_{r2} = 10\ , \sum x_r y_r = 7.$

The normal equation are

$$na + b\sum x_r = \sum y_r \ ,$$

$$a\sum x_r + b\sum x_r^2 = \sum x_r y_r \ .$$

Putting the value of $n\ , \sum x_r\ , \sum y_r\ , \sum x_r^2\ and\ \sum x_r y_r$ in the above equation we get

$$5a = 13\ , 10b = 7\ .$$

Solving

$$a = 2.6\ , b = 0.7\ .$$

∴The required line of fit is

$$y = 2.6 + b = (0.7)x\ .$$

Example 3:

Find a formula for the line of the form $y = a + bx + cx^2$ Which will fit the following data

X	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Y	3.1950	3.2299	3.2532	3.2516	3.2516	3.2282	3.1807	3.1266	3.0594	2.9759

Solution :

The normal equation are

$$na = b\sum x_r + c\sum x_r^2 = \sum y_r^2 \ ,$$

$$a\sum x_r + b\sum x_r^2 + c\sum x_r^3 = \sum x_r y_r$$

$$a\sum x_r^2 + b\sum x_r^3 + c\sum x_r^4 = \sum x_r^2 y_r \ .$$

Substituting the value of $\sum x_r\ , \sum x_r^2\ , \sum x_r^3, \sum x_r y_r, \sum x_r^2\ y_r, \sum y_r, and\ n, we\ get$

$$10a + 4.5b + 2.85c = 31.7616\ ,$$

$$4.5a + 2.85b + 2.025c = 14.0896 \ ,$$

$$2.85a + 2.025b + 1.5333 = 8.82881 \ ,$$

And solving these equations we obtain

$a = 3.1951 \, , b = 0.44254 \, , c = -0.76531.$

$\therefore \ The \ requied \ equation \ is$

$$y = 3.1951 + 0.44254x - 0.76531x^2.$$

Example 4:

Fit second degree parabola to the following data

X	0	1	2	3	4
y	0	1.8	1.3	2.5	6.3

Solution : The values of x are 0,1,2,3,4

The number of value is odd. Shifting the origin to the middle value 2 of x, and making the substituting

$$u = x - 2, v = y \, ,$$

The curve of fit as

$$v = a + bu + cu^2 \ ,$$

We obtain

x	v	u	v	uv	u^2	u^2v	u^3	u^4
0	1	-1	1	-2	4	4	-8	16
1	1.8	-2	1.8	1.6	1	1.8	-1	1
2	1.2	0	1.3	0	0	0	0	0
3	2.5	1	2.5	2.5	1	2.5	1	1
4	6.3	2	6.3	12.6	4	25.2	8	16
	0		12.9	11.3	10	33.5	0	34

The normal equation are

$$\sum u = na + b\sum u + c\sum u_r^2 \quad ,$$

$$\sum uv = a\sum u + bu^2 + c\sum u^3 \quad ,$$

$$\sum u^2 v = a\sum u^2 + b\sum u^3 + c\sum u^4 \quad .$$

Putting the values of n ,$\sum u$, $\sum v$, $\sum uv$, … … … . , etc,

12.9=5a+40c,

$11.3 = 10b$,

33.5=10a+34c,

Solving these normal equations we get

$a = 1.48, b = 1.13, c = 0.550.$

∴The required equation is

$$u = 1.48 + 1.13u + 1.55u^2 \quad ,$$

Substituting $u = x - 2$, $v = y$ in the above equation we get,

$y = 1.42 + 1.13(x - 2) + (0.55)(x - 2)^2$.

$\therefore The\ requird\ curve\ of\ fit\ is\ the\ parabola$

$y = 1.42 - 1.07x + 0.55x^2.$

5.4 Solved problems:

1)Find the lest square line $y = a + bx$ for the data

X	-4	-2	0	2	4
Y	1.2	2.8	6.2	7.8	13.2

A)	X	y	x^2	xy
-4	1.2	16	-4.8	
-2	2.8	4	-5.6	
0	6.2	0	0	

	2	7.8	4	15.6
4	13.2	16	52.8	
0	31.2	4.	58	

The normal equations are

$$\sum y = na + b\sum xi$$

$$\sum xy = a\sum x_i + b\sum x_i^2$$

31.2=5a

$$a = \frac{31.2}{5}$$

= 6.24

$$58 = b(40)$$

$$\therefore y = 6.24 + 1.45 \ .$$

2) **Fit a straight line to the following data regarding x as the independent variable**

X	0	1	2	3	4
Y	1	1.8	3.3	4.5	6.3

A)	x	y	x^2	xy
0	1	0	0	
1	1.8	1	1.8	
2	3.3	4	6.6	
3	4.5	4	13.5	
4	6.3	16	25.2	
	10	16.9	30	47.1

$$na + b\sum xi = \sum y$$

$$5a + b(10) = 16.9 \qquad (1)$$

$$a\sum x + b\sum x^2 = \sum xy$$

$$a(10) + b(30) = 47.1$$

$1*2 \rightarrow 10a + 20b = 33.8$

$$\underline{2 \rightarrow 10a + 30b = 47.1}$$

$$-10b = -11.3$$

$$b = \frac{13.3}{10}$$

=1.33

Substituting b in (1)

$$5a + (1.33)10 = 16.9$$

$$5a = 16.9 - 13.3$$

$$a = \frac{3.6}{5}$$

=0.72

$$\therefore y = 0.72 + 1.33x$$

3) **Find the least square line $y = a + bx$**

	X	-2	0	2	4	6
Y	1	3	6	8	13	

A)	X	y	x^2	xy
	-2	1	4	-2
	0	3	0	-2
	2	6	4	12
	4	8	16	12
	6	13	36	78
	10	31	60	78

$$na + b\sum x_i = \sum xy$$

$$5a + 10b = 31 \qquad (1)$$

$$a\sum x + b\sum x^2 = \sum xy$$

$$10a + 60b = 102 \qquad (2)$$

$$1*2 \rightarrow 10a + 20b = 62$$

10a+60b=120

$$-40b = -58$$

$$b = 1.45$$

Substituting b in (1)

$$5a + (1.45)10 = 31$$

$$5a = 31 - 14.5$$

$$a = \frac{16.5}{5}$$

=3.3

$$a = 3.3 \ , b = 1.45$$

$$\therefore y = 3.3 + 1.45x$$

4) Find the least squares parabolic lift $y = a + bx + cx^2$

A) x	y	x^2	x^3	x^4	xy	x^2y
-3	15	9	-27	81	-45	135
-1	5	1	-1	1	-5	5
1	1	1	1	1	1	1
3	5	9	27	81	15	45
0	26	20	0	164	-34	186

$$\sum y = na + b\sum x + c\sum x^2$$

$$26 = 4a + 0 + 20c$$

$$4a + 20c = 26 \qquad (1)$$

$$\sum xy = a\sum ax + b\sum x^2 + c\sum x^3$$

$$-34 = 0 + b(20) + c(0)$$

$$20b = -34$$

$$b = -\frac{34}{20}$$

$$= -1.7$$

$$\sum x^2y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

$$186 = 20a + 164c \qquad (2)$$

$$1*5 \quad \rightarrow \quad 20a + 100c = 130$$

$$\underline{2 \quad \rightarrow \quad 20a + 164c = 186}$$

$$-64c = -56$$

$$c = 0.875$$

$$4a + 20(0.875) = 26$$

$$4a = 36 - 17.5$$

$$a = \frac{8.5}{4}$$

$$= 2.125$$

$$y = a + bx + cx^2$$

$$\therefore y = 2.215 - 1.70x + 0.875x^2$$

5.5 APPLICATIONS OF CURVE FITTING

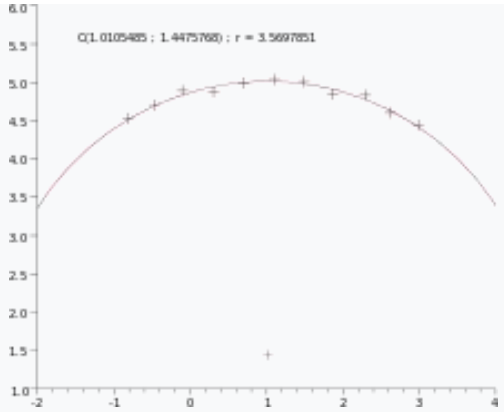
Fitting plane curves to data points :

If a function of the form cannot be postulated, one can still try to fit a plane curve.

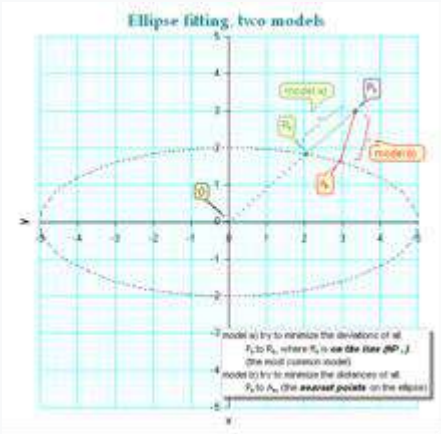
Other types of curves, such as conic sections (circular, elliptical, parabolic, and hyperbolic arcs) or trigonometric functions (such as sine and cosine), may also be used, in certain cases. For example, trajectories of objects under the influence of gravity follow a parabolic path, when air resistance is ignored. Hence, matching trajectory data points to a parabolic curve would make sense. Tides follow sinusoidal patterns, hence tidal data points should be matched to a sine wave, or the sum of two sine waves of different periods, if the effects of the Moon and Sun are both considered.

For a parametric curve, it is effective to fit each of its coordinates as a separate function of arc length; assuming that data points can be ordered, the chord distance may be used.

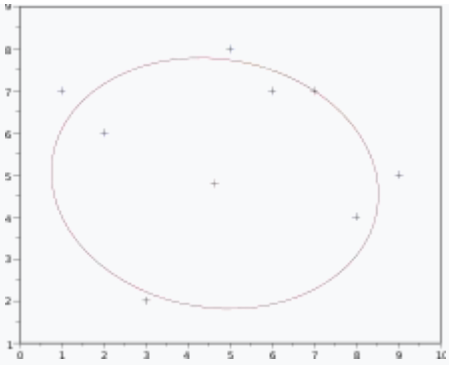
Fitting a circle by geometric fit



Circle fitting with the Coope method, the points describing a circle arc, centre (1 ; 1), radius 4.



different models of ellipse fitting



Ellipse fitting minimizing the algebraic distance (Fitzgibbon method).

Cooper approaches the problem of trying to find the best visual fit of circle to a set of 2D data points. The method elegantly transforms the ordinarily non-linear problem into a linear problem that can be solved without using iterative numerical methods, and is hence an order of magnitude faster than previous techniques.

Fitting an ellipse by geometric fit

The above technique is extended to general ellipses by adding a non-linear step, resulting in a method that is fast, yet finds visually pleasing ellipses of arbitrary orientation and displacement.

Application to surfaces

Note that while this discussion was in terms of 2D curves, much of this logic also extends to 3D surfaces, each patch of which is defined by a net of curves in two parametric directions, typically called **u** and **v**. A surface may be composed of one or more surface patches in each direction.

APPLICATIONS OF CURVE FITTING

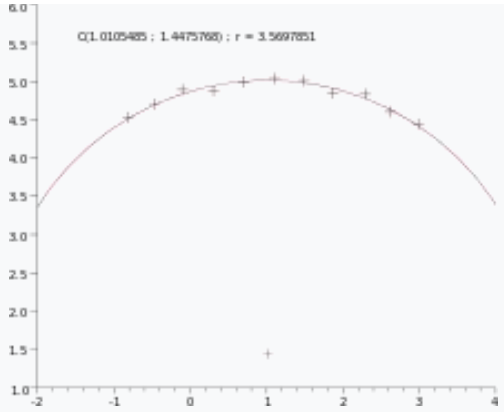
Fitting plane curves to data points :

If a function of the form cannot be postulated, one can still try to fit a plane curve.

Other types of curves, such as conic sections (circular, elliptical, parabolic, and hyperbolic arcs) or trigonometric functions (such as sine and cosine), may also be used, in certain cases. For example, trajectories of objects under the influence of gravity follow a parabolic path, when air resistance is ignored. Hence, matching trajectory data points to a parabolic curve would make sense. Tides follow sinusoidal patterns, hence tidal data points should be matched to a sine wave, or the sum of two sine waves of different periods, if the effects of the Moon and Sun are both considered.

For a parametric curve, it is effective to fit each of its coordinates as a separate function of arc length; assuming that data points can be ordered, the chord distance may be used.

Fitting a circle by geometric fit



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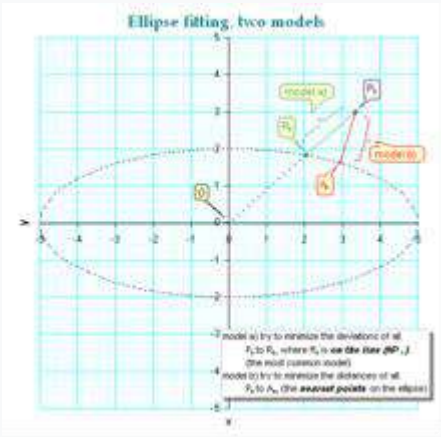
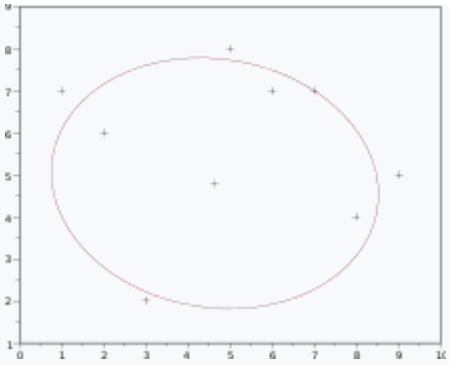


different models of ellipse fitting



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Fitting an ellipse by geometric fit

The above technique is extended to general ellipses by adding a non-linear step, resulting in a method that is fast, yet finds visually pleasing ellipses of arbitrary orientation and displacement.

Application to surfaces

Further information: Computer representation of surfaces

Note that while this discussion was in terms of 2D curves, much of this logic also extends to 3D surfaces, each patch of which is defined by a net of curves in two parametric directions, typically called **u** and **v**. A surface may be composed of one or more surface patches in each direction.

A STUDY ON

APPLICATIONS OF NUMERICAL ANALYSIS

A Group Project submitted to KRISHNA UNIVERSITY

KRISHNA UNIVERSITY

In partial fulfilment of the requirement For the award of the degree of

BACHELOR OF SCIENCE

Submitted by

G.RAJESH 18303256

UNDER THE GUIDENCE OF

SK.PARVEEN

LECTURER IN MATHEMATICS.

S.R.R & C.V.R GOVT DEGREE COLLEGE.

MACHAVARAM,VIJAYAWADA-520004

2018-2021

S.R.R & C.V.R GOVT. DEGREE COLLEGE (AUTONOMUS)

CERTIFICATE

This is to certificate that the project entitled "APPLICATIONS OF NUMERICAL ANALYSIS" bonafied workdone by G.RAJESH(18303256) belongs to department of bachelor science studies,S.R.R & C.V.R GOVT.DEGREE COLLEGE VIJAYAWADA under my guidance in partial fulfilment of the award for the degree in bachelor of science

K.V.NAGALAKSHMI SK.PARVEEN

EXTERNAL EXAMINAR (H.O.D Of Mathematics) (PROJECT GUIDE)

DECLARATION

I **G.RAJESH (18303256)** here by declare that the project entitled **"APPLICATIONS OF NUMERICAL ANALYSIS "**has been prepared by in a partial fulfilment of the requirement for the award of the degree of bachelor of science submitted at **S.R.R & C.V.R GOVT DEGREE COLLEGE** affiliated to **KRISHNA UNIVERSITY i**s an authentic project record of our own work .

The matter presented as not be submitted to any University or Institute for the award of bachelor of science Degree .

G.RAJESH 18303256

ACKNOWLEDGMENT

We would like to convey my deep felt regards to all of my well wishers,

Who have contributed their share of cooperation and help me to complete this project work

.

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G.RAJESH 18303256

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1.Interpolation With Equal Intervals

1.1 INTRODUCTION :

Interpolation means insertion or filling up intermediate terms of the series. It is the technique of getting the value of a function for any intermediate value of the argument from the given set of values of function is called interpolation, while the process of computing the value of the function outside of the given range is called interpolation. The study of interpolation is based on the calculus of finite differences which
The following assumptions are to be kept in mind for interpolation.
(1) The values of the function should be either increasing or decreasing
(2) The rise or fall in the values should be uniform plays a vital role in numerical analysis manner.

1.2 INTERPOLATION METHODS :

(i) Graphical method
(i) Method of curve fitting
(iii) Applications of the calculus of finite differences formula.

1.2.1 Graphical Method :

Let y =f(x), then plot a graph for different values of x corresponding to the value of y. From the graph, we can find the value of y for given value of x.
For example

Year(x)	1891	1901	1911	1921	1931
Population (Y)(in 1000's)	46	66	81	93	101

Interpolate population for the year 1919.
The following steps are to be maintained
1. Take a suitable scale for x and y plot the different points on the graph for given data.
2. Join the point by hand curve.

3. Find the point on the curve corresponding to x = 1919 and find corresponding value of y

Remarks : This is simply an approximate method of evaluating the value of y and is not reliable in number of cases.

1.2.2 Method of Curve Fitting :

This method is applicable in some cases in which the form of the function is known. In such cases, by the method of least squares, we can fit the curve of known form to given set of values

and afterwards we can find the unknown value with the help of fitted curve.

Remarks :

(1) This is not exact method.

(2) In case of number of values are sufficiently large this method is complicated

(3) The form of the function in this method for given set of observations is assumed to be known.

1.2.3 Applications of the Calculus of Finite Differences :

(1) Newton-Gregory formula for interpolation. If n is a integer and h is the interval differencing, then $f(a+ nh) = f(a)+{}^{n}c_1 \Delta f(a)+ {}^{n}c_2\Delta^2 f(a)+....+ {}^{n}c_n \Delta^{n}{}_{f(a)}$

We prove the theorem by using the method of induction.

we know that $\Delta f(a) = f(a + h) - f(a)$

$f(a+h)= f(a) +\Delta f(a)$

The theorem is true for n = 1.

Assume that the theorem is true for n=k.

$f(a+kh)=f(a)+{}^{k}c_1 \Delta^2 f(a)+....+{}^{k}c_n \Delta^{k} f(a)$

We have $\Delta f(a+kh) = f(a+(k+ 1)h)-f(a+kh)$

$f(a + (k + 1)h) = f(a+kh) + h + \Delta f(a + kh)$

$= [f(a)+{}^{k}c_1\Delta f(a)+ {}^{k}c_2\Delta^{r}f(a)+....+ \Delta^{k}f(a)]$

$[\Delta^{r}f(a)+{}^{k}c_1\Delta^{r}f(a)+ {}^{k}c_1\Delta^{3}f(a)+.....+ \Delta^{k+1}f(a)]$

$= f(a)+({}^{k}c_1+1) \Delta f(a)+({}^{k}c_1+ {}^{k}c_2) \Delta^2 f(a)+.....+ \Delta^{k+1}f(a)$

$= f(a)+{}^{k+1}c_1+ \Delta f(a)+{}^{k+1}c_2 \Delta^2 f(a)+.....+ \Delta^{k+1}f(a)$

The theorem is true for n=k+1

By the method of induction, the theorem is true for any integer.

$f(a+ nh) = f(a)+{}^{n}c_1 \Delta f(a)+ {}^{n}c_2\Delta^2 f(a)+....+ {}^{n}c_n \Delta^{n}{}_{f(a)}$

Note :

1. $f(a+nh) = f(a)+{}^{n}c_1\,\Delta f(a)+ {}^{n}c_2\Delta^2 f(a)+....+ {}^{n}c_n\,\Delta^{n}{}_{f(a)}$

2. Put a=0, h=1

$f(h)=f(0)+{}^{n}c_1\,\Delta f(0)+{}^{n}c_2+ {}^{n}c_2\,\Delta^2 f(0)+\ldots\ldots$

$f(x)=f(0)+\frac{x}{1!}\,\Delta f(0)+ \frac{x(x-1)}{2!}\,\Delta^2 f(0)+\ldots\ldots.$

This formula is used only when the first term a =0 and h =1.

Alter : Let y=f(x) represent a function which assumes the values f(a),f(a+h)....f(a+nh) for (n +1) equidistant values a, a+ h, a +2h, a+nh of the independent variable x and let $P_n(x)$ be a polynomial in x of degree n.

This polynomial can be expressed in the form.

$P_n(x) = Bo +B\,(x-a) + B2(x-a)(x-a-h)+\ldots\ldots\ldots+ B_n(x-a)(x-a-h)(x-a-2h)....(x-a-(n-1)h)$ ....(1)

where B_o. B_1, B_n are constants and are choosen such that

$P_n(a) = f(a)$, $P_n(a + h) = f(a + h)$, . . . $P_n(a + nh) = f(a + nh)$

By substituting successively the values a, a+ h, a +2h . a+nh in equation(1) and writing the values of $P_n(a)$, $P_n(a+h)$... $P(a + nh)$, we get $f(a)= B_0$ B1=f(a + h)-f(a)

$f(a) = B_0$, $f(a + h) = B_0+ hB_1$

B_0 =f(a) or $B_1 = \frac{f(a+h)-f(a)}{h} = \frac{\Delta f(a)}{h}$

$f(a + 2h) = B_0 + 2hB_1 + h^rB_2$ (or)

A2 = $f(a + 2h)-2\{f(a+ h)-f(a)\}-f(a)/2h^2$

$= \frac{1}{2!h^r}\,\Delta^2 f(a)$

Similarly, we get $B_3 = \frac{1}{3!h^3}\,\Delta^3 f(a)$..... $B_n = \frac{1}{n!h^n}\,\Delta^n f(a)$

Substituting the values of B_0, B_1,....., B_n, we get

$P_n(x) = f(a) +\frac{\Delta f(a)}{h}\,(x-a)+ \frac{\Delta^2 \mathbf{f(a)}}{\mathbf{2!}\boldsymbol{h^r}}\,(x-a)(x-a-h)+....+ \frac{\Delta^n \mathbf{f(a)}}{\boldsymbol{n!h^n}}\,(x-a)(x-a-h)...(x-a-(n-1)h)$

This is Newton-Gregory formula for forward interpolation putting $\frac{x-a}{h}$ = u or x=a+hu then(1) can be expressed as

$P_n(x)=P_n(a+nh) = f(a)+u\Delta f(a)+\frac{u(u-1)}{2!}\,\Delta^2 f(a)+....+\frac{u(u-1).....(u-n+1)}{n!}\,\Delta^n f(a)$

Newton-Gregory Formula for Backward Interpolation

In this case , we write

$P_n(x) = C_0 +C_1\,(x-a-nh)+ C_2(x-a-nh)+....+C_n(x-a-nh)(\,x-a-nh+h).....(x-a-h)$ .....(1)

where Co, C_1 ,$C_{2...}$ C_n are constants and are choosen such that $f_{n(}a+ nh) = f(a+ nh)$,.., $P_n(a) = f(a)$

Substituting in (1), x=a + nh, a + nh-h, we have

$C_0 = f(a + nh)$, $f(a + nh-h) = C_o + C_1\,(-h)$

$C_1 = \frac{f(a + nh)-f(a + nh-h)}{h} = \frac{\Delta f(a + nh)}{h}$

Similarly $C_2 = \frac{1}{2!h^r}\,\nabla^2 f(a+nh)$

$C_3 = \frac{1}{3!h^3} \nabla^3 f(a+nh)$

……

$C_n = \frac{1}{n!h^n} \nabla^n f(a+nh)$

Substituting these values of $C_o, C_1, \ldots, C_n$ in (1), we obtain

$P_n(x) = f(a+nh) + \frac{\nabla f(a+nh)}{1!h}(x-a+nh) + \frac{\nabla^2 f(a+nh))}{2!h}(x-a+nh)\,(x-(a+nh-h)) + \ldots + \frac{(\nabla^n f(a+nh)}{n!h^n}(x-a+nh)\,(x-(a+nh-h))\ldots\ldots(x-(a+h))$ ……(2)

This is Newton-Gregory formula for backward interpolation. Substituting $n = \frac{x-(a+nh)}{h}$ or

$x = a + nh + hl$ in eq. (2), then we obtain $P_n(x) = P_n(a + nh + h) = f(a + nh) + u\nabla f(a+nh) + \frac{u(u+1)}{2!}\nabla^2 f(a+nh) + \ldots\ldots\ldots + \frac{u(u+1)\ldots\ldots(u+n-1)}{n!}\nabla^n f(a+nh)$ Where $P_n(x) = f(x)$.

1.3 SOLVED PROBLEMS

1. Given log x for x = 40, 45, 50, 55, 60, 65 for the following data :

x	40	45	50	55	60	65
logx	1.60206	1.65321	1.69877	1.74036	1.7785	1.81291

Estimate the value of log (58.75) by using Newton backward interpolation formula.

Solution.

X	10^5xlogx=f(x)	10^5x∇f(x)	10^5x∇^2f(x)	10^5x∇^3f(x)	10^5x∇^4f(x)
40	160206				
		5115			
45	165321		-539		
		4516		102	
50	169892		-437		-25
		4319		77	
55	174036		-360		-20
		3729		57	
60	177815		-330		
		3476			
65	181291				

$(a + nh) + uh = 58.75$

$65 + 5u = 58.75$

$u = 1.25$

Newton backward interpolation formula

$f(a+nh)+nh) = f(a+nh) + \frac{h}{1!}\Delta f(a+nh) + \frac{u(u+1)}{2!}\Delta^r f(a+nh) + \frac{u(u+1)(u+2)}{3!}\Delta^r f(a+nh)$

$10^5 f(58.75) = 181291 - 1.25x(3476) + \frac{(-1.25)(-1.25u)x(-30.5)}{2!} + \frac{(-1.25)(-1.25+1)x(-1.25+2)}{6}x(5)$

$= 181291 - 4345 - 47.3438 + 2.2267 = 176900.89$

$F(58.75) = 1.7690089$

2. Find the cubic polynomial for the following data hence find f(u)

X	0	1	2	3
F(x)	-2	1	4	6

Solution :

x	**F(x)**	Δf(x)	Δ^2**f(x)**	Δ^3**f(x)**
0	-2			
		3		
1	1		0	-1
		3		
2	4		-1	
		2		
3	6			

Newton's Gregory's formula is

$f(x) = f(0) + \frac{x}{1!}\Delta f(0) + \frac{x(x-1)}{2!}\Delta^2 f(0) + \frac{x(x-1)(x-2)}{3!}\Delta^3 f(0)$

$= -2 + \frac{x}{1!}(3) + \frac{x(x-1)}{2!}(0) + \frac{x(x-1)(x-2)}{3!}(-1)$

$= -2 + 3x - \frac{(x^3+3x^2+2x)}{6} = \frac{-12-18x-x^3+3x^2-2x}{6}$

$= \frac{x^3+3x^2+16x-12}{6} = \frac{-(4)^3+3(4)^2+16(4)-12}{6}$

$= \frac{-64+48+64-12}{6} = \frac{36}{6} = 6$

$f(4) = 6$

3. Find f(1.5) by using Newton's advancing difference formula given that f(o)= +2, f(1)=4, f(2)=7, f(3)=8

x	F(x)	$\Delta f(x)$	Δ^2**f(x)**	Δ^3**f(x)**
0	**2**			
		2		
1	4		1	
		3		**-3**
2	7		-2	
		1		

3	8			

Solution. Newton's advancing formula is

$$f(x) = f(0)+\frac{x}{1!}\Delta f(0)+\frac{x(x-1)}{2!}\Delta^2 f(0)+\frac{x(x-1)(x-2)}{3!}\Delta^3 f(0)$$

$$= 2+\frac{x}{1!}(2)+\frac{x(x-1)}{2!}(1)+\frac{x(x-1)(x-2)}{3!}(-3)$$

$$= 2+2x+\frac{x^2-x}{2!}+\frac{x^3-3x^2+2x}{2!}(-1)$$

$$= \frac{4+4x+x^2-x-x^3+3x^2-2x}{2}$$

$$F(x) = \frac{-(1.5)^3+4(1.5)^2+(1.5)+4}{2}$$

$$= \frac{-3.375+9+1.5+4}{2} = \frac{11-125}{2} = 5.5625$$

$$f(1.5) = 5.5625$$

4. Find the polynomial and hence find f(5) from the following data by using Newton's Gregory's formula.

x	**0**	**1**	**2**	**3**	**4**
F(x)	**0**	**7**	**26**	**63**	**124**

Solution.

X	F(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	0				
		7			
1	7		12		
		19		6	
2	26		18		0
		37		6	
3	63		24		
		61			
4	124				

$$F(x) = f(0)+\frac{x}{1!}\Delta f(0)+\frac{x(x-1)}{2!}\Delta^2 f(0)+\frac{x(x-1)(x-2)}{3!}\Delta^3 f(0)$$

$$= 0+\frac{x}{1!}(7)+\frac{x(x-1)}{2!}(12)+\frac{x(x-1)(x-2)}{3!}(6)$$

$$= 7x+6x^2-6x+x^3-3x^2+2x = x^3+3x^2+7x-4x$$

$$F(x) = x^3-3x^2+2x$$

$$F(5) = 5^3+3(5)^2+3(5) = 125+75+15 = 215$$

$$F(5) = 215$$

5. Find f from the given data by using Newton's Gregory's formula

X	10	20	30	40
F(x)	9	30	35	42

Solution .

X	F(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
10	9			
		21		
20	30		-16	
		5		18
30	35		2	
		7		
40	42			

a=10 , h= 10

Let a+uh=1.5

10+10u = 1.5

$u = \frac{5}{10} = \frac{1}{2}$

Newton's Gregory's formula is

$F(a+uh) = f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)$

$F(52) = 0.7071+\frac{7}{5}(0.0589)+\frac{\frac{7}{5}(\frac{7}{5}-1)}{2!}(-0.0057)+\frac{\frac{7}{5}(\frac{7}{5}-1)(\frac{7}{5}-2)}{3!}(-0.0007)$

= 7071+824.6-15.96+0.392 = 7880

$10^4 f(52) = 7880$

$F(52) = \frac{7880}{10^4} = 0.788 \qquad \sin 52^0 = 0.7880$

7. Using Newton's formula for interpolation estimate population for the year 1905 from the table.

Year (x)	Population(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1891	98752				
		33533			
1901	132285		2258		
		35791		-10435	
1911	168076		-8177		41358
		27614		30923	
1921	195690		22746		
		50360			
1931	246050				

Solution.

a = 1891, h = 10

Let a+ uh = 1905

u(10) = 1905-1891 = 14

$u = \frac{14}{10} = \frac{7}{6}$

Newton's Gregory's formula is

$$f(a+uh) = f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)+\frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(a)$$

$$f(1905) = 98752+\frac{7}{5}(33533)+\frac{\frac{7}{5}(\frac{7}{5}-1)}{2}(2258)+\frac{\frac{7}{5}(\frac{7}{5}-1)(\frac{7}{5}-2)}{6}(-10435)+\frac{\frac{7}{5}(\frac{7}{5}-1)(\frac{7}{5}-2)(\frac{7}{5}-3)}{24}(41358)$$

= 98,752 + 48946·2+ 632.24 + 584.36 + 926.4192

= 147841.2192

The population for the year 1905 is 14,784 approximately

8 . From the following table find the no. of students who obtained less than 45 marks.

x	30-40	40-50	50-60	60-70	70-80
F(x)	31	42	51	35	31

Solution .

Marks less than x	No.of students	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
40	31				
		42			
50	73		9		
		51		-25	
60	124		-16		37
		35		12	
70	159		-4		
		31			
80	190				

a=40,h=10

let a +uh = 45

u(10) = 45-40

$u = \frac{5}{10} = \frac{1}{2}$

Newton's advancing formula is

F(a+uh) = f(a)

$$+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)+\frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(a)$$

$$= 31+\frac{1}{2}(42)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(9)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(-25)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\left(\frac{1}{2}-3\right)}{4!}(37)$$

$$= 31+21-\frac{9}{8}+\left(-\frac{75}{48}\right)-\frac{15}{16}\left(\frac{37}{21}\right)$$

$$= 52-1.125-1.5625-1.4453125$$

$$= 47.8671875 = 47$$

: The no. of students getting less than 45 marks is 47 and the no. of students who getting more than 45 is 190-47-143

9. Find the no. of men getting the wages less than is rupees from the following :

0-10	10-20	20-30	30-40
9	30	35	42

Solution .

Wages x	No. of Men	$\Delta\mathbf{f(x)}$	$\Delta^2\mathbf{f(x)}$	$\Delta^3\mathbf{f(x)}$
0-10	9			
		21		
10-20	30		-16	
		5		18
20-30	35		2	
		7		
30-40	42			

Newton's advancing formula is

$$f(a+uh) = f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)$$

$$f(15) = 9+\frac{1}{2}(21)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(-16)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(18)$$

$$= 9+\frac{21}{2}+2+\frac{3}{8}\left(\frac{18}{16}\right) = 11+\frac{21}{2}+\frac{9}{8}$$

$$= 11+10.5+1.125- 22.625 = 23$$

11. Find the second degree polynomial from the data is then find f(5)

0	1	2	3	4
3	6	11	17	27

Solution .

X	F(x)	$\Delta\mathbf{f(x)}$	$\Delta^2\mathbf{f(x)}$	$\Delta^3\mathbf{f(x)}$
0	3			
		3		
1	6		2	
		5		0
2	11		2	

3 4	18 27	7 9	2	0

Newton's Gregory's formula is

$f(x) = f(0)+\frac{x}{1!}\Delta f(0)+\frac{x(x-1)}{2!}\Delta^2 f(0)$

a=0 , h=1

a+uh = 5 u =5

$f(x) = 3+x(3)+\frac{x(x-1)}{2}(2)$

$= 3+3x + x^2-x = x^2 + 2x + 3$

$f(5) = (5)^2 + 2(5) + 3=25+10+ 3 = 38.$

12. Second degree polynomial passes through the points (o, 1), (1, 3), (2, 7),(3, 13) then find the polynomial or find f(x). Also find the value of f(1.5).

Solution.

X	Y = f(x)	$\Delta\mathbf{f(x)}$	$\Delta^2\mathbf{f(x)}$	$\Delta^3\mathbf{f(x)}$
0 1 2 3	1 3 7 13	2 4 6	2 2	0

Newton's Gregory's formula is

$f(x) = f(0)+\frac{x}{1!}\Delta f(0)+\frac{x(x-1)}{2!}\Delta^2 f(0)$

$= 1+x(2)+\frac{x(x-1)}{2}(2)$

$f(x) = 1 + 2x + x^2-x$

$= x^2 + x + 1$

$f(1.5) = (1.5)^2+(1.5)+1$

$= 2.25+1.5+1 = 4.75$

$f(1.5) = 4.75$

13. Find the polynomial fits the data in the following table then also find f(13).

X	3	5	7	9	11
Y	6	24	58	109	174

solution. a=3, h = 2

Let a+uh = x = 13 3+2u = x

$$u = \frac{x-3}{2}$$

X	Y =f(x)	$\Delta \mathbf{f(x)}$	$\Delta^2 \mathbf{f(x)}$	$\Delta^3 \mathbf{f(x)}$	$\Delta^4 \mathbf{f(x)}$
3	6				
		18			
5	24		16		
		34		0	
7	58		16		0
		50		0	
9	109		16		
		66			
11	174				

Newton's Gregory's formula is

$$f(a+uh) = f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)$$

$$= 6+\frac{x-3}{1!}(18)+\frac{\left(\frac{x-3}{2}\right)\left(\frac{x-3}{2}-1\right)}{2!}(16)$$

$f(a+uh) = 6+9x-27+2(x-3)(x-5)=9x-21+2x^2-16x+30$

$f(x) = 2x^2-7x+9$

$f(13) = 2(13)^2-7(13)+9=2(169)-91+9=338+9-91$

$f(13) = 2.56$

14. Find the value of f(35) from the following table :

X	Y=f(x)	$\Delta \mathbf{f(x)}$	$\Delta^2 \mathbf{f(x)}$	$\Delta^3 \mathbf{f(x)}$
20	512			
		-73		
30	439		-20	
		-93		10
40	346		-10	
		-103		
50	243			

Solution .

Newton's advancing formula is

$$F(a+uh) = f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)+\frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(a)$$

a=20 , h=10 => a+uh=35a=30, h=10

20+10h=35 (or) 30+10u=35

h= $\frac{3}{2}$,u= $\frac{1}{2}$

f(35)=439+$\frac{1}{2}$(-93)+$\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}$(-10)+$\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}$(-10)

=439-$\frac{93}{2}$+$\frac{10}{8}$+$\frac{30}{48}$

=439-46.5+1,25+6.625=440.875-46.5=394.375

f(35)=394.

Central Difference Interpolation Formulae

1.3.1 INTRODUCTION :

In the previous chapter, we derived and discussed Newton forward and backward interpolation formula, which are applicable for interpolation, near the beginning and end respectively of tabulated data. We shall now discuss in the present chapter, about the central difference formula, which are suitable for interpolation near the middle of a tabulated data.

The central difference operator (δ) is defined as $\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$

The first central difference operator δf(x) defined by

δ f(x) =f(x+$\frac{h}{2}$) - f(x-$\frac{h}{2}$)

δ f(x) =($E^{\frac{1}{2}} - E^{-\frac{1}{2}}$) f(x)

We have E = 1 + Δ

$\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}} = E^{-\frac{1}{2}}(E-1) = E^{-\frac{1}{2}}\Delta$ ..... (1)

Again by the first backward difference of f(x)

We have ∇f(x) = f(x)-f(x-h) = f(x)-f(x-h) =E(x)- E^{-1}f(x)

∇ f(x) = (1- E^{-1})f(x)

∇ = (1- E^{-1})

$$\nabla E^{\frac{1}{2}} = E^{\frac{1}{2}} - E^{-\frac{1}{2}} = \delta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

From(1) and (2) $\delta = \Delta E^{-\frac{1}{2}} = \nabla E^{\frac{1}{2}}$ ……(3)

The average operator u is defined as

$$U = \frac{1}{2}[E^{\frac{1}{2}} + E^{-\frac{1}{2}}]$$

Results :

(1) $U^2 = 1 + \frac{1}{4}\delta^2 u^2 f(x) = \frac{1}{2}(E^{\frac{1}{2}} + E^{-\frac{1}{2}})^2$

$= \frac{1}{4}(E^{\frac{1}{2}} + E^{-\frac{1}{2}})^2 = \frac{1}{4}(E + E^{-1} + 2) = \frac{1}{4}(E + E^{-1} - 2 + 4)$

$= \frac{(u+k-1)^{(2k+1)}}{(2k+1)!} = -\frac{(k-v)^{(2k+1)}}{(2k+1)!}$

$= \frac{v(v^2-1)(v^2-4)\ldots\ldots((v^2-k^2)}{(2k+1)!}$

From (3), we obtain

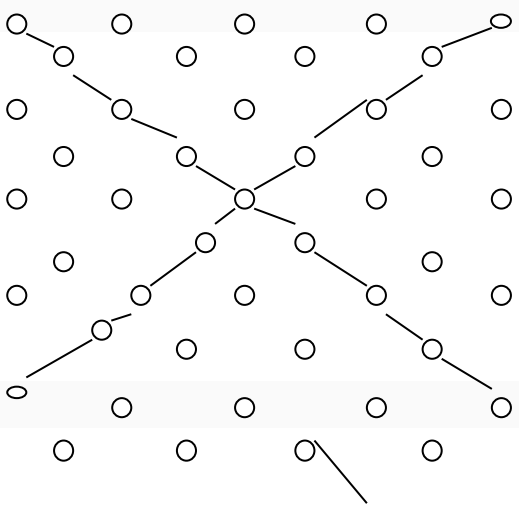
$y_u = vy_o + \frac{v(v^2-1)}{3!}\Delta^2 y_{-1} + \frac{v(v^2-1)(v^2-4)}{5!}\Delta^4 y_{-2} + \ldots\ldots uy_1 + \frac{u(u^2-1)}{3!}\Delta^2 y_0 + \frac{u(u^2-1)(u^2-4)}{5!}\Delta^4 y_{-2}$

$+\ldots\ldots\ldots + \frac{u(u^2-1)(u^2-4)(u^2-k^2)}{(2k+1)!}\Delta^{2k} y_{1-k} + \ldots\ldots\ldots\ldots$ ……..(5)

this can be written as

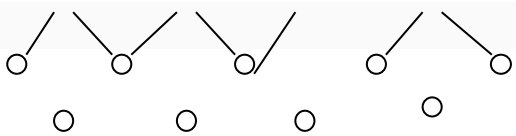
$f(u) = vf(0) + \frac{v(v^2-1)}{3!}\delta^2 f(0) + \frac{v(v^2-1)(v^2-4)}{5!}\delta^2 f(0) + \ldots\ldots + uf(1) + \frac{u(u^2-1)}{3!}\delta^2 f(1) + \frac{u(u^2-1)(u^2-4)}{5!}\delta^2 f(1) + \ldots$

this is known as Laplace-Everette's formula

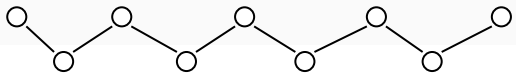


Newtons backward formula

Newtons backward formula

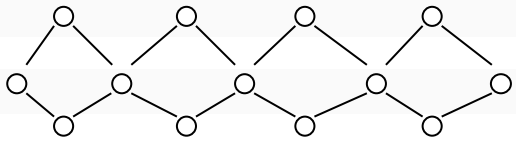


Gauss backward formula

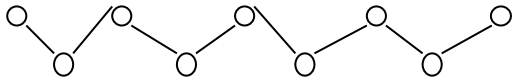


Gauss forward formula

Stirling's formula



Bessel's formula



Note 1: This formula is useful especially when using the tables in which only even order difference are tabulated.

All the difference formula can be explained with the help of following diagram.

Results : (1) $\delta = \Delta E^{-\frac{1}{2}} = \nabla E^{\frac{1}{2}}$

(2) $\Delta\nabla = \nabla\Delta = \Delta - \nabla = \delta^2$

(3) u= $\delta = \frac{1}{2}(\nabla + \Delta) = \frac{1}{2}\Delta E^{-1} + \frac{1}{2}\Delta$ (4) $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$)

Proof :

(1) $\delta = \Delta E^{-\frac{1}{2}} = \nabla E^{\frac{1}{2}}$

$\delta f(x) = f(x+\frac{h}{2}) - f(x-\frac{h}{2}) = \nabla f(x+\frac{h}{2}) = \nabla E^{\frac{1}{2}} f(x)$

$\delta = \nabla E^{\frac{1}{2}}$

(2) (a) $\Delta\nabla f(x) = \Delta\Delta E^{-1}f(x) = \Delta^2 E^{-1}f(x)$

(b) $\Delta\nabla f(x) = \Delta E^{-1}f(x) = \Delta^2 E^{-1}f(x)$

(c) $(\Delta - \nabla)f(x) = (\Delta - \nabla E^{-1})f(x) = \Delta(1-\frac{1}{E})f(x)$

$= \frac{\Delta(E-1)}{E} f(x) = \Delta.\Delta E^{-1}f(x) = \Delta^2 E^{-1}f(x)$

(d) $\delta^2 f(x) = (\Delta E^{-\frac{1}{2}})^2 f(x) = \Delta^2 E^{-1}f(x)$

$\Delta\nabla = \nabla\Delta = \Delta - \nabla = \delta^2$

(3) $u\delta f(x) = \frac{(E^{\frac{1}{2}}+E^{-\frac{1}{2})}}{2}(E^{\frac{1}{2}}-E^{-\frac{1}{2}})f(x)$

$= \frac{1}{2}(e-e^{-1})f(x) = \frac{1}{2}(e-\frac{1}{E})f(x) = \frac{1}{2}(E^2-\frac{1}{E})f(x)$

$= \frac{(E-1)(E+1)}{2E}f(x) = \frac{\Delta}{2}(1+\frac{1}{E})f(x)$

We know that $\nabla = \Delta E^{-1} = \frac{1}{2}(\Delta+\nabla)f(x) = \frac{1}{2}(\Delta+\nabla E^{-1})f(x)$

$= \frac{1}{2}\nabla E^{-1} + \frac{1}{2}\Delta$

$U\delta = \frac{1}{2}(\Delta+\nabla E^{-1})$

(4) $\Delta+\nabla = \frac{\Delta}{\nabla}-\frac{\nabla}{\Delta}$

$(\frac{\Delta}{\nabla}-\frac{\nabla}{\Delta})f(x) = (\frac{\Delta}{\Delta E^{-1}}-\frac{\Delta E^{-1}}{\Delta})f(x) = (E-\frac{1}{E})f(x) = (\frac{E^2 1}{E})f(x)$

$= (E-1)\frac{(E+1)}{E}f(x) = \Delta(1+E^{-1})f(x) =$

$(\Delta+\nabla E^{-1})f(x)$

$= (\Delta+\nabla)f(x)$

$(\frac{\Delta}{\nabla}-\frac{\nabla}{\Delta}) = \Delta+\nabla$

1.4 SOLVED PROBLEMS

1. Using Gauss forward formula find the value of y when x = 0.5 from the following table.

X	**-2**	**-1**	**0**	**1**	**2**
F(x)	**22**	**30**	**82**	**106**	**216**

Solution .

X	**F(x)**	$\delta f(x)$	$\delta^2 f(x)$	$\delta^3 f(x)$	$\delta^4 f(x)$
-2	22				
		8			
-1	30		44		
		52		-72	
0	82		-28		186
		24		114	
1	106		86		
		110			
2	216				

Gauss forward interpolation formula is

$f(x) = f(0) + \frac{x}{1!}\delta f(1/2) + \frac{x(x-1)}{2!}\delta^2 f(0) + \frac{x(x-1)(x+1)}{3!}\delta^3 f(1/2) + \frac{x(x-1)(x+1)(x-2)}{4!}\delta^4 f(0)$

$f(x) = 82 + \frac{x}{1!}(24) + \frac{x(x-1)}{2!}(-28) + \frac{x(x-1)(x+1)}{6}(114) + \frac{x(x-1)(x+1)(x-2)}{24}(186)$

$f(0.5) = 82 + (0.5)(24) + \frac{(0.5)(0.5-1)}{2}(-28)$

$+ \frac{(0.5)(0.5-1)(0.5+1)}{6}(114) + \frac{(0.5)(0.5-1)(0.5+1)(0.5-2)}{24}(186)$

$= 82 + 12 + \frac{7}{2} - \frac{57}{24} + 4.359 = 94 + 4.359 + 3.5 - 7.125$

$f(1/2) = 94.734$

2. Using Gauss backward interpretation formula find the value at y when x=0.5[0,-1,+1,-2,+2]

Solution.

x	$f(x)$	$\delta f(x)$	$\delta^2 f(x)$	$\delta^3 f(x)$	$\delta^4 f(x)$
-2	22				
		8			
-1	30		44		
		52		-72	
0	82		-28		186
		24		114	
1	106		86		
		110			
2	216				

Gauss backward interpolation formula is

$F(x) = f(0) + \frac{x}{1!}\delta f(-1/2) + \frac{x(x+1)}{2!}\delta^2 f(0) + \frac{x(x-1)(x+1)}{3!}\delta^2 f(-1/2) + \frac{x(x-1)(x+1)(x+2)}{4!}(186)$

$f(0.5) = f(\frac{1}{2}) = 82 + (0.5)(52) + \frac{(0.5)(0.5+1)}{2}(28) + \frac{(0.5)(0.5+1)(0.5-1)}{6}(-72) +$

$\frac{(0.5)(0.5+1)(0.5-1)(0.5+2)}{24}(186)$

$= 82 + 26 - 10.5 + 4.5 - 7.2656 = 112.5 - 17.7656$

$f(0.5) = 94.7344$

a. Using Gauss backward interpolation formula and the value of f(-1/2) from the following data

X	F(x)	$\delta f(x)$	$\delta^2 f(x)$	$\delta^3 f(x)$	$\delta^4 f(x)$
-2	22				
		8			
-1	30		14		
		22		18	
0	52		32		6
		54		24	
1	106		56		
		110			
2	216				

Solution. Gauss backward interpolation formula is

F(x) = f(0)+ $\frac{x}{1!}\delta f$(-1/2)+ $\frac{x(x+1)}{2!}$ δ^2f(0) +$\frac{x(x-1)(x+1)}{3!}$ δ^2f(-1/2)+ $\frac{x(x-1)(x+1)(x+2)}{4!}$ δ^4f(0)

= 52+$\frac{x}{1!}$(22)+ $\frac{x(x+1)}{2}$(32)+ $\frac{x(x^2-1)}{6}$(18)+ $\frac{x(x^2-1)(x+2)}{4!}$(16)

= 52+11+ $\frac{\frac{1}{2}(\frac{1}{2}+1)}{2}$(32)+ $\frac{\frac{1}{2}(\frac{1}{4}-1)}{6}$(18)+)+ $\frac{\frac{1}{2}(\frac{1}{4}-1)(\frac{1}{2}+2)}{24}$(16)

f(1/2) = 63+ 12-1.125-0.234375 = 75-1.359375 = 73.64

4. Apply Gauss forward formula to obtain f(33) given that

X	25	30	35	40
F(x)	0.2707	0.3027	0.3386	0.3794

Solution .

X	10^4 F(x)	$10^4\delta f(x)$	$10^4\delta^2 f(x)$	$10^4\ \delta^3 f(x)$
-1	2707			
		320		
0	3027		39	
		359		10
1	3386		49	
		408		
2	3794			

Gauss forward interpolation formula is

f (x) = f(0) +$\frac{x}{1!}\delta f$(1/2)+ $\frac{x(x-1)}{2!}$ δ^2f(0) +$\frac{x(x-1)(x+1)}{3!}$ δ^3f(1/2)

let us take a = 30 as origin h= 5

put a+uh = 32 30+5u = 32 => u = $\frac{2}{5}$

$$10^4 f(2/5) = 3027 + \frac{2}{5}(359) + \frac{\frac{2}{5}(\frac{2}{5}-1)}{2}(39) + \frac{\frac{2}{5}(\frac{2}{5}-1)(\frac{2}{5}+1)}{6}(10)$$

$$10^4 f(2/5) = 3027+143.6-4.68-0.045 = 3170.6-4.105 = 3165.36$$

$$f(\frac{2}{5}) = 0.3165$$

f(x) = 0.3165 when x =32

5. Use Gauss forward formula to find the value of y when y=3.75 from the following table.

X	2.5	3.0	3.5	4.0	4.5	5.0
Y	24.145	22.043	20.225	18.644	17.262	16.047

Solution. Taking 3.2 as the origin and 0.5 as the unit, the value of y required will be value for

$$u = \frac{3.75-3.5}{0.5} = 0.5$$

U	$\mathbf{Y_u}$	$\boldsymbol{\delta Y_u}$	$\boldsymbol{\delta^2 Y_u}$	$\boldsymbol{\delta^3 Y_u}$	$\boldsymbol{\delta^4 Y_u}$	$\boldsymbol{\delta^5 Y_u}$
-2	24.145					
		-2.102				
-1	22.043		0.284			
		-1.818		-0.047		
0	20.225		0.237		0.009	
		-1.581		-0.038		-0.003
1	18.644		0.99		0.006	
		-1.382		-0.032		
2	17.262		0.167			
		-1.215				
3	16.047					

Using the Gauss forward formula we have

$$F(u) = f(0) + u\delta f(0) + \frac{u(u-1)}{2!}\delta f(1) + \frac{u(u-1)(u+1)}{6}\delta^2 f(-1/2) + \frac{u(u-1)(u+1)(u-2)}{24}\delta^4 f(1/2)$$

$$Y_5 = 20.225+5(-1.5881)+\frac{0.5(0.5-1)}{2}(0.237)+\frac{0.5(0.5-1)(0.5+1)}{6}(-0.38)+\frac{0.5(0.5-1)(0.5+1)(0.5-2)}{24}(0.009)$$

$$+\frac{0.5(0.5-1)(0.5+1)(0.5-2)(0.5+2)}{120}(-0.003)$$

$$= 20.225-0.7905-0.029625+ 0.00238 + 0.9023750 + 0.0002106$$

$$= 19.40 \text{ approx}$$

Thus value of y for x = 3.75 is 19.40 approximately.

6. Use Gauss forward formula to find y_{30} given that $y_{21} = 18.4708$, $y_{25} = 17.8144$, $y_{29} = 17.1070$, $y_{33} = 16.3432$, $y_{37} = 15.5154$.

X	21	25	29	33	37
Y	18.4708	17.8144	17.1070	16.3432	15.5154

Solution. Arrange the above data in the following form :

Take the origin x = 29 and h = u as the unit; then the value of y for u = $\frac{30-29}{4}$ = 0.25 is to be found. Construct the following difference table.

X	$u = \frac{x-29}{4}$	Y_u	δY_u	$\delta^2 Y_u$	$\delta^3 Y_u$	$\delta^4 Y_u$
21	-2	18.4708				
			-0.6564			
25	-1	17.8144		-0.0510		
			-0.7074		-0.0054	
29	0	17.1070		-0.0564		-
			-0.7638		-00076	0.0022
33	1	16.3432		-0.640		
			-0.8278			
37	2	15.5154`				

Now applying the gauss forward interpolation formula.

We get

f(0.4) = 17.1070+ (0.25) (-0.7638)+$\frac{(0.25)(-0.75)}{2}(-0.0564)+\frac{(1.25)(0.25)(-0.75)}{6}(-0.0076)+$

$\frac{(1.25)(0.25)(-0.75)(-1.75)}{24}(-0.0022)$

= 17.1070-0.19095+0.0052875+0.00002968-0.00000375

= 16.9216 approx.

7. Given that $\sqrt{12500}$ = 111.803399; $\sqrt{12510}$ = 111.848111; $\sqrt{12520}$ = 111.892806; $\sqrt{12530}$ = 111.937483 Show by Gauss's backward formula that $\sqrt{12516}$ = 111.874930

Solution. Taking 12520 as the origin h 10 as the unit, the value of y required is for

u = $\frac{12516-12520}{10}$ = -0.4

U	Y_u	δY_u	$\delta^2 Y_u$	$\delta^3 Y_u$
-2	111.803399			
		0.044712		
-1	111.848111		-0.00017	
		0.044695		-0.000001

0	111.892806		-0.000018	
		0.044677		
1	111.937483			

Now by using Gauss backward formula, we have

$y_{-0.4}$ = 111.8928+ (-0.4)(0.044695)-+$\frac{(0.6)(-0.4)}{2!}$(-0.000018) +

$\frac{(0.6)(-0.4)(-1.4)}{3!}$(-0.000001)

= 111.8928-0.017878 + negligible quantities = 111.874930

i.e., $\sqrt{12516}$ = 111.874930.

8. Apply Gauss forward formula to find the value of f(x) at x = 3.75 from the following table.

X	**2.5**	**3.0**	**3.5**	**4.0**	**4.5**
F(x)	**24.145**	**22.043**	**20.225**	**18.644**	**17.262**

Solution. Let us take a = 3.5 as origin, h = 0.5

Put a + uh = 3.75 => 3.5+ 0.5u = 3.75

0.5u = 0.25

u = 0.5 = ½

u	**10^3 f(u)**	**10^3 $\delta f(u)$**	**10^3 δ^2 f(u)**	**10^3 δ^3f(u)**	**10^3 δ^4 f(u)**
-2	24145	-2102			
			284		
-1	22043	-1818		-47	
			237		9
0	20225	-1581		-38	
			199		6
1	18644	-1382		-32	
			167		
2	17262	-1215			

$10^3 f(\frac{1}{2})$ = 20225+$\frac{1}{2}$(-1581)+$\frac{\frac{1}{2}(\frac{1}{2}-1)}{2}$(237)+ $\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}+1)}{6}$ (-38) +

$\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}+1)((\frac{1}{2}-2)}{24}$ (9)

= 20225-790.5-29.625+2.375+0.2709 = 19407.46

f($\frac{1}{2}$) = 19.408

f(x) = 19.408 when x= 375.

9**. Find the value of cos 51^0. 42' by Gauss backward formula given that**

X	$\mathbf{50^0}$	$\mathbf{51^0}$	$\mathbf{52^0}$	$\mathbf{53^0}$	$\mathbf{54^0}$
Cosx	**0.6428**	**0.6293**	**0.6157**	**0.6018**	**0.5878**

Solution : Let us take a =50^0 as origin h =1^0

Put a+uh = 51^042^1,52^0 +u = $51^042^|$ => u= -18$^|$ = $\frac{-18}{60}$ = 0.3, u = -0.3

u	$\mathbf{10^4}$ **f(u)**	$\mathbf{10^4}$ $\boldsymbol{\delta f(u)}$	$\mathbf{10^4}$ $\boldsymbol{\delta^2}$ **f(u)**	$\mathbf{10^4}$ $\boldsymbol{\delta}$**f(u)**	$\mathbf{10^4}$ $\boldsymbol{\delta^4}$ **f(u)**
-2	6428	-135			
-1	6293	-136	-1	-2	4
0	6157	-139	-3	2	
1	6018	-140	-1		
2	5878				

Gauss backward interpolation formula is

f(u) = f(0)+$\frac{u}{1!}$ $\boldsymbol{\delta}$ f($\frac{1}{2}$) +$\frac{u(u+1)}{2}$ $\boldsymbol{\delta^2}$ f(0)+ $\frac{u(u+1)(u-1)}{3!}$ δ^3(-$\frac{1}{2}$)+

$\frac{u(u+1)(u-1)(u+2)}{4!}$ δ^4f(0)

10^4f(-0.3) = 6157 + (-0.3)(-136)+$\frac{(-0.3)(0.7)}{2}$(-3) + $\frac{(-0.3)(0.7)(-1.3)}{6}$(-2) +

$\frac{(-0.3)(0.7)(-1.3)(1.7)}{24}$(2)

= 6157 + 40.8 + 0.315 -0,091 + 0.07735 = 6198.1013

f(-0.3) = 0.61981013

cos $57^042^|$ = 0.6198.

10. Find by Gauss backward formula the sales of the conceived by the year 1936.

Given that

Year	1901	1911	1921	1931	1941	1951
Sales	12	15	20	27	29	52

(in thousand of rupees)

Solution : let us take a = 1931 as origin

H = 10

a + uh = 1936 => 1931 + 10u = 1936 => 10u = 5

=> $U=\frac{1}{2}[0,-1,1,-2,2,-3]$

U	f(u)	$\delta f(u)$	$\delta^2 f(u)$	$\delta^3 f(u)$	$\delta^4 f(u)$
-3	12				
		3			
-2	15		2		
		5		0	
-1	20		2		3
		7		3	
0	27		5		-7
		12		-4	
1	39		1		
		13			
2	52				

Gauss backward interpolation formula is

$f(u = f(0) + \frac{u}{1!}\delta f(\frac{1}{2}) + \frac{u(u+1)}{2}\delta^{2}f(0) + \frac{u(u+1)(u-1)}{3!}\delta^3 f(\frac{1}{2}) +$

$\frac{u(u+1)(u-1)(u+2)}{4!}\delta^4 f(0$

$f(\frac{1}{2}) = 27 + (0.5)7 + \frac{(0.5)(0.5+1)}{2}(5) + \frac{(-0.5)(0.5+1)(0.5-1)}{6}(3) +$

$\frac{(-0.5)(0.5+1)(0.5-1)(0.5-1)}{24}(-7)$

= 27 + 3.5 + 1.875 -0.1875 + 0.2734 = 32.46 thousand 32.7

In the year 1936 = 32.7 thousands. Difference table:

X	$U=\frac{x-30}{5}$	Y_u	δy_u	$\delta^2 y_u$	$\delta^3 y_u$	$\delta^4 y_u$

20	-2	49225				
			-909			
25	-1	48316		-171		
			-1080		-59	
30	0	47236		-230		-21
			-1310		-80	
35	1	45926		-310		
			-1620			
40	2	44306				

Using Stirlings formula

Putting u = -0.4 we get difference

$y_{-0.4} = 47236+(-0.4)\frac{-1310-1080}{2}+\frac{0.16}{2}(-230)+\frac{(-0.4)(0.16-1)}{6}(\frac{80-59}{2})$

$+\frac{(0.16)(0.16-1)}{24}(-21)$

$= 47326+ 478 -18.4 – 3.8920 + 1176$

$= 47692$

$Y_{28} = 47692$

12. Apply Bessel's formula to obtain y_{25}, given $y_{20} = 2854$, $y_{24} = 3162$, $y_{32} = 3992$

Solution. take 24 as origin and 4 $u = \frac{25-24}{4} = \frac{1}{4} = 0.25$

Difference table is

X	$\mathbf{u = \frac{x-24}{4}}$	$\mathbf{Y_u}$	$\boldsymbol{\delta y_u}$	$\boldsymbol{\delta^2 y_u}$	$\boldsymbol{\delta^3 y_u}$
20	-1	2854			
			308		
24	0	3162		74	
			382		-8
28	1	3544		64	
			448		
32	2	3992			

Using bessel's formula, we get

$Y_{1/4} = \frac{1}{2}(3162+3544)+(\frac{1}{4}-\frac{1}{2})382+\frac{\frac{1}{4}(\frac{1}{4}-\frac{1}{2})}{2!}(\frac{14+66}{2})+\frac{(\frac{1}{4}-\frac{1}{2})(\frac{1}{4})(\frac{1}{4}-1)}{3!}(-8)$

$= 3353 – 95.5 – 6.5625 – 0.0625 = 3250.875.$

13.Given y_{20} =24 , y_{24} =32, y_{28} =35, y_{32} =40 find y_{25} by Bessel's formula.

Solution : Taking origin at x =24 and u as the unit.

Find y_u for u = $\frac{25-24}{4} = \frac{1}{4} = 0.25$

The difference table is

X	U = $\frac{x-24}{4}$	Y_u	δy_u	$\delta^2 y_u$	$\delta^3 y_u$
20	-1	24			
			8		
24	0	32		-5	
			3		7
28	1	35		2	
			5		
32	2	40			

By using Bessel's formula we have

$$y_{\frac{1}{4}} = \frac{32+35}{2} + (\frac{1}{4} - \frac{1}{2})3 + \frac{\frac{1}{4}(\frac{1}{4}-1)(\frac{-5+2}{2})}{2} + \frac{(\frac{1}{4}-\frac{1}{2})\frac{1}{4}(\frac{1}{4}-1)}{6}(7)$$

= 33.5-0.75+ 0.1406+0.54687 = 32.945287.

14.Given that

0	0^0	5^0	10^0	15^0	20^0	25^0	30^0
$\tan\theta$	0.0000	0.0875	0.1763	0.2679	0.3640	0.4663	0.5774

Show that $\tan 16^0$ = 0.2867 (use Stirilling's formula)

Solution : Taking 15^0 as origin and 5^0 as the unit , the value required will be for

$$U = \frac{16^0-15^0}{15^0} = 0.2^0$$

θ	U	Y_u	δy_u	$\delta^2 y_u$	$\delta^3 y_u$	$\delta^4 y_u$	$\delta^5 y_u$	$\delta^6 y_u$
0^0	-3	0.0000						
			0.0872					
5^0	-2	0.0875		0.0013				
			0.0888		0.0015			
10^0	-1	0.1763		0.0028		0.0002		
			0.916		0.0017		-0.0002	
15^0	0	0.2679		0.0045		0.0000		0.0011
			0.961		0.0017		0.0009	
20^0	1			0.0062		0.0009		
		0.3640	0.1023		0.0026			
25^0	2							

30^0	3	0.4663 0.5774	0.1111	0.0088				

By using Stirlings formula, we have

Tan 16^0 = $y_{0.2}$ = $0.2679+\frac{0.2}{2}(0.0961+0.0916)\{+\frac{0.04}{2}(0.0045)+\frac{0.2(0.4-1)}{2}\frac{1}{2}(0.0017+0.0017)+$

$\frac{0.4(0.4-1)}{24}(0.0000)+\frac{0.2(0.4-1)(0.04-1)}{120}\frac{1}{2}(-0.0002+0.0009)+$

$\frac{0.2(0.04-1)(0.04-4)}{720}(0.0011)$

= 0.2679 + 0.01877 + 0.00009 + negligible

= 0.2867.

1.5 Applications in engineering :

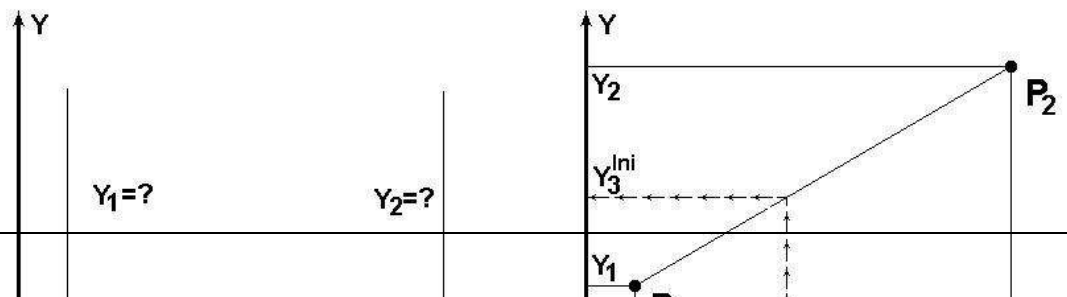
General Ideas Regarding the Use of the Interpolation in the Numerical Analysis

There are certain cases when approximation solutions are useful within the numerical methods field itself.

Some of these cases are presented in the next sections.

Approximation of the Initial Solution for Iterative Methods

Let us consider a theoretical problem where the formulation of the problem depends on some parameters and an iterative solution is possible. It is well known that the final solution depends on the initial approximation in terms of accuracy and of computer runtime.



Calculus of the Integrals :

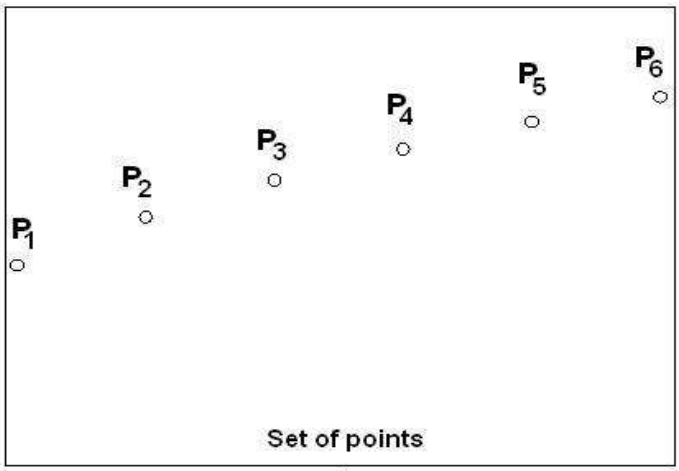
Let us consider that a set of points are known, that is $P_j\ x_j\ ,\ y_j\ ,\ j\ 1,\ N$. Usually these points result from experimental studies. There must be computed the integral of the function approximated by this set of points.

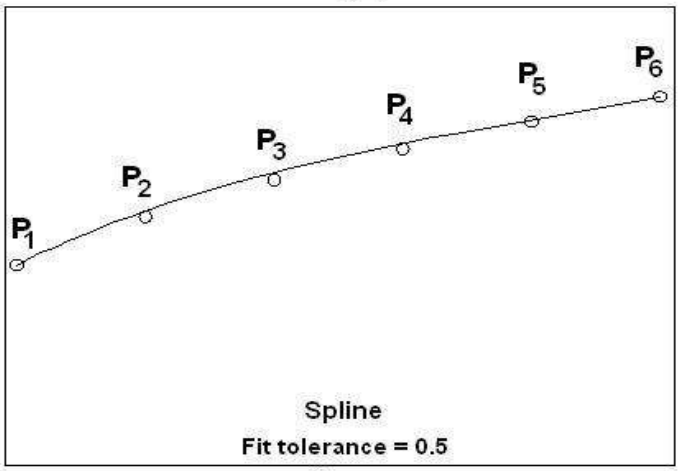
A first method to compute the integral is to use the summation of the areas of the trapezoids defined by two consecutive points. This is a coarse approximation, but if the points are close one to the other the accuracy may be considered satisfactory.

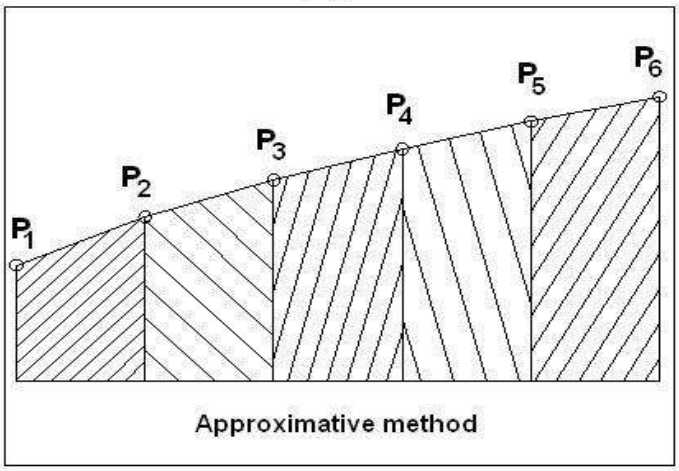
The second method has two stages. The first one is to approximate the set of points using one of the previously mentioned interpolation methods, and it results a set of equations like (6). The second stage is to analytically integrate the functions and to calculate the result. In his case the source of errors is in the interpolation stage.

A similar approach can be used to compute the derivative of the functions, either using the initial points, or using the appropriate interpolation functions.

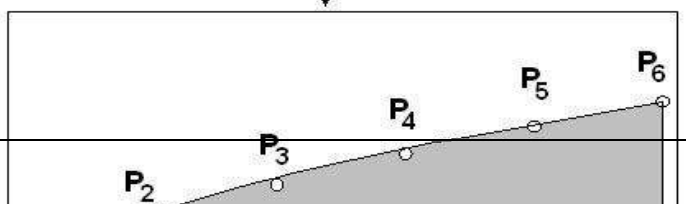
The error for each of the methods may be evaluated for a certain dataset. Basically, it is necessary to use as many methods exist, in order to have several results which, ideally, must be close one to the other.







$$\sum_{j=1}^{N} \underbrace{\frac{\left(Y_j + Y_{j+1}\right) \cdot \left(X_{j+1} - X_j\right)}{2}}_{Area\ A_j}$$



2. NUMERICAL INTEGRATION

2.1 INTRODUCTION :

The numerical integration is the process of evaluating the approximate

Value of the integration $\int_a^b f(x)dx$ from a set of numerical values of f(x). If the problem is to be applied to the integration of function of a single variable, then the process is called as quadrature.

The problem of numerical integration is evolved by first approximating the f(x) by a polynomial with the help of interpolation formula and then integrating this expression between the limits.

$\therefore$ To find the value of $\int_a^b f(x)\,dx$, first express the function f(x) by interpolation formula, say p(x) and then integrate between the limits.

$$\therefore \int_a^b f(x)dx = \int_a^b p(x)dx$$

The error is given by $\int_a^b f(x)dx - \int_a^b p(x)dx$

2.2 A GENERAL QUADRATURE FORMULA FOR NUMERICAL INTEGRATION

Consider the integral $\int_a^b f(x)dx$. Let f(x) be given for some equally spaced values of the orguments say ,x_0 , x_0+h ,x_0+2h,…. Let the range (b-a) be divided into n equal parts, each of which is of width h.

b-a = nh

h = $\frac{b-a}{n}$

let x_0 = a,x_1 = x_0+h = a+h , x_2 = a+2h ….. x_n = a+nh = b

I = $\int_a^{a+nh} f(x)dx$

Putting x = a+uh

dx = hdu

x -> a -> u ->0

x ->a+uh -> u->n

I = $\int_a^n f(a+uh)du$ = h $\int_a^n f(a+uh)du$

No write as f(a+uh) on the Newton's interpolation formula as

= h $\int_0^n [f(a) + \frac{u}{1!}\Delta f(a) + \frac{u(u-1)}{2!}\Delta^2 f(a) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(a) + \cdots +$ $\frac{u(u-1)\ldots(u-n+1)}{n!}\Delta^n f(a)\ ]\, du$

= h [f(a).u + $\frac{u^2}{2}\Delta f(a)$+($\frac{u^3}{3} - \frac{u^2}{2}$) $\Delta^2 f(a)$+ $\frac{(\frac{u^4}{4} - \frac{3u^3}{3} + \frac{2u^2}{2})}{6}\Delta^3 f(a) + \cdots +]$

= h[nf(a) + $\frac{n^2}{2}\Delta f(a)$ + ($\frac{n^2}{6} - \frac{n^2}{4}$) $\Delta^2 f(a)$ +($\frac{n^4}{24} - \frac{n^3}{6} + \frac{n^2}{6}$)$\Delta^3 f(a)$ +]

$\int_a^b f(x)dx$ = h[ny_0 + $\frac{n^2}{2}$(y_1-y_0)+ ($\frac{n^3}{6} - \frac{n^2}{4}$)($y_2-2y_1$+$y_0$)+$\left(\frac{n^4}{24} - \frac{n^3}{6} + \frac{n^2}{6}\right)$

(y_3-3y_2+3y_1-y_0)+……..] …….(1)

$\int_a^b f(x)dx$ = $\int_a^{a+nh} f(x)dx$

This is known as general quadrature formula we deduce the following important quadratic formulas by taking n= 1,2,3….

2.3 TRAPEZOIDAL RULE :

Put n=1 eq(1) and neglect the second and higher order differences when n=1 , the argument are a, a+h and its corresponding functional values are y_0 and y_1 thus we have

$\int_a^{a+nh} f(x)dx$ = h[$y_0 + \frac{1}{2}(y_1 - y_0)$] = $\frac{h}{2}(y_0+y_1)$

$I_1 = \int_a^{a+nh} f(x)dx = \frac{h}{2}(y_0+y_1)$

$I_2 = \int_a^{a+nh} f(x)dx = \frac{h}{2}(y_0+y_1)$

$I_3 = \int_{a+2h}^{a+3h} f(x)dx = \frac{h}{2}(y_2+y_3)$

……..

……..

$I_n = \int_{a+(n-1)h}^{a+nh} f(x)dx = \frac{h}{2}(y_{n-1}+y_n)$

Adding these n integrals, we have

$\int_a^{a+nh=b} f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

This rule is called the trapezoidal rule.

2.4 SIMPSON'S $\frac{1}{3}$ RULE :

Put n=2 in the equation (1) and neglect the third and higher order difference. In this case the arguments are a, a+h, a+2h the corresponding entries are y_0,y_1 and y_2, thus we have

$\int_a^{a+2h} f(x)dx$ = h[$2y_0 + 2(y_1-y_0)+(\frac{y}{3}-1)(y_2-2y_1+y_0)$]

= $\frac{h}{3}[y_0+4y_1+y_2]$

Similarly $I_2 = \int_a^{a+2h} f(x)dx = \frac{h}{3}(y_0+4y_1+y_2)$

$I_3 = \int_{a+4h}^{a+6h} f(x)dx = \frac{h}{3}(y_4+4y_5+y_6)$

……..

……..

$I_n = \int_{a+(n-2)h}^{a+6h} f(x)dx = \frac{h}{3}(y_{n-2}+4y_{n-1}+y_n)$

Adding these integrals, we have $I = \int_a^b f(x)dx$

= $\frac{h}{3}[(y_0+y_n)+(y_1+y_3+y_5+\ldots+y_{n-1})+2(y_2+y_4+\ldots.+y_{n-2})]$

This is known as SIMPSON'S $\frac{1}{3}$ RULE

2.5 SIMPSON'S $\frac{3}{8}$ RULE :

Put n=3 in equation(1) and neglect fourth and higher order differences. In this case, the arguats are a, a+h, a+2h and a+3h and the corresponding fractional values are y_0, y_1, y_2 and y_3, thus we have

$\int_a^{a+3h} f(x)dx$ = n[$3y_0+\frac{9}{2}(y_1-y_0)+(\frac{9}{2}-\frac{9}{4})(y_2-2y_1+y_0)+(\frac{27}{8}-\frac{27}{6}-\frac{9}{6})(y_3-3y_2+3y_1-y_0)$]

= h[$3y_0+\frac{9}{2}(y_1-y_0)+(\frac{9}{2}-\frac{9}{4})(y_2-2y_1+y_0)+(\frac{27}{8}-\frac{27}{6}-\frac{9}{6})(y_3-3y_2+3y_1-y_0)$]

= h[$3y_0+\frac{9}{2}(y_1-y_0)+(\frac{9}{4})(y_2-2y_1+y_0)+(\frac{3}{8})(y_3-3y_2+3y_1-y_0)$]

$I_1 = \int_a^{a+3h} f(x)dx = \frac{3h}{8}(y_0+3y_1+3y_2+y_3)$

$I_2 = \int_{a+4h}^{a+6h} f(x)dx = \frac{3h}{8}(y_3+3y_4+3y_5+y_6)$

……..

……..

$I_n = \int_{a+(n-2)h}^{a+6h} f(x)dx = \frac{3h}{8}(y_{n-3}+3y_{n-2}+3y_{n-1}+y_n)$

Adding all these integrals we get

$\int_a^{a+nh} f(x)dx = \frac{3h}{8}[(y_0+y_n)+3(y_1+y_2+y_4+\ldots+y_{n-1})+2(y_3+y_6+\ldots.+y_{n-3})]$

This formulas is called the Simpsons' rule.

Note : Applying this formula, the number of sub-internals should be taken as multiple of three

2.6 WEDDLE'S RULE :

put n=6 in equation (1) and neglect the seventh and higher order differences. in this case the arguments are from a to a+6h and there will be only seven functional values, thus we get

$\int_a^{a+6h} f(x)dx$ = h[$6y_0+18\Delta y_0+27\Delta^2 y_0+24\Delta^3 y_0+\frac{123}{10}\Delta^4 y_0+\frac{33}{10}\Delta^5 y_0+\frac{41}{140}\Delta^6 y_0$]

Replacing $\frac{41}{140}\Delta^6 y_0$ by $\frac{3}{10}\Delta^6 y_0$,

Since the error involved is negligible.

Thus above equation can be expressed as

$\int_a^{a+6h} f(x)dx$ = h[$6y_0+18\Delta y_0+27\Delta^2 y_0+24\Delta^3 y_0+\frac{123}{10}\Delta^4 y_0+\frac{33}{10}\Delta^5 y_0+\frac{3}{10}\Delta^6 y_0$]

$= \frac{3h}{10}(y_0+5y_1+y_2+ 6y_3+y_4+5y_5+y_6)$

Similarly $\int_{a+6h}^{a+12h} f(x)dx = \frac{3h}{10}(y_6+5y_7+y_8+ 6y_9+y_{10}+5y_{11}+y_{12})$

…….

…….

$\int_{a+(n-6)h}^{a+nh} f(x)dx = \frac{3h}{10}(y_{n-6}+5y_{n-5}+y_{n-4}+ 6y_{n-3}+y_{n-2}+5y_{n-1}+y_n)$

Adding these integrals, we have

$\int_a^{a+nh} f(x)dx = \frac{3h}{10}(y_0+5y_1+y_2+ 6y_3+y_4+5y_5+2y_6+ 5y_7+y_8 +……)$

This formula is known as Weddle's rule.

Note:

1. This rule is more accurate than the Simpson's rule, but it requires atleast seven consecutive values of the function
2. This method is applicable when n is a multiple of 6.

2.7 ERROR IN QUADRATURE FORMULA :

The error terms in the quadrature formula is given by

$$E = \int_a^b y dx - \int_a^b p(x)dx$$

where P(x) is the polynomial representing the function y = f(x) in the interval [a, b].

(i) Error in Trapezoidal Rule : Expanding y = f(x), about $x = x_o$ by Taylor's series, we obtain

$$y = y_0 + (x-x_0)y_0 + \frac{(x-x0)^2}{2!} y_0^{11} + \ldots\ldots\ldots\ldots(1)$$

$$\int_{x_0}^{x_0+h} y\, dx = \int_{x_0}^{x_0+h} [y_0 + (x - x_0)y_0 + \frac{(x-x_0)^2}{2!} y_0 + \cdots]dx$$

$$= y_0 n + \frac{h^2}{2!} y_0 + \frac{h^2}{3!} y_0 + \ldots\ldots\ldots\ldots(2)$$

But area of the first trapezium in the interval $[x_o, x_1] = \frac{1}{2}h(y_0 + y_1)$ ….(3)

Then putting $x = x_o + h$ and $y = y_1$ in (1), we obtain

$$y_1 = y_0 + h\, y_0 + \frac{h^2}{2!} y_0 + \ldots\ldots\ldots\ldots$$

Putting this value of y, in (3), we have

$$B = \frac{1}{2} h[y_0 + y_0 + h\, y_0 + \frac{h^2}{2!} y_0 + \ldots\ldots\ldots] = h\, y_0 + \frac{h^2}{2!} y_0 + \frac{h^3}{2.2!} y_0 + \ldots\ldots \quad \ldots\ldots(4)$$

Hence the error in the interval $[x_o, x_1]$

$= \int_{x_0}^{x_1} y\,dx$ - B_1 = $[\frac{1}{3!} - \frac{1}{2.2!}]\ h^3y_0$+…… = $-\frac{h^3}{12}\ y_0$+…….

Principal part of the error in $[x_o,x_1]$ = $-\frac{h^3}{12}\ y_0$

And similarly principal part of the error in $[x_1, x_2]$ = $-\frac{h^3}{12}\ y_1$ etc.,

Thus, the total error = $-\frac{h^3}{12}\ [y_0 + y_1 + y_{n-1}]$ = E , say

Then assuming that y''(x) is the largest of the n quantities y_o , y_1 ,.. . ., y_{n-1} ,

we can say that

E< $-\frac{nh^3}{12}\ y^{11}(x)$ = $-\frac{(b-a)h^2}{12}\ y^{11}(x)$ as b-a = nh ……(5)

Thus the error in the trapezoidal rule is of the order h^2.

(ii) Error in Simpson's $\frac{1}{3}$ Rule : Expanding y = f(x) about x = x_o by Taylor's series, we have

$y = y_0 + (x-x_0)y_0 + \frac{(x-x0)^2}{2!}\ y_0$+……

$\int_{x_0}^{x^2} y\,dx = \int_{x_0}^{x_0+2h} [y_0 - (x - x_0) + \frac{(x-x_0)^2}{2!}\ y_{0+\cdots}]$ dr

$2hy_0 + \frac{4h^2}{2!}y_0 + \frac{8h^3}{3!}y_0 + \frac{16h^4}{4!}y_0 + \cdots + \frac{32h^5}{5!}y_0 + \cdots\cdots$ ……..(2)

and area over the first double strip by Simpson's 1/3 rule

= $\frac{1}{3}\ h(y_0 + 4y_1 + y_2)$ = A_1 say

Then putting x = x_0+ h and y = y_1 in (1), we get

$y_2 = y_0 + 2hy_0 + \frac{4h^2}{2!}y_0 + \frac{8h^3}{3!}y_0$ + ……..

Further putting x = x_o+ 2h and y = 1/2 in (1), we get

$y_2 = y_0 + 2hy_0 + \frac{4h^2}{2!}y_0 + \frac{8h^3}{3!}y_0$ + ……..

(3) → $A_1 = \frac{h}{3}\ [y_0 + 4(y_0 + hy_0 + \frac{h^2}{2!}\ y_{0+\cdots\cdots}) + (y_0 + 2h\ y_0 + \frac{4h^2}{2!}\ y_0 + \frac{8h^3}{3!}\ y_0 + \cdots\cdots)]$

= $2hy_0 + 2h^2y_0 + \frac{4h^3}{3}\ y_0 + \frac{2h^2}{8}\ y_0 + \frac{5h^5}{18}\ y_0$ +……. ……(4)

Hence the error in the interval $[x_o, x_2] = \int_{x_0}^{x} y\,dx - A_1$

= $[\frac{4}{15} - \frac{5}{18}]\ h^5y_0^5$ +……. ……..(5)

and the principal part of the error in $[x_o, x_2]$

$= [\frac{4}{15} - \frac{5}{18}] h^5 y_0^5 = - -\frac{h^5}{90} y_0^5$

On the same lines principal part of the error in $[x_2, x_4]$

$= \frac{h^5}{90} y_2^5$, etc.,

Total error = $- -\frac{x^5}{90} [y_0^4 + y_2^4 + \ldots\ldots\ldots + y_2^4 (n-1)] = E$, Say ……(6)

The assuming $y_{1v}(x)$ as the largest of $y_0^{1V}, y_2^{1V} \ldots\ldots, y_{2n-2}^{1V}$

We readily obtain $E < -\frac{n/5}{90} y_0^{1V}(x) = - \frac{(b-a)h^y y^5(x)}{180}$ ……(7) [2nh = b-a]

(iii) Error in Simpson's 3/8 rule : Proceeding as above, we have the principal part of the error in the interval $[x_o, x_3]$ as $= -\frac{3h^5}{80} y^5$

(iv) Error in Weddle's rule: Principal part of the error in the interval$[x_0, x_6] = -(h^7/140) y_0^5$

2.8 SOLVED PROBLEMS ::

1.Evalaute the value of the integral using $\int_2^{1.4} (sinx - logx + e^x)$dx

(a) The Trapezoidal rule, (b) Simpson's one-third rule,

(c) Simpson's three eight rule.

Solution. In this case divide the range of integration into twelve equal parts by taking h = -1 and then the values of the function y = sin x $-\log_e x$ +e are computed for each point of subdivision which are as under

x	Sinx	$\text{Log}_e x = \log_{10} x x 2.30258$	e^x	$Y = \text{sinx} - \log_e x + e^x$
0.2	0.19867	-1.60943	1.22140	3.02950
0.3	0.29552	-1.20397	1.34986	2.84935
0.4	0.38942	-0.91629	1.49182	2.79753
0.5	0.47943	-0.69315	1.64872	2.82130
0.6	0.5646	-0.51083	1.82212	2.89759
0.7	0.64422	-0.35667	2.01375	3.01464
0.8	0.71736	-0.22314	2.22554	3.16604
0.9	0.78333	-0.10537	2.45960	3.34830
1.0	0.84147	+0.00000	2.71828	3.55975
1.1	0.89121	0.09530	3.00417	3.80008
1.2	0.93204	0.18232	3.32012	4.06984
1.3	0.96356	0.26236	3.66930	4.37050
1.4	0.98545	0.33647	4.05520	4.70418

(a)By Trapezoidal rule, we have

$\int_2^{1.4}$(sin x -$\log_e{}^x$ + e^x)dx = $\int_2^{2+12x\ 0.1} y\,dx$

Here n = 12; h = 0.1, x_o = 0.2, y = sin x -$\log_e{}^r$+ e^x

Again w.k.t. $\int_{x0}^{xo\ +nh} y\,dx$ = $\frac{h}{2}$[y_0+y_{n+2}(y_1+y_2+…+y_{n-1})]

Substituting these values of, h_1, y_o, y_n…, in the above formula we get

$\int_2^{1.4}(\sin x - \log ex + e^x)\,dx$ = $\frac{1}{20}$[3.02950+ 4.70418+2(2.8493+…+4.37050)]

= $\frac{1}{20}$[7.73368+2(36.69492)]

= 4.05617

(b) By Simpson's rule $\frac{1}{3}$ we have

$\int_{x0}^{xo\ +nh} y\,dx$ = $\frac{h}{3}$[y_0+y_6+4(y_1+y_3+…)+2(y_2+y_4+…)]

Put the values of h, y_o, $y_{1..}$.. from the table, we have

= $\int_2^{1.4}(\sin x - \log ex + e^x)\,dx$

= $\frac{1}{3}$[7.73368+ 4(20.20417)-3(16.49075)]

= $\frac{1}{3}$ [121.53186] = 4.05106

(c) By Simpson's 3/8 rule we have

$\int_{x0}^{xo\ +nh} y\,dx$ = $\frac{3h}{8}$[(y_0+ y_n)+3(y_1+y_2+y_4+…+y_{n-1})+2(y_3+y_6+….+y_{n-3})]

= $\frac{3(0.1)}{8}$[7.73368+ (26.90750)+ 2(9.78742)] = 4.05116

2. A curve is drawn to pass through the points given by the following table.

x	1	1.5	2	2.5	3	3.5	4
y	2	2.4	2.7	2.8	3	2.6	2.1

Estimate the area bounded by the curve, the x-axis and the lines x=1,x=4.

Solution. By Simpson's rule, we have required area

= $\frac{h}{3}$[y_0+4y_1+2y_2+4y_3+…….+4y_{n-1}+y_n]

= $\frac{0.5}{3}$ [2+ 9.6+5.4+11.2+10.4+ 2.1]

= $\frac{50}{30}$x46.7 = $\frac{23.35}{3}$ = 7.783 unit of area

By Weddle's rule, we have required area

= $\frac{3h}{10}$[y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6]

$= \frac{3x0.5}{10}[2+12.0+2.7+16.8+3.13.0+2.1] = 7.74$ units of area

3. Calculate by Simpson's rule an approximate value of by taking seven equidistant ordinates. Compare it with the exact value and the value obtained by using the Trapezoidal rule.

Solution. Let us divide the range of integration (-3, 3) into six equal parts each of width $\frac{3(-3)}{6}$ = 1 unit. Hence h-1, value of the function for each point of sub division are as given below.

x	$y= x^4$
-3	$(-3)^4 = 81$
-2	$(-2)^4 = 16$
-1	$(-1)^4 = 1$
0	$(0)^4 = 0$
1	$(1)^4 = 1$
2	$(2)^4 = 16$
3	$(3)^4 = 81$

Now by Simpson's rule, we obtain

$$\int_{-3}^{3} x^4 \, dx = \frac{h}{3}[y_0+y_6+4(y_1+y_3+y_5)+2(y_2+y_4)]$$

$$= \frac{1}{3}[162+4x32+ 2x2] = \frac{1}{3}x294 = 98$$

But the exact value of

$$\int_{-3}^{3} x^4 \, dx = \left[\frac{x^5}{5}\right]_{-3}^{3} = \frac{1}{5}[(3)^5 - (-3)^5] = \frac{1}{5}x486 = 97.2$$

Also by Trapezoidal rule, we have

$$\int_{-3}^{3} x^4 \, dx = \frac{h}{2}[y_0+y_6+2(y_1+ y_2+y_3+ y_4+y_5)] = \frac{1}{2}[162+2x34]$$

Thus it is obvious that the Trapezoidal rule does not give an accurate result. **4.Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rule. Hence obtain the approximate value of π in each case.**

Solution. Divide the range into six equal parts each of with h = $\frac{1}{6}$ and compute

The value of y = $\frac{1}{1+x^2}$ at each point of sub division.

These values are given below.

X	$Y_x = 1/(1+x^2)$
0	1.0000
1/6	36/37 = 0.97297
2/6	36/40 = 0.90000
3/6	36/45 = -0.80000
4/6	36/52 = 0.69231
5/6	36/61 = 0.59016
1	1 / 2 = 0.5000

By Simpson's rule, we have

$\int_{x_0}^{x_0+6h} y\,dx = \frac{h}{3}[y_0+y_6+4(y_1+y_3+y_5)+2(y_2+y_4)]$

$= \frac{1}{18}$ [1.00 + 0.500 + 4(0.97292 + 0.50000 + 0.59016) +

2(0.900+ 0.69231)] ……(1)

= 0.785395

By Simpson's 3/8 rule we have

$\int_{x_0}^{x_0+6h} y\,dx = \frac{3h}{8}[y_0+y_6+3(y_1+y_2+ y_4+y_5)+2(y_3)]$

$= \frac{1}{16}$ [1.5000 + 3 (0.97297 + 0.900 + 0.69231+0.59016 +

2 x (0.800))] = 0.785395 ……(2)

Again, $\int_0^1 \frac{dx}{1+x^2} = [\tan^{-1} x]^1_0 = \pi/4$ ……(3)

From (1) and (2), we get π = 3.14188 and from (2) and (3) we get

π = 3.141580

5. Evaluate $\int_2^{10} \frac{dx}{x}$ by dividing the range into '8' equal parts by using Trapezoidal rule.

Solution. Given integral, $\int_2^{10} \frac{dx}{x}$

Let f(x) = $\frac{1}{4}$

a = 2, b = 10;n = 8

h = $\frac{b-a}{n} = \frac{10-2}{8} = 1$

x	y = f(x) = $\frac{1}{x}$

2	1/2 = 0.5000
3	1/3 = 0.3333
4	1/4 = 0.2500
5	1/5 = 0.2000
6	1/6 = 0.1666
7	1/7 = 0.1428
8	1/8 = 0.1250
9	1/9 = 0.1000
10	1/10 = 0.1000

Trapezoidal rule is $\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

$\int_2^{10} \frac{dx}{x} = \frac{h}{2}[(y_0+y_8)+2(y_1+y_2+y_3+ y_{4+} y_{5+} y_6+y_7)]$

$= \frac{1}{2}[(0.5+ 0.1)+ 2(0.333+ 0.25)+ 0.2+ 0.125+ 0.166 + 0.1428 +0.1111]$

$= \frac{1}{2}[(0.6)+2(1.3288)]$

$= 0.3+1.3288$

$= 1.62880$

6. Evaluate $\int_2^{10} \frac{dx}{1+x}$ by dividing the range into '8' equal parts by using Trapezoidal rule.

Solution. Let $f(x) = \frac{1}{1+x}$

a = 2, b = 10, n = number of equal parts = 8

$h = \frac{b-a}{n} = \frac{10-2}{8} = 1$

h = 1

x	f(x) = $\frac{1}{1+x}$

2	1/3 = 0.333
3	1/4 = 0.2500
4	1/5 = 0.2000
5	1/6 = 0.1666
6	1/7 = 0.1428
7	1/8 = 0.1250
8	1/9 = 0.1111
9	1/10 = 0.1000
10	1/11 = 0.0909

Trapezoidal rule is $\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

$\int_2^{10} \frac{dx}{1+x} = \frac{h}{2}[(y_0+y_8)+2(y_1+y_2+y_3+ y_{4+} y_{5+} y_6+y_7)]$

$= \frac{1}{2}[(0.333+ 0.090)+ 2(0.25+ 0.2+ 0.1661 + 0.1425+ 0.125+$

$0.111+ 0.1)]$

$= \frac{1}{2}[0.4242+2(1.0955)] = 0.2121 + 1.0955 = 1.3076$

8. Evaluate $\int_4^{5.2} f(x)dx$ by using Trapezoidal rule from the following.

X	**4**	**4.2**	**4.4**	**4.6**	**4.8**	**5.0**	**5.2**
Y	**1.4**	**1.4**	**1.5**	**1.5**	**1.6**	**1.6**	**1.6**

Where h = 0.2

Solution. Trapezoidal rule is, $\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

$\int_4^{5.2} f(x)dx = \frac{0.2}{2}[(1.4+ 1.6)+ 2(1.4+1.5+ 1.5+1.6+ 1.6)]$

$= 0.1[3+2(7.6)]$

$= 0.1[18.2] = 1.82$

9. Evaluate $\log_e 7$ from the integral $\int_1^7 \frac{dx}{x}$

Solution. Given f(x) = $\frac{1}{x}$ a = 1, b = 7, n = 6, h = $\frac{b-a}{n} = \frac{7-1}{6} = \frac{6}{6} = 1$

x	y= f(x)
1	1.0000
2	0.5000
3	0.333
4	0.2500
5	0.2000
6	0.1666
7	0.1428

Trapezoidal rule is, $\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

$= \frac{h}{2}[(1+ 0.1428)+ 20.5+ 0.333+0.25+ 0.2+ 0.1666)]$

$= \frac{1}{2}[1.1428+ 2(1.4499)]$

$= \frac{1}{2}[4.0426]$ and $\int_1^7 \frac{dx}{x} = [\log_e x]_1^7$

$= \log_e 7 = \log_e 1 = \log_e 7$

$\log_e 7 = 2.0113$

10. A curve is drawn to passes through the point given by the following table.

X	1	1.5	2	2.5	3	3.5	4
Y	2	2.4	2.7	2.8	3	2.6	2

Solution. Estimate the area bounded by the curve, the axis-x and the lines x=1,x=4by using Trapezoidal rule.

Trapezoidal rule is h= $\frac{4-1}{6}$=0.5

$\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+y_4+y_5)]$

$= \frac{0.5}{2}[2+2.1+2+ (2.4 +2.7+2.8+2.6+3)]$

$\frac{0.5}{2}(4.1+27.0) = \frac{15.35}{2} = 7.775$ units of area

11. calculate e $\int_2^{10} \frac{dx}{1+x}$by dividing the arrange into 8 equal parts by using Simpson's 1/3 rule.

Solution.: Simpson's 1/3 rule is

$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_5+\ldots..)+2(y_2+ y_4+\ldots....)]$

$\int_2^{10} \frac{dx}{1+x}$ = $\frac{h}{3}$[(O.333+ 0.0909)+ 4(0.25+ 0.1666+ 0.125+0.1)

+ 200.2+ 0.142+ 0.111)]

= $\frac{1}{3}$ [0.4242+ 4(0.6416)+ 2(0.4539))= $\frac{1}{3}$ [3.8984] =1.2994666

12. calculate $\int_a^b f(x)dx$ by using Simpson's $\frac{1}{3}$ rule

X	4	4.2	4.4	4.6	4.8	5.0	5.2
Y	1.4	1.4	1.5	1.5	1.6	1.6	1.6

where h = 0.2

Solution: Simpson's $\frac{1}{3}$ rule is

$\int_a^b f(x)dx$ = $\frac{h}{3}$[(y_0+y_n)+4(y_1+ y_3+ y_5+.....)+2(y_2+ y_4+.....)]

$\int_4^{5.2} f(x)dx$ =$\frac{0.2}{3}$[(1.4+1.6)+ 4(1.4+1.5+ 1.6)+ 2(1.5+ 1.6)]

= $\frac{0.2}{3}$[3+4(4.5)+2(3.1)] = $\frac{0.2}{3}$[27.2]

= 1.813333.....

13. Evaluate $\int_2^{10} \frac{dx}{1+x}$ by dividing the range into 8 equal parts by using Simpson's $\frac{1}{3}$ rule.

Solution:. Let f(x)= $\frac{1}{x}$

a = 2, b = 10, n =number of equal parts = 8

h = $\frac{b-a}{n}$ = $\frac{8}{8}$ = 1

X	Y = f(x) =$\frac{1}{x}$
2	0.5000
3	0.3333
4	0.2500
5	0.2000
6	0.1666
7	0.1428
8	0.1250

9	0.1111
10	0.1000

Simpson's $\frac{1}{3}$ rule is

$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_5+.....+y_{n-1})+2(y_2+ y_4+y_6.....+y_{n-2})]$

$= \frac{1}{3}$ [I0.6)+ 4(0.3333+ 0.2+ 0.1428+ 0.1111)] +2(0.25+0.1666+0.125)]

$=\frac{1}{3}$[0.6+4(0.7872+ 1.0832)]

$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_6+.....)+2(y_2+ y_4+.....)]$

$\int_0^1 \frac{dx}{1+x^2} = \frac{1}{6*3}$ [(1 +0.5)+ 4(0. 9729+0.8+0.3+ 2(09+0.6923)]

$= \frac{1}{18}$[1.5+9.452+3.1846] = $\frac{1}{18}$[14.1366]

= 0.7853666

On the other hand $\int_0^1 \frac{1}{1+x^2}dx = (\tan^{-1})^1_0$

$= \tan^{-1}(1)-\tan^{-1}(0)$

$= \frac{\pi}{4}-0 = \frac{\pi}{4}$

$= \frac{\pi}{4} = 0.7853$

$= \pi = 3142$

14. Evaluate $\int_0^4 e^x\,dx$ by using Simpson's rule using the data
e = 2°.72, e2= 739, e3=20.09, e4=54.60 and compare it will the actual value.

Solution.: Divide the whole range (0,4) into four equal parts by taking h=1. By Simpson's rule 1/3 we have

$\int_0^4 e^x\,dx = \int_{x_0}^{x_0+4h} y\,dx = \frac{h}{3}(y_0+y_4+4(y_1+y_3)+242)$

$= \frac{1}{3}[1+54.60+4(2.72+20.09)+2(7.39)]$

$= \frac{1}{3}[55.60+91.24+14.78] = \frac{1}{3}[161.62]$

= 553.373

The actual value of the integral is

$$\int_0^4 e^x \, dx = e^4 - e^0 = 54.60 - 1.0 = 53.60$$

15. Evaluate $\int_4^{5.2} \log, x \, dx$ by (a) Simpson's $\frac{1}{3}$ (b) $\frac{3}{8}$ rule . After finding the true value of the integral.

Solution. Divide the whole range of integration ie., (4, 5, 2) into six equal parts taking h 0.2 and then compute the values of the function f(x)= $\log^x_e$ for each of sub division. These computed values are as follows.

X	$Y = \log^x_e x$	X	$Y = \log^x_e x$
Xo = 4.0	1.38629436	Xo+ 4h= 4.8	1.56861592
Xo+ h =4.2	1.43508453	Xo+5h= 5.0	1.60943791
X_0 + 2h= 4.4	1.48160454	Xo+ 6h= 5.2	1.64865863
Xo +3h= 4.6	1.52605630		

(a) By Simpson's $\frac{1}{3}$ rule, we have

$$\int_4^{5.2} \log, x \, dx = \frac{h}{3} [y_0+y_6+(y_1+y_3++y_5)+2(y_2+y_4)]$$

$$= \frac{2}{8} [3.03495299+ 4(4.57056874)+ 2(3.05022046)]$$

$$=1.82784726.$$

(b) By Simpson's 3/8 rule, we have

$$\int_4^{5.2} \log e, x \, dx = \frac{3h}{8} [y_0+y_2+3(y_1+y_2++y_4+ y_5)+2 y_3]$$

$$= \frac{3(0.2)}{8}[03495299+ 3(6.09424290)+ 2 (1.52605630)]$$

$$= \frac{0.3}{4}[24.371294291]$$

$$= = 1.82784707$$

Again exact value of the integral is

$$\int_4^{5.2} \log e, x \, dx = x(\log_e x-1)\int_{4.0}^{5.4} 1.82784741$$

16. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ rule.

Solution : let f(x) = $\frac{1}{1+x^2}$ let n =6 , a= -0 , b = 1 ,h = $\frac{b-a}{n} = \frac{1-0}{6} = \frac{1-0}{6} = \frac{1}{6}$ = 0.1666

X	f(x) = $\frac{1}{1+x^2}$
0	1
1/6	0.9729
2/6	0.9000
3/6	0.8000
4/6	0.6923
5/6	0.5901
$\frac{6}{6}$=1	0.5000

Simpson's $\frac{1}{3}$ rule is

a = 0, b = 6, n= 6

h = $\frac{b-a}{n} = \frac{6-0}{6}$ = 1

y_0 = f(0) -= $\frac{1}{1+0^2} = \frac{1}{1}$ = 1

y_1 = f(1) = $\frac{1}{1+1^2} = \frac{1}{2}$ = 0.5000

y_2 = f(2) =$\frac{1}{1+2^2} = \frac{1}{5}$ = 0.2000

y_3 = f(3)= $\frac{1}{1+3^2}$ =$\frac{1}{10}$ = 0.1000

y_4= f(4) = $\frac{1}{1+4^2}$ =$\frac{1}{17}$ = 0.0588

y_5 = f(5)= $\frac{1}{1+5^2}$ = $\frac{1}{26}$ = 0.0385

y_6 = f(6) = $\frac{1}{1+6^2}$ = $\frac{1}{37}$ = 0.0270

Simpson's $\frac{1}{3}$ rule is

$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_5+.....)+2(y_2+ y_4+.....)]$

$= \frac{1}{3}[(1 + 0.027)+ 4(0.5+ 0.1 + 0.0385)+ 2(0.2+ 0.0588)]$

$=\frac{1}{3}$ [3.7104]= 1.2368

17. Find the value of $\int_0^1 \frac{x^2}{1+x^2}$ dx by using Simpson's $\frac{1}{3}$ rule and also find the error

Solution. : Dividing the whole range (o, 1) into 4 equal parts nd making use of Simpson's $\frac{1}{3}$ rule we have

$\int_0^1 \frac{x^2}{1+x^2}$ dx = $\frac{h}{3}[(y_0+y_4)+4(y_1+ y_3+)+2y_2)]$

h = $\frac{1}{4}$, a =0 , b= 1 ,n = $\frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4}$

X	F(x)
0	0
0.25	0.0615
0.5	0.2222
0.75	0.3956
1.0	0.5000

$\int_0^1 \frac{x^2}{1+x^2}$ dx = $\frac{1}{12}$ [(0+ 0.5)+ 4C0.0615+ 0.3956)+ 2(0.222)]

= $\frac{1}{12}$ [0.5+ 1.8284+ 0.444]

=$\frac{2.7728}{12}$ = 0.2310

18. Find $\int_4^{5.2} \log x_e$ dx by using Simpson's $\frac{1}{3}$ rule.

solution.: Given integral is $\int_4^{5.2} \log x_e$ dx

A = 4, b = 52 let n= 6 = $\frac{b-a}{n}$ = $\frac{5.2-2}{6}$ = 0.2

X	F(x)
4	1.3862
4.2	1.4350
4.4	1.4816
4.6	1.5260
4.8	1.5686
5.0	1.6094
5.2	1.6486

$\int_4^{5.2} \log x_e$ = $\frac{h}{3}$[(y_0+y_6)+4(y_1+ y_3+ y_5)+2(y_2+ y_4)]]

= $\frac{0.2}{3}$[(1.3862+1.6481+4(1.435071 +1.5260+1.6094)

+2(1.4816+1.5686)]

= $\frac{0.2}{3}$ [(3.0348)+ 4 (4.5704)+ 2(3.05027)]

= $\frac{0.2}{3}$[3.0348+18.2816+ 6.1004] = $\frac{5.48336}{3}$ = 1.8277866.

19. Evaluate $\int_{-3}^{3} x^4$dx by using Simpson's $\frac{1}{3}$ rule.

Solution. We know the integral is $\int_{-3}^{3} x^4$dx

Where h = $\frac{b-a}{h}$ = $\frac{3-(-3)}{6}$ = $\frac{6}{6}$ = 1

x	F(x)
-3	81
-2	16
-1	1
0	0

1	1
2	16
3	81

Simpson's $\frac{1}{3}$ rule is

$$\int_b^a f(x)dx = \frac{h}{3}[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)]$$

$$= \frac{1}{3}[(81+81)+4(16+0+16)+2(1+1)]$$

a = 3.b = 3 , n = 6

$$h = \frac{b-a}{h} = \frac{3-(-3)}{6} = 1$$

let $f(x) = x^2$

$y_0 = f(-3) = (-3)^2 = 9$

$y_1 = f(-2) = (-2)^2 = 4$

$y_2 = f(-1) = (-1)^2 = 1$

$y_3 = f(0) = (0)^2 = 0$

$y_4 = f(1) = (1)^2 = 1$

$y_5 = f(2) = (2)^2 = 4$

$y_6 = f(3) = (3)^2 = 9$

Simpson's $\frac{1}{3}$ rule

$$\int_a^b f(x) = \frac{h}{3}[(y_0+y_n)+4(y_1+y_{3+\ldots})+2(y_2+y_{4+\ldots})]$$

$$= \frac{1}{3}[(9+9)+4(4+0+4)+2(1+1)]$$

$$= \frac{54}{3} = 18 = \frac{1}{3}[162+128+4] = \frac{1}{3}[294] = 98$$

20 .calculate an approximate value of $\int_a^{\pi/2} sinx\ dx$ by (a) Trapezoidal rule (b) Simpson's rule using 11 ordinates

Solution : Divide the range of integration into a ten equal parts taking h = $\pi/20$ and then compute the values of the function f(x) = sin x for each point of sub-division. The computed values are as follows :

X	Y=sin x
0	0.00000
$\pi/20$	0.15663
$2\pi/20$	0.30902
$3\,\pi/20$	0.45399
$4\pi/20$	0.58778
$5\pi/20$	0.707111
$6\,\pi/20$	0.80900
$7\,\pi/20$	0.89101
$8\,\pi/20$	0.95106
$9\pi/20$	0.98769
$10\,\pi/20$	1.00000

(a) By the trapezoidal rule, we have

$$\int_0^{\pi/2} sinx\ \mathrm{dx} = \frac{h}{3}[(y_0+y_{10})+2(y_1+y_{2+\ldots})]$$
$$= \frac{\pi/20}{2}[1.0000+2(5.85311)]$$
$$= \frac{\pi}{40}(12.70622) = 0.9981$$

(b) By Simpson's $\frac{1}{3}$ rule , we have

$$\int_0^{\pi/2} sinx\ \mathrm{dx} = \frac{h}{3}[(y_0+y_n)+4(y_1+y_{3+\ldots})+2(y_2+y_{4+\ldots})]$$
$$= \frac{\pi}{60}[1.00000+4(3.19623)+2(2.65688)]$$
$$= \frac{\pi}{60}[1.00000+12.78492+5.31376]$$
$$= \frac{\pi}{60}[19.09868] = 1.0006$$

Again the exact value of the integral

$$\int_0^{\pi/2} sinx\ \mathrm{dx} = [-\cos x]_0^{\pi/2} = 1.0000$$

The error (i) Due to Simpson's $\frac{1}{3}$ rule , we have = -0.0006

(ii) Due to trapezoidal rule = 0.0019

21.Evaluate $\int_a^b f(x)dx$,by Simpson's $\frac{3}{8}$ rule from the following

X	4	4.2	4.4	4.6	4.8	5.0	5.2
Y	1.4	1.4	1.5	1.5	1.6	1.6	1.6

Where

Solution : Simpson's $\frac{3}{8}$ rule is

$\int_a^b f(x)dx$ = $\frac{3h}{8}$[y_0+3y_1+2y_2+3y_2+2y_3+3y_4+3y_5+y_6]

= $\frac{3(0.2)}{8}$[1.4+3(1.4)+3(1.5)+2(1.5)+3(1.6)+3(1.6)+1.6]

= $\frac{0.6}{8}$[1.4+4.2+4.5+3.0+4.8+4.8+1.6]

= $\frac{0.6}{8}$(24.3) = 1.8225

22. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's $\frac{3}{8}$ rule.

Solution : By the above problem we have y_0 =1 ,y_1 = 0.5000,y_2 = 0.1000,

y_4 = 0.0588 , y_5 = 0.0385, y_6 = 0.0270 and h = 1

Simpson's $3/8\ rule\ is$

$\int_a^b f(x)dx$ = $\frac{3h}{8}$[(y_0+ y_6)+3y_1+2y_3+3y_4+3y_5]

$\int_0^6 \frac{dx}{1+x^2}$ = $\frac{3}{8}$[(1+0.027)+3(0.5+0.2+0.0588+0.0385+2(0.1))]

= $\frac{3}{8}$[1.027+2.3919+0.2] = 13.09437/8 = 1.6770

23.Evaluate $\int_{-3}^{3} x^2 dx$, using by Simpson's $\frac{3}{8}$ rule we know by the above problem

Solution. y_0=9 ,y_1= 4,y_2= 1 , y_3 = 0 ,y_4 = 1 , y_5 = 4 ,y_6 = 9

Simpson's $3/8\ rule\ is$

$\int_a^b f(x)dx$ = $\frac{3h}{8}$[(y_0+ y_6)+3y_1+2y_3+3y_4+3y_5]

= $\frac{3h}{8}$[(9+9)+3(4+1+4+1)+0]

= $\frac{3}{8}$[48] = 18

24. Evaluate $\int_{-3}^{3} x^4 dx$,by using Simpson's $\frac{3}{8}$ rule

Solution. we know by the above problem.

Let f(x) = x^4

X	F(X)
-3	81
-2	16
-1	1
0	0
1	1
2	16
3	81

Where h=1

Simpson's 3/8 rule is

$\int_a^b f(x)dx = \frac{3h}{8}[(y_0+ y_6)+3y_1+2y_3+3y_4+3y_5]$

$= \frac{3}{8}[81+81+3(16+1+1+16)]$

$= \frac{3}{8}[162+102] = \frac{3}{8}(264) = 99$

25. A curve is drawn to passes through the part given by the following .

X	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
Y	**2**	**2.4**	**2.7**	**2.8**	**3**	**2.6**	**2.1**

Estimate the area bounded by the curve the x- axis and the lines x =1, x=4 by using Simpson's $\frac{3}{8}$rule.

Solution. h = $\frac{4-1}{6}$ = 0.5

Simpson's 3/8 rule is

$\int_a^b f(x)dx = \frac{3h}{8}[(y_0+ y_6)+3y_1+2y_3+3y_4+3y_5]$

$= \frac{3(0.5)}{8}[2+2.1+3(2.4+2.7+3+2.6)+2(2.8)]$

$= \frac{6.35}{8}$ = 8.04375 units of area

26. Evaluate $\int_a^b f(x)$ dx by using weddles formula from the following.

x	4	4.2	4.4	4.6	4.8	5.0	5.2
F(x)	1.4	1.4	1.5	1.5	1.6	1.6	1.6

Where h=0.4

Weddle's formula is

$\int_a^b f(x)dx$ = $\frac{3h}{10}$[y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6]

= $\frac{3(0.2)}{10}$[(1.4+1.6)+5(1.4+1.6)+(1.5+1.6)+6(1.5)]

= $\frac{0.6}{10}$[3+15+3.1+9]

= $\frac{0.6}{10}(30.1)$

= 1.806

27. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using weddle's formula.

Solution. a =0 , b= 1 ,h = $\frac{1-0}{6}$ = $\frac{1}{6}$ = 0.1666

X	f(x)
0	1
1/6	0.9729
2/6	0.9000
3/6	0.8000
4/6	0.6923
5/6	0.5901
$\frac{6}{6}$=1	0.5000

Weddle's formula is

$\int_a^b f(x)dx$ = $\frac{3h}{10}$[y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6]

= $\frac{3}{6x10}$[(1+0.5)+5(0.972+0.590)+(0.9+0.6923)+6(0.8)]

= $\frac{1}{20}$[1.5+7.815+1.5923+4.8]

$= \frac{1}{20}[15.703]$

$= 0.785365$

2.9 Applications of Numerical Methods

Considering the following model equation:

$$(\mu u_y)_y + y^n u_y = C, \qquad (1)$$

defined in a domain $\Omega = [0\ 1]$.

Where $\mu = (1 - exp(-y/\varepsilon) + \delta)/Re$, $\varepsilon << 1$, $\delta << 1$, $Re >> 1$, $n > 0$.

The first term simulates dissipative terms in the Navier-Stokes equations, whereas the second term models the contribution of convective terms and the right hand side represents the pressure gradient term or source in the transport equations. The "viscosity" coefficient μ corresponds to the effective viscosity coefficient. The coefficient is rapidly changed from a relatively small value $\mu_l = \delta /Re$ (laminar viscosity) to a "turbulent" viscosity $\mu_t \approx 1/Re$.

The equation simulates the low-Reynolds-number effects and can be considered as a model equation for the LR model. The left hand side point in the domain Ω will be treated as a "wall". The low-Reynolds-number effects occur nearby this point. If we set $\mu = (1+ \delta)/Re$ in (1), we have the HR approach.

Furthermore, we will consider the following values for the constants: $Re = 10^2$, $\varepsilon = 3*10^{-2}$, $\delta = 10^{-2}$. For simplicity the right-hand side will be considered as the constant $C = -1$

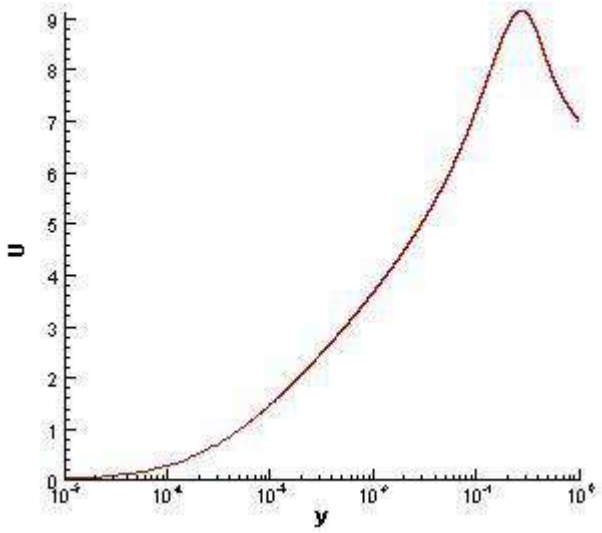
although this assumption is not important for our consideration.

Consider the following boundary-value Dirichlet problem:

$$(\mu u_{y})_y + y^n u_y = C \qquad (2)$$

$$u(0) = u_0 \quad u(1) = u_1$$

In the case of $n = 2$, $u_0 = 0$ *and* $u_1 = 7$, the solution is shown in Figure 1. The profile includes both the linear near-wall and logarithmic parts. Near the wall, $u = \varepsilon \ln(1+y/(\varepsilon\delta))$. The thickness of the viscous "sub-layer", where u is a linear function, can be approximated as $y_v \approx \varepsilon\delta$.



In the calculations, the compact scheme [4] is used. The scheme allows us to calculate both the function and its derivative with a fourth-order of approximation. In Table 1, the results of calculations of the "friction" $\tau_w = \mu_w du/dy(0)$ (here $\mu_w = \mu(0)$) are given on different meshes. To exclude questions concerned with a mesh adaptation, a uniform mesh is used

Grid	10^2	$5*10^2$	10^3	$5*10^3$	10^4

τ_w	$3.1*10^{-1}$	$3.2*10^{-1}$	$3.3*10^{-1}$	$3.36*10^{-1}$	$3.36*10^{-1}$

Table 1. Calculation of τ_w on different meshes.

If we use the HR model with the same boundary conditions, then $\tau_w = 4.9*10^{-1}$, which shows the importance of the sub-layer.

3. Linear System of Equations

Solutions of simultaneous linear equations:

Rate, Distance and Time:

You can calculate the best routes for your running or cycling schedule by creating a mathematical expression that takes into account the distance and your average speed for various parts of the route. You can use the

equations to set different goals, such as to maximize time for build endurance, or to maximize speed for performance.

Planes, Trains and Automobiles:

The same formula used to calculate running times can be used to determine speed, distances and time duration when traveling by car, plane or train and you want to know the values for the unknown variables in your travel situations.

The Best Deal:

You want to find out the better deal when renting a car, and you're comparing two rental companies. By putting the variable and fixed costs, such as the per - mile and daily rate, into an algebraic expression, then solving for the total cost, you ca see which company saves you money for different amounts of driving.

The Best Plan:

You can use this same process with a system of equations when trying to decide on the best call phone plan, determining at how many minutes both companies charge the same amount and deciding from there which is the best plan for you and your intended usage.

Deciding on a Loan:

Simultaneous equations can be used to determine the best loan

choice to make when buying a car or a house when you consider the duration of the loan, the interest rate and the monthly payment of the loan. Other variables may be involved as well, with the information at hand, you can calculate which loan is the best choice for you.

Cost and Demand:

Simultaneous equations can be used when considering the relationship between the price of a commodity and the quantities of the commodity people want to buy at a certain price. An equation can be written that describes the relationship between quantity, price and other variables, such

as income. These relationship equations can be solved simultaneously to determine the best way to price the commodity and sell it.

In the Air:

An air traffic controller can use Simultaneous equations to ensure two airplanes don't intersect at the same time.

The Best job for the Money:

System of equations can be used when trying to determine if you'll make more money at one job or another, taking multiple variables into account, such as salary, benefits and commissions.

Investing Wisely:

You can use Simultaneous equations to decide on your best investment option, taking into account the duration of the investment, the interest it will accrue, as well as other variables that will affect the end result. If you know the amount you'd like to accrue, you can set the options equal to each other and figure out which option is best for your future.

Mixing it up:

With respect to mixtures Simultaneous equations can be used for achieving a certain consistency in a resultant product, which is dependent on the consistency of the compounds mixed together to produce it.

Rates :

Linear equations can be useful tool for comparing rates of pay. For example, if one company offers to pay you $450 per week and other offers $10 per hour, and both ask you to work 40 hours per week, which company is offering the better rate of pay? A linear equation can help you figure it out! The first company's offer expressed as 450=40x. the second company's offer is expressed as y=10(40). After computing the two offers, the equations tell you that the first company is offering the better rate of pay at $11.25 per hour.

Variable costs:

Imagine that you taking a taxi while on vacation. You know that the taxi service charges $9 to pick your family up form your hotel and another $0.15 per mile for the trip. Without knowing how many miles it will be to each destination, you can set up a linear equation that can be used to find the cost of any taxi trip you take on your trip. By using "x" to represent the number of miles to your destination and "y" to represent the cost of that taxi ride, the linear equation would be: y= 0.15x+9.

3.1 MATRIX INVERSION METHOD:

* Solve x + 2y = 10, 2x - y = 15 by matrix inverse method?

Solution:-

Let $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 10 \\ 15 \end{bmatrix}$

Given equations are equivalent to AX = B

=> $X = A^{-1} B$.

$$A^{-1} = \frac{1}{-1-4}\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix} \frac{1}{-1-4} = -\frac{-1}{5}\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix}$$

$$\therefore X = A^{-1} B = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} \frac{1}{5} \begin{bmatrix} 10 \\ 15 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 10 + 30 \\ 20 - 15 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 40 \\ 5 \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

$\therefore$ x = 18, y = 1 is the solution.

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* Solve the equations 3x + 2y + 4z = 7; 2x + y + z = 7; x + 3y + 5z = 2. by matrix inversion method ?

Solution:-

Let $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 7 \\ 7 \\ 2 \end{bmatrix}$

Given equations are equivalent to AX = B

det A = 3(5 - 3) - 2(10 - 1) +4(6 - 1) = 6 - 18 + 20 = 8 ≠ 0.

$\therefore$ $X = A^{-1} B$ is the solution.

Now adj $A = \begin{bmatrix} 2 & 2 & -2 \\ -9 & 11 & 5 \\ 5 & -7 & -1 \end{bmatrix}$

$\Rightarrow A^{-1} = \frac{AdjA}{\det A} = \frac{1}{8} \begin{bmatrix} 2 & 2 & -2 \\ -9 & 11 & 5 \\ 5 & -7 & -1 \end{bmatrix}$

$\therefore X = A^{-1} B = \frac{1}{8} \begin{bmatrix} 2 & 2 & -2 \\ -9 & 11 & 5 \\ 5 & -7 & -1 \end{bmatrix} \begin{bmatrix} 7 \\ 7 \\ 2 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 24 \\ 24 \\ -16 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ -2 \end{bmatrix}$

$\therefore$ x = 3, y = 3, z = -2 is the solution.

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3.2 GAUSS ELIMINATION METHOD:

* Solve the equations 2x - y + 4z = 12; 8x - 3y =2z = 23; 4x + 11y - z = 33 by Gauss Elimination Method ?

Solution:-

Given equations are 2x - y + 4z = 12, 8x - 3y + 2z = 23, 4x + 11y - z = 33.

Dividing first equation by 2, we get x - $\frac{1}{2}$y+ 2z = 6 ----------(1)

Now subtract (1) after multiplying by 8 and 4 respectively from second and third equations we get y - 14z = -25----->(2), 13y - 9z = 9 ------(3)

Subtract (2) after multiplied by 13 from (3) we get 173z = 334

=> z = $\frac{334}{173}$ = 1.931

From (3); 13y - 9$\left(\frac{334}{173}\right)$ = 9 => 13y = 9 + $\frac{3006}{173}$ => y = $\frac{4563}{2249}$ = 2.029

From (1); x = 6+$\frac{1}{2}\left(\frac{4563}{2249}\right)$ - 2$\left(\frac{334}{173}\right)$ = 6+$\frac{1}{2}$(2.029) - 2(1.931)

= 6 + 1.0145 - 3.862 = 3.153.

$\therefore$ The solution is x = 3.153, y = 2.029, z = 1.931.

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* Solve the system of equations 2x + y + z = 10, 3x + 2y + 3z = 18, x + 4y + 9z = 16 by Gauss Elimination Method.

Solution:-

We have 2x + y + z = 10------(1), 3x + 2y + 3z = 18------(2), x + 4y + 9z = 16-----(3)

Divide (1) by (2) and subtract multiplied by 3 from (2) then subtract from (3), we get

$x + \frac{1}{2}y + \frac{1}{2}z = 5$ -----(4), $\frac{1}{2}y + \frac{3}{2}z = 3$ -----(5), $\frac{7}{2}y + \frac{17}{2}z = 11$ ----(6)

Now divide (5) by $\frac{1}{2}$ and then subtract after multiplied by $\frac{7}{2}$ from (6) we get

$x + \frac{1}{2}y + \frac{1}{2}z = 5$ ------(7), y + 3z = 6 ----(8), -2z = -10 -----(9)

From back substitution form (9), (8) & (7) we get z = 5 and y + 3z = 6 => y + 3(5) = 6

=> y = 6 - 15 => y = -9 and

$x + \frac{1}{2}(-9) + \frac{1}{2}(5) = 5 \Rightarrow x = 5 + \frac{9}{2} - \frac{5}{2} = 7$

$\therefore$ Hence the solution is x = 7, y = -9, z = 5.

3.3GAUSS JORDAN METHOD:

* Solve the system of equations 2x + y + z = 1, x -2y - 3z = 1, 3x + 2y + 4z = 5 by Gauss Jordan Method.

Solution:-

The augmented matrix is $\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & -2 & -3 & 1 \\ 3 & 2 & 4 & 5 \end{bmatrix}$

$2R_2 - R_1, 2R_3 - 3R_1 \sim \begin{bmatrix} 2 & 1 & 1 & 1 \\ 0 & -5 & -7 & 1 \\ 0 & 1 & 5 & 7 \end{bmatrix}$

$5R_1 + R_2, 5R_3 + R_2 \sim \begin{bmatrix} 10 & 0 & -2 & 6 \\ 0 & -5 & -7 & 1 \\ 0 & 0 & 18 & 36 \end{bmatrix}$

$9R_1 + R_3, 18R_2 + 7R_3 \sim \begin{bmatrix} 90 & 0 & 0 & 90 \\ 0 & -90 & 0 & 270 \\ 0 & 0 & 18 & 36 \end{bmatrix}$

$\frac{1}{90}R_1, \frac{1}{90}R_2, \frac{1}{18}R_3 \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

$\therefore$ **By** Gauss Jordan method, x = 1, y = -3, z = 2.

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* Solve the equations by Gauss Jordan Method . 5x + 2y + z = 12, x + 4y + 2z = 15, x + 2y + 5z = 20.

Solution:-

The augmented matrix is $\begin{bmatrix} 5 & 2 & 1 & 12 \\ 1 & 4 & 2 & 11 \\ 1 & 2 & 5 & 20 \end{bmatrix}$

$5R_2 - R_1, 5R_3 - R_1 \sim \begin{bmatrix} 5 & 2 & 1 & 12 \\ 0 & 18 & 9 & 63 \\ 0 & 8 & 24 & 88 \end{bmatrix}$

$9R_1 - R_2, 9R_3 - 4R_1 \sim \begin{bmatrix} 45 & 0 & 0 & 45 \\ 0 & 18 & 9 & 63 \\ 0 & 0 & 180 & 540 \end{bmatrix}$

$180R_3 \sim \begin{bmatrix} 45 & 0 & 0 & 45 \\ 0 & 18 & 9 & 63 \\ 0 & 0 & 1 & 3 \end{bmatrix}$ $R_2 - 9R_3 \sim \begin{bmatrix} 45 & 0 & 0 & 45 \\ 0 & 18 & 0 & 36 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

$\frac{1}{45}R_1, \frac{1}{18}R_2 \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

$\therefore$ **By** Gauss Jordan method, x = 1, y = 2, z = 3.

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3.4 LU - DECOMPOSITION METHOD:

* Solve the system of equations $2x_1 + x_2 + x_3 = 2$, $x_1 + 3x_2 + 2x_3 = 2$, $3x_1 + x_2 + 2x_3 = 2$ by LU Decomposition Method?

Solution:-

Given equations are $2x_1 + x_2 + x_3 = 2$----------(1)

$$x_1 + 3x_2 + 2x_3 = 2\text{---------(2)}$$

$$3x_1 + x_2 + 2x_3 = 2\text{---------(3)}$$

Here the coefficient matrix A is given by $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 3 & 1 & 2 \end{bmatrix}$ and

$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$

$\therefore AX = B$

Now $A = LU \Rightarrow \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$

$$= \begin{bmatrix} u_{11} & u_{12} & u_{12} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix}$$

On comparing two matrices, we get $U_{11} = 2$, $U_{12} = 1$, $U_{13} = 1$;

$L_{21}U_{11} = 1 \Rightarrow L_{21} = \frac{1}{U_{11}} = \frac{1}{2}$; $L_{31}U_{11} = 3 \Rightarrow L_{31} = \frac{3}{U_{11}} = \frac{3}{2}$;

$L_{21}U_{12}+U_{22} = 3 \Rightarrow 3 - L_{21}U_{12} = 3 - \frac{1}{2}(1) = \frac{5}{2}$

$L_{21}U_{13}+U_{2} = 3 \Rightarrow 2 - L_{21}U_{12} = 2 - \frac{1}{2}(1) = \frac{3}{2}$

$L_{31}U_{12}+L_{32}U_{22} = 1 \Rightarrow L_{32} = \frac{1-L_{31}U_{12}}{U_{22}} = \frac{1-\frac{3}{2}}{\frac{5}{2}}(1) = \frac{-1}{5}$

$L_{31}U_{13}+L_{32}U_{23}+U_{33} = 2$

$\Rightarrow U_{33} = 2 - L_{31}U_{13} - L_{32}U_{23} = 2 - \frac{3}{2} + \frac{1}{5}\left(\frac{3}{2}\right) = 2 - \frac{3}{2} + \frac{3}{10} = \frac{8}{10} = \frac{4}{5}$

Thus $L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{3}{2} & \frac{-1}{5} & 1 \end{bmatrix}$ and

$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{bmatrix} 2 & \frac{1}{5} & \frac{1}{3} \\ 0 & \frac{5}{2} & \frac{3}{2} \\ 0 & 0 & \frac{4}{5} \end{bmatrix}$

$LY = B \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{3}{2} & \frac{-1}{5} & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$

From this we get $y_1 = 1$, $\frac{1}{2}y_1 + y_2 = 2$, $\frac{3}{2}y_1 - \frac{1}{5}y_2 + y_3 = 2$.

On solving, we get $y_1 = 2$, $y_2 = 1$, $y_3 = \frac{-4}{5}$.

And since, we have UX =Y => $\begin{bmatrix} 2 & 1 & 1 \\ 0 & \frac{5}{2} & \frac{3}{2} \\ 0 & 0 & \frac{4}{5} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ \frac{-4}{5} \end{bmatrix}$

$2x_1 + x_2 + x_3 = 2$, $\frac{5}{2}x_2 + \frac{3}{2}x_3 = 1$, $\frac{4}{5}x_3 = \frac{-4}{5}$

By back substitution, we get $x_1 = 1$, $x_2 = 1$, $x_3 = -1$.

$\therefore$ Hence, solution is $x_1 = 1$, $x_2 = 1$, $x_3 = -1$

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* Solve the equations $2x + 3y + z = 9$, $x + 2y + 3z = 6$, $3x + y\ 2z = 8$ by LU Decomposition method.

<u>Solution:-</u>

Given equations can be written as AX= B where A = $\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$, X= $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, B = $\begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$

Let A = LU

$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{12} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{bmatrix}$$

Comparing, we get $U_{11}= 2$, $U_{12}= 3$, $U_{13}= 1$ and $L_{21}U_{11}= 1 \Rightarrow L_{21}= \frac{1}{2}$;

$L_{21}U_{12}+U_{22}= 2 \Rightarrow U_{22}= 2 - L_{21}U_{12} = 2 - \frac{3}{2} = \frac{1}{2}$;

$L_{21}U_{13}+U_{23}= 3 \Rightarrow U_{23}= 3 - L_{21}U_{13} = 3 - \frac{1}{2} = \frac{5}{2}$;

$L_{31}U_{11} = 3 \Rightarrow L_{31}= \frac{3}{U_{11}} = \frac{3}{2}$ and

$L_{31}U_{12} +L_{32}U_{22} = 1 \Rightarrow L_{32}= \frac{1-L_{31}U_{12}}{U_{22}} = \frac{1-\frac{9}{2}}{\frac{1}{2}} = -7$

$L_{31}U_{13} +L_{32}U_{23} = 2 \Rightarrow U_{33}= 1 - L_{31}U_{13} - L_{32}U_{23} = 2 - \frac{3}{2} - (-7)\left(\frac{5}{2}\right) = 2 - \frac{3}{2} + \frac{35}{2} = \frac{36}{2}$

$= 18.$

Thus L and U are given by $L = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -7 & 1 \end{bmatrix}$, $U = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1/2 & 5/2 \\ 0 & 0 & 18 \end{bmatrix}$

$$LY = B \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -7 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$$

From this we get $y_1 = 9$, $\frac{1}{2}y_1+y_2 =6$, $\frac{3}{2}y_1- 7y_2 + y_3 = 8$.

On solving, we get $y_1 = 9$, $y_2 = \frac{3}{2}$, $y_3 = 5$.

$$\text{Further since } UX = Y \Rightarrow \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1/2 & 5/2 \\ 0 & 0 & 18 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 3/2 \\ 8 \end{bmatrix}$$

$2x + 3y + z = 9$, $1/2y + 5/2z = 3/2$, $18z = 5$.

By back substitution, we get $z = 5/18$ and $1/2y + 5/2(5/18) = 3/2$

$\Rightarrow y = 3 - 25/18 = 29/18$

And $2x + 3(29/18) + (5/18) = 9 \Rightarrow 2x = 9 - 29/6 - 5/18 = 9 - 92/18$

$\Rightarrow 2x = 70/18 \Rightarrow x = 35/18$

Hence, the solution is $x = 35/18$, $y = 29/18$, $z = 5/18$.

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3.5 ILL CONDITIONED OR WELL CONDITIONED:

* Establish whether the following matrix is singular, ill conditioned or well conditioned: $\begin{bmatrix} 3 & -1 & 3 \\ 1 & 0 & 4 \\ 2 & 2 & 1 \end{bmatrix}$?

Solution:-

Let A = $\begin{bmatrix} 3 & -1 & 3 \\ 1 & 0 & 4 \\ 2 & 2 & 1 \end{bmatrix}$

det A = $3(0 - 8) + 1(1 - 8) + 3(2 - 0) = -24 - 7 + 6 = -25$

$\therefore$ A is not singular.

$\begin{vmatrix} 3.01 & -1 & 3 \\ 1 & 0 & 4 \\ 2 & 2 & 1 \end{vmatrix}$ = $(3.01)(0 - 8) + 1(1 - 8) + 3(2 - 0) = -24.08-7+6 = -25.08$.

$\therefore$ The matrix is well conditioned.

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* Show that A = $\begin{bmatrix} -73 & 78 & 24 \\ 92 & 66 & 25 \\ -80 & 37 & 10 \end{bmatrix}$ is ill- conditioned?

Solution :-

The ,matrix A = $\begin{bmatrix} -73 & 78 & 24 \\ 92 & 66 & 25 \\ -80 & 37 & 10 \end{bmatrix}$ is well conditioned, since $|A| = 1$ whereas

$\begin{vmatrix} -73 & 78 & 24 \\ 92.01 & 66 & 25 \\ -80 & 37 & 10 \end{vmatrix} = 2.08,$ $\begin{vmatrix} -73 & 78.01 & 24 \\ 92 & 66 & 25 \\ -80 & 37 & 10 \end{vmatrix} = -28.20$

and $\begin{vmatrix} -73 & 78 & 24 \\ 92 & 66 & 25 \\ -80 & 37 & 10.01 \end{vmatrix} = -118.94$

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3.6 GAUSS - JACOBI METHOD:

* Solve the following equations by Jacobi method 20x + y -2z = 17, 3x + 20y - z = -18, 2x - 3y + 20z = 25?

Solution:-

Given equations can be written as $x = \frac{1}{20}(17 - y + 2z)$---------(1)

$y = \frac{1}{20}(-18 - 3x + z)$--------(2)

$z = \frac{1}{20}(25- 2x + 3y)$--------(3)

Let us take first approximation $x^{(1)} = 0$, $y^{(1)} = 0$, $z^{(1)} = 0$.

Then the second approximation using (1), we get

$x^{(2)} = \frac{1}{20}(17 - y^{(1)} + 2z^{(1)}) = \frac{17}{20} = 0.85$

$y^{(2)} = \frac{1}{20}(-18 - 3x^{(1)} + z^{(1)}) = \frac{1}{20}(-18 - 0 + 0) = -\frac{18}{20} = -0.9.$

$z^{(2)} = \frac{1}{20}(25 - 2x^{(1)} + 3y^{(1)}) = \frac{1}{20}(25 - 0 + 0) = \frac{25}{20} = 1.25.$

Substituting these values in R.H.S of (1), we get third approximation

$x^{(3)} = \frac{1}{20}(17 - y^{(2)} + 2z^{(2)}) = \frac{1}{20}(17 + 0.9 + 2(1.25)) = \frac{20.4}{20} = 1.02.$

$y^{(3)} = \frac{1}{20}(-18 - 3x^{(2)} + z^{(2)}) = \frac{1}{20}(-18 - 3(0.85) + 1.25) = \frac{-19.3}{20} = -0.965.$

$z^{(3)} = \frac{1}{20}(25 - 2x^{(2)} + 3y^{(2)}) = \frac{1}{20}(25 - 2(0.85) + 3(-0.9)) = \frac{20.6}{20} = 1.03.$

Now Substitute these values in (1), we get fourth approximation

$x^{(4)} = \frac{1}{20}(17 - y^{(3)} + 2z^{(3)}) = \frac{1}{20}(17 + 0.965 + 2(1.03)) = \frac{20.025}{20} = 1.0013.$

$y^{(4)} = \frac{1}{20}(-18 - 3x^{(3)} + z^{(3)}) = \frac{1}{20}(-18 - 3(1.02) + 1.03) = -\frac{20.03}{20} = -1.0015.$

$z^{(4)} = \frac{1}{20}(25 - 2x^{(3)} + 3y^{(3)}) = \frac{1}{20}(25 - 2(1.02) + 3(-0.965)) = \frac{20.065}{20} = 1.0032.$

Substitute these values in (1), we get fifth approximation

$x^{(5)} = \frac{1}{20}(17 - y^{(4)} + 2z^{(4)}) = \frac{1}{20}(17 + 1.0015 + 2(1.0032)) = \frac{20.0079}{20} =$

1.0004.

$y^{(5)} = \frac{1}{20}(-18 - 3x^{(4)} + z^{(4)}) = \frac{1}{20}(-18 - 3(1.0013) + 1.0032) =$

$- \quad = \quad -\frac{20.0007}{20} = -1.00003.$

$z^{(5)} = \frac{1}{20}(25 - 2x^{(4)} + 3y^{(4)}) = \frac{1}{20}(25 - 2(1.0013) + 3(1.0015)) = \frac{19.9929}{20} =$

1.9996.

Substitute these values in (1), we get sixth approximation

$x^{(6)} = \frac{1}{20}(17 - y^{(5)} + 2z^{(5)}) = \frac{1}{20}(17 + 1.00003 + 2(0.9996)) = \frac{19.99923}{20} =$ 0.99996.

$y^{(6)} = \frac{1}{20}(-18 - 3x^{(5)} + z^{(3)}) = \frac{1}{20}(-18 - 3(1.0004) + 0.9996) = \frac{-20.0016}{20} =$ -1.00008.

$z^{(6)} = \frac{1}{20}(25 - 2x^{(5)} + 3y^{(5)}) = \frac{1}{20}(25 - 2(1.0004) + 3(-1.00003)) = \frac{19.99911}{20} =$ 0.99995.

The values in the fifth and sixth approximation are nearly same.

$\therefore$ Hence the solution is x = 1, y = -1, z = 1.

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* Solve the equations by Jacobi Method 83x + 11y - 4z = 95; 7x + 52y + 13z = 104; 3x +8y + 29z = 71?

Solution:- Given equation can be written as

$x = \frac{1}{83}(95 - 11y + 4z)$, $y = \frac{1}{52}(104 - 7x - 13z)$, $z = \frac{1}{83}(71 - 3x - 8y)$------(1).

Let us take the first approximation to be $x^{(1)} = 0$, $y^{(1)} = 0$, $z^{(1)} = 0$.

Then the second approximation using (1), we get

$x^{(2)} = \frac{1}{83}(95 - 11y^{(1)} + 4z^{(1)}) = \frac{1}{83}(95 - 0 + 0) = 1.14$

$y^{(2)} = \frac{1}{52}(104 - 7x^{(1)} - 13z^{(1)}) = \frac{1}{52}(104 - 0 - 0) = \frac{104}{52} = 2.$

$z^{(2)} = \frac{1}{29}(71 - 3x^{(1)} - 8y^{(1)}) = \frac{1}{29}(71 - 0 - 0) = \frac{71}{23} = 2.4.$

Substituting these values in R.H.S of (1), we get third approximation

$x^{(3)} = \frac{1}{83}(95 - 11y^{(2)} + 4z^{(2)}) = \frac{1}{83}(95 - 11(2) + 4(2.4)) = \frac{82.6}{83} = 0.995 \sim 1.$

$y^{(3)} = \frac{1}{52}(104 - 7x^{(2)} - 13z^{(2)}) = \frac{1}{52}(104 - 7(1.14) - 13(2.4)) = \frac{64.82}{52} = 1.25.$

$z^{(3)} = \frac{1}{29}(71- 3x^{(2)} - 8y^{(2)}) = \frac{1}{29}(71 - 3(1.14) - 8(2)) = \frac{51.58}{29} = 1.78.$

Now Substitute these values in (1), we get fourth approximation

$x^{(4)} = \frac{1}{83}(95 - 11y^{(3)} + 4z^{(3)}) = \frac{1}{83}(95 -11(1.25)+ 4(1.78)) = \frac{88.37}{83} = 1.1.$

$y^{(4)} = \frac{1}{52}(104 - 7x^{(3)} - 13z^{(3)}) = \frac{1}{52}(104 - 7(1) - 13(1.78)) = \frac{73.86}{52} = 1.42.$

$z^{(4)} = \frac{1}{29}(71- 3x^{(3)} - 8y^{(3)}) = \frac{1}{29}(71 - 3(1) - 8(1.25)) = \frac{58}{29} = 2.$

Substitute these values in (1), we get fifth approximation

$x^{(5)} = \frac{1}{83}(95 - 11y^{(4)} + 4z^{(4)}) = \frac{1}{83}(95 -11(1.42)+ 4(2)) = \frac{-87.38}{83} = 1.1.$

$y^{(5)} = \frac{1}{52}(104 - 7x^{(4)} - 13z^{(4)}) = \frac{1}{52}(104 - 7(1.1) - 13(2)) = \frac{70.3}{52} = 1.4.$

$z^{(5)} = \frac{1}{29}(71- 3x^{(4)} - 8y^{(4)}) = \frac{1}{29}(71 - 3(1.1) - 8(1.42)) = \frac{51.7}{29} = 1.78.$

$\therefore$ Hence the solution is x = 1.1, y = 1.4, z = 1.78.

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3.7 GAUSS SEIDEL METHOD:

* Find the solution of the system by Gauss Seidel Method 83x + 11y - 4z = 95; 7x + 52y + 13z = 104; 3x +8y + 29z = 71?

Solution:-

Given equations can be written as $x = \frac{1}{83}(95 - 11y + 4z)$,

$y = \frac{1}{52}(104 - 7x - 13z)$,

$z = \frac{1}{83}(71 - 3x - 8y)$------(1).

Let us take the first approximation to be $x^{(1)} = 0$, $y^{(1)} = 0$, $z^{(1)} = 0$.

Then the second approximation using (1), we get

$$x^{(2)} = \frac{1}{83}(95 - 11y^{(1)} + 4z^{(1)}) = \frac{1}{83}(95 - 0 + 0) = 1.14$$

Put $x^{(2)} = 1.14$, $y^{(1)} = 1.85$, $z^{(1)} = 0$ in (3), we get

$$y^{(2)} = \frac{1}{52}(104 - 7x^{(2)} - 13z^{(1)}) = \frac{1}{52}(104 - 7(1.14) - 0) = \frac{96.02}{52} = 1.85.$$

Put $x^{(2)} = 1.14$, $y^{(2)} = 1.85$, $z^{(1)} = 0$ in (3), we get

$$z^{(2)} = \frac{1}{83}(71 - 3x^{(2)} - 8y^{(2)}) = \frac{1}{83}(71 - 3(1.14) - 8(1.85)) = \frac{52.78}{29} = 1.82.$$

Put $x^{(2)} = 1.14$, $y^{(2)} = 1.85$, $z^{(2)} = 1.82$ in (3), we get

$$x^{(3)} = \frac{1}{83}(95 - 11y^{(2)} + 4z^{(2)}) = \frac{1}{83}(95 - 11(1.85) + 4(1.82)) = \frac{81.93}{83} = 0.99.$$

Put $x^{(3)} = 1.14$, $y^{(2)} = 1.85$, $z^{(2)} = 1.82$ in (3), we get

$$y^{(3)} = \frac{1}{52}(104 - 7x^{(3)} - 13z^{(2)}) = \frac{1}{52}(104 - 7(0.99) - 13(1.82)) = \frac{73.41}{52} = 1.41$$

Put $x^{(3)} = 0.99$, $y^{(3)} = 1.41$, in (3) we get

$$z^{(3)} = \frac{1}{83}(71 - 3x^{(3)} - 8y^{(3)}) = \frac{1}{83}(71 - 3(0.99) - 8(1.41)) = \frac{56.75}{29} = 1.95.$$

Again put $x^{(3)} = 0.99$, $y^{(3)} = 1.41$, $z^{(3)} = 1.95$ in (3) we get

$$x^{(4)} = \frac{1}{83}(95 - 11y^{(3)} + 4z^{(3)}) = \frac{1}{83}(95 - 11(0.99) + 4(1.95)) = \frac{87.29}{83} = 1.05.$$

Put $x^{(4)} = 1.05$ $z^{(3)} = 1.95$ in (2) we get $y^{(4)} = \frac{1}{52}(104 - 7x^{(3)} - 13z^{(2)}) =$

$$\frac{1}{52}(104 - 7(1.05) - 13(1.95)) = \frac{71.3}{52} = 1.37.$$

Put $x^{(4)} = 1.05$, $y^{(4)} = 1.37$ in (3), we get

$$z^{(4)} = \frac{1}{83}(71 - 3x^{(4)} - 8y^{(4)}) = \frac{1}{83}(71 - 3(1.05) - 8(1.37)) = \frac{56.89}{29} = 1.96.$$

Thus $x^{(4)}, y^{(4)}, z^{(4)}$ are sufficiently close to $x^{(3)}, y^{(3)}, z^{(3)}$ respectively

$\therefore$ Hence the solution is $x = 1.05$, $y = 1.37$, $z = 1.96$.

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* Solve by Gauss Seidel method $10x_1 + x_2 + x_3 = 12$, $2x_1 + 10x_2 + x_3 = 13$, $2x_1 + 2x_2 + 10x_3 = 14$.

Solution:-

From the given equations we have

$$X_1 = \frac{1}{10}[12 - x_1 - x_2] \text{---------(1)}$$

$$X_2 = \frac{1}{10}[13 - 2x_1 - x_3] \text{---------(2)}$$

$$X_3 = \frac{1}{10}[14 - 2x_1 - 2x_2] \text{---------(3)}$$

Putting x = 0 in right side of the equation (1), we get $x_1^{(1)} = \frac{12}{10} = 1.2$

Putting $x_1 = x_1^{(1)} = 1.2$, $x_3 = 0$ in (2), we get $x_2^{(1)} = \frac{1}{10}(13 - 2.4 - 0) = 10.6/10 = 1.06$

Putting $x_1 = x_1^{(1)} = 1.2$, and $x_2 = x_2^{(1)} = 1.06$ in (3), we get

$x_3^{(2)} = \frac{1}{10}[14 - 2.4 - 2.12] = 0.948.$

For the second iteration we have $x_1^{(2)} = \frac{1}{10}[12 - 1.06 - 0.948] = 0.9992.$

$x_2^{(2)} = \frac{1}{10}[13 - 2(0.9992) - 0.948] = 1.00536.$

$x_3^{(2)} = \frac{1}{10}[14 - 2(0.9992) - 2(1.00536)] = 0.999098.$

Thus the iteration process is continued

The results are tabulated as follows correcting to four decimal places.

i	$x_1^{(i)}$	$x_2^{(i)}$	$x_3^{(i)}$
0	1.2000	0.0000	0.0000
1	1.2000	1.0600	0.9480
2	0.9992	1.0054	0.9991
3	0.9996	1.0000	1.0000
4	1.0000	1.0000	1.0000
5	1.0000	1.0000	1.0000

$\therefore$ The exact value of the roots are $x_1 = 1$, $x_2 = 1$, $x_3 = 1$.

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Network Analysis in linear system of Equations:

Set up a system of linear equations to represent the network shown in the figure. Then solve the system.

Solution:-

Each of the network's five junctions give rise to a linear equations as follows:

$X_1 + X_2 = 20$ Junction 1

$X_3 - X_4 = -20$ Junction

$X_2 + X_3 = 20$ Junction 3

$X_1 - X_5 = -10$ Junction 4

$-X_4 + X_5 = -10$ Junction 5

The augmented matrix for this system is

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 20 \\ 0 & 0 & 1 & -1 & 0 & -20 \\ 0 & 1 & 1 & 0 & 0 & 20 \\ 1 & 0 & 0 & 0 & -1 & -10 \\ 0 & 0 & 0 & -1 & 1 & -10 \end{bmatrix}$$

Gauss Jordan elimination produces the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & -10 \\ 0 & 1 & 0 & 0 & 1 & 30 \\ 0 & 0 & 1 & 0 & -1 & -10 \\ 0 & 0 & 0 & 1 & -1 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

From the matrix above, you can see that

$X_1 - X_5 = -10$, $X_2 - X_5 = 30$,

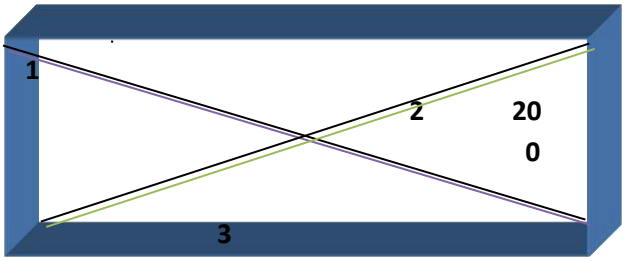
$X_3 - X_5 = -10$ and $X_4 - X_5 = -10$

Letting $t = X_5$, we have

$X_1 = t - 10$, $X_2 = -t + 30$, $X_3 = t + 30$, $X_4 = t + 10$, $X_5 = t$

Where t is the real number, so this system has infinitely many solutions.

1



2

4

5

Figure

In example 5, suppose you could control the mount of flow along the branch labeled X_5. using the solution of example 5 you could then control the flow represented by each of the other variables. For instance, letting t =10 would reduces the flow of X_1 and X_3 to zero, as shown In the figure.

4.LAPLACE TRANSFORM

4.1:Definition of Laplace Transform:It is piecewise continuous function,the Laplace Transform method of solving initial value problems

The method of Laplace transforms is a system that relies on algebra (rather than calculus-based methods) to solve linear differential equations. While it might seem to be a somewhat cumbersome method at times, it is a very powerful tool that enables us to readily deal with linear differential equations with discontinuous forcing functions.

***Definition*:** Let $f(t)$ be defined for $t \geq 0$. The Laplace transform of $f(t)$, denoted by $F(s)$ or L[f(t)] is an *integral transform* given by the *Laplace integral*:

$$L[f(t)]=F(s)=\int_0^\infty e^{-st} f(t)\, dt .$$

Provided that this (improper) integral exists, i.e. that the integral is convergent.

The Laplace transform is an operation that transforms a function of t (i.e., a function of *time domain*), defined on $[0, \infty)$, to a function of s (i.e., of *frequency domain*)*. $F(s)$ is the *Laplace transform*, or simply *transform*, of $f(t)$.

Together the two functions $f(t)$ and $F(s)$ are called a *Laplace transform pair*.

For functions of t continuous on $[0, \infty)$, the above transformation to the frequency domain is *one-to-one*. That is, different continuous functions will have different transforms.

History of Laplace Transforms

Transformation in mathematics deals with the conversion of one function to another function that may not be in the same domain. The transform method finds its application in those problems which can't be solved directly. This transform is named after the mathematician and renowned astronomer Pierre Simon Laplace who lived in France.

He used a similar transform on his additions to the probability theory. It became popular after World War Two. This transform was made popular by Oliver Heaviside, an English Electrical Engineer. Other famous scientists such as Niels Abel, Mathias Lerch, and Thomas Bromwich used it in the 19th century.The complete history of the Laplace Transforms can be tracked a little more to the past, more specifically 1744. This is when another great mathematician called Leonhard Euler was researching on other types of integrals. Euler however did not pursue it very far and left it. An admirer of Euler called Joseph Lagrange; made some modifications to Euler's work and did further work. LaGrange's work got Laplace's attention 38 years later, in 1782 where he continued to pick up where Euler left off. But it was not 3 years later; in 1785 where Laplace had a stroke of genius and changed the way we solve differential equations forever. He continued to work on it and continued to

unlock the true power of the Laplace transform until 1809, where he started to use infinity as a integral condition.

4.2 Properties of Laplace Transform

Some of the Laplace transformation properties are:

If $f_1(t) \leftrightarrow F_1(s)$ and [note: $\leftrightarrow$ implies Laplace Transform]

$f_2(t) \leftrightarrow F_2(s)$, then

Linearity Property	$A\ f_1(t) + B\ f_2(t) \leftrightarrow A\ F_1(s) + B\ F_2(s)$
Frequency Shifting Property	$\int_0^s e\ d\lambda \leftrightarrow F(s - s_0)$
Integration	$\int_0^t f(\lambda)\ d\lambda \leftrightarrow \frac{1}{s}F(s)$
Multiplication by Time	$T\ f(t) \leftrightarrow (-d\ \frac{F(s)}{ds})$
Complex Shift Property	$f(t)\ e^{-at} \leftrightarrow F(s + a)$
Time Reversal Property	$f(-t) \leftrightarrow F(-s)$
Time Scaling Property	$f(\frac{t}{a}) \leftrightarrow a\ F(as)$

4.3 Laplace Transform Table

The following Laplace transform table helps to solve the differential equations for different functions:

SI.No.	f(t)	L(f(t)) = F(p)
1	1	$\frac{1}{p}$
2	t^n at t = 1,2,3,…	$\frac{n!}{p(n+1)}$
3	√(t)	$\frac{\sqrt{\pi}}{2p^{\frac{3}{2}}}$
4	sin(at)	$\frac{a}{(p^2+a^2)^2}$
5	sinht(at)	$\frac{a}{(p^2-a^2)}$
6	$e^{(at)}$ sin(bt)	$\frac{b}{(p-a)^2+b^2}$

SI.No.	f(t)	L(f(t)) = F(p)
7	$e^{(ct)}f(t)$	F(p-c)
8	f'(t)	pF(p) – f(0)
9	$e^{(at)}$	$\frac{1}{(p-a)}$
10	t^nat p>-1	$\frac{\Gamma(p+1)}{p^{(p+1)}}$
11	$t^{(n-\frac{1}{2})}$ at n = 1,2,..	$\frac{(1.2.3..(2n-1)}{2^n\, p^{(n+\frac{1}{2})}}$
12	cos(at)	$\frac{p}{(p^2+a^2)}$
13	sin(at)	$\frac{a}{p^2+a^2}$
14	t cos(at)	$\frac{p^2-a^2}{(p^2+a^2)^2}$
15	sinht	$\frac{a}{p^2-a^2}$
16	cosh(at)	$\frac{p}{p^2-a^2}$

SI.No.	f(t)	L(f(t)) = F(p)
17	$e^{(at)}\cos(bt)$	$\frac{(p-a)}{(p-a)^2+b^2}$
18	t^nf(t) at n = 1,2,3..	$(-1)^n F^n$ s
19	f'(t)	$p^2F(p) - pf(0) - f'(0)$
20	$e^{at}t^n$	$\frac{n!}{(p-a)^{n+1}}$

Laplace transforms examples:

Example:1

Find $\mathcal{L}\{\sinh at\}$

Solution: $\mathcal{L}\{\sinh at\} = \mathcal{L}\left\{\frac{e^{at} - e^{-at}}{2}\right\}$

$= \frac{1}{2}\mathcal{L}\{e^{at}\} - \frac{1}{2}\mathcal{L}\{e^{-at}\}$

$= \frac{1}{2}\left\{\frac{1}{s-a} - \frac{1}{s+a}\right\} = \frac{a}{s^2-a^2}$

Example:2

Find $\mathcal{L}\{(t^3+1)^3\}$

Solution:Given,

$\mathcal{L}\{(t^3+1)^3\}$

$\Rightarrow \mathcal{L}\{t^6\} + L\{1\} + 2L\{t^3\}$

$$= \frac{6!}{p^{6+1}} + \frac{1}{p} + 2.\frac{3!}{p^{3+1}}$$

$$= \frac{6.5.4.3.2.1}{p^7} + \frac{1}{p} + 2.\frac{3.2}{p^4}$$

$$= \frac{720}{p^7} + \frac{1}{p} + \frac{12}{p^4}$$

$$= \frac{720}{p^7} + \frac{12}{p^4} + \frac{1}{p}$$

Example:3

Find $\mathbf{L\{\sin^2 4t\}}$

Solution: We know that,

$$\text{Cos2t} = 1 - 2\sin^2 t$$

$$\text{Cos6t} = 1 - 2\sin^2 3t$$

$$\text{Cos8t} = 1 - 2\sin^2 4t$$

$$\text{L}\left\{\frac{(1-\cos 8t)}{2}\right\}$$

$$\Rightarrow \text{L}\left\{\frac{1}{2}\right\} - \frac{1}{2}\text{L}\{cos8t\}$$

$$= \frac{1}{2}\{1\} - \frac{1}{2}\text{L}\{\text{cos8t}\}$$

$$= \frac{1}{2}.\frac{1}{p} - \frac{1}{2}\left[\frac{p}{p^2+8^2}\right]$$

$$\mathrm{L}\{\sin^2 4t\} = \frac{1}{2}\left[\frac{1}{p} - \frac{p}{p^2+64}\right]$$

Example:4

Find $\mathcal{L}\{\mathbf{sint + cost}\}^2$

Solution: Given $\mathcal{L}\{\mathrm{sint} + \mathrm{cost}\}^2$

$$\mathcal{L}\{\sin^2 t + \cos^2 t + 2sintcost\}$$

$$\Rightarrow \mathcal{L}\{1 + 2sintcost\}$$

$$\Rightarrow \mathcal{L}\{1 + sin2t\}$$

$$\Rightarrow \mathcal{L}\{1\} + \mathcal{L}\{\, sin2t\}$$

$$\mathcal{L}\{\mathrm{sint} + \mathrm{cost}\}^2 = \frac{1}{p} + \frac{2}{p^2+4}$$

Example:5

Find $\mathcal{L}\{\boldsymbol{sinhat.cosat}\}$

Solution: Given $\mathcal{L}\{sinht.cosat\}$

Sinht=($e^{ax}-e^{-ax}$ 、

$$\Rightarrow \mathrm{L}\{(\frac{e^{ax}-e^{-ax}}{2})cosat\}$$

$$=> \frac{1}{2}[\mathrm{L}\{e^{ax}cosat\} - \mathrm{L}\{e^{-ax}\}]$$

$$\mathrm{L}\{sinht.cosat\} = \frac{1}{2}\left\{\frac{p-a}{(p-a)^2+a^2} - \frac{p+a}{(p+a)^2+a^2}\right\}$$

4.4:Theorem:

FIRST SHIFTING THEOREM OF LAPLACE TRANSFORM:

STATEMENT: If $\mathcal{L}\{F(t)\}=f(p)$ then $\mathcal{L}\{e^{at}F(t)\} = f(p-a)$

Proof:- By the definition of Laplace Transform

$\mathcal{L}\{F(t)\}=f(p)$

$\Rightarrow \mathcal{L}\{F(t)\}=\int_0^\infty e^{-pt}F(t)dt$

$\Rightarrow \mathcal{L}\{e^{at}F(t)\} = \int_0^\infty e^{at}\, e^{-pt}F(t)dt$

$\Rightarrow \mathcal{L}\{e^{at}F(t)\} = \int_0^\infty e^{(a-p)t}F(t)dt$

Example:1

Find $\mathcal{L}\{e^{3t}\sin^2 t\}$

Solution: Given $\mathcal{L}\{e^{3t}\sin^2 t\}$

We know $\mathcal{L}\{\sin^2 t\} = \mathcal{L}\left\{\frac{1-cos2t}{2}\right\}$

$= \frac{1}{2}[\mathcal{L}\{1\} - \mathcal{L}\{\cos 2t\}]$

$$= \frac{1}{2}\left[\frac{1}{p} - \frac{p}{p^2+4}\right]$$

$$= \frac{2}{p(p^2+4)} = f(p), (say)$$

By First Shifting Theorem, $\mathcal{L}\{e^{3t}\sin^2 t\} = f(p-3)$

$L\{e^{-2t}\sin 2t.\sin 3t\} = \left[\frac{2}{p(p^2+4)}\right]_{p\to p-3}$

$$= \frac{2}{(p-3)[(p-3)^2+4]}$$

$\mathcal{L}\{e^{3t}\sin^2 t\} = \frac{2}{(p-3)(p^2-6p+13)}$

Example:**2**

Evaluate $\mathrm{L}\left\{e^t\left(\boldsymbol{cos2t}+\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{sinh2t}\right)\right\}$

Solution: Given $\mathrm{L}\left\{e^t\left(cos2t+\frac{1}{2}sinh2t\right)\right\}$

$\mathrm{L}\left\{cos2t+\frac{1}{2}sinh2t\right\}$

$\mathrm{L}\{cos2t\}+\mathrm{L}\{\frac{1}{2}sinh2t\}$

$$=\frac{p}{p^2+2^2}+\frac{1}{2}.\frac{2}{p^2-2^2}$$

$$=\frac{p}{p^2+4}+\frac{1}{p^2-4}=f(p),say$$

$By\ first\ shifting\ theroem,\ \mathrm{L}\ \left\{e^t\left(cos2t+\frac{1}{2}sinh2t\right)\right\}=f(p-1)$

$=\left(\frac{p}{p^2+4}+\frac{1}{p^2-4}\right)_{\mathrm{p}\to\mathrm{p\text{-}1}}$

$$=\frac{p-1}{(p-1)^2+4}+\frac{1}{(p-1)^2-4}$$

$\mathcal{L}\left\{\mathrm{e}^{\mathrm{t}}\left(\mathrm{cos2t}+\frac{1}{2}\mathrm{sinh2t}\right)\right\}=\frac{\mathrm{p-1}}{\mathrm{p^2-2p+5}}+\frac{1}{\mathrm{p^2-2p-3}}$

Example:**3**

Find $\mathbf{L\left\{e^{-2t}sin2t.\,sin3t\right\}}$.

Solution: Given $\mathrm{L}\{\mathrm{e}^{-2\mathrm{t}}\mathrm{sin2t.\,sin3t}\}$

[2sinA.sinB=cos(A-B)-cos(A+B)]

$$L\left[\frac{cost-cos5t}{2}\right]$$

$$= \frac{1}{2}[L(cost) - L(cos5t)]$$

$$= \frac{1}{2}\left[\frac{p}{p^2+1} - \frac{p}{p^2+25}\right]$$

$$= \frac{12p}{(p^2+1)(p^2+25)}$$

By first shifting theorem, $L\{e^{-2t}sin2t.sin3t\} = \left[\frac{12p}{(p^2+1)(p^2+25)}\right]_{p\to p+2}$

$$= \frac{12(p+2)}{\{(p+2)^2+1\}\{(p+2)^2+25\}}$$

$\rightarrow L\{e^{-2t}sin2t.sin3t\} = \frac{12p+24}{(p^2+4p+5)(p^2+4p+29)}$

4.5: Inverse Laplace Transform

In the inverse Laplace transform, we are provided with the transform F(s) and asked to find what function we have initially. The inverse transform of the function F(s) is given by:

$f(t) = L^{-1}\{F(s)\}$

For example, for the two Laplace transform, say F(s) and G(s), the inverse Laplace transform is defined by:

$L^{-1}\{aF(s)+bG(s)\}= a\ L^{-1}\{F(s)\}+bL^{-1}\ \{G(s)\}$

Where a and b are constants.

In this case, we can take the inverse transform for the individual transforms, and add their constant values in their respective places, and perform the operation to get the result.

Example: 1

Find $\mathbf{L^{-1}\left\{\frac{3(p^2-1)^2}{2p^5}\right\}}$

Solution: Given $L^{-1}\left\{\frac{3(p^2-1)^2}{2p^5}\right\}$

$$L^{-1}\left\{3\frac{(p^4+1-2p^2)}{2p^5}\right\}$$

$$=>\frac{3}{2}\left[L^{-1}\left\{\frac{p^4}{p^5}+\frac{1}{p^5}-2.\frac{p^2}{p^5}\right\}\right]$$

$$=>\frac{3}{2}\left[L^{-1}\left\{\frac{1}{p}\right\}+L^{-1}\left\{\frac{1}{p^5}\right\}-2L^{-1}\left\{\frac{1}{p^3}\right\}\right]$$

$$=\frac{3}{2}[1+\frac{t^4}{4!}-2.\frac{t^2}{2!}$$

$$=\frac{3}{2}\left[1+\frac{t^4}{24}-t^2\right]$$

$$=\frac{3}{2}+\frac{3}{2}.\frac{t^4}{24}-\frac{3}{2}t^2$$

$$L^{-1}\left\{\frac{3(p^2-1)^2}{2p^5}\right\}=\frac{3}{2}+\frac{t^4}{16}-\frac{3}{2}t^2$$

Example:2

Find $\boldsymbol{L^{-1}\left\{\frac{1}{(p+1)(p+2)}\right\}}$

Solution: Given $L^{-1}\left\{\frac{1}{(p+1)(p+2)}\right\}$

Take $\left\{\frac{1}{(p-1)(p+2)}\right\}=\frac{A}{p+1}+\frac{B}{p+2}\ldots\ldots\ldots(1)$

$$\left\{\frac{1}{(\mathrm{p}+1)(\mathrm{p}+2)}\right\} = \frac{\mathrm{A}(\mathrm{p}+2)+\mathrm{B}(\mathrm{p}+1)}{(\mathrm{p}+1)(\mathrm{p}+2)}$$

$$=> 1 = A(p+2)+B(p+1) \ldots\ldots\ldots (2)$$

Put p=-2 in (2),we get

1=0+B(-1)

B=-1

Put p=-1 in (2),we get

1=A+0

A=1

From (1)

$$\left\{\frac{1}{(p-1)(p+2)}\right\} = \frac{1}{p+1} - \frac{1}{p+2} \ldots\ldots\ldots (1)$$

Apply inverse Laplace on both sides we get

$$L^{-1}\left\{\frac{1}{(p-1)(p+2)}\right\} = L^{-1}\left\{\frac{1}{p+1}\right\} - L^{-1}\left\{\frac{1}{p+2}\right\}$$

$$= e^{-t} - e^{-2t}$$

Example:3

Find $\boldsymbol{L^{-1}\left\{\frac{1}{p-2}+\frac{2}{p+5}+\frac{6}{p^4}\right\}}$

Solution: Given $L^{-1}\left\{\frac{1}{p-2}+\frac{2}{p+5}+\frac{6}{p^4}\right\}$

$$=> L^{-1}\left\{\frac{1}{p-2}\right\} + L^{-1}\left\{\frac{2}{p+5}\right\} + L^{-1}\left\{\frac{3!}{p^{3+1}}\right\}$$

$$= e^{2t} + 2e^{5t} + t^n$$

4.6:THEOREM

SECOND SHIFTING THEOREM OF INVERSE LAPLCE TRANSFORM:

STATEMENT: If $L^{-1}\{f(p)\} = F(t), then\ L^{-1}\{e^{-ap}f(p)\} = G(t)$

$$G(t)= \begin{cases} F(t-a), t > a \\ 0 \qquad , t < a \end{cases}$$

Proof: $L\{G(t) = \int_0^\infty e^{-pt}G(t)dt$

$$= \int_0^a e^{-pt}G(t)dt + \int_a^\infty e^{-pt}G(t)dt$$

$$= \int_0^a e^{-pt}\,0\,dt + \int_a^\infty e^{-pt}F(t-a)dt$$

$$= 0 + \int_a^\infty e^{-pt}F(t-a)dt$$

$$= \int_a^\infty e^{-p(a+u)}F(u)du\,, t-a = u$$

$$= \int_a^\infty e^{-ap}\,e^{-pu}F(u)du$$

$$= e^{-ap}\int_a^\infty e^{-pu}F(u)du$$

$$= e^{-ap} L\{F(t)\}$$

$$= e^{-ap} f(p)$$

$$Thus, L\{G(t)\} = e^{-ap} f(p)$$

$$\rightarrow G(t) = L^{-1}\{e^{-ap} f(p)$$

Example:1

Find the inverse Lapalce transform of $\frac{e^{-2p}}{(p-2)^2}$

Solution: Given $\frac{e^{-2p}}{(p-2)^2}$

Take $L^{-1}\left[\frac{1}{(p-2)^2}\right]$

$$=> e^{2t} L^{-1}\left[\frac{1}{p^2}\right]$$

$$= e^{2t} t$$

$$By\ \ second\ shifting\ theorem, we\ have\ L^{-1}\left[\frac{e^{-2p}}{(p-2)^2}\right]$$

$$= \begin{cases} e^{2(t-2)}(t-2), t > 2 \\ 0 \qquad\qquad\quad , t < 2 \end{cases}$$

4.7: Laplace Transform of Differential Equation

The Laplace transform is a well established mathematical technique for solving a differential equation. Many mathematical problems are solved using transformations. The

idea is to transform the problem into another problem that is easier to solve. On the other side, the inverse transform is helpful to calculate the solution to the given problem.

For better understanding, let us solve a first-order differential equation with the help of Laplace transformation.

Example:1

Consider $y' - 2y = e^{3x}$ and $y(0) = -5$. Find the value of L(y).

First step of the equation can be solved with the help of the linearity equation:

Sol: $L[(y' - 2y)] = L(e^{3x})$

$L(y') - L(2y) = \frac{1}{(s-3)}$

(because $L(e^{ax}) = \frac{1}{(s-a)}$

$L(y') - 2s(y) = \frac{1}{(s-3)}$

$sL(y) - y(0) - 2L(y) = \frac{1}{(s-3)}$

(Using Linearity property of the Laplace transform)

$L(y)(s-2) + 5 = \frac{1}{(s-3)}$ (Use value of y(0) ie -5 (given))

$L(y)(s-2) = \frac{1}{(s-3)} - 5$

$L(y) = \frac{(-5s+16)}{(s-2)(s-3)}$.....(1)

Here $\frac{(-5s+16)}{(s-2)(s-3)}$ can be written as $\frac{-6}{(s-2)} + \frac{1}{(s-3)}$ using partial fraction method

(1) implies $L(y) = \frac{-6}{(s-2)} + \frac{1}{(s-3)}$

$L(y) = -6e^{2x} + e^{3x}$

Example: 2

$y'' - 4y = 0, \quad y(0) = 1, \; y'(0) = 2$

Solution: Take Laplace Transform on both sides,

$$L\{ y'' - 4y \} = L\{ 0 \}$$

$$\text{or} \quad L\{ y'' \} - 4 L\{ y \} = 0$$

$$s^2 L\{ y \} - s\, y(0) - y'(0) - 4 L\{ y \} = 0$$

$$\text{or} \quad s^2 L\{ y \} - s - 2 - 4 L\{ y \} = 0$$

$$\Rightarrow L\{ y \} = \frac{s + 2}{s^2 - 4} = \frac{1}{s - 2}$$

$$\Rightarrow y(t) = e^{2t}$$

Example:3

$y'' - 3y' + 2y = 4t - 6, \quad y(0) = 1, \; y'(0) = 3$

Solution: $(s^2 \bar{y} - s - 3) - 3 (s \bar{y} - 1) + 2 \bar{y} = \frac{4}{s^2} - \frac{6}{s}$

$$\Rightarrow \bar{y} = \frac{s^2 + 2s - 2}{s^2 (s - 1)} = \frac{1}{s - 1} + \frac{2}{s^2}$$

$$\therefore \quad y = \mathcal{L}^{-1}\left\{ \frac{s^2 + 2s - 2}{s^2 (s - 1)} \right\}$$

$$= \mathcal{L}^{-1}\left\{ \frac{1}{s - 1} + \frac{2}{s^2} \right\} = e^t + 2t$$

Solving initial value problems using the method of Laplace transforms

To solve a linear differential equation using Laplace transforms, there are only 3 basic steps:

1.Take the Laplace transforms of both sides of an equation.

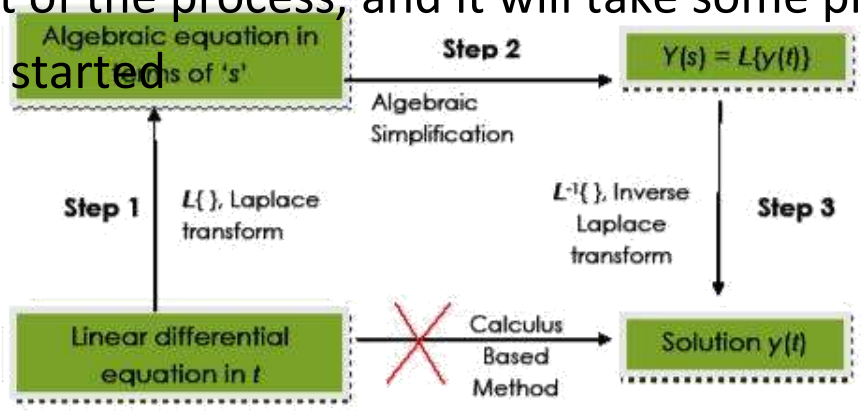
2.Simplify algebraically the result to solve for $\mathcal{L}\{y\} = Y(s)$ in terms of s.

3.Find the inverse transform of $Y(s)$. (Or, rather, find a function $y(t)$ whose Laplace transform matches the expression of $Y(s)$.) This inverse transform, $y(t)$, is the solution of the given differential equation.

The nice thing is that the same 3-step procedure works whether or not the differential equation is homogeneous or nonhomogeneous. The first two steps in the procedure are rather mechanical. The last step is the heart of the process, and it will take some practice. Let's get started

The Laplace Transform Method of Solving an Initial Value Problem

Flow Chart



Applications of Laplace transform

This section describes the applications of laplace transform in the domain of engineering. Laplace transform has several applications in almost all Engineering disciplines such as System Modelling, Analysis of Electrical and Electronic Circuits, Digital Signal Processing and Process Controls. Laplace transforms are critical for process controls. It helps analyze the variables, which when altered, produces desired manipulations in the result. For example, while study heat experiments, Laplace transform is used to find out to what extent the given input can be altered by changing temperature, hence one can alter temperature to get desired output for a while. This is an

efficient and easier way to control processes that are guided by differential equations

Electric Circuit theory

Example 1 :: The switching transient phenomenon in the RL , RC or RLC circuits can be solved by laplace

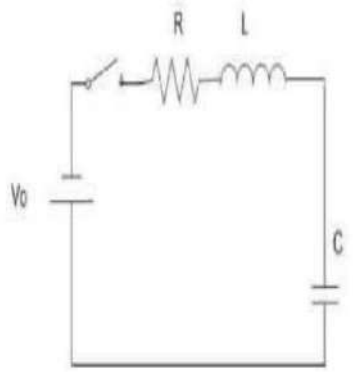


FIG. 1: Series RLC circuit

transform. Let us consider a series RLC circuit as shown figure 1 to which a dc voltage V 0 is suddenly applied.

Now applying kirchoff‘s voltage law (KVL) to the circuit , we

have Differentiating both sides,

$$L\frac{d^2i}{dt^2}+\frac{1}{C\,i}+R\frac{d\,i}{d\,t}=0\ \ or$$

Applying laplace transform to this equation, let us assume

$$\frac{d^2i}{d\,t^2}+\frac{R}{L}\frac{d\,i}{d\,t}+\frac{1}{L\,C}\,i=0\quad\rightarrow(1)$$

that the solution of this equation is

$$\mathbf{i(\,t\,)}=e^{sT}$$

$$L\,K\,s^2\,e^{s\,T}+R\,K\,e^{s\,T}+\frac{1}{C\,K}\,e^{s\,T}=0$$

5.CURVE FITTING

5.1 INTRODUCTION :

In this chapter we are concerned with the problem of fitting

an equation or a curve to data involving period values. An approximate non-mathematical relationship between the two variables , can be established by a diagram called **scatter diagram .** The exact mathematical relationship between the two variables is given by simple algebra expression called **curve fitting .** Though there are infinite variety of curve in mathematics , the curve used for the purpose of curve fitting are relatively limited in type. The straight line is the simple and one of the most important curve used.

The straight line :

The equation $y = a + bx$…. (1) Is an equation of the degree in x and y .It represents a straight line .

The difference $y_i - (a + bx_i)$….. (2)

Is zero if and only if the point (x_i, y_i) lies on the line given by (1)

Fitting a straight line:

Usually fitting a straight line means finding the value of the parameter a and b of the straight line given by (1) , as well as actually constructing the line itself .The Graphic method and the method of least square are two useful method for fitting a straight line .

The graphical method:

The method can be used whenever the given formula can be plotted as a straight line either directly or after suitable transformation. The straight line drawn after a careful visual estimate of its position has been made with the aid of a ruler. The coordinated of any two points on the measures and substitute in equation (1). The resulting equations in a and b are then solved for their *parameters.* Graphical method , whatever its theoretical attractions , suffers from the disadvantage that it is difficult apply in practice except for the straight line .This method will give fairly good results when finely divided coordinate paper is used , but in general it is recommended .

An alternative method, which id in almost universal use at present time, is known as the of **Least Squares** and we proceed to discuss it at length.

The method of lest squares:

This method of curve fitting was suggested early in the nineteenth century by the French mathematician Adrian Legendre. The method of least squares assumes that the best fitting line in the curve for which the sum of the squares of the vertical distances of the points (x_r , y_r) from the line is minimum. This method is more accurate that the graphical method.

Let $y = a + bx$ Refer (1)

Be the equation of the line. The ordinate of any point Q_r on the line vertically above or below a given point P_r , can be found by substituting the abscissa x_r , in the right-hand side of (1) . The two coordinates of Q_r will be $(x_r , a + bx_r)$ (see in the above figure). The vertical distances e_r from the line of any point p_r with coordinates (x_r , y_r) will therefore be given by the equation

$$e_r = y_r - (a + x_r) \ldots\ldots (3)$$

We may say that e_r represents the difference between the actual ordinate y_r , of a pint and its theoretical ordinate $a + bx_r$.

Let $q = \sum_{r=1}^{n} e_r^2 = \sum_{r=1}^{n} (y_r - (a + bx_r))^2$ ……. (4)

The best fitting line is that line for which the sum of the square, $q = \sum e_r^2$ is a minimum. We find the value of a and b which make q minimum as follows

:Differentiating q partially with respect to a and b equally these partially to zero , we obtain

$$\therefore \frac{\partial_q}{\partial_a} = \sum_{r=1}^{n}(-2)[y_r - (a + bx_r)] = 0 \ ,$$

And $\frac{\partial_q}{\partial_o} = \sum_{r=1}^{n}(-2)x_r[y_r - (a + bx_r)] = 0$,

Which yield the so – called **System of normal equations,**

$$\sum_{r=1}^{n} y_r = an + b\sum_{r=1}^{n} x_r \quad ,\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

And $\sum_{r=1}^{n} x_r\, y_r \ = a\ \sum_{r=1}^{n} x_r + \ b\ \sum_{r=1}^{n} x_r^2$ ……..(6)

Solving this system of equations we get

$$a = x = \frac{\sum_1^n x_r^2\ \sum_1^n y_r - \sum_1^n x_r \sum x_r y_r}{n\sum_1^n x_r^2 - [\sum_1^n x_r}$$ …….. (7)

$$b = \frac{n[\sum_{r=1}^{n} x_r y_r] - [\sum_{r=1}^{n} x_r][\sum_{r=1}^{n} x_r^2]}{n[\sum_{r=1}^{n} x_r^2] - [\sum_{r=1}^{n} x_r]^2}$$,……..(8)

Where n is the number of points $(x_r\ , y_r)$.

To simplify the for formulae, we get

$$t_r = \frac{x_r - \bar{x}}{h} \ ,$$ (9)

Where h is the interval between successive value of x assumed to be equally spaced. Shifting the origin to the mean $\bar{x}$ and compressing the intervals to unity we get transformed equation of (2) referred to the new (t, y) axes as

$$Y = \bar{a} + \bar{b}t \ ,$$ (10)

Where $\bar{a}\ and\ \bar{b}$ are the parameter in the new equation . In terms of the new coordinate formula (7) and (8) becomes

$$\bar{a} = \frac{\sum_1^n t_r^2 \sum_1^n y_r - \sum_1^n t_r \sum t_r y_r}{n \sum t_r^2 - (\sum t_r)^2} \quad , \qquad (11)$$

$$\bar{b} = \frac{n \sum_1^n t_r y_1 - \sum_1^n t_r \sum t_r y_r}{n \sum_1^n t_r^2 - [\sum_1^n t_r]^2} \quad , \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (12)$$

Transformation (9) replace the x_r $\grave{s}$ by unit deviations t_r from the mean $\bar{x}$.

Since

$$\sum_1^n t_r = 0$$

The formula (11)and (12) reduce to

$$\bar{a} = \frac{\sum_1^n t_r^2 \sum_1^n y_r - 0}{n \sum t_r^2 - 0}$$

And

$$\bar{b} = \frac{n \sum_1^n t_r y_r - 0}{n \sum t_r^2 - 0}$$

Which further simplify to the form

5.2 Fitting a parabola:

When a set of point exhibits a parabolic trend , the fitting of a quadratic functions

$$y = a + bx + cx^2 \ ,$$

To the data may be carried out by the method of least square , Which leads to the three equations

$$na + b\sum_1^n x_r + c\sum x_r^2 \;=\; \sum_1^n y_r$$

$$a\sum_1^n x_r + b\sum_1^n x_r^2 + c\sum_1^n x_r^3 = \sum_1^n x_r y_r \,.$$

$$a\sum_1^n x_r^2 + b\sum_1^n x_r^3 + c\sum_1^n x_r^4 = \sum_1^n x_r^2 y_r \,.$$

The above equations can be solve for $a\,, b, c$.

5.3 EXPONETIAL FUNCTION Y= Ae^{bx}

Transforming the exponential equation $y = ae^{bx}$ by taking logarithms on both sides we get

$$\log y = \log a + bx \log e \,.$$

If we replace $\log y$ by y and the constant $\log a$ and b $\log e$ by $\grave{a}$ and $\acute{b}$

Respectively , we obtain

$$y = \acute{a} + b\acute{x} \;,$$

Which defines a straight line .

Example:

1.Find the lest square line fir the data points (-1,10) ,(0,9),(1,7),(,4,5), (,2,5),(3,4),(4,3),(5,0) and (6,-1).

Solution: Here

x_r	y_r	x_r^2	y_r^2
-1	10	1	-10
0	9	0	0
1	7	1	7
2	5	4	10
3	4	9	12
4	3	9	12
5	0	25	0
6	-1	36	-6
20	37	92	25

From the table we have

$$n = 8\,, \sum x_r = 20\,, \sum y_r = 37, \sum x_r^2 = 92\,, and\ \sum x_r y_r = 25\,.$$

The normal equation are

$$na + b\sum x_r = \sum y_r\,,$$

$$a\sum x_r + \sum x_r^2 = \sum x_r y_r\,.$$

Putting the value in normal equations

$$8a + 20b = 37\,,$$

And

$$20a + 97b + 25\,,$$

And solving these equations we get

$$a = 8.6428571\,,$$

$$b = -1.6071429 \ .$$

Therefore the least square line is

$$y = 8.6428571 + (-1)1.6071429x \ ,$$

i.e.,

$$y = -1.6071429x + 8.6428571 \ .$$

Example 2: **Find the least square line $y = a + bx$ for the data**

X	**-2**	**-1**	**0**	**1**	**2**
Y	**1**	**2**	**3**	**3**	**4**

Solution:

x_r	y_r	x_r^2	$x_r y_r$
-2	1	4	-2
-1	2	1	-2
0	3	0	0
1	3	1	3
2	4	4	8
0	13	10	7

$$\therefore In\ this\ case\ \ n = 5 \ , \sum x_r = 0 \ , \sum y_r = 13 \ , \sum x_{r2} = 10 \ , \sum x_r y_r = 7.$$

The normal equation are

$$na + b\sum x_r = \sum y_r \ ,$$

$$a\sum x_r + b\sum x_r^2 = \sum x_r y_r \ .$$

Putting the value of $n\ , \sum x_r\ , \sum y_r\ , \sum x_r^2\ and\ \sum x_r y_r$ in the above equation we get

$$5a = 13\ , 10b = 7\ .$$

Solving

$$a = 2.6\ , b = 0.7\ .$$

∴The required line of fit is

$$y = 2.6 + b = (0.7)x\ .$$

Example 3:

Find a formula for the line of the form $y = a + bx + cx^2$ Which will fit the following data

X	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Y	3.1950	3.2299	3.2532	3.2516	3.2516	3.2282	3.1807	3.1266	3.0594	2.9759

Solution :

The normal equation are

$$na = b\sum x_r + c\sum x_r^2 = \sum y_r^2 \ ,$$

$$a\sum x_r + b\sum x_r^2 + c\sum x_r^3 = \sum x_r y_r$$

$$a\sum x_r^2 + b\sum x_r^3 + c\sum x_r^4 = \sum x_r^2 y_r \ .$$

Substituting the value of $\sum x_r\ , \sum x_r^2\ , \sum x_r^3, \sum x_r y_r, \sum x_r^2\ y_r, \sum y_r, and\ n, we\ get$

$$10a + 4.5b + 2.85c = 31.7616 \ ,$$

$$4.5a + 2.85b + 2.025c = 14.0896 \ ,$$

$$2.85a + 2.025b + 1.5333 = 8.82881 \ ,$$

And solving these equations we obtain

$$a = 3.1951 \, , b = 0.44254 \, , c = -0.76531.$$

$\therefore \ The \ requied \ equation \ is$

$$y = 3.1951 + 0.44254x - 0.76531x^2.$$

Example 4:

Fit second degree parabola to the following data

X	0	1	2	3	4
y	0	1.8	1.3	2.5	6.3

Solution : The values of x are 0,1,2,3,4

The number of value is odd. Shifting the origin to the middle value 2 of x, and making the substituting

$$u = x - 2, v = y \, ,$$

The curve of fit as

$$v = a + bu + cu^2 \ ,$$

We obtain

x	v	u	v	uv	u^2	u^2v	u^3	u^4
0	1	-1	1	-2	4	4	-8	16
1	1.8	-2	1.8	1.6	1	1.8	-1	1
2	1.2	0	1.3	0	0	0	0	0

3	2.5	1	2.5	2.5	1	2.5	1	1
4	6.3	2	6.3	12.6	4	25.2	8	16
		0	12.9	11.3	10	33.5	0	34

The normal equation are

$$\sum u = na + b\sum u + c\sum u_r^2 \quad ,$$

$$\sum uv = a\sum u + bu^2 + c\sum u^3 \quad ,$$

$$\sum u^2 v = a\sum u^2 + b\sum u^3 + c\sum u^4 \quad .$$

Putting the values of n ,$\sum u$, $\sum v$, $\sum uv$, … … … . , etc,

$$12.9=5a+40c,$$

$$11.3 = 10b \quad ,$$

$$33.5=10a+34c,$$

Solving these normal equations we get

$$a = 1.48, b = 1.13, c = 0.550.$$

∴The required equation is

$$u = 1.48 + 1.13u + 1.55u^2 \quad ,$$

Substituting $u = x - 2$, $v = y$ in the above equation we get,

$$y = 1.42 + 1.13(x - 2) + (0.55)(x - 2)^2 \quad .$$

$$\therefore The\ requird\ curve\ of\ fit\ is\ the\ parabola$$

$$y = 1.42 - 1.07x + 0.55x^2.$$

5.4 Solved problems:

1)Find the lest square line $y = a + bx$ *for the data*

X	-4	-2	0	2	4
Y	1.2	2.8	6.2	7.8	13.2

A)	X	y	x^2	xy
	-4	1.2	16	-4.8
	-2	2.8	4	-5.6
	0	6.2	0	0
	2	7.8	4	15.6
	4	13.2	16	52.8
	0	31.2	4.	58

The normal equations are

$$\sum y = na + b\sum xi$$

$$\sum xy = a\sum x_i + b\sum x_i^2$$

31.2=5a

$$a = \frac{31.2}{5}$$

= 6.24

$$58 = b(40)$$

$\therefore y = 6.24 + 1.45$.

2) **Fit a straight line to the following data regarding x as the independent variable**

X	0	1	2	3	4
Y	1	1.8	3.3	4.5	6.3

A)	x	y	x^2	xy
	0	1	0	0
	1	1.8	1	1.8
	2	3.3	4	6.6
	3	4.5	4	13.5
	4	6.3	16	25.2
	10	16.9	30	47.1

$$na + b\sum xi = \sum y$$

$$5a + b(10) = 16.9 \qquad (1)$$

$$a\sum x + b\sum x^2 = \sum xy$$

$$a(10) + b(30) = 47.1$$

$$1*2 \rightarrow 10a + 20b = 33.8$$

$$\underline{2 \rightarrow 10a + 30b = 47.1}$$

$$-10b = -11.3$$

$$b = \frac{13.3}{10}$$

$$=1.33$$

Substituting b in (1)

$$5a + (1.33)10 = 16.9$$

$5a = 16.9 - 13.3$

$a = \frac{3.6}{5}$

=0.72

$\therefore y = 0.72 + 1.33x$

3) Find the least square line $y = a + bx$

X	-2	0	2	4	6
Y	1	3	6	8	13

A)	X	y	x^2	xy
	-2	1	4	-2
	0	3	0	-2
	2	6	4	12
	4	8	16	12
	6	13	36	78
	10	31	60	78

$na + b\sum x_i = \sum xy$

$5a + 10b = 31$ (1)

$a\sum x + b\sum x^2 = \sum xy$

$10a + 60b = 102$ (2)

1*2→ $10a + 20b = 62$

10a+60b=120

$-40b = -58$

$$b = 1.45$$

Substituting b in (1)

$$5a + (1.45)10 = 31$$

$$5a = 31 - 14.5$$

$$a = \frac{16.5}{5}$$

$$=3.3$$

$$a = 3.3 \ , b = 1.45$$

$$\therefore y = 3.3 + 1.45x$$

4) Find the least squares parabolic lift $y = a + bx + cx^2$

A)	x	y	x^2	x^3	x^4	xy	x^2y
	-3	15	9	-27	81	-45	135
	-1	5	1	-1	1	-5	5
	1	1	1	1	1	1	1
	3	5	9	27	81	15	45
	0	26	20	0	164	-34	186

$$\sum y = na + b\sum x + c\sum x^2$$

$$26 = 4a + 0 + 20c$$

$$4a + 20c = 26 \qquad (1)$$

$$\sum xy = a\sum ax + b\sum x^2 + c\sum x^3$$

$$-34 = 0 + b(20) + c(0)$$

$$20b = -34$$

$$b = -\frac{34}{20}$$

$$= -1.7$$

$$\sum x^2 y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

$$186 = 20a + 164c \qquad (2)$$

$$1*5 \quad \rightarrow \quad 20a + 100c = 130$$

$$2 \quad \rightarrow \quad 20a + 164c = 186$$

$$-64c = -56$$

$$c = 0.875$$

$$4a + 20(0.875) = 26$$

$$4a = 36 - 17.5$$

$$a = \frac{8.5}{4}$$

$$= 2.125$$

$$y = a + bx + cx^2$$

$$\therefore y = 2.215 - 1.70x + 0.875x^2$$

5.5 APPLICATIONS OF CURVE FITTING

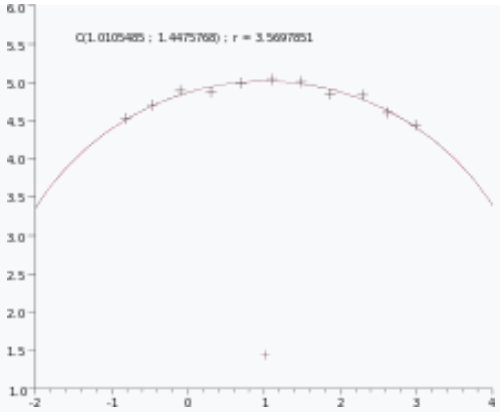
Fitting plane curves to data points :

If a function of the form cannot be postulated, one can still try to fit a plane curve.

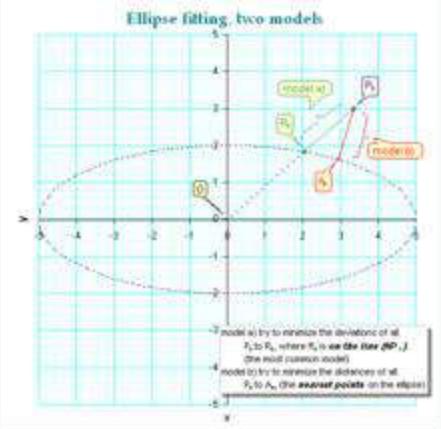
Other types of curves, such as conic sections (circular, elliptical, parabolic, and hyperbolic arcs) or trigonometric functions (such as sine and cosine), may also be used, in certain cases. For example, trajectories of objects under the influence of gravity follow a parabolic path, when air resistance is ignored. Hence, matching trajectory data points to a parabolic curve would make sense. Tides follow sinusoidal patterns, hence tidal data points should be matched to a sine wave, or the sum of two sine waves of different periods, if the effects of the Moon and Sun are both considered.

For a parametric curve, it is effective to fit each of its coordinates as a separate function of arc length; assuming that data points can be ordered, the chord distance may be used.

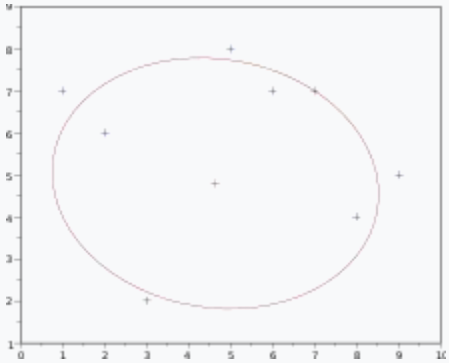
Fitting a circle by geometric fit



Circle fitting with the Coope method, the points describing a circle arc, centre (1 ; 1), radius 4.



different models of ellipse fitting



Ellipse fitting minimizing the algebraic distance (Fitzgibbon method).

Cooper[22] approaches the problem of trying to find the best visual fit of circle to a set of 2D data points. The method elegantly transforms the ordinarily non-linear problem into a linear problem that can be solved without using iterative numerical methods, and is hence an order of magnitude faster than previous techniques.

Fitting an ellipse by geometric fit

The above technique is extended to general ellipses[23] by adding a non-linear step, resulting in a method that is fast, yet finds visually pleasing ellipses of arbitrary orientation and displacement.

Application to surfaces

Further information: Computer representation of surfaces

Note that while this discussion was in terms of 2D curves, much of this logic also extends to 3D surfaces, each patch of which is defined by a net of curves in two parametric directions, typically called **u** and **v**. A surface may be composed of one or more surface patches in each direction.

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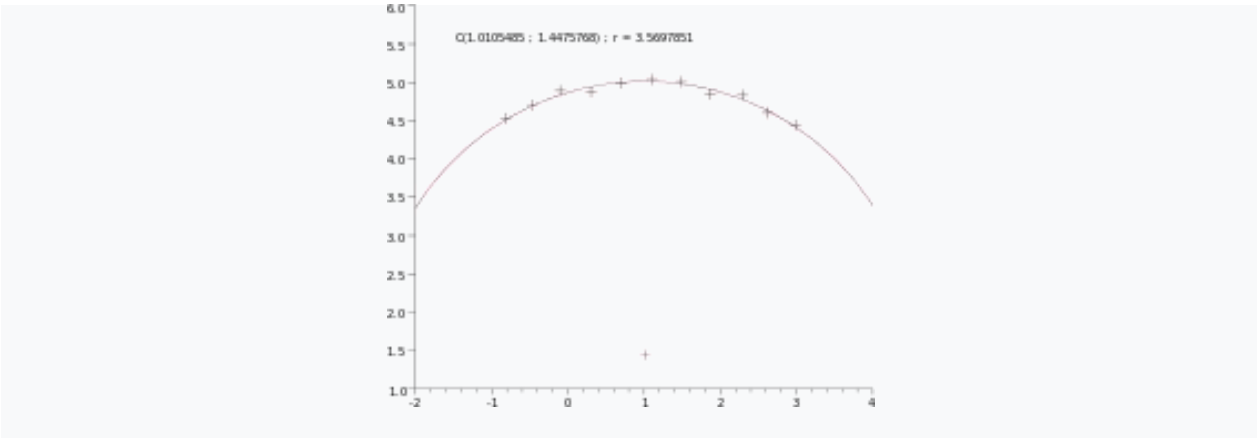

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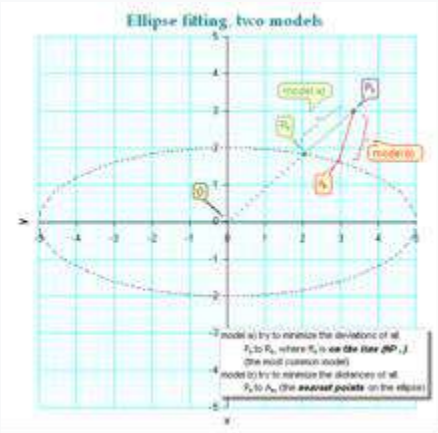

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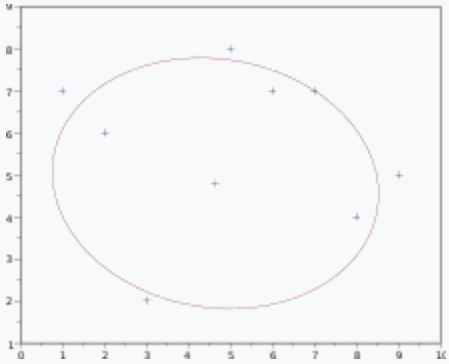

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A STUDY ON

APPLICATIONS OF NUMERICAL ANALYSIS

A Group Project submitted to KRISHNA UNIVERSITY

KRISHNA UNIVERSITY

In partial fulfilment of the requirement For the award of the degree of

BACHELOR OF SCIENCE

Submitted by

G.RAJESH 18303256

UNDER THE GUIDENCE OF

SK.PARVEEN

LECTURER IN MATHEMATICS.

S.R.R & C.V.R GOVT DEGREE COLLEGE.

MACHAVARAM,VIJAYAWADA-520004

2018-2021

S.R.R & C.V.R GOVT. DEGREE COLLEGE (AUTONOMUS)

CERTIFICATE

This is to certificate that the project entitled "APPLICATIONS OF NUMERICAL ANALYSIS" bonafied workdone by G.RAJESH(18303256) belongs to department of bachelor science studies,S.R.R & C.V.R GOVT.DEGREE COLLEGE VIJAYAWADA under my guidance in partial fulfilment of the award for the degree in bachelor of science

K.V.NAGALAKSHMI SK.PARVEEN

EXTERNAL EXAMINAR (H.O.D Of Mathematics) (PROJECT GUIDE)

DECLARATION

I **G.RAJESH (18303256)** here by declare that the project entitled **"APPLICATIONS OF NUMERICAL ANALYSIS "**has been prepared by in a partial fulfilment of the requirement for the award of the degree of bachelor of science submitted at **S.R.R & C.V.R GOVT DEGREE COLLEGE** affiliated to **KRISHNA UNIVERSITY i**s an authentic project record of our own work .

The matter presented as not be submitted to any University or Institute for the award of bachelor of science Degree .

G.RAJESH 18303256

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We would like to convey my deep felt regards to all of my well wishers,

Who have contributed their share of cooperation and help me to complete this project work

.

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G.RAJESH 18303256

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1.Interpolation With Equal Intervals

1.1 INTRODUCTION :

Interpolation means insertion or filling up intermediate terms of the series. It is the technique of getting the value of a function for any intermediate value of the argument from the given set of values of function is called interpolation, while the process of computing the value of the function outside of the given range is called interpolation. The study of interpolation is based on the calculus of finite differences which
The following assumptions are to be kept in mind for interpolation.
(1) The values of the function should be either increasing or decreasing
(2) The rise or fall in the values should be uniform plays a vital role in numerical analysis manner.

1.2 INTERPOLATION METHODS :

(i) Graphical method
(i) Method of curve fitting
(iii) Applications of the calculus of finite differences formula.

1.2.1 Graphical Method :

Let y =f(x), then plot a graph for different values of x corresponding to the value of y. From the graph, we can find the value of y for given value of x.
For example

Year(x)	1891	1901	1911	1921	1931
Population (Y)(in 1000's)	46	66	81	93	101

Interpolate population for the year 1919.
The following steps are to be maintained
1. Take a suitable scale for x and y plot the different points on the graph for given data.
2. Join the point by hand curve.

3. Find the point on the curve corresponding to x = 1919 and find corresponding value of y

Remarks : This is simply an approximate method of evaluating the value of y and is not reliable in number of cases.

1.2.2 Method of Curve Fitting :

This method is applicable in some cases in which the form of the function is known. In such cases, by the method of least squares, we can fit the curve of known form to given set of values

and afterwards we can find the unknown value with the help of fitted curve.

Remarks :

(1) This is not exact method.

(2) In case of number of values are sufficiently large this method is complicated

(3) The form of the function in this method for given set of observations is assumed to be known.

1.2.3 Applications of the Calculus of Finite Differences :

(1) Newton-Gregory formula for interpolation. If n is a integer and h is the interval differencing, then $f(a+ nh) = f(a)+{}^{n}c_1 \Delta f(a)+ {}^{n}c_2\Delta^2 f(a)+....+ {}^{n}c_n \Delta^{n}{}_{f(a)}$

We prove the theorem by using the method of induction.

we know that $\Delta f(a) = f(a + h) - f(a)$

$f(a+h)= f(a) +\Delta f(a)$

The theorem is true for n = 1.

Assume that the theorem is true for n=k.

$f(a+kh)=f(a)+{}^{k}c_1 \Delta^2 f(a)+....+{}^{k}c_n \Delta^{k} f(a)$

We have $\Delta f(a+kh) = f(a+(k+ 1)h)-f(a+kh)$

$f(a + (k + 1)h) = f(a+kh) + h + \Delta f(a + kh)$

$= [f(a)+{}^{k}c_1\Delta f(a)+ {}^{k}c_2\Delta^{r}f(a)+....+ \Delta^{k}f(a)]$

$[\Delta^{r}f(a)+{}^{k}c_1\Delta^{r}f(a)+ {}^{k}c_1\Delta^{3}f(a)+.....+ \Delta^{k+1}f(a)]$

$= f(a)+({}^{k}c_1+1) \Delta f(a)+({}^{k}c_1+ {}^{k}c_2) \Delta^2 f(a)+.....+ \Delta^{k+1}f(a)$

$= f(a)+{}^{k+1}c_1+ \Delta f(a)+{}^{k+1}c_2 \Delta^2 f(a)+.....+ \Delta^{k+1}f(a)$

The theorem is true for n=k+1

By the method of induction, the theorem is true for any integer.

$f(a+ nh) = f(a)+{}^{n}c_1 \Delta f(a)+ {}^{n}c_2\Delta^2 f(a)+....+ {}^{n}c_n \Delta^{n}{}_{f(a)}$

Note :

1. $f(a+nh) = f(a)+{}^{n}c_1\,\Delta f(a)+ {}^{n}c_2\Delta^2 f(a)+....+ {}^{n}c_n\,\Delta^{n}{}_{f(a)}$

2. Put a=0, h=1

$f(h)=f(0)+{}^{n}c_1\,\Delta f(0)+{}^{n}c_2+ {}^{n}c_2\,\Delta^2 f(0)+\ldots\ldots$

$f(x)=f(0)+\frac{x}{1!}\,\Delta f(0)+ \frac{x(x-1)}{2!}\,\Delta^2 f(0)+\ldots\ldots.$

This formula is used only when the first term a =0 and h =1.

Alter : Let y=f(x) represent a function which assumes the values f(a),f(a+h)....f(a+nh) for (n +1) equidistant values a, a+ h, a +2h, a+nh of the independent variable x and let $P_n(x)$ be a polynomial in x of degree n.

This polynomial can be expressed in the form.

$P_n(x) = Bo +B\,(x-a) + B2(x-a)(x-a-h)+\ldots\ldots\ldots+ B_n(x-a)(x-a-h)(x-a-2h)....(x-a-(n-1)h)$ ....(1)

where B_o. B_1, B_n are constants and are choosen such that

$P_n(a) = f(a)$, $P_n(a + h) = f(a + h)$, . . . $P_n(a + nh) = f(a + nh)$

By substituting successively the values a, a+ h, a +2h . a+nh in equation(1) and writing the values of $P_n(a)$, $P_n(a+h)$... $P(a + nh)$, we get $f(a)= B_0$ B1=f(a + h)-f(a)

$f(a) = B_0$, $f(a + h) = B_0+ hB_1$

B_0 =f(a) or $B_1 = \frac{f(a+h)-f(a)}{h} = \frac{\Delta f(a)}{h}$

$f(a + 2h) = B_0 + 2hB_1 + h^rB_2$ (or)

A2 = $f(a + 2h)-2\{f(a+ h)-f(a)\}-f(a)/2h^2$

$= \frac{1}{2!h^r}\,\Delta^2 f(a)$

Similarly, we get $B_3 = \frac{1}{3!h^3}\,\Delta^3 f(a)$..... $B_n = \frac{1}{n!h^n}\,\Delta^n f(a)$

Substituting the values of B_0, B_1,....., B_n, we get

$P_n(x) = f(a) +\frac{\Delta f(a)}{h}\,(x-a)+ \frac{\Delta^2 \mathbf{f(a)}}{\mathbf{2!}\boldsymbol{h^r}}\,(x-a)(x-a-h)+....+ \frac{\Delta^n \mathbf{f(a)}}{\boldsymbol{n!h^n}}\,(x-a)(x-a-h)...(x-a-(n-1)h)$

This is Newton-Gregory formula for forward interpolation putting $\frac{x-a}{h}$ = u or x=a+hu then(1) can be expressed as

$P_n(x)=P_n(a+nh) = f(a)+u\Delta f(a)+\frac{u(u-1)}{2!}\,\Delta^2 f(a)+....+\frac{u(u-1).....(u-n+1)}{n!}\,\Delta^n f(a)$

Newton-Gregory Formula for Backward Interpolation

In this case , we write

$P_n(x) = C_0 +C_1\,(x-a-nh)+ C_2(x-a-nh)+....+C_n(x-a-nh)(\,x-a-nh+h).....(x-a-h)$ .....(1)

where Co, C_1 ,$C_{2...}$ C_n are constants and are choosen such that $f_{n(}a+ nh) = f(a+ nh)$,.., $P_n(a) = f(a)$

Substituting in (1), x=a + nh, a + nh-h, we have

$C_0 = f(a + nh)$, $f(a + nh-h) = C_o + C_1\,(-h)$

$C_1 = \frac{f(a + nh)-f(a + nh-h)}{h} = \frac{\Delta f(a + nh)}{h}$

Similarly $C_2 = \frac{1}{2!h^r}\,\nabla^2 f(a+nh)$

$C_3 = \frac{1}{3!h^3} \nabla^3 f(a+nh)$

……

$C_n = \frac{1}{n!h^n} \nabla^n f(a+nh)$

Substituting these values of $C_o, C_1, \ldots, C_n$ in (1), we obtain

$P_n(x) = f(a+nh) + \frac{\nabla f(a+nh)}{1!h}(x-a+nh) + \frac{\nabla^2 f(a+nh))}{2!h}(x-a+nh)\,(x-(a+nh-h)) + \ldots + \frac{(\nabla^n f(a+nh)}{n!h^n}(x-a+nh)\,(x-(a+nh-h))\ldots\ldots(x-(a+h))$ ……(2)

This is Newton-Gregory formula for backward interpolation. Substituting $n = \frac{x-(a+nh)}{h}$ or

$x = a + nh + hl$ in eq. (2), then we obtain $P_n(x) = P_n(a + nh + h) = f(a + nh) + u\nabla f(a+nh) + \frac{u(u+1)}{2!}\nabla^2 f(a+nh) + \ldots\ldots\ldots + \frac{u(u+1)\ldots\ldots(u+n-1)}{n!}\nabla^n f(a+nh)$ Where $P_n(x) = f(x)$.

1.3 SOLVED PROBLEMS

1. Given log x for x = 40, 45, 50, 55, 60, 65 for the following data :

x	40	45	50	55	60	65
logx	1.60206	1.65321	1.69877	1.74036	1.7785	1.81291

Estimate the value of log (58.75) by using Newton backward interpolation formula.

Solution.

X	10^5xlogx=f(x)	10^5x∇f(x)	10^5x∇^2f(x)	10^5x∇^3f(x)	10^5x∇^4f(x)
40	160206				
		5115			
45	165321		-539		
		4516		102	
50	169892		-437		-25
		4319		77	
55	174036		-360		-20
		3729		57	
60	177815		-330		
		3476			
65	181291				

$(a + nh) + uh = 58.75$

$65 + 5u = 58.75$

$u = 1.25$

Newton backward interpolation formula

$f(a+nh)+nh) = f(a+nh) + \frac{h}{1!}\Delta f(a+nh) + \frac{u(u+1)}{2!}\Delta^r f(a+nh) + \frac{u(u+1)(u+2)}{3!}\Delta^r f(a+nh)$

$10^5 f(58.75) = 181291 - 1.25x(3476) + \frac{(-1.25)(-1.25u)x(-30.5)}{2!} + \frac{(-1.25)(-1.25+1)x(-1.25+2)}{6}x(5)$

$= 181291 - 4345 - 47.3438 + 2.2267 = 176900.89$

$F(58.75) = 1.7690089$

2. Find the cubic polynomial for the following data hence find f(u)

X	0	1	2	3
F(x)	-2	1	4	6

Solution :

x	**F(x)**	Δf(x)	Δ^2**f(x)**	Δ^3**f(x)**
0	-2			
		3		
1	1		0	-1
		3		
2	4		-1	
		2		
3	6			

Newton's Gregory's formula is

$f(x) = f(0) + \frac{x}{1!}\Delta f(0) + \frac{x(x-1)}{2!}\Delta^2 f(0) + \frac{x(x-1)(x-2)}{3!}\Delta^3 f(0)$

$= -2 + \frac{x}{1!}(3) + \frac{x(x-1)}{2!}(0) + \frac{x(x-1)(x-2)}{3!}(-1)$

$= -2 + 3x - \frac{(x^3+3x^2+2x)}{6} = \frac{-12-18x-x^3+3x^2-2x}{6}$

$= \frac{x^3+3x^2+16x-12}{6} = \frac{-(4)^3+3(4)^2+16(4)-12}{6}$

$= \frac{-64+48+64-12}{6} = \frac{36}{6} = 6$

$f(4) = 6$

3. Find f(1.5) by using Newton's advancing difference formula given that f(o)= +2, f(1)=4, f(2)=7, f(3)=8

x	F(x)	$\Delta f(x)$	Δ^2**f(x)**	Δ^3**f(x)**
0	**2**			
		2		
1	4		1	
		3		**-3**
2	7		-2	
		1		

3	8			

Solution. Newton's advancing formula is

$$f(x) = f(0)+\frac{x}{1!}\Delta f(0)+\frac{x(x-1)}{2!}\Delta^2 f(0)+\frac{x(x-1)(x-2)}{3!}\Delta^3 f(0)$$

$$= 2+\frac{x}{1!}(2)+\frac{x(x-1)}{2!}(1)+\frac{x(x-1)(x-2)}{3!}(-3)$$

$$= 2+2x+\frac{x^2-x}{2!}+\frac{x^3-3x^2+2x}{2!}(-1)$$

$$= \frac{4+4x+x^2-x-x^3+3x^2-2x}{2}$$

$$F(x) = \frac{-(1.5)^3+4(1.5)^2+(1.5)+4}{2}$$

$$= \frac{-3.375+9+1.5+4}{2} = \frac{11-125}{2} = 5.5625$$

$$f(1.5) = 5.5625$$

4. Find the polynomial and hence find f(5) from the following data by using Newton's Gregory's formula.

x	**0**	**1**	**2**	**3**	**4**
F(x)	**0**	**7**	**26**	**63**	**124**

Solution.

X	F(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	0				
		7			
1	7		12		
		19		6	
2	26		18		0
		37		6	
3	63		24		
		61			
4	124				

$$F(x) = f(0)+\frac{x}{1!}\Delta f(0)+\frac{x(x-1)}{2!}\Delta^2 f(0)+\frac{x(x-1)(x-2)}{3!}\Delta^3 f(0)$$

$$= 0+\frac{x}{1!}(7)+\frac{x(x-1)}{2!}(12)+\frac{x(x-1)(x-2)}{3!}(6)$$

$$= 7x+6x^2-6x+x^3-3x^2+2x = x^3+3x^2+7x-4x$$

$$F(x) = x^3-3x^2+2x$$

$$F(5) = 5^3+3(5)^2+3(5) = 125+75+15 = 215$$

$$F(5) = 215$$

5. Find f from the given data by using Newton's Gregory's formula

X	10	20	30	40
F(x)	9	30	35	42

Solution .

X	F(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
10	9			
		21		
20	30		-16	
		5		18
30	35		2	
		7		
40	42			

a=10 , h= 10

Let a+uh=1.5

10+10u = 1.5

$u = \frac{5}{10} = \frac{1}{2}$

Newton's Gregory's formula is

$F(a+uh) = f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)$

$F(52) = 0.7071+\frac{7}{5}(0.0589)+\frac{\frac{7}{5}(\frac{7}{5}-1)}{2!}(-0.0057)+\frac{\frac{7}{5}(\frac{7}{5}-1)(\frac{7}{5}-2)}{3!}(-0.0007)$

= 7071+824.6-15.96+0.392 = 7880

$10^4 f(52) = 7880$

$F(52) = \frac{7880}{10^4} = 0.788 \qquad \sin 52^0 = 0.7880$

7. Using Newton's formula for interpolation estimate population for the year 1905 from the table.

Year (x)	Population(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1891	98752				
		33533			
1901	132285		2258		
		35791		-10435	
1911	168076		-8177		41358
		27614		30923	
1921	195690		22746		
		50360			
1931	246050				

Solution.

a = 1891, h = 10

Let a+ uh = 1905

u(10) = 1905-1891 = 14

$u = \frac{14}{10} = \frac{7}{6}$

Newton's Gregory's formula is

$$f(a+uh) = f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)+\frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(a)$$

$$f(1905) = 98752+\frac{7}{5}(33533)+\frac{\frac{7}{5}(\frac{7}{5}-1)}{2}(2258)+\frac{\frac{7}{5}(\frac{7}{5}-1)(\frac{7}{5}-2)}{6}(-10435)+\frac{\frac{7}{5}(\frac{7}{5}-1)(\frac{7}{5}-2)(\frac{7}{5}-3)}{24}(41358)$$

= 98,752 + 48946·2+ 632.24 + 584.36 + 926.4192

= 147841.2192

The population for the year 1905 is 14,784 approximately

8 . From the following table find the no. of students who obtained less than 45 marks.

x	30-40	40-50	50-60	60-70	70-80
F(x)	31	42	51	35	31

Solution .

Marks less than x	No.of students	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
40	31				
		42			
50	73		9		
		51		-25	
60	124		-16		37
		35		12	
70	159		-4		
		31			
80	190				

a=40,h=10

let a +uh = 45

u(10) = 45-40

$u = \frac{5}{10} = \frac{1}{2}$

Newton's advancing formula is

F(a+uh) = f(a)

$$+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)+\frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(a)$$

$$= 31+\frac{1}{2}(42)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(9)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(-25)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\left(\frac{1}{2}-3\right)}{4!}(37)$$

$$= 31+21-\frac{9}{8}+\left(-\frac{75}{48}\right)-\frac{15}{16}\left(\frac{37}{21}\right)$$

$$= 52-1.125-1.5625-1.4453125$$

$$= 47.8671875 = 47$$

: The no. of students getting less than 45 marks is 47 and the no. of students who getting more than 45 is 190-47-143

9. Find the no. of men getting the wages less than is rupees from the following :

0-10	10-20	20-30	30-40
9	30	35	42

Solution .

Wages x	No. of Men	$\Delta\mathbf{f(x)}$	$\Delta^2\mathbf{f(x)}$	$\Delta^3\mathbf{f(x)}$
0-10	9			
		21		
10-20	30		-16	
		5		18
20-30	35		2	
		7		
30-40	42			

Newton's advancing formula is

$$f(a+uh) = f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)$$

$$f(15) = 9+\frac{1}{2}(21)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(-16)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(18)$$

$$= 9+\frac{21}{2}+2+\frac{3}{8}\left(\frac{18}{16}\right) = 11+\frac{21}{2}+\frac{9}{8}$$

$$= 11+10.5+1.125- 22.625 = 23$$

11. Find the second degree polynomial from the data is then find f(5)

0	1	2	3	4
3	6	11	17	27

Solution .

X	F(x)	$\Delta\mathbf{f(x)}$	$\Delta^2\mathbf{f(x)}$	$\Delta^3\mathbf{f(x)}$
0	3			
		3		
1	6		2	
		5		0
2	11		2	

3 4	18 27	7 9	2	0

Newton's Gregory's formula is

$f(x) = f(0)+\frac{x}{1!}\Delta f(0)+\frac{x(x-1)}{2!}\Delta^2 f(0)$

a=0 , h=1

a+uh = 5 u =5

$f(x) = 3+x(3)+\frac{x(x-1)}{2}(2)$

$= 3+3x + x^2-x = x^2 + 2x + 3$

$f(5) = (5)^2 + 2(5) + 3=25+10+ 3 = 38.$

12. Second degree polynomial passes through the points (o, 1), (1, 3), (2, 7),(3, 13) then find the polynomial or find f(x). Also find the value of f(1.5).

Solution.

X	Y = f(x)	$\Delta\mathbf{f(x)}$	$\Delta^2\mathbf{f(x)}$	$\Delta^3\mathbf{f(x)}$
0 1 2 3	1 3 7 13	2 4 6	2 2	0

Newton's Gregory's formula is

$f(x) = f(0)+\frac{x}{1!}\Delta f(0)+\frac{x(x-1)}{2!}\Delta^2 f(0)$

$= 1+x(2)+\frac{x(x-1)}{2}(2)$

$f(x) = 1 + 2x + x^2-x$

$= x^2 + x + 1$

$f(1.5) = (1.5)^2+(1.5)+1$

$= 2.25+1.5+1 = 4.75$

$f(1.5) = 4.75$

13. Find the polynomial fits the data in the following table then also find f(13).

X	3	5	7	9	11
Y	6	24	58	109	174

solution. a=3, h = 2

Let a+uh = x = 13 3+2u = x

$$u = \frac{x-3}{2}$$

X	Y =f(x)	$\Delta \mathbf{f(x)}$	$\Delta^2 \mathbf{f(x)}$	$\Delta^3 \mathbf{f(x)}$	$\Delta^4 \mathbf{f(x)}$
3	6				
		18			
5	24		16		
		34		0	
7	58		16		0
		50		0	
9	109		16		
		66			
11	174				

Newton's Gregory's formula is

$$f(a+uh) = f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)$$

$$= 6+\frac{x-3}{1!}(18)+\frac{\left(\frac{x-3}{2}\right)\left(\frac{x-3}{2}-1\right)}{2!}(16)$$

$f(a+uh) = 6+9x-27+2(x-3)(x-5)=9x-21+2x^2-16x+30$

$f(x) = 2x^2-7x+9$

$f(13) = 2(13)^2-7(13)+9=2(169)-91+9=338+9-91$

$f(13) = 2.56$

14. Find the value of f(35) from the following table :

X	Y=f(x)	$\Delta \mathbf{f(x)}$	$\Delta^2 \mathbf{f(x)}$	$\Delta^3 \mathbf{f(x)}$
20	512			
		-73		
30	439		-20	
		-93		10
40	346		-10	
		-103		
50	243			

Solution .

Newton's advancing formula is

$$F(a+uh) = f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)+\frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(a)$$

a=20 , h=10 => a+uh=35a=30, h=10

20+10h=35 (or) 30+10u=35

h= $\frac{3}{2}$,u= $\frac{1}{2}$

f(35)=439+$\frac{1}{2}$(-93)+$\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}$(-10)+$\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}$(-10)

=439-$\frac{93}{2}$+$\frac{10}{8}$+$\frac{30}{48}$

=439-46.5+1,25+6.625=440.875-46.5=394.375

f(35)=394.

Central Difference Interpolation Formulae

1.3.1 INTRODUCTION :

In the previous chapter, we derived and discussed Newton forward and backward interpolation formula, which are applicable for interpolation, near the beginning and end respectively of tabulated data. We shall now discuss in the present chapter, about the central difference formula, which are suitable for interpolation near the middle of a tabulated data.

The central difference operator (δ) is defined as $\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$

The first central difference operator δf(x) defined by

δ f(x) =f(x+$\frac{h}{2}$) - f(x-$\frac{h}{2}$)

δ f(x) =($E^{\frac{1}{2}} - E^{-\frac{1}{2}}$) f(x)

We have E = 1 + Δ

$\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}} = E^{-\frac{1}{2}}(E-1) = E^{-\frac{1}{2}}\Delta$ ..... (1)

Again by the first backward difference of f(x)

We have ∇f(x) = f(x)-f(x-h) = f(x)-f(x-h) =E(x)- E^{-1}f(x)

∇ f(x) = (1- E^{-1})f(x)

∇ = (1- E^{-1})

$$\nabla E^{\frac{1}{2}} = E^{\frac{1}{2}} - E^{-\frac{1}{2}} = \delta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

From(1) and (2) $\delta = \Delta E^{-\frac{1}{2}} = \nabla E^{\frac{1}{2}}$ ……(3)

The average operator u is defined as

$$U = \frac{1}{2}[E^{\frac{1}{2}} + E^{-\frac{1}{2}}]$$

Results :

(1) $U^2 = 1 + \frac{1}{4}\delta^2 u^2 f(x) = \frac{1}{2}(E^{\frac{1}{2}} + E^{-\frac{1}{2}})^2$

$= \frac{1}{4}(E^{\frac{1}{2}} + E^{-\frac{1}{2}})^2 = \frac{1}{4}(E + E^{-1} + 2) = \frac{1}{4}(E + E^{-1} - 2 + 4)$

$= \frac{(u+k-1)^{(2k+1)}}{(2k+1)!} = -\frac{(k-v)^{(2k+1)}}{(2k+1)!}$

$= \frac{v(v^2-1)(v^2-4)\ldots\ldots((v^2-k^2)}{(2k+1)!}$

From (3), we obtain

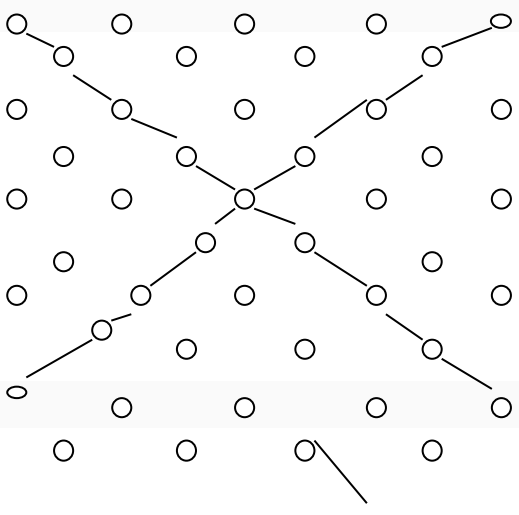
$y_u = vy_o + \frac{v(v^2-1)}{3!}\Delta^2 y_{-1} + \frac{v(v^2-1)(v^2-4)}{5!}\Delta^4 y_{-2} + \ldots\ldots uy_1 + \frac{u(u^2-1)}{3!}\Delta^2 y_0 + \frac{u(u^2-1)(u^2-4)}{5!}\Delta^4 y_{-2}$

$+\ldots\ldots\ldots + \frac{u(u^2-1)(u^2-4)(u^2-k^2)}{(2k+1)!}\Delta^{2k} y_{1-k} + \ldots\ldots\ldots\ldots$ ……..(5)

this can be written as

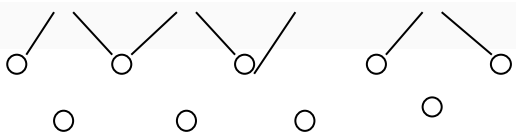
$f(u) = vf(0) + \frac{v(v^2-1)}{3!}\delta^2 f(0) + \frac{v(v^2-1)(v^2-4)}{5!}\delta^2 f(0) + \ldots\ldots + uf(1) + \frac{u(u^2-1)}{3!}\delta^2 f(1) + \frac{u(u^2-1)(u^2-4)}{5!}\delta^2 f(1) + \ldots$

this is known as Laplace-Everette's formula

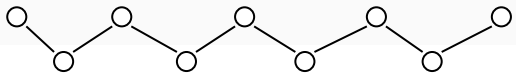


Newtons backward formula

Newtons backward formula

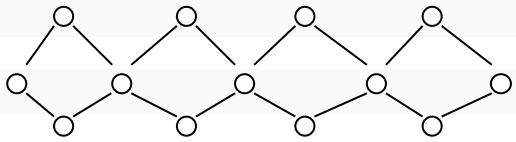


Gauss backward formula

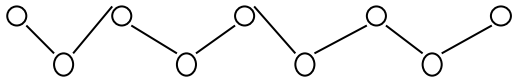


Gauss forward formula

Stirling's formula



Bessel's formula



Note 1: This formula is useful especially when using the tables in which only even order difference are tabulated.

All the difference formula can be explained with the help of following diagram.

Results : (1) $\delta = \Delta E^{-\frac{1}{2}} = \nabla E^{\frac{1}{2}}$

(2) $\Delta\nabla = \nabla\Delta = \Delta - \nabla = \delta^2$

(3) u= $\delta = \frac{1}{2}(\nabla + \Delta) = \frac{1}{2}\Delta E^{-1} + \frac{1}{2}\Delta$ (4) $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$)

Proof :

(1) $\delta = \Delta E^{-\frac{1}{2}} = \nabla E^{\frac{1}{2}}$

$\delta f(x) = f(x+\frac{h}{2}) - f(x-\frac{h}{2}) = \nabla f(x+\frac{h}{2}) = \nabla E^{\frac{1}{2}} f(x)$

$\delta = \nabla E^{\frac{1}{2}}$

(2) (a) $\Delta\nabla f(x) = \Delta\Delta E^{-1}f(x) = \Delta^2 E^{-1}f(x)$

(b) $\Delta\nabla f(x) = \Delta E^{-1}f(x) = \Delta^2 E^{-1}f(x)$

(c) $(\Delta - \nabla)f(x) = (\Delta - \nabla E^{-1})f(x) = \Delta(1-\frac{1}{E})f(x)$

$= \frac{\Delta(E-1)}{E} f(x) = \Delta.\Delta E^{-1}f(x) = \Delta^2 E^{-1}f(x)$

(d) $\delta^2 f(x) = (\Delta E^{-\frac{1}{2}})^2 f(x) = \Delta^2 E^{-1}f(x)$

$\Delta\nabla = \nabla\Delta = \Delta - \nabla = \delta^2$

(3) $u\delta f(x) = \frac{(E^{\frac{1}{2}}+E^{-\frac{1}{2})}}{2}(E^{\frac{1}{2}}-E^{-\frac{1}{2}})f(x)$

$= \frac{1}{2}(e-e^{-1})f(x) = \frac{1}{2}(e-\frac{1}{E})f(x) = \frac{1}{2}(E^2-\frac{1}{E})f(x)$

$= \frac{(E-1)(E+1)}{2E}f(x) = \frac{\Delta}{2}(1+\frac{1}{E})f(x)$

We know that $\nabla = \Delta E^{-1} = \frac{1}{2}(\Delta+\nabla)f(x) = \frac{1}{2}(\Delta+\nabla E^{-1})f(x)$

$= \frac{1}{2}\nabla E^{-1} + \frac{1}{2}\Delta$

$U\delta = \frac{1}{2}(\Delta+\nabla E^{-1})$

(4) $\Delta+\nabla = \frac{\Delta}{\nabla}-\frac{\nabla}{\Delta}$

$(\frac{\Delta}{\nabla}-\frac{\nabla}{\Delta})f(x) = (\frac{\Delta}{\Delta E^{-1}}-\frac{\Delta E^{-1}}{\Delta})f(x) = (E-\frac{1}{E})f(x) = (\frac{E^2 1}{E})f(x)$

$= (E-1)\frac{(E+1)}{E}f(x) = \Delta(1+E^{-1})f(x) =$

$(\Delta+\nabla E^{-1})f(x)$

$= (\Delta+\nabla)f(x)$

$(\frac{\Delta}{\nabla}-\frac{\nabla}{\Delta}) = \Delta+\nabla$

1.4 SOLVED PROBLEMS

1. Using Gauss forward formula find the value of y when x = 0.5 from the following table.

X	**-2**	**-1**	**0**	**1**	**2**
F(x)	**22**	**30**	**82**	**106**	**216**

Solution .

X	**F(x)**	$\delta f(x)$	$\delta^2 f(x)$	$\delta^3 f(x)$	$\delta^4 f(x)$
-2	22				
		8			
-1	30		44		
		52		-72	
0	82		-28		186
		24		114	
1	106		86		
		110			
2	216				

Gauss forward interpolation formula is

$f(x) = f(0) + \frac{x}{1!}\delta f(1/2) + \frac{x(x-1)}{2!}\delta^2 f(0) + \frac{x(x-1)(x+1)}{3!}\delta^3 f(1/2) + \frac{x(x-1)(x+1)(x-2)}{4!}\delta^4 f(0)$

$f(x) = 82 + \frac{x}{1!}(24) + \frac{x(x-1)}{2!}(-28) + \frac{x(x-1)(x+1)}{6}(114) + \frac{x(x-1)(x+1)(x-2)}{24}(186)$

$f(0.5) = 82 + (0.5)(24) + \frac{(0.5)(0.5-1)}{2}(-28)$

$+ \frac{(0.5)(0.5-1)(0.5+1)}{6}(114) + \frac{(0.5)(0.5-1)(0.5+1)(0.5-2)}{24}(186)$

$= 82 + 12 + \frac{7}{2} - \frac{57}{24} + 4.359 = 94 + 4.359 + 3.5 - 7.125$

$f(1/2) = 94.734$

2. Using Gauss backward interpretation formula find the value at y when x=0.5[0,-1,+1,-2,+2]

Solution.

x	$f(x)$	$\delta f(x)$	$\delta^2 f(x)$	$\delta^3 f(x)$	$\delta^4 f(x)$
-2	22				
		8			
-1	30		44		
		52		-72	
0	82		-28		186
		24		114	
1	106		86		
		110			
2	216				

Gauss backward interpolation formula is

$F(x) = f(0) + \frac{x}{1!}\delta f(-1/2) + \frac{x(x+1)}{2!}\delta^2 f(0) + \frac{x(x-1)(x+1)}{3!}\delta^2 f(-1/2) + \frac{x(x-1)(x+1)(x+2)}{4!}(186)$

$f(0.5) = f(\frac{1}{2}) = 82 + (0.5)(52) + \frac{(0.5)(0.5+1)}{2}(28) + \frac{(0.5)(0.5+1)(0.5-1)}{6}(-72) +$

$\frac{(0.5)(0.5+1)(0.5-1)(0.5+2)}{24}(186)$

$= 82 + 26 - 10.5 + 4.5 - 7.2656 = 112.5 - 17.7656$

$f(0.5) = 94.7344$

a. Using Gauss backward interpolation formula and the value of f(-1/2) from the following data

X	F(x)	$\delta f(x)$	$\delta^2 f(x)$	$\delta^3 f(x)$	$\delta^4 f(x)$
-2	22				
		8			
-1	30		14		
		22		18	
0	52		32		6
		54		24	
1	106		56		
		110			
2	216				

Solution. Gauss backward interpolation formula is

F(x) = f(0)+ $\frac{x}{1!}\delta f$(-1/2)+ $\frac{x(x+1)}{2!}$ δ^2f(0) +$\frac{x(x-1)(x+1)}{3!}$ δ^2f(-1/2)+ $\frac{x(x-1)(x+1)(x+2)}{4!}$ δ^4f(0)

= 52+$\frac{x}{1!}$(22)+ $\frac{x(x+1)}{2}$(32)+ $\frac{x(x^2-1)}{6}$(18)+ $\frac{x(x^2-1)(x+2)}{4!}$(16)

= 52+11+ $\frac{\frac{1}{2}(\frac{1}{2}+1)}{2}$(32)+ $\frac{\frac{1}{2}(\frac{1}{4}-1)}{6}$(18)+)+ $\frac{\frac{1}{2}(\frac{1}{4}-1)(\frac{1}{2}+2)}{24}$(16)

f(1/2) = 63+ 12-1.125-0.234375 = 75-1.359375 = 73.64

4. Apply Gauss forward formula to obtain f(33) given that

X	25	30	35	40
F(x)	0.2707	0.3027	0.3386	0.3794

Solution .

X	10^4 F(x)	$10^4\delta f(x)$	$10^4\delta^2 f(x)$	$10^4\ \delta^3 f(x)$
-1	2707			
		320		
0	3027		39	
		359		10
1	3386		49	
		408		
2	3794			

Gauss forward interpolation formula is

f (x) = f(0) +$\frac{x}{1!}\delta f$(1/2)+ $\frac{x(x-1)}{2!}$ δ^2f(0) +$\frac{x(x-1)(x+1)}{3!}$ δ^3f(1/2)

let us take a = 30 as origin h= 5

put a+uh = 32 30+5u = 32 => u = $\frac{2}{5}$

$$10^4 f(2/5) = 3027 + \frac{2}{5}(359) + \frac{\frac{2}{5}(\frac{2}{5}-1)}{2}(39) + \frac{\frac{2}{5}(\frac{2}{5}-1)(\frac{2}{5}+1)}{6}(10)$$

$$10^4 f(2/5) = 3027+143.6-4.68-0.045 = 3170.6-4.105 = 3165.36$$

$$f(\frac{2}{5}) = 0.3165$$

f(x) = 0.3165 when x =32

5. Use Gauss forward formula to find the value of y when y=3.75 from the following table.

X	2.5	3.0	3.5	4.0	4.5	5.0
Y	24.145	22.043	20.225	18.644	17.262	16.047

Solution. Taking 3.2 as the origin and 0.5 as the unit, the value of y required will be value for

$$u = \frac{3.75-3.5}{0.5} = 0.5$$

U	$\mathbf{Y_u}$	$\boldsymbol{\delta Y_u}$	$\boldsymbol{\delta^2 Y_u}$	$\boldsymbol{\delta^3 Y_u}$	$\boldsymbol{\delta^4 Y_u}$	$\boldsymbol{\delta^5 Y_u}$
-2	24.145					
		-2.102				
-1	22.043		0.284			
		-1.818		-0.047		
0	20.225		0.237		0.009	
		-1.581		-0.038		-0.003
1	18.644		0.99		0.006	
		-1.382		-0.032		
2	17.262		0.167			
		-1.215				
3	16.047					

Using the Gauss forward formula we have

$$F(u) = f(0) + u\delta f(0) + \frac{u(u-1)}{2!}\delta f(1) + \frac{u(u-1)(u+1)}{6}\delta^2 f(-1/2) + \frac{u(u-1)(u+1)(u-2)}{24}\delta^4 f(1/2)$$

$$Y_5 = 20.225+5(-1.5881)+\frac{0.5(0.5-1)}{2}(0.237)+\frac{0.5(0.5-1)(0.5+1)}{6}(-0.38)+\frac{0.5(0.5-1)(0.5+1)(0.5-2)}{24}(0.009)$$

$$+\frac{0.5(0.5-1)(0.5+1)(0.5-2)(0.5+2)}{120}(-0.003)$$

$$= 20.225-0.7905-0.029625+ 0.00238 + 0.9023750 + 0.0002106$$

$$= 19.40 \text{ approx}$$

Thus value of y for x = 3.75 is 19.40 approximately.

6. Use Gauss forward formula to find y_{30} given that $y_{21} = 18.4708$, $y_{25} = 17.8144$, $y_{29} = 17.1070$, $y_{33} = 16.3432$, $y_{37} = 15.5154$.

X	21	25	29	33	37
Y	18.4708	17.8144	17.1070	16.3432	15.5154

Solution. Arrange the above data in the following form :

Take the origin x = 29 and h = u as the unit; then the value of y for u = $\frac{30-29}{4}$ = 0.25 is to be found. Construct the following difference table.

X	$u = \frac{x-29}{4}$	Y_u	δY_u	$\delta^2 Y_u$	$\delta^3 Y_u$	$\delta^4 Y_u$
21	-2	18.4708				
			-0.6564			
25	-1	17.8144		-0.0510		
			-0.7074		-0.0054	
29	0	17.1070		-0.0564		-
			-0.7638		-00076	0.0022
33	1	16.3432		-0.640		
			-0.8278			
37	2	15.5154`				

Now applying the gauss forward interpolation formula.

We get

f(0.4) = 17.1070+ (0.25) (-0.7638)+$\frac{(0.25)(-0.75)}{2}(-0.0564)+\frac{(1.25)(0.25)(-0.75)}{6}(-0.0076)+$

$\frac{(1.25)(0.25)(-0.75)(-1.75)}{24}(-0.0022)$

= 17.1070-0.19095+0.0052875+0.00002968-0.00000375

= 16.9216 approx.

7. Given that $\sqrt{12500}$ = 111.803399; $\sqrt{12510}$ = 111.848111; $\sqrt{12520}$ = 111.892806; $\sqrt{12530}$ = 111.937483 Show by Gauss's backward formula that $\sqrt{12516}$ = 111.874930

Solution. Taking 12520 as the origin h 10 as the unit, the value of y required is for

u = $\frac{12516-12520}{10}$ = -0.4

U	Y_u	δY_u	$\delta^2 Y_u$	$\delta^3 Y_u$
-2	111.803399			
		0.044712		
-1	111.848111		-0.00017	
		0.044695		-0.000001

0	111.892806		-0.000018	
		0.044677		
1	111.937483			

Now by using Gauss backward formula, we have

$y_{-0.4}$ = 111.8928+ (-0.4)(0.044695)-+$\frac{(0.6)(-0.4)}{2!}$(-0.000018) +

$\frac{(0.6)(-0.4)(-1.4)}{3!}$(-0.000001)

= 111.8928-0.017878 + negligible quantities = 111.874930

i.e., $\sqrt{12516}$ = 111.874930.

8. Apply Gauss forward formula to find the value of f(x) at x = 3.75 from the following table.

X	**2.5**	**3.0**	**3.5**	**4.0**	**4.5**
F(x)	**24.145**	**22.043**	**20.225**	**18.644**	**17.262**

Solution. Let us take a = 3.5 as origin, h = 0.5

Put a + uh = 3.75 => 3.5+ 0.5u = 3.75

0.5u = 0.25

u = 0.5 = ½

u	**10^3 f(u)**	**10^3 $\delta f(u)$**	**10^3 δ^2 f(u)**	**10^3 δ^3f(u)**	**10^3 δ^4 f(u)**
-2	24145	-2102			
			284		
-1	22043	-1818		-47	
			237		9
0	20225	-1581		-38	
			199		6
1	18644	-1382		-32	
			167		
2	17262	-1215			

$10^3 f(\frac{1}{2})$ = 20225+$\frac{1}{2}$(-1581)+$\frac{\frac{1}{2}(\frac{1}{2}-1)}{2}$(237)+ $\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}+1)}{6}$ (-38) +

$\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}+1)((\frac{1}{2}-2)}{24}$ (9)

= 20225-790.5-29.625+2.375+0.2709 = 19407.46

f($\frac{1}{2}$) = 19.408

f(x) = 19.408 when x= 375.

9**. Find the value of cos 51^0. 42' by Gauss backward formula given that**

X	$\mathbf{50^0}$	$\mathbf{51^0}$	$\mathbf{52^0}$	$\mathbf{53^0}$	$\mathbf{54^0}$
Cosx	**0.6428**	**0.6293**	**0.6157**	**0.6018**	**0.5878**

Solution : Let us take a =50^0 as origin h =1^0

Put a+uh = 51^042^1,52^0 +u = $51^042^|$ => u= -18$^|$ = $\frac{-18}{60}$ = 0.3, u = -0.3

u	$\mathbf{10^4}$ **f(u)**	$\mathbf{10^4}$ $\boldsymbol{\delta f(u)}$	$\mathbf{10^4}$ $\boldsymbol{\delta^2}$ **f(u)**	$\mathbf{10^4}$ $\boldsymbol{\delta}$**f(u)**	$\mathbf{10^4}$ $\boldsymbol{\delta^4}$ **f(u)**
-2	6428	-135			
-1	6293	-136	-1	-2	4
0	6157	-139	-3	2	
1	6018	-140	-1		
2	5878				

Gauss backward interpolation formula is

f(u) = f(0)+$\frac{u}{1!}$ $\boldsymbol{\delta}$ f($\frac{1}{2}$) +$\frac{u(u+1)}{2}$ $\boldsymbol{\delta^2}$ f(0)+ $\frac{u(u+1)(u-1)}{3!}$ δ^3(-$\frac{1}{2}$)+

$\frac{u(u+1)(u-1)(u+2)}{4!}$ δ^4f(0)

10^4f(-0.3) = 6157 + (-0.3)(-136)+$\frac{(-0.3)(0.7)}{2}$(-3) + $\frac{(-0.3)(0.7)(-1.3)}{6}$(-2) +

$\frac{(-0.3)(0.7)(-1.3)(1.7)}{24}$(2)

= 6157 + 40.8 + 0.315 -0,091 + 0.07735 = 6198.1013

f(-0.3) = 0.61981013

cos $57^042^|$ = 0.6198.

10. Find by Gauss backward formula the sales of the conceived by the year 1936.

Given that

Year	1901	1911	1921	1931	1941	1951
Sales	12	15	20	27	29	52

(in thousand of rupees)

Solution : let us take a = 1931 as origin

H = 10

a + uh = 1936 => 1931 + 10u = 1936 => 10u = 5

=> $U=\frac{1}{2}[0,-1,1,-2,2,-3]$

U	f(u)	$\delta f(u)$	$\delta^2 f(u)$	$\delta^3 f(u)$	$\delta^4 f(u)$
-3	12				
		3			
-2	15		2		
		5		0	
-1	20		2		3
		7		3	
0	27		5		-7
		12		-4	
1	39		1		
		13			
2	52				

Gauss backward interpolation formula is

$f(u = f(0) + \frac{u}{1!}\delta f(\frac{1}{2}) + \frac{u(u+1)}{2}\delta^{2}f(0) + \frac{u(u+1)(u-1)}{3!}\delta^3 f(\frac{1}{2}) +$

$\frac{u(u+1)(u-1)(u+2)}{4!}\delta^4 f(0$

$f(\frac{1}{2}) = 27 + (0.5)7 + \frac{(0.5)(0.5+1)}{2}(5) + \frac{(-0.5)(0.5+1)(0.5-1)}{6}(3) +$

$\frac{(-0.5)(0.5+1)(0.5-1)(0.5-1)}{24}(-7)$

= 27 + 3.5 + 1.875 -0.1875 + 0.2734 = 32.46 thousand 32.7

In the year 1936 = 32.7 thousands. Difference table:

X	$U=\frac{x-30}{5}$	Y_u	δy_u	$\delta^2 y_u$	$\delta^3 y_u$	$\delta^4 y_u$

20	-2	49225				
			-909			
25	-1	48316		-171		
			-1080		-59	
30	0	47236		-230		-21
			-1310		-80	
35	1	45926		-310		
			-1620			
40	2	44306				

Using Stirlings formula

Putting u = -0.4 we get difference

$y_{-0.4} = 47236+(-0.4)\frac{-1310-1080}{2}+\frac{0.16}{2}(-230)+\frac{(-0.4)(0.16-1)}{6}(\frac{80-59}{2})$

$+\frac{(0.16)(0.16-1)}{24}(-21)$

$= 47326+ 478 -18.4 – 3.8920 + 1176$

$= 47692$

$Y_{28} = 47692$

12. Apply Bessel's formula to obtain y_{25}, given $y_{20} = 2854$, $y_{24} = 3162$, $y_{32} = 3992$

Solution. take 24 as origin and 4 $u = \frac{25-24}{4} = \frac{1}{4} = 0.25$

Difference table is

X	$\mathbf{u = \frac{x-24}{4}}$	$\mathbf{Y_u}$	$\boldsymbol{\delta y_u}$	$\boldsymbol{\delta^2 y_u}$	$\boldsymbol{\delta^3 y_u}$
20	-1	2854			
			308		
24	0	3162		74	
			382		-8
28	1	3544		64	
			448		
32	2	3992			

Using bessel's formula, we get

$Y_{1/4} = \frac{1}{2}(3162+3544)+(\frac{1}{4}-\frac{1}{2})382+\frac{\frac{1}{4}(\frac{1}{4}-\frac{1}{2})}{2!}(\frac{14+66}{2})+\frac{(\frac{1}{4}-\frac{1}{2})(\frac{1}{4})(\frac{1}{4}-1)}{3!}(-8)$

$= 3353 – 95.5 – 6.5625 – 0.0625 = 3250.875.$

13.Given y_{20} =24 , y_{24} =32, y_{28} =35, y_{32} =40 find y_{25} by Bessel's formula.

Solution : Taking origin at x =24 and u as the unit.

Find y_u for u = $\frac{25-24}{4} = \frac{1}{4} = 0.25$

The difference table is

X	U = $\frac{x-24}{4}$	Y_u	δy_u	$\delta^2 y_u$	$\delta^3 y_u$
20	-1	24			
			8		
24	0	32		-5	
			3		7
28	1	35		2	
			5		
32	2	40			

By using Bessel's formula we have

$$y_{\frac{1}{4}} = \frac{32+35}{2} + (\frac{1}{4} - \frac{1}{2})3 + \frac{\frac{1}{4}(\frac{1}{4}-1)(\frac{-5+2}{2})}{2} + \frac{(\frac{1}{4}-\frac{1}{2})\frac{1}{4}(\frac{1}{4}-1)}{6}(7)$$

= 33.5-0.75+ 0.1406+0.54687 = 32.945287.

14.Given that

0	0^0	5^0	10^0	15^0	20^0	25^0	30^0
$\tan\theta$	0.0000	0.0875	0.1763	0.2679	0.3640	0.4663	0.5774

Show that $\tan 16^0$ = 0.2867 (use Stirilling's formula)

Solution : Taking 15^0 as origin and 5^0 as the unit , the value required will be for

$$U = \frac{16^0-15^0}{15^0} = 0.2^0$$

θ	U	Y_u	δy_u	$\delta^2 y_u$	$\delta^3 y_u$	$\delta^4 y_u$	$\delta^5 y_u$	$\delta^6 y_u$
0^0	-3	0.0000						
			0.0872					
5^0	-2	0.0875		0.0013				
			0.0888		0.0015			
10^0	-1	0.1763		0.0028		0.0002		
			0.916		0.0017		-0.0002	
15^0	0	0.2679		0.0045		0.0000		0.0011
			0.961		0.0017		0.0009	
20^0	1			0.0062		0.0009		
		0.3640	0.1023		0.0026			
25^0	2							

30^0	3	0.4663 0.5774	0.1111	0.0088				

By using Stirlings formula, we have

Tan 16^0 = $y_{0.2}$ = $0.2679+\frac{0.2}{2}(0.0961+0.0916)\{+\frac{0.04}{2}(0.0045)+\frac{0.2(0.4-1)}{2}\frac{1}{2}(0.0017+0.0017)+$

$\frac{0.4(0.4-1)}{24}(0.0000)+\frac{0.2(0.4-1)(0.04-1)}{120}\frac{1}{2}(-0.0002+0.0009)+$

$\frac{0.2(0.04-1)(0.04-4)}{720}(0.0011)$

= 0.2679 + 0.01877 + 0.00009 + negligible

= 0.2867.

1.5 Applications in engineering :

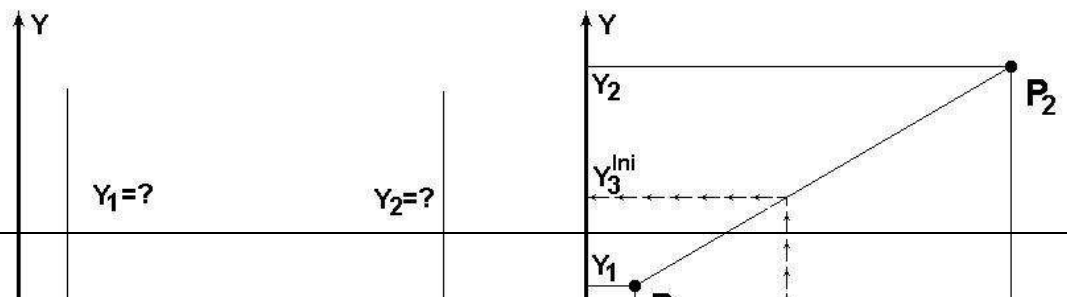
General Ideas Regarding the Use of the Interpolation in the Numerical Analysis

There are certain cases when approximation solutions are useful within the numerical methods field itself.

Some of these cases are presented in the next sections.

Approximation of the Initial Solution for Iterative Methods

Let us consider a theoretical problem where the formulation of the problem depends on some parameters and an iterative solution is possible. It is well known that the final solution depends on the initial approximation in terms of accuracy and of computer runtime.



Calculus of the Integrals :

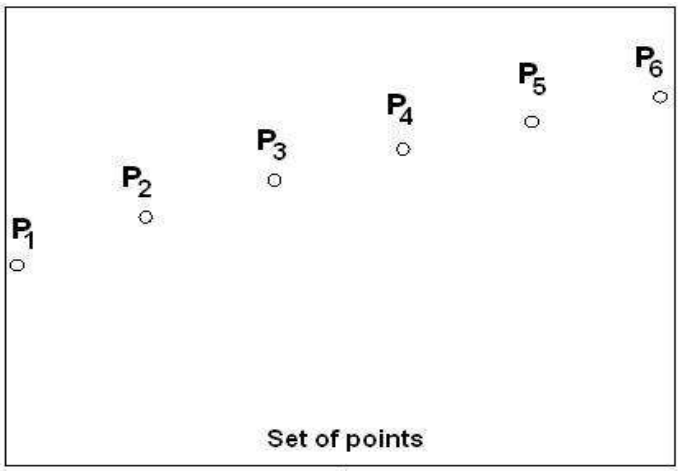
Let us consider that a set of points are known, that is $P_j\ x_j\ ,\ y_j\ ,\ j\ 1,\ N$. Usually these points result from experimental studies. There must be computed the integral of the function approximated by this set of points.

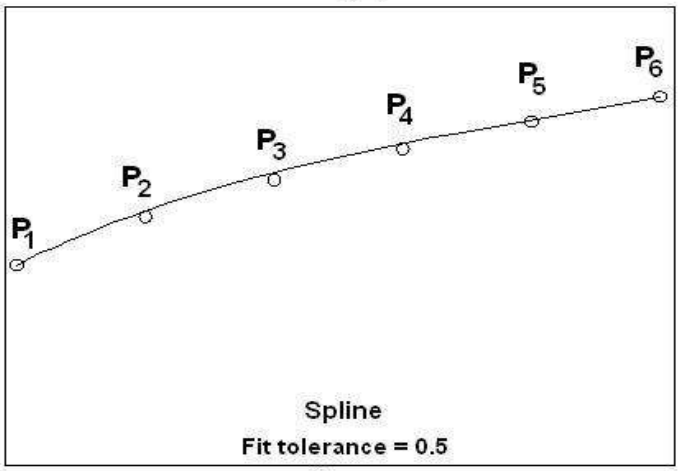
A first method to compute the integral is to use the summation of the areas of the trapezoids defined by two consecutive points. This is a coarse approximation, but if the points are close one to the other the accuracy may be considered satisfactory.

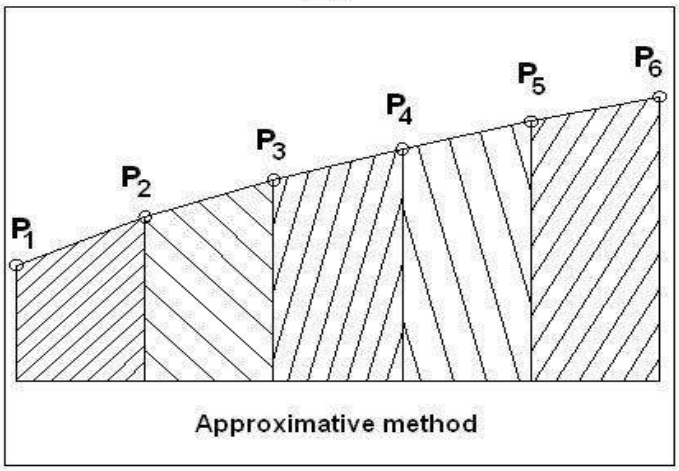
The second method has two stages. The first one is to approximate the set of points using one of the previously mentioned interpolation methods, and it results a set of equations like (6). The second stage is to analytically integrate the functions and to calculate the result. In his case the source of errors is in the interpolation stage.

A similar approach can be used to compute the derivative of the functions, either using the initial points, or using the appropriate interpolation functions.

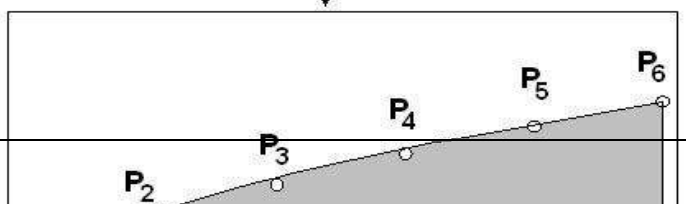
The error for each of the methods may be evaluated for a certain dataset. Basically, it is necessary to use as many methods exist, in order to have several results which, ideally, must be close one to the other.







$$\sum_{j=1}^{N} \underbrace{\frac{\left(Y_j + Y_{j+1}\right) \cdot \left(X_{j+1} - X_j\right)}{2}}_{Area\ A_j}$$



2. NUMERICAL INTEGRATION

2.1 INTRODUCTION :

The numerical integration is the process of evaluating the approximate

Value of the integration $\int_a^b f(x)dx$ from a set of numerical values of f(x). If the problem is to be applied to the integration of function of a single variable, then the process is called as quadrature.

The problem of numerical integration is evolved by first approximating the f(x) by a polynomial with the help of interpolation formula and then integrating this expression between the limits.

$\therefore$ To find the value of $\int_a^b f(x)\,dx$, first express the function f(x) by interpolation formula, say p(x) and then integrate between the limits.

$$\therefore \int_a^b f(x)dx = \int_a^b p(x)dx$$

The error is given by $\int_a^b f(x)dx - \int_a^b p(x)dx$

2.2 A GENERAL QUADRATURE FORMULA FOR NUMERICAL INTEGRATION

Consider the integral $\int_a^b f(x)dx$. Let f(x) be given for some equally spaced values of the orguments say ,x_0 , x_0+h ,x_0+2h,…. Let the range (b-a) be divided into n equal parts, each of which is of width h.

b-a = nh

h = $\frac{b-a}{n}$

let x_0 = a,x_1 = x_0+h = a+h , x_2 = a+2h ….. x_n = a+nh = b

I = $\int_a^{a+nh} f(x)dx$

Putting x = a+uh

dx = hdu

x -> a -> u ->0

x ->a+uh -> u->n

I = $\int_a^n f(a+uh)du$ = h $\int_a^n f(a+uh)du$

No write as f(a+uh) on the Newton's interpolation formula as

= h $\int_0^n [f(a) + \frac{u}{1!}\Delta f(a) + \frac{u(u-1)}{2!}\Delta^2 f(a) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(a) + \cdots +$ $\frac{u(u-1)\ldots(u-n+1)}{n!}\Delta^n f(a)\ ]\, du$

= h [f(a).u + $\frac{u^2}{2}\Delta f(a)$+($\frac{u^3}{3} - \frac{u^2}{2}$) $\Delta^2 f(a)$+ $\frac{(\frac{u^4}{4} - \frac{3u^3}{3} + \frac{2u^2}{2})}{6}\Delta^3 f(a) + \cdots +]$

= h[nf(a) + $\frac{n^2}{2}\Delta f(a)$ + ($\frac{n^2}{6} - \frac{n^2}{4}$) $\Delta^2 f(a)$ +($\frac{n^4}{24} - \frac{n^3}{6} + \frac{n^2}{6}$)$\Delta^3 f(a)$ +]

$\int_a^b f(x)dx$ = h[ny_0 + $\frac{n^2}{2}$(y_1-y_0)+ ($\frac{n^3}{6} - \frac{n^2}{4}$)($y_2-2y_1$+$y_0$)+$\left(\frac{n^4}{24} - \frac{n^3}{6} + \frac{n^2}{6}\right)$

(y_3-3y_2+3y_1-y_0)+……..] …….(1)

$\int_a^b f(x)dx$ = $\int_a^{a+nh} f(x)dx$

This is known as general quadrature formula we deduce the following important quadratic formulas by taking n= 1,2,3….

2.3 TRAPEZOIDAL RULE :

Put n=1 eq(1) and neglect the second and higher order differences when n=1 , the argument are a, a+h and its corresponding functional values are y_0 and y_1 thus we have

$\int_a^{a+nh} f(x)dx$ = h[$y_0 + \frac{1}{2}(y_1 - y_0)$] = $\frac{h}{2}(y_0+y_1)$

$I_1 = \int_a^{a+nh} f(x)dx = \frac{h}{2}(y_0+y_1)$

$I_2 = \int_a^{a+nh} f(x)dx = \frac{h}{2}(y_0+y_1)$

$I_3 = \int_{a+2h}^{a+3h} f(x)dx = \frac{h}{2}(y_2+y_3)$

……..

……..

$I_n = \int_{a+(n-1)h}^{a+nh} f(x)dx = \frac{h}{2}(y_{n-1}+y_n)$

Adding these n integrals, we have

$\int_a^{a+nh=b} f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

This rule is called the trapezoidal rule.

2.4 SIMPSON'S $\frac{1}{3}$ RULE :

Put n=2 in the equation (1) and neglect the third and higher order difference. In this case the arguments are a, a+h, a+2h the corresponding entries are y_0,y_1 and y_2, thus we have

$\int_a^{a+2h} f(x)dx$ = h[$2y_0 + 2(y_1-y_0)+(\frac{y}{3}-1)(y_2-2y_1+y_0)$]

= $\frac{h}{3}[y_0+4y_1+y_2]$

Similarly $I_2 = \int_a^{a+2h} f(x)dx = \frac{h}{3}(y_0+4y_1+y_2)$

$I_3 = \int_{a+4h}^{a+6h} f(x)dx = \frac{h}{3}(y_4+4y_5+y_6)$

……..

……..

$I_n = \int_{a+(n-2)h}^{a+6h} f(x)dx = \frac{h}{3}(y_{n-2}+4y_{n-1}+y_n)$

Adding these integrals, we have $I = \int_a^b f(x)dx$

= $\frac{h}{3}[(y_0+y_n)+(y_1+y_3+y_5+\ldots+y_{n-1})+2(y_2+y_4+\ldots.+y_{n-2})]$

This is known as SIMPSON'S $\frac{1}{3}$ RULE

2.5 SIMPSON'S $\frac{3}{8}$ RULE :

Put n=3 in equation(1) and neglect fourth and higher order differences. In this case, the arguats are a, a+h, a+2h and a+3h and the corresponding fractional values are y_0, y_1, y_2 and y_3, thus we have

$\int_a^{a+3h} f(x)dx$ = n[$3y_0+\frac{9}{2}(y_1-y_0)+(\frac{9}{2}-\frac{9}{4})(y_2-2y_1+y_0)+(\frac{27}{8}-\frac{27}{6}-\frac{9}{6})(y_3-3y_2+3y_1-y_0)$]

= h[$3y_0+\frac{9}{2}(y_1-y_0)+(\frac{9}{2}-\frac{9}{4})(y_2-2y_1+y_0)+(\frac{27}{8}-\frac{27}{6}-\frac{9}{6})(y_3-3y_2+3y_1-y_0)$]

= h[$3y_0+\frac{9}{2}(y_1-y_0)+(\frac{9}{4})(y_2-2y_1+y_0)+(\frac{3}{8})(y_3-3y_2+3y_1-y_0)$]

$I_1 = \int_a^{a+3h} f(x)dx = \frac{3h}{8}(y_0+3y_1+3y_2+y_3)$

$I_2 = \int_{a+4h}^{a+6h} f(x)dx = \frac{3h}{8}(y_3+3y_4+3y_5+y_6)$

……..

……..

$I_n = \int_{a+(n-2)h}^{a+6h} f(x)dx = \frac{3h}{8}(y_{n-3}+3y_{n-2}+3y_{n-1}+y_n)$

Adding all these integrals we get

$\int_a^{a+nh} f(x)dx = \frac{3h}{8}[(y_0+y_n)+3(y_1+y_2+y_4+\ldots+y_{n-1})+2(y_3+y_6+\ldots.+y_{n-3})]$

This formulas is called the Simpsons' rule.

Note : Applying this formula, the number of sub-internals should be taken as multiple of three

2.6 WEDDLE'S RULE :

put n=6 in equation (1) and neglect the seventh and higher order differences. in this case the arguments are from a to a+6h and there will be only seven functional values, thus we get

$\int_a^{a+6h} f(x)dx$ = h[$6y_0+18\Delta y_0+27\Delta^2 y_0+24\Delta^3 y_0+\frac{123}{10}\Delta^4 y_0+\frac{33}{10}\Delta^5 y_0+\frac{41}{140}\Delta^6 y_0$]

Replacing $\frac{41}{140}\Delta^6 y_0$ by $\frac{3}{10}\Delta^6 y_0$,

Since the error involved is negligible.

Thus above equation can be expressed as

$\int_a^{a+6h} f(x)dx$ = h[$6y_0+18\Delta y_0+27\Delta^2 y_0+24\Delta^3 y_0+\frac{123}{10}\Delta^4 y_0+\frac{33}{10}\Delta^5 y_0+\frac{3}{10}\Delta^6 y_0$]

$= \frac{3h}{10}(y_0+5y_1+y_2+ 6y_3+y_4+5y_5+y_6)$

Similarly $\int_{a+6h}^{a+12h} f(x)dx = \frac{3h}{10}(y_6+5y_7+y_8+ 6y_9+y_{10}+5y_{11}+y_{12})$

…….

…….

$\int_{a+(n-6)h}^{a+nh} f(x)dx = \frac{3h}{10}(y_{n-6}+5y_{n-5}+y_{n-4}+ 6y_{n-3}+y_{n-2}+5y_{n-1}+y_n)$

Adding these integrals, we have

$\int_a^{a+nh} f(x)dx = \frac{3h}{10}(y_0+5y_1+y_2+ 6y_3+y_4+5y_5+2y_6+ 5y_7+y_8 +……)$

This formula is known as Weddle's rule.

Note:

1. This rule is more accurate than the Simpson's rule, but it requires atleast seven consecutive values of the function
2. This method is applicable when n is a multiple of 6.

2.7 ERROR IN QUADRATURE FORMULA :

The error terms in the quadrature formula is given by

$$E = \int_a^b y dx - \int_a^b p(x)dx$$

where P(x) is the polynomial representing the function y = f(x) in the interval [a, b].

(i) Error in Trapezoidal Rule : Expanding y = f(x), about $x = x_o$ by Taylor's series, we obtain

$$y = y_0 + (x-x_0)y_0 + \frac{(x-x0)^2}{2!} y_0^{11} + \ldots\ldots\ldots\ldots(1)$$

$$\int_{x_0}^{x_0+h} y\, dx = \int_{x_0}^{x_0+h} [y_0 + (x - x_0)y_0 + \frac{(x-x_0)^2}{2!} y_0 + \cdots]dx$$

$$= y_0 n + \frac{h^2}{2!} y_0 + \frac{h^2}{3!} y_0 + \ldots\ldots\ldots\ldots(2)$$

But area of the first trapezium in the interval $[x_o, x_1] = \frac{1}{2}h(y_0 + y_1)$ ….(3)

Then putting $x = x_o + h$ and $y = y_1$ in (1), we obtain

$$y_1 = y_0 + h\, y_0 + \frac{h^2}{2!} y_0 + \ldots\ldots\ldots\ldots$$

Putting this value of y, in (3), we have

$$B = \frac{1}{2} h[y_0 + y_0 + h\, y_0 + \frac{h^2}{2!} y_0 + \ldots\ldots\ldots] = h\, y_0 + \frac{h^2}{2!} y_0 + \frac{h^3}{2.2!} y_0 + \ldots\ldots \quad \ldots\ldots(4)$$

Hence the error in the interval $[x_o, x_1]$

$= \int_{x_0}^{x_1} y\,dx$ - B_1 = $[\frac{1}{3!} - \frac{1}{2.2!}]\ h^3y_0$+…… = $-\frac{h^3}{12}\ y_0$+…….

Principal part of the error in $[x_o,x_1]$ = $-\frac{h^3}{12}\ y_0$

And similarly principal part of the error in $[x_1, x_2]$ = $-\frac{h^3}{12}\ y_1$ etc.,

Thus, the total error = $-\frac{h^3}{12}\ [y_0 + y_1 + y_{n-1}]$ = E , say

Then assuming that y''(x) is the largest of the n quantities y_o , y_1 ,.. . ., y_{n-1} ,

we can say that

E< $-\frac{nh^3}{12}\ y^{11}(x)$ = $-\frac{(b-a)h^2}{12}\ y^{11}(x)$ as b-a = nh ……(5)

Thus the error in the trapezoidal rule is of the order h^2.

(ii) Error in Simpson's $\frac{1}{3}$ Rule : Expanding y = f(x) about x = x_o by Taylor's series, we have

$y = y_0 + (x-x_0)y_0 + \frac{(x-x0)^2}{2!}\ y_0$+……

$\int_{x_0}^{x^2} y\,dx = \int_{x_0}^{x_0+2h} [y_0 - (x - x_0) + \frac{(x-x_0)^2}{2!}\ y_{0+\cdots}]$ dr

$2hy_0 + \frac{4h^2}{2!}y_0 + \frac{8h^3}{3!}y_0 + \frac{16h^4}{4!}y_0 + \cdots + \frac{32h^5}{5!}y_0 + \cdots\cdots$ ……..(2)

and area over the first double strip by Simpson's 1/3 rule

= $\frac{1}{3}\ h(y_0 + 4y_1 + y_2)$ = A_1 say

Then putting x = x_0+ h and y = y_1 in (1), we get

$y_2 = y_0 + 2hy_0 + \frac{4h^2}{2!}y_0 + \frac{8h^3}{3!}y_0$ + ……..

Further putting x = x_o+ 2h and y = 1/2 in (1), we get

$y_2 = y_0 + 2hy_0 + \frac{4h^2}{2!}y_0 + \frac{8h^3}{3!}y_0$ + ……..

(3) → $A_1 = \frac{h}{3}\ [y_0 + 4(y_0 + hy_0 + \frac{h^2}{2!}\ y_{0+\cdots\cdots}) + (y_0 + 2h\ y_0 + \frac{4h^2}{2!}\ y_0 + \frac{8h^3}{3!}\ y_0 + \cdots\cdots)]$

= $2hy_0 + 2h^2y_0 + \frac{4h^3}{3}\ y_0 + \frac{2h^2}{8}\ y_0 + \frac{5h^5}{18}\ y_0$ +……. ……(4)

Hence the error in the interval $[x_o, x_2] = \int_{x_0}^{x} y\,dx - A_1$

= $[\frac{4}{15} - \frac{5}{18}]\ h^5y_0^5$ +……. ……..(5)

and the principal part of the error in $[x_o, x_2]$

$= [\frac{4}{15} - \frac{5}{18}] h^5 y_0^5 = - -\frac{h^5}{90} y_0^5$

On the same lines principal part of the error in $[x_2, x_4]$

$= \frac{h^5}{90} y_2^5$, etc.,

Total error = $- -\frac{x^5}{90} [y_0^4 + y_2^4 + \ldots\ldots\ldots + y_2^4 (n-1)] = E$, Say ……(6)

The assuming $y_{1v}(x)$ as the largest of $y_0^{1V}, y_2^{1V} \ldots\ldots, y_{2n-2}^{1V}$

We readily obtain $E < -\frac{n/5}{90} y_0^{1V}(x) = - \frac{(b-a)h^y y^5(x)}{180}$ ……(7) [2nh = b-a]

(iii) Error in Simpson's 3/8 rule : Proceeding as above, we have the principal part of the error in the interval $[x_o, x_3]$ as $= -\frac{3h^5}{80} y^5$

(iv) Error in Weddle's rule: Principal part of the error in the interval$[x_0, x_6] = -(h^7/140) y_0^5$

2.8 SOLVED PROBLEMS ::

1.Evalaute the value of the integral using $\int_2^{1.4} (sinx - logx + e^x)$dx

(a) The Trapezoidal rule, (b) Simpson's one-third rule,

(c) Simpson's three eight rule.

Solution. In this case divide the range of integration into twelve equal parts by taking h = -1 and then the values of the function y = sin x $-\log_e x$ +e are computed for each point of subdivision which are as under

x	Sinx	$\text{Log}_e x = \log_{10} x x 2.30258$	e^x	$Y = \text{sinx} - \log_e x + e^x$
0.2	0.19867	-1.60943	1.22140	3.02950
0.3	0.29552	-1.20397	1.34986	2.84935
0.4	0.38942	-0.91629	1.49182	2.79753
0.5	0.47943	-0.69315	1.64872	2.82130
0.6	0.5646	-0.51083	1.82212	2.89759
0.7	0.64422	-0.35667	2.01375	3.01464
0.8	0.71736	-0.22314	2.22554	3.16604
0.9	0.78333	-0.10537	2.45960	3.34830
1.0	0.84147	+0.00000	2.71828	3.55975
1.1	0.89121	0.09530	3.00417	3.80008
1.2	0.93204	0.18232	3.32012	4.06984
1.3	0.96356	0.26236	3.66930	4.37050
1.4	0.98545	0.33647	4.05520	4.70418

(a)By Trapezoidal rule, we have

$\int_2^{1.4}$(sin x -$\log_e{}^x$ + e^x)dx = $\int_2^{2+12x\ 0.1} y\,dx$

Here n = 12; h = 0.1, x_o = 0.2, y = sin x -$\log_e{}^r$+ e^x

Again w.k.t. $\int_{x0}^{xo\ +nh} y\,dx$ = $\frac{h}{2}$[y_0+y_{n+2}(y_1+y_2+…+y_{n-1})]

Substituting these values of, h_1, y_o, y_n…, in the above formula we get

$\int_2^{1.4}(\sin x - \log ex + e^x)\,dx$ = $\frac{1}{20}$[3.02950+ 4.70418+2(2.8493+…+4.37050)]

= $\frac{1}{20}$[7.73368+2(36.69492)]

= 4.05617

(b) By Simpson's rule $\frac{1}{3}$ we have

$\int_{x0}^{xo\ +nh} y\,dx$ = $\frac{h}{3}$[y_0+y_6+4(y_1+y_3+…)+2(y_2+y_4+…)]

Put the values of h, y_o, $y_{1..}$.. from the table, we have

= $\int_2^{1.4}(\sin x - \log ex + e^x)\,dx$

= $\frac{1}{3}$[7.73368+ 4(20.20417)-3(16.49075)]

= $\frac{1}{3}$ [121.53186] = 4.05106

(c) By Simpson's 3/8 rule we have

$\int_{x0}^{xo\ +nh} y\,dx$ = $\frac{3h}{8}$[(y_0+ y_n)+3(y_1+y_2+y_4+…+y_{n-1})+2(y_3+y_6+….+y_{n-3})]

= $\frac{3(0.1)}{8}$[7.73368+ (26.90750)+ 2(9.78742)] = 4.05116

2. A curve is drawn to pass through the points given by the following table.

x	1	1.5	2	2.5	3	3.5	4
y	2	2.4	2.7	2.8	3	2.6	2.1

Estimate the area bounded by the curve, the x-axis and the lines x=1,x=4.

Solution. By Simpson's rule, we have required area

= $\frac{h}{3}$[y_0+4y_1+2y_2+4y_3+…….+4y_{n-1}+y_n]

= $\frac{0.5}{3}$ [2+ 9.6+5.4+11.2+10.4+ 2.1]

= $\frac{50}{30}$x46.7 = $\frac{23.35}{3}$ = 7.783 unit of area

By Weddle's rule, we have required area

= $\frac{3h}{10}$[y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6]

$= \frac{3x0.5}{10}[2+12.0+2.7+16.8+3.13.0+2.1] = 7.74$ units of area

3. Calculate by Simpson's rule an approximate value of by taking seven equidistant ordinates. Compare it with the exact value and the value obtained by using the Trapezoidal rule.

Solution. Let us divide the range of integration (-3, 3) into six equal parts each of width $\frac{3(-3)}{6}$ = 1 unit. Hence h-1, value of the function for each point of sub division are as given below.

x	$y= x^4$
-3	$(-3)^4 = 81$
-2	$(-2)^4 = 16$
-1	$(-1)^4 = 1$
0	$(0)^4 = 0$
1	$(1)^4 = 1$
2	$(2)^4 = 16$
3	$(3)^4 = 81$

Now by Simpson's rule, we obtain

$$\int_{-3}^{3} x^4 \, dx = \frac{h}{3}[y_0+y_6+4(y_1+y_3+y_5)+2(y_2+y_4)]$$

$$= \frac{1}{3}[162+4x32+ 2x2] = \frac{1}{3}x294 = 98$$

But the exact value of

$$\int_{-3}^{3} x^4 \, dx = \left[\frac{x^5}{5}\right]_{-3}^{3} = \frac{1}{5}[(3)^5 - (-3)^5] = \frac{1}{5}x486 = 97.2$$

Also by Trapezoidal rule, we have

$$\int_{-3}^{3} x^4 \, dx = \frac{h}{2}[y_0+y_6+2(y_1+ y_2+y_3+ y_4+y_5)] = \frac{1}{2}[162+2x34]$$

Thus it is obvious that the Trapezoidal rule does not give an accurate result. **4.Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rule. Hence obtain the approximate value of π in each case.**

Solution. Divide the range into six equal parts each of with h = $\frac{1}{6}$ and compute

The value of y = $\frac{1}{1+x^2}$ at each point of sub division.

These values are given below.

X	$Y_x = 1/(1+x^2)$
0	1.0000
1/6	36/37 = 0.97297
2/6	36/40 = 0.90000
3/6	36/45 = -0.80000
4/6	36/52 = 0.69231
5/6	36/61 = 0.59016
1	1 / 2 = 0.5000

By Simpson's rule, we have

$\int_{x_0}^{x_0+6h} y\,dx = \frac{h}{3}[y_0+y_6+4(y_1+y_3+y_5)+2(y_2+y_4)]$

$= \frac{1}{18}$ [1.00 + 0.500 + 4(0.97292 + 0.50000 + 0.59016) +

2(0.900+ 0.69231)] ……(1)

= 0.785395

By Simpson's 3/8 rule we have

$\int_{x_0}^{x_0+6h} y\,dx = \frac{3h}{8}[y_0+y_6+3(y_1+y_2+ y_4+y_5)+2(y_3)]$

$= \frac{1}{16}$ [1.5000 + 3 (0.97297 + 0.900 + 0.69231+0.59016 +

2 x (0.800))] = 0.785395 ……(2)

Again, $\int_0^1 \frac{dx}{1+x^2} = [\tan^{-1} x]^1_0 = \pi/4$ ……(3)

From (1) and (2), we get π = 3.14188 and from (2) and (3) we get

π = 3.141580

5. Evaluate $\int_2^{10} \frac{dx}{x}$ by dividing the range into '8' equal parts by using Trapezoidal rule.

Solution. Given integral, $\int_2^{10} \frac{dx}{x}$

Let f(x) = $\frac{1}{4}$

a = 2, b = 10;n = 8

h = $\frac{b-a}{n} = \frac{10-2}{8} = 1$

x	y = f(x) = $\frac{1}{x}$

2	1/2 = 0.5000
3	1/3 = 0.3333
4	1/4 = 0.2500
5	1/5 = 0.2000
6	1/6 = 0.1666
7	1/7 = 0.1428
8	1/8 = 0.1250
9	1/9 = 0.1000
10	1/10 = 0.1000

Trapezoidal rule is $\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

$\int_2^{10} \frac{dx}{x} = \frac{h}{2}[(y_0+y_8)+2(y_1+y_2+y_3+ y_{4+} y_{5+} y_6+y_7)]$

$= \frac{1}{2}[(0.5+ 0.1)+ 2(0.333+ 0.25)+ 0.2+ 0.125+ 0.166 + 0.1428 +0.1111]$

$= \frac{1}{2}[(0.6)+2(1.3288)]$

$= 0.3+1.3288$

$= 1.62880$

6. Evaluate $\int_2^{10} \frac{dx}{1+x}$ by dividing the range into '8' equal parts by using Trapezoidal rule.

Solution. Let $f(x) = \frac{1}{1+x}$

a = 2, b = 10, n = number of equal parts = 8

$h = \frac{b-a}{n} = \frac{10-2}{8} = 1$

h = 1

x	f(x) = $\frac{1}{1+x}$

2	1/3 = 0.333
3	1/4 = 0.2500
4	1/5 = 0.2000
5	1/6 = 0.1666
6	1/7 = 0.1428
7	1/8 = 0.1250
8	1/9 = 0.1111
9	1/10 = 0.1000
10	1/11 = 0.0909

Trapezoidal rule is $\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

$\int_2^{10} \frac{dx}{1+x} = \frac{h}{2}[(y_0+y_8)+2(y_1+y_2+y_3+ y_{4+} y_{5+} y_6+y_7)]$

$= \frac{1}{2}[(0.333+ 0.090)+ 2(0.25+ 0.2+ 0.1661 + 0.1425+ 0.125+$

$0.111+ 0.1)]$

$= \frac{1}{2}[0.4242+2(1.0955)] = 0.2121 + 1.0955 = 1.3076$

8. Evaluate $\int_4^{5.2} f(x)dx$ by using Trapezoidal rule from the following.

X	**4**	**4.2**	**4.4**	**4.6**	**4.8**	**5.0**	**5.2**
Y	**1.4**	**1.4**	**1.5**	**1.5**	**1.6**	**1.6**	**1.6**

Where h = 0.2

Solution. Trapezoidal rule is, $\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

$\int_4^{5.2} f(x)dx = \frac{0.2}{2}[(1.4+ 1.6)+ 2(1.4+1.5+ 1.5+1.6+ 1.6)]$

$= 0.1[3+2(7.6)]$

$= 0.1[18.2] = 1.82$

9. Evaluate $\log_e 7$ from the integral $\int_1^7 \frac{dx}{x}$

Solution. Given f(x) = $\frac{1}{x}$ a = 1, b = 7, n = 6, h = $\frac{b-a}{n} = \frac{7-1}{6} = \frac{6}{6} = 1$

x	y= f(x)
1	1.0000
2	0.5000
3	0.333
4	0.2500
5	0.2000
6	0.1666
7	0.1428

Trapezoidal rule is, $\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

$= \frac{h}{2}[(1+ 0.1428)+ 20.5+ 0.333+0.25+ 0.2+ 0.1666)]$

$= \frac{1}{2}[1.1428+ 2(1.4499)]$

$= \frac{1}{2}[4.0426]$ and $\int_1^7 \frac{dx}{x} = [\log_e x]_1^7$

$= \log_e 7 = \log_e 1 = \log_e 7$

$\log_e 7 = 2.0113$

10. A curve is drawn to passes through the point given by the following table.

X	1	1.5	2	2.5	3	3.5	4
Y	2	2.4	2.7	2.8	3	2.6	2

Solution. Estimate the area bounded by the curve, the axis-x and the lines x=1,x=4by using Trapezoidal rule.

Trapezoidal rule is h= $\frac{4-1}{6}$=0.5

$\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+y_4+y_5)]$

$= \frac{0.5}{2}[2+2.1+2+ (2.4 +2.7+2.8+2.6+3)]$

$\frac{0.5}{2}(4.1+27.0) = \frac{15.35}{2} = 7.775$ units of area

11. calculate e $\int_2^{10} \frac{dx}{1+x}$by dividing the arrange into 8 equal parts by using Simpson's 1/3 rule.

Solution.: Simpson's 1/3 rule is

$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_5+\ldots..)+2(y_2+ y_4+\ldots....)]$

$\int_2^{10} \frac{dx}{1+x}$ = $\frac{h}{3}$[(O.333+ 0.0909)+ 4(0.25+ 0.1666+ 0.125+0.1)

+ 200.2+ 0.142+ 0.111)]

= $\frac{1}{3}$ [0.4242+ 4(0.6416)+ 2(0.4539))= $\frac{1}{3}$ [3.8984] =1.2994666

12. calculate $\int_a^b f(x)dx$ by using Simpson's $\frac{1}{3}$ rule

X	4	4.2	4.4	4.6	4.8	5.0	5.2
Y	1.4	1.4	1.5	1.5	1.6	1.6	1.6

where h = 0.2

Solution: Simpson's $\frac{1}{3}$ rule is

$\int_a^b f(x)dx$ = $\frac{h}{3}$[(y_0+y_n)+4(y_1+ y_3+ y_5+.....)+2(y_2+ y_4+.....)]

$\int_4^{5.2} f(x)dx$ =$\frac{0.2}{3}$[(1.4+1.6)+ 4(1.4+1.5+ 1.6)+ 2(1.5+ 1.6)]

= $\frac{0.2}{3}$[3+4(4.5)+2(3.1)] = $\frac{0.2}{3}$[27.2]

= 1.813333.....

13. Evaluate $\int_2^{10} \frac{dx}{1+x}$ by dividing the range into 8 equal parts by using Simpson's $\frac{1}{3}$ rule.

Solution:. Let f(x)= $\frac{1}{x}$

a = 2, b = 10, n =number of equal parts = 8

h = $\frac{b-a}{n}$ = $\frac{8}{8}$ = 1

X	Y = f(x) =$\frac{1}{x}$
2	0.5000
3	0.3333
4	0.2500
5	0.2000
6	0.1666
7	0.1428
8	0.1250

9	0.1111
10	0.1000

Simpson's $\frac{1}{3}$ rule is

$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_5+.....+y_{n-1})+2(y_2+ y_4+y_6.....+y_{n-2})]$

$= \frac{1}{3}$ [I0.6)+ 4(0.3333+ 0.2+ 0.1428+ 0.1111)] +2(0.25+0.1666+0.125)]

$=\frac{1}{3}$[0.6+4(0.7872+ 1.0832)]

$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_6+.....)+2(y_2+ y_4+.....)]$

$\int_0^1 \frac{dx}{1+x^2} = \frac{1}{6*3}$ [(1 +0.5)+ 4(0. 9729+0.8+0.3+ 2(09+0.6923)]

$= \frac{1}{18}$[1.5+9.452+3.1846] = $\frac{1}{18}$[14.1366]

= 0.7853666

On the other hand $\int_0^1 \frac{1}{1+x^2}dx = (\tan^{-1})^1_0$

$= \tan^{-1}(1)-\tan^{-1}(0)$

$= \frac{\pi}{4}-0 = \frac{\pi}{4}$

$= \frac{\pi}{4} = 0.7853$

$= \pi = 3142$

14. Evaluate $\int_0^4 e^x\,dx$ by using Simpson's rule using the data
e = 2°.72, e2= 739, e3=20.09, e4=54.60 and compare it will the actual value.

Solution.: Divide the whole range (0,4) into four equal parts by taking h=1. By Simpson's rule 1/3 we have

$\int_0^4 e^x\,dx = \int_{x_0}^{x_0+4h} y\,dx = \frac{h}{3}(y_0+y_4+4(y_1+y_3)+242)$

$= \frac{1}{3}[1+54.60+4(2.72+20.09)+2(7.39)]$

$= \frac{1}{3}[55.60+91.24+14.78] = \frac{1}{3}[161.62]$

= 553.373

The actual value of the integral is

$$\int_0^4 e^x \, dx = e^4 - e^0 = 54.60 - 1.0 = 53.60$$

15. Evaluate $\int_4^{5.2} \log, x \, dx$ by (a) Simpson's $\frac{1}{3}$ (b) $\frac{3}{8}$ rule . After finding the true value of the integral.

Solution. Divide the whole range of integration ie., (4, 5, 2) into six equal parts taking h 0.2 and then compute the values of the function f(x)= $\log^x_e$ for each of sub division. These computed values are as follows.

X	$Y = \log^x_e x$	X	$Y = \log^x_e x$
Xo = 4.0	1.38629436	Xo+ 4h= 4.8	1.56861592
Xo+ h =4.2	1.43508453	Xo+5h= 5.0	1.60943791
X_0 + 2h= 4.4	1.48160454	Xo+ 6h= 5.2	1.64865863
Xo +3h= 4.6	1.52605630		

(a) By Simpson's $\frac{1}{3}$ rule, we have

$$\int_4^{5.2} \log, x \, dx = \frac{h}{3} [y_0+y_6+(y_1+y_3++y_5)+2(y_2+y_4)]$$

$$= \frac{2}{8} [3.03495299+ 4(4.57056874)+ 2(3.05022046)]$$

$$=1.82784726.$$

(b) By Simpson's 3/8 rule, we have

$$\int_4^{5.2} \log e, x \, dx = \frac{3h}{8} [y_0+y_2+3(y_1+y_2++y_4+ y_5)+2 y_3]$$

$$= \frac{3(0.2)}{8}[03495299+ 3(6.09424290)+ 2 (1.52605630)]$$

$$= \frac{0.3}{4}[24.371294291]$$

$$= = 1.82784707$$

Again exact value of the integral is

$$\int_4^{5.2} \log e, x \, dx = x(\log_e x-1)\int_{4.0}^{5.4} 1.82784741$$

16. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ rule.

Solution : let f(x) = $\frac{1}{1+x^2}$ let n =6 , a= -0 , b = 1 ,h = $\frac{b-a}{n} = \frac{1-0}{6} = \frac{1-0}{6} = \frac{1}{6}$ = 0.1666

X	f(x) = $\frac{1}{1+x^2}$
0	1
1/6	0.9729
2/6	0.9000
3/6	0.8000
4/6	0.6923
5/6	0.5901
$\frac{6}{6}$=1	0.5000

Simpson's $\frac{1}{3}$ rule is

a = 0, b = 6, n= 6

h = $\frac{b-a}{n} = \frac{6-0}{6}$ = 1

y_0 = f(0) -= $\frac{1}{1+0^2} = \frac{1}{1}$ = 1

y_1 = f(1) = $\frac{1}{1+1^2} = \frac{1}{2}$ = 0.5000

y_2 = f(2) =$\frac{1}{1+2^2} = \frac{1}{5}$ = 0.2000

y_3 = f(3)= $\frac{1}{1+3^2}$ =$\frac{1}{10}$ = 0.1000

y_4= f(4) = $\frac{1}{1+4^2}$ =$\frac{1}{17}$ = 0.0588

y_5 = f(5)= $\frac{1}{1+5^2}$ = $\frac{1}{26}$ = 0.0385

y_6 = f(6) = $\frac{1}{1+6^2}$ = $\frac{1}{37}$ = 0.0270

Simpson's $\frac{1}{3}$ rule is

$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_5+.....)+2(y_2+ y_4+.....)]$

$= \frac{1}{3}[(1 + 0.027)+ 4(0.5+ 0.1 + 0.0385)+ 2(0.2+ 0.0588)]$

$=\frac{1}{3}$ [3.7104]= 1.2368

17. Find the value of $\int_0^1 \frac{x^2}{1+x^2}$ dx by using Simpson's $\frac{1}{3}$ rule and also find the error

Solution. : Dividing the whole range (o, 1) into 4 equal parts nd making use of Simpson's $\frac{1}{3}$ rule we have

$\int_0^1 \frac{x^2}{1+x^2}$ dx = $\frac{h}{3}[(y_0+y_4)+4(y_1+ y_3+)+2y_2)]$

h = $\frac{1}{4}$, a =0 , b= 1 ,n = $\frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4}$

X	F(x)
0	0
0.25	0.0615
0.5	0.2222
0.75	0.3956
1.0	0.5000

$\int_0^1 \frac{x^2}{1+x^2}$ dx = $\frac{1}{12}$ [(0+ 0.5)+ 4C0.0615+ 0.3956)+ 2(0.222)]

= $\frac{1}{12}$ [0.5+ 1.8284+ 0.444]

=$\frac{2.7728}{12}$ = 0.2310

18. Find $\int_4^{5.2} \log x_e$ dx by using Simpson's $\frac{1}{3}$ rule.

solution.: Given integral is $\int_4^{5.2} \log x_e$ dx

A = 4, b = 52 let n= 6 = $\frac{b-a}{n}$ = $\frac{5.2-2}{6}$ = 0.2

X	F(x)
4	1.3862
4.2	1.4350
4.4	1.4816
4.6	1.5260
4.8	1.5686
5.0	1.6094
5.2	1.6486

$\int_4^{5.2} \log x_e$ = $\frac{h}{3}$[(y_0+y_6)+4(y_1+ y_3+ y_5)+2(y_2+ y_4)]]

= $\frac{0.2}{3}$[(1.3862+1.6481+4(1.435071 +1.5260+1.6094)

+2(1.4816+1.5686)]

= $\frac{0.2}{3}$ [(3.0348)+ 4 (4.5704)+ 2(3.05027)]

= $\frac{0.2}{3}$[3.0348+18.2816+ 6.1004] = $\frac{5.48336}{3}$ = 1.8277866.

19. Evaluate $\int_{-3}^{3} x^4$dx by using Simpson's $\frac{1}{3}$ rule.

Solution. We know the integral is $\int_{-3}^{3} x^4$dx

Where h = $\frac{b-a}{h}$ = $\frac{3-(-3)}{6}$ = $\frac{6}{6}$ = 1

x	F(x)
-3	81
-2	16
-1	1
0	0

1	1
2	16
3	81

Simpson's $\frac{1}{3}$ rule is

$$\int_b^a f(x)dx = \frac{h}{3}[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)]$$

$$= \frac{1}{3}[(81+81)+4(16+0+16)+2(1+1)]$$

a = 3.b = 3 , n = 6

$$h = \frac{b-a}{h} = \frac{3-(-3)}{6} = 1$$

let $f(x) = x^2$

$y_0 = f(-3) = (-3)^2 = 9$

$y_1 = f(-2) = (-2)^2 = 4$

$y_2 = f(-1) = (-1)^2 = 1$

$y_3 = f(0) = (0)^2 = 0$

$y_4 = f(1) = (1)^2 = 1$

$y_5 = f(2) = (2)^2 = 4$

$y_6 = f(3) = (3)^2 = 9$

Simpson's $\frac{1}{3}$ rule

$$\int_a^b f(x) = \frac{h}{3}[(y_0+y_n)+4(y_1+y_{3+\ldots})+2(y_2+y_{4+\ldots})]$$

$$= \frac{1}{3}[(9+9)+4(4+0+4)+2(1+1)]$$

$$= \frac{54}{3} = 18 = \frac{1}{3}[162+128+4] = \frac{1}{3}[294] = 98$$

20 .calculate an approximate value of $\int_a^{\pi/2} sinx\ dx$ by (a) Trapezoidal rule (b) Simpson's rule using 11 ordinates

Solution : Divide the range of integration into a ten equal parts taking h = $\pi/20$ and then compute the values of the function f(x) = sin x for each point of sub-division. The computed values are as follows :

X	Y=sin x
0	0.00000
$\pi/20$	0.15663
$2\pi/20$	0.30902
$3\,\pi/20$	0.45399
$4\pi/20$	0.58778
$5\pi/20$	0.707111
$6\,\pi/20$	0.80900
$7\,\pi/20$	0.89101
$8\,\pi/20$	0.95106
$9\pi/20$	0.98769
$10\,\pi/20$	1.00000

(a) By the trapezoidal rule, we have

$$\int_0^{\pi/2} sinx\ \mathrm{dx} = \frac{h}{3}[(y_0+y_{10})+2(y_1+y_{2+\ldots})]$$
$$= \frac{\pi/20}{2}[1.0000+2(5.85311)]$$
$$= \frac{\pi}{40}(12.70622) = 0.9981$$

(b) By Simpson's $\frac{1}{3}$ rule , we have

$$\int_0^{\pi/2} sinx\ \mathrm{dx} = \frac{h}{3}[(y_0+y_n)+4(y_1+y_{3+\ldots})+2(y_2+y_{4+\ldots})]$$
$$= \frac{\pi}{60}[1.00000+4(3.19623)+2(2.65688)]$$
$$= \frac{\pi}{60}[1.00000+12.78492+5.31376]$$
$$= \frac{\pi}{60}[19.09868] = 1.0006$$

Again the exact value of the integral

$$\int_0^{\pi/2} sinx\ \mathrm{dx} = [-\cos x]_0^{\pi/2} = 1.0000$$

The error (i) Due to Simpson's $\frac{1}{3}$ rule , we have = -0.0006

(ii) Due to trapezoidal rule = 0.0019

21.Evaluate $\int_a^b f(x)dx$,by Simpson's $\frac{3}{8}$ rule from the following

X	4	4.2	4.4	4.6	4.8	5.0	5.2
Y	1.4	1.4	1.5	1.5	1.6	1.6	1.6

Where

Solution : Simpson's $\frac{3}{8}$ rule is

$\int_a^b f(x)dx$ = $\frac{3h}{8}$[y_0+3y_1+2y_2+3y_2+2y_3+3y_4+3y_5+y_6]

= $\frac{3(0.2)}{8}$[1.4+3(1.4)+3(1.5)+2(1.5)+3(1.6)+3(1.6)+1.6]

= $\frac{0.6}{8}$[1.4+4.2+4.5+3.0+4.8+4.8+1.6]

= $\frac{0.6}{8}$(24.3) = 1.8225

22. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's $\frac{3}{8}$ rule.

Solution : By the above problem we have y_0 =1 ,y_1 = 0.5000,y_2 = 0.1000,

y_4 = 0.0588 , y_5 = 0.0385, y_6 = 0.0270 and h = 1

Simpson's $3/8\ rule\ is$

$\int_a^b f(x)dx$ = $\frac{3h}{8}$[(y_0+ y_6)+3y_1+2y_3+3y_4+3y_5]

$\int_0^6 \frac{dx}{1+x^2}$ = $\frac{3}{8}$[(1+0.027)+3(0.5+0.2+0.0588+0.0385+2(0.1))]

= $\frac{3}{8}$[1.027+2.3919+0.2] = 13.09437/8 = 1.6770

23.Evaluate $\int_{-3}^{3} x^2 dx$, using by Simpson's $\frac{3}{8}$ rule we know by the above problem

Solution. y_0=9 ,y_1= 4,y_2= 1 , y_3 = 0 ,y_4 = 1 , y_5 = 4 ,y_6 = 9

Simpson's $3/8\ rule\ is$

$\int_a^b f(x)dx$ = $\frac{3h}{8}$[(y_0+ y_6)+3y_1+2y_3+3y_4+3y_5]

= $\frac{3h}{8}$[(9+9)+3(4+1+4+1)+0]

= $\frac{3}{8}$[48] = 18

24. Evaluate $\int_{-3}^{3} x^4 dx$,by using Simpson's $\frac{3}{8}$ rule

Solution. we know by the above problem.

Let f(x) = x^4

X	F(X)
-3	81
-2	16
-1	1
0	0
1	1
2	16
3	81

Where h=1

Simpson's 3/8 rule is

$\int_a^b f(x)dx = \frac{3h}{8}[(y_0+ y_6)+3y_1+2y_3+3y_4+3y_5]$

$= \frac{3}{8}[81+81+3(16+1+1+16)]$

$= \frac{3}{8}[162+102] = \frac{3}{8}(264) = 99$

25. A curve is drawn to passes through the part given by the following .

X	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
Y	**2**	**2.4**	**2.7**	**2.8**	**3**	**2.6**	**2.1**

Estimate the area bounded by the curve the x- axis and the lines x =1, x=4 by using Simpson's $\frac{3}{8}$rule.

Solution. h = $\frac{4-1}{6}$ = 0.5

Simpson's 3/8 rule is

$\int_a^b f(x)dx = \frac{3h}{8}[(y_0+ y_6)+3y_1+2y_3+3y_4+3y_5]$

$= \frac{3(0.5)}{8}[2+2.1+3(2.4+2.7+3+2.6)+2(2.8)]$

$= \frac{6.35}{8}$ = 8.04375 units of area

26. Evaluate $\int_a^b f(x)$ dx by using weddles formula from the following.

x	4	4.2	4.4	4.6	4.8	5.0	5.2
F(x)	1.4	1.4	1.5	1.5	1.6	1.6	1.6

Where h=0.4

Weddle's formula is

$\int_a^b f(x)dx$ = $\frac{3h}{10}$[y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6]

= $\frac{3(0.2)}{10}$[(1.4+1.6)+5(1.4+1.6)+(1.5+1.6)+6(1.5)]

= $\frac{0.6}{10}$[3+15+3.1+9]

= $\frac{0.6}{10}(30.1)$

= 1.806

27. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using weddle's formula.

Solution. a =0 , b= 1 ,h = $\frac{1-0}{6}$ = $\frac{1}{6}$ = 0.1666

X	f(x)
0	1
1/6	0.9729
2/6	0.9000
3/6	0.8000
4/6	0.6923
5/6	0.5901
$\frac{6}{6}$=1	0.5000

Weddle's formula is

$\int_a^b f(x)dx$ = $\frac{3h}{10}$[y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6]

= $\frac{3}{6x10}$[(1+0.5)+5(0.972+0.590)+(0.9+0.6923)+6(0.8)]

= $\frac{1}{20}$[1.5+7.815+1.5923+4.8]

$= \frac{1}{20}[15.703]$

$= 0.785365$

2.9 Applications of Numerical Methods

Considering the following model equation:

$$(\mu u_y)_y + y^n u_y = C, \qquad (1)$$

defined in a domain $\Omega = [0\ 1]$.

Where $\mu = (1 - exp(-y/\varepsilon) + \delta)/Re$, $\varepsilon << 1$, $\delta << 1$, $Re >> 1$, $n > 0$.

The first term simulates dissipative terms in the Navier-Stokes equations, whereas the second term models the contribution of convective terms and the right hand side represents the pressure gradient term or source in the transport equations. The "viscosity" coefficient μ corresponds to the effective viscosity coefficient. The coefficient is rapidly changed from a relatively small value $\mu_l = \delta /Re$ (laminar viscosity) to a "turbulent" viscosity $\mu_t \approx 1/Re$.

The equation simulates the low-Reynolds-number effects and can be considered as a model equation for the LR model. The left hand side point in the domain Ω will be treated as a "wall". The low-Reynolds-number effects occur nearby this point. If we set $\mu = (1+ \delta)/Re$ in (1), we have the HR approach.

Furthermore, we will consider the following values for the constants: $Re = 10^2$, $\varepsilon = 3*10^{-2}$, $\delta = 10^{-2}$. For simplicity the right-hand side will be considered as the constant $C = -1$

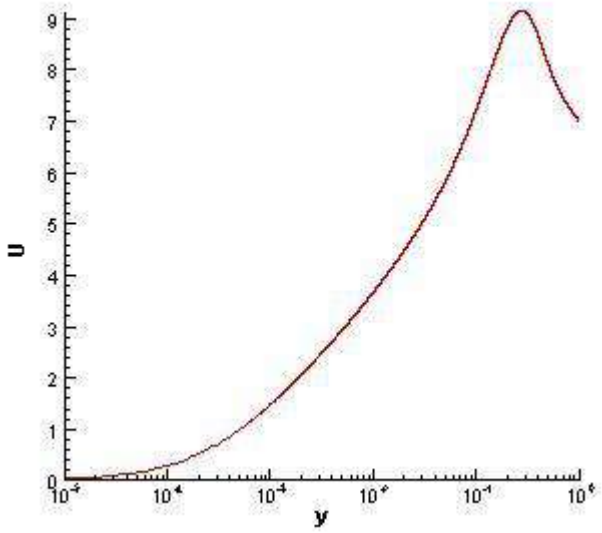
although this assumption is not important for our consideration.

Consider the following boundary-value Dirichlet problem:

$$(\mu u_{y})_y + y^n u_y = C \qquad (2)$$

$$u(0) = u_0 \quad u(1) = u_1$$

In the case of $n = 2$, $u_0 = 0$ *and* $u_1 = 7$, the solution is shown in Figure 1. The profile includes both the linear near-wall and logarithmic parts. Near the wall, $u = \varepsilon \ln(1+y/(\varepsilon\delta))$. The thickness of the viscous "sub-layer", where u is a linear function, can be approximated as $y_v \approx \varepsilon\delta$.



In the calculations, the compact scheme [4] is used. The scheme allows us to calculate both the function and its derivative with a fourth-order of approximation. In Table 1, the results of calculations of the "friction" $\tau_w = \mu_w du/dy(0)$ (here $\mu_w = \mu(0)$) are given on different meshes. To exclude questions concerned with a mesh adaptation, a uniform mesh is used

Grid	10^2	$5*10^2$	10^3	$5*10^3$	10^4

τ_w	$3.1*10^{-1}$	$3.2*10^{-1}$	$3.3*10^{-1}$	$3.36*10^{-1}$	$3.36*10^{-1}$

Table 1. Calculation of τ_w on different meshes.

If we use the HR model with the same boundary conditions, then $\tau_w = 4.9*10^{-1}$, which shows the importance of the sub-layer.

3. Linear System of Equations

Solutions of simultaneous linear equations:

Rate, Distance and Time:

You can calculate the best routes for your running or cycling schedule by creating a mathematical expression that takes into account the distance and your average speed for various parts of the route. You can use the

equations to set different goals, such as to maximize time for build endurance, or to maximize speed for performance.

Planes, Trains and Automobiles:

The same formula used to calculate running times can be used to determine speed, distances and time duration when traveling by car, plane or train and you want to know the values for the unknown variables in your travel situations.

The Best Deal:

You want to find out the better deal when renting a car, and you're comparing two rental companies. By putting the variable and fixed costs, such as the per - mile and daily rate, into an algebraic expression, then solving for the total cost, you ca see which company saves you money for different amounts of driving.

The Best Plan:

You can use this same process with a system of equations when trying to decide on the best call phone plan, determining at how many minutes both companies charge the same amount and deciding from there which is the best plan for you and your intended usage.

Deciding on a Loan:

Simultaneous equations can be used to determine the best loan

choice to make when buying a car or a house when you consider the duration of the loan, the interest rate and the monthly payment of the loan. Other variables may be involved as well, with the information at hand, you can calculate which loan is the best choice for you.

Cost and Demand:

Simultaneous equations can be used when considering the relationship between the price of a commodity and the quantities of the commodity people want to buy at a certain price. An equation can be written that describes the relationship between quantity, price and other variables, such

as income. These relationship equations can be solved simultaneously to determine the best way to price the commodity and sell it.

In the Air:

An air traffic controller can use Simultaneous equations to ensure two airplanes don't intersect at the same time.

The Best job for the Money:

System of equations can be used when trying to determine if you'll make more money at one job or another, taking multiple variables into account, such as salary, benefits and commissions.

Investing Wisely:

You can use Simultaneous equations to decide on your best investment option, taking into account the duration of the investment, the interest it will accrue, as well as other variables that will affect the end result. If you know the amount you'd like to accrue, you can set the options equal to each other and figure out which option is best for your future.

Mixing it up:

With respect to mixtures Simultaneous equations can be used for achieving a certain consistency in a resultant product, which is dependent on the consistency of the compounds mixed together to produce it.

Rates :

Linear equations can be useful tool for comparing rates of pay. For example, if one company offers to pay you $450 per week and other offers $10 per hour, and both ask you to work 40 hours per week, which company is offering the better rate of pay? A linear equation can help you figure it out! The first company's offer expressed as 450=40x. the second company's offer is expressed as y=10(40). After computing the two offers, the equations tell you that the first company is offering the better rate of pay at $11.25 per hour.

Variable costs:

Imagine that you taking a taxi while on vacation. You know that the taxi service charges $9 to pick your family up form your hotel and another $0.15 per mile for the trip. Without knowing how many miles it will be to each destination, you can set up a linear equation that can be used to find the cost of any taxi trip you take on your trip. By using "x" to represent the number of miles to your destination and "y" to represent the cost of that taxi ride, the linear equation would be: y= 0.15x+9.

3.1 MATRIX INVERSION METHOD:

* Solve x + 2y = 10, 2x - y = 15 by matrix inverse method?

Solution:-

Let $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 10 \\ 15 \end{bmatrix}$

Given equations are equivalent to AX = B

=> $X = A^{-1} B$.

$$A^{-1} = \frac{1}{-1-4}\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix} \frac{1}{-1-4} = -\frac{-1}{5}\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix}$$

$$\therefore X = A^{-1} B = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} \frac{1}{5} \begin{bmatrix} 10 \\ 15 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 10 + 30 \\ 20 - 15 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 40 \\ 5 \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

$\therefore$ x = 18, y = 1 is the solution.

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* Solve the equations 3x + 2y + 4z = 7; 2x + y + z = 7; x + 3y + 5z = 2. by matrix inversion method ?

Solution:-

Let $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 7 \\ 7 \\ 2 \end{bmatrix}$

Given equations are equivalent to AX = B

det A = 3(5 - 3) - 2(10 - 1) +4(6 - 1) = 6 - 18 + 20 = 8 ≠ 0.

$\therefore$ $X = A^{-1} B$ is the solution.

Now adj $A = \begin{bmatrix} 2 & 2 & -2 \\ -9 & 11 & 5 \\ 5 & -7 & -1 \end{bmatrix}$

$\Rightarrow A^{-1} = \frac{AdjA}{\det A} = \frac{1}{8} \begin{bmatrix} 2 & 2 & -2 \\ -9 & 11 & 5 \\ 5 & -7 & -1 \end{bmatrix}$

$\therefore X = A^{-1} B = \frac{1}{8} \begin{bmatrix} 2 & 2 & -2 \\ -9 & 11 & 5 \\ 5 & -7 & -1 \end{bmatrix} \begin{bmatrix} 7 \\ 7 \\ 2 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 24 \\ 24 \\ -16 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ -2 \end{bmatrix}$

$\therefore$ x = 3, y = 3, z = -2 is the solution.

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3.2 GAUSS ELIMINATION METHOD:

* Solve the equations 2x - y + 4z = 12; 8x - 3y =2z = 23; 4x + 11y - z = 33 by Gauss Elimination Method ?

Solution:-

Given equations are 2x - y + 4z = 12, 8x - 3y + 2z = 23, 4x + 11y - z = 33.

Dividing first equation by 2, we get x - $\frac{1}{2}$y+ 2z = 6 ----------(1)

Now subtract (1) after multiplying by 8 and 4 respectively from second and third equations we get y - 14z = -25----->(2), 13y - 9z = 9 ------(3)

Subtract (2) after multiplied by 13 from (3) we get 173z = 334

=> z = $\frac{334}{173}$ = 1.931

From (3); 13y - 9$\left(\frac{334}{173}\right)$ = 9 => 13y = 9 + $\frac{3006}{173}$ => y = $\frac{4563}{2249}$ = 2.029

From (1); x = 6+$\frac{1}{2}\left(\frac{4563}{2249}\right)$ - 2$\left(\frac{334}{173}\right)$ = 6+$\frac{1}{2}$(2.029) - 2(1.931)

= 6 + 1.0145 - 3.862 = 3.153.

$\therefore$ The solution is x = 3.153, y = 2.029, z = 1.931.

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* Solve the system of equations 2x + y + z = 10, 3x + 2y + 3z = 18, x + 4y + 9z = 16 by Gauss Elimination Method.

Solution:-

We have 2x + y + z = 10------(1), 3x + 2y + 3z = 18------(2), x + 4y + 9z = 16-----(3)

Divide (1) by (2) and subtract multiplied by 3 from (2) then subtract from (3), we get

$x + \frac{1}{2}y + \frac{1}{2}z = 5$ -----(4), $\frac{1}{2}y + \frac{3}{2}z = 3$ -----(5), $\frac{7}{2}y + \frac{17}{2}z = 11$ ----(6)

Now divide (5) by $\frac{1}{2}$ and then subtract after multiplied by $\frac{7}{2}$ from (6) we get

$x + \frac{1}{2}y + \frac{1}{2}z = 5$ ------(7), y + 3z = 6 ----(8), -2z = -10 -----(9)

From back substitution form (9), (8) & (7) we get z = 5 and y + 3z = 6 => y + 3(5) = 6

=> y = 6 - 15 => y = -9 and

$x + \frac{1}{2}(-9) + \frac{1}{2}(5) = 5 \Rightarrow x = 5 + \frac{9}{2} - \frac{5}{2} = 7$

$\therefore$ Hence the solution is x = 7, y = -9, z = 5.

3.3GAUSS JORDAN METHOD:

* Solve the system of equations 2x + y + z = 1, x -2y - 3z = 1, 3x + 2y + 4z = 5 by Gauss Jordan Method.

Solution:-

The augmented matrix is $\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & -2 & -3 & 1 \\ 3 & 2 & 4 & 5 \end{bmatrix}$

$2R_2 - R_1, 2R_3 - 3R_1 \sim \begin{bmatrix} 2 & 1 & 1 & 1 \\ 0 & -5 & -7 & 1 \\ 0 & 1 & 5 & 7 \end{bmatrix}$

$5R_1 + R_2, 5R_3 + R_2 \sim \begin{bmatrix} 10 & 0 & -2 & 6 \\ 0 & -5 & -7 & 1 \\ 0 & 0 & 18 & 36 \end{bmatrix}$

$9R_1 + R_3, 18R_2 + 7R_3 \sim \begin{bmatrix} 90 & 0 & 0 & 90 \\ 0 & -90 & 0 & 270 \\ 0 & 0 & 18 & 36 \end{bmatrix}$

$\frac{1}{90}R_1, \frac{1}{90}R_2, \frac{1}{18}R_3 \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

$\therefore$ **By** Gauss Jordan method, x = 1, y = -3, z = 2.

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* Solve the equations by Gauss Jordan Method . 5x + 2y + z = 12, x + 4y + 2z = 15, x + 2y + 5z = 20.

Solution:-

The augmented matrix is $\begin{bmatrix} 5 & 2 & 1 & 12 \\ 1 & 4 & 2 & 11 \\ 1 & 2 & 5 & 20 \end{bmatrix}$

$5R_2 - R_1, 5R_3 - R_1 \sim \begin{bmatrix} 5 & 2 & 1 & 12 \\ 0 & 18 & 9 & 63 \\ 0 & 8 & 24 & 88 \end{bmatrix}$

$9R_1 - R_2, 9R_3 - 4R_1 \sim \begin{bmatrix} 45 & 0 & 0 & 45 \\ 0 & 18 & 9 & 63 \\ 0 & 0 & 180 & 540 \end{bmatrix}$

$180R_3 \sim \begin{bmatrix} 45 & 0 & 0 & 45 \\ 0 & 18 & 9 & 63 \\ 0 & 0 & 1 & 3 \end{bmatrix}$ $R_2 - 9R_3 \sim \begin{bmatrix} 45 & 0 & 0 & 45 \\ 0 & 18 & 0 & 36 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

$\frac{1}{45}R_1, \frac{1}{18}R_2 \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

$\therefore$ **By** Gauss Jordan method, x = 1, y = 2, z = 3.

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3.4 LU - DECOMPOSITION METHOD:

* Solve the system of equations $2x_1 + x_2 + x_3 = 2$, $x_1 + 3x_2 + 2x_3 = 2$, $3x_1 + x_2 + 2x_3 = 2$ by LU Decomposition Method?

Solution:-

Given equations are $2x_1 + x_2 + x_3 = 2$----------(1)

$$x_1 + 3x_2 + 2x_3 = 2\text{---------(2)}$$

$$3x_1 + x_2 + 2x_3 = 2\text{---------(3)}$$

Here the coefficient matrix A is given by $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 3 & 1 & 2 \end{bmatrix}$ and

$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$

$\therefore AX = B$

Now $A = LU \Rightarrow \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$

$$= \begin{bmatrix} u_{11} & u_{12} & u_{12} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix}$$

On comparing two matrices, we get $U_{11} = 2$, $U_{12} = 1$, $U_{13} = 1$;

$L_{21}U_{11} = 1 \Rightarrow L_{21} = \frac{1}{U_{11}} = \frac{1}{2}$; $L_{31}U_{11} = 3 \Rightarrow L_{31} = \frac{3}{U_{11}} = \frac{3}{2}$;

$L_{21}U_{12}+U_{22} = 3 \Rightarrow 3 - L_{21}U_{12} = 3 - \frac{1}{2}(1) = \frac{5}{2}$

$L_{21}U_{13}+U_{2} = 3 \Rightarrow 2 - L_{21}U_{12} = 2 - \frac{1}{2}(1) = \frac{3}{2}$

$L_{31}U_{12}+L_{32}U_{22} = 1 \Rightarrow L_{32} = \frac{1-L_{31}U_{12}}{U_{22}} = \frac{1-\frac{3}{2}}{\frac{5}{2}}(1) = \frac{-1}{5}$

$L_{31}U_{13}+L_{32}U_{23}+U_{33} = 2$

$\Rightarrow U_{33} = 2 - L_{31}U_{13} - L_{32}U_{23} = 2 - \frac{3}{2} + \frac{1}{5}\left(\frac{3}{2}\right) = 2 - \frac{3}{2} + \frac{3}{10} = \frac{8}{10} = \frac{4}{5}$

Thus $L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{3}{2} & \frac{-1}{5} & 1 \end{bmatrix}$ and

$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{bmatrix} 2 & \frac{1}{5} & \frac{1}{3} \\ 0 & \frac{5}{2} & \frac{3}{2} \\ 0 & 0 & \frac{4}{5} \end{bmatrix}$

$LY = B \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{3}{2} & \frac{-1}{5} & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$

From this we get $y_1 = 1$, $\frac{1}{2}y_1 + y_2 = 2$, $\frac{3}{2}y_1 - \frac{1}{5}y_2 + y_3 = 2$.

On solving, we get $y_1 = 2$, $y_2 = 1$, $y_3 = \frac{-4}{5}$.

And since, we have UX =Y => $\begin{bmatrix} 2 & 1 & 1 \\ 0 & \frac{5}{2} & \frac{3}{2} \\ 0 & 0 & \frac{4}{5} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ \frac{-4}{5} \end{bmatrix}$

$2x_1 + x_2 + x_3 = 2$, $\frac{5}{2}x_2 + \frac{3}{2}x_3 = 1$, $\frac{4}{5}x_3 = \frac{-4}{5}$

By back substitution, we get $x_1 = 1$, $x_2 = 1$, $x_3 = -1$.

$\therefore$ Hence, solution is $x_1 = 1$, $x_2 = 1$, $x_3 = -1$

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* Solve the equations $2x + 3y + z = 9$, $x + 2y + 3z = 6$, $3x + y\ 2z = 8$ by LU Decomposition method.

<u>Solution:-</u>

Given equations can be written as AX= B where A = $\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$, X= $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, B = $\begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$

Let A = LU

$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{12} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{bmatrix}$$

Comparing, we get $U_{11}= 2$, $U_{12}= 3$, $U_{13}= 1$ and $L_{21}U_{11}= 1 \Rightarrow L_{21}= \frac{1}{2}$;

$L_{21}U_{12}+U_{22}= 2 \Rightarrow U_{22}= 2 - L_{21}U_{12} = 2 - \frac{3}{2} = \frac{1}{2}$;

$L_{21}U_{13}+U_{23}= 3 \Rightarrow U_{23}= 3 - L_{21}U_{13} = 3 - \frac{1}{2} = \frac{5}{2}$;

$L_{31}U_{11} = 3 \Rightarrow L_{31}= \frac{3}{U_{11}} = \frac{3}{2}$ and

$L_{31}U_{12} +L_{32}U_{22} = 1 \Rightarrow L_{32}= \frac{1-L_{31}U_{12}}{U_{22}} = \frac{1-\frac{9}{2}}{\frac{1}{2}} = -7$

$L_{31}U_{13} +L_{32}U_{23} = 2 \Rightarrow U_{33}= 1 - L_{31}U_{13} - L_{32}U_{23} = 2 - \frac{3}{2} - (-7)\left(\frac{5}{2}\right) = 2 - \frac{3}{2} + \frac{35}{2} = \frac{36}{2}$

$= 18.$

Thus L and U are given by $L = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -7 & 1 \end{bmatrix}$, $U = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1/2 & 5/2 \\ 0 & 0 & 18 \end{bmatrix}$

$$LY = B \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -7 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$$

From this we get $y_1 = 9$, $\frac{1}{2}y_1+y_2 =6$, $\frac{3}{2}y_1- 7y_2 + y_3 = 8$.

On solving, we get $y_1 = 9$, $y_2 = \frac{3}{2}$, $y_3 = 5$.

$$\text{Further since } UX = Y \Rightarrow \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1/2 & 5/2 \\ 0 & 0 & 18 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 3/2 \\ 8 \end{bmatrix}$$

$2x + 3y + z = 9$, $1/2y + 5/2z = 3/2$, $18z = 5$.

By back substitution, we get $z = 5/18$ and $1/2y + 5/2(5/18) = 3/2$

$\Rightarrow y = 3 - 25/18 = 29/18$

And $2x + 3(29/18) + (5/18) = 9 \Rightarrow 2x = 9 - 29/6 - 5/18 = 9 - 92/18$

$\Rightarrow 2x = 70/18 \Rightarrow x = 35/18$

Hence, the solution is $x = 35/18$, $y = 29/18$, $z = 5/18$.

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3.5 ILL CONDITIONED OR WELL CONDITIONED:

* Establish whether the following matrix is singular, ill conditioned or well conditioned: $\begin{bmatrix} 3 & -1 & 3 \\ 1 & 0 & 4 \\ 2 & 2 & 1 \end{bmatrix}$?

Solution:-

Let A = $\begin{bmatrix} 3 & -1 & 3 \\ 1 & 0 & 4 \\ 2 & 2 & 1 \end{bmatrix}$

det A = $3(0 - 8) + 1(1 - 8) + 3(2 - 0) = -24 - 7 + 6 = -25$

$\therefore$ A is not singular.

$\begin{vmatrix} 3.01 & -1 & 3 \\ 1 & 0 & 4 \\ 2 & 2 & 1 \end{vmatrix}$ = $(3.01)(0 - 8) + 1(1 - 8) + 3(2 - 0) = -24.08-7+6 = -25.08$.

$\therefore$ The matrix is well conditioned.

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* Show that A = $\begin{bmatrix} -73 & 78 & 24 \\ 92 & 66 & 25 \\ -80 & 37 & 10 \end{bmatrix}$ is ill- conditioned?

Solution :-

The ,matrix A = $\begin{bmatrix} -73 & 78 & 24 \\ 92 & 66 & 25 \\ -80 & 37 & 10 \end{bmatrix}$ is well conditioned, since $|A| = 1$ whereas

$\begin{vmatrix} -73 & 78 & 24 \\ 92.01 & 66 & 25 \\ -80 & 37 & 10 \end{vmatrix} = 2.08,$ $\begin{vmatrix} -73 & 78.01 & 24 \\ 92 & 66 & 25 \\ -80 & 37 & 10 \end{vmatrix} = -28.20$

and $\begin{vmatrix} -73 & 78 & 24 \\ 92 & 66 & 25 \\ -80 & 37 & 10.01 \end{vmatrix} = -118.94$

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3.6 GAUSS - JACOBI METHOD:

* Solve the following equations by Jacobi method 20x + y -2z = 17, 3x + 20y - z = -18, 2x - 3y + 20z = 25?

Solution:-

Given equations can be written as $x = \frac{1}{20}(17 - y + 2z)$---------(1)

$y = \frac{1}{20}(-18 - 3x + z)$--------(2)

$z = \frac{1}{20}(25- 2x + 3y)$--------(3)

Let us take first approximation $x^{(1)} = 0$, $y^{(1)} = 0$, $z^{(1)} = 0$.

Then the second approximation using (1), we get

$x^{(2)} = \frac{1}{20}(17 - y^{(1)} + 2z^{(1)}) = \frac{17}{20} = 0.85$

$y^{(2)} = \frac{1}{20}(-18 - 3x^{(1)} + z^{(1)}) = \frac{1}{20}(-18 - 0 + 0) = -\frac{18}{20} = -0.9.$

$z^{(2)} = \frac{1}{20}(25 - 2x^{(1)} + 3y^{(1)}) = \frac{1}{20}(25 - 0 + 0) = \frac{25}{20} = 1.25.$

Substituting these values in R.H.S of (1), we get third approximation

$x^{(3)} = \frac{1}{20}(17 - y^{(2)} + 2z^{(2)}) = \frac{1}{20}(17 + 0.9 + 2(1.25)) = \frac{20.4}{20} = 1.02.$

$y^{(3)} = \frac{1}{20}(-18 - 3x^{(2)} + z^{(2)}) = \frac{1}{20}(-18 - 3(0.85) + 1.25) = \frac{-19.3}{20} = -0.965.$

$z^{(3)} = \frac{1}{20}(25 - 2x^{(2)} + 3y^{(2)}) = \frac{1}{20}(25 - 2(0.85) + 3(-0.9)) = \frac{20.6}{20} = 1.03.$

Now Substitute these values in (1), we get fourth approximation

$x^{(4)} = \frac{1}{20}(17 - y^{(3)} + 2z^{(3)}) = \frac{1}{20}(17 + 0.965 + 2(1.03)) = \frac{20.025}{20} = 1.0013.$

$y^{(4)} = \frac{1}{20}(-18 - 3x^{(3)} + z^{(3)}) = \frac{1}{20}(-18 - 3(1.02) + 1.03) = -\frac{20.03}{20} = -1.0015.$

$z^{(4)} = \frac{1}{20}(25 - 2x^{(3)} + 3y^{(3)}) = \frac{1}{20}(25 - 2(1.02) + 3(-0.965)) = \frac{20.065}{20} = 1.0032.$

Substitute these values in (1), we get fifth approximation

$x^{(5)} = \frac{1}{20}(17 - y^{(4)} + 2z^{(4)}) = \frac{1}{20}(17 + 1.0015 + 2(1.0032)) = \frac{20.0079}{20} =$

1.0004.

$y^{(5)} = \frac{1}{20}(-18 - 3x^{(4)} + z^{(4)}) = \frac{1}{20}(-18 - 3(1.0013) + 1.0032) =$

$- \quad = \quad -\frac{20.0007}{20} = -1.00003.$

$z^{(5)} = \frac{1}{20}(25 - 2x^{(4)} + 3y^{(4)}) = \frac{1}{20}(25 - 2(1.0013) + 3(1.0015)) = \frac{19.9929}{20} =$

1.9996.

Substitute these values in (1), we get sixth approximation

$x^{(6)} = \frac{1}{20}(17 - y^{(5)} + 2z^{(5)}) = \frac{1}{20}(17 + 1.00003 + 2(0.9996)) = \frac{19.99923}{20} =$ 0.99996.

$y^{(6)} = \frac{1}{20}(-18 - 3x^{(5)} + z^{(3)}) = \frac{1}{20}(-18 - 3(1.0004) + 0.9996) = \frac{-20.0016}{20} =$ -1.00008.

$z^{(6)} = \frac{1}{20}(25 - 2x^{(5)} + 3y^{(5)}) = \frac{1}{20}(25 - 2(1.0004) + 3(-1.00003)) = \frac{19.99911}{20} =$ 0.99995.

The values in the fifth and sixth approximation are nearly same.

$\therefore$ Hence the solution is x = 1, y = -1, z = 1.

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* Solve the equations by Jacobi Method 83x + 11y - 4z = 95; 7x + 52y + 13z = 104; 3x +8y + 29z = 71?

Solution:- Given equation can be written as

$x = \frac{1}{83}(95 - 11y + 4z)$, $y = \frac{1}{52}(104 - 7x - 13z)$, $z = \frac{1}{83}(71 - 3x - 8y)$------(1).

Let us take the first approximation to be $x^{(1)} = 0$, $y^{(1)} = 0$, $z^{(1)} = 0$.

Then the second approximation using (1), we get

$x^{(2)} = \frac{1}{83}(95 - 11y^{(1)} + 4z^{(1)}) = \frac{1}{83}(95 - 0 + 0) = 1.14$

$y^{(2)} = \frac{1}{52}(104 - 7x^{(1)} - 13z^{(1)}) = \frac{1}{52}(104 - 0 - 0) = \frac{104}{52} = 2.$

$z^{(2)} = \frac{1}{29}(71 - 3x^{(1)} - 8y^{(1)}) = \frac{1}{29}(71 - 0 - 0) = \frac{71}{23} = 2.4.$

Substituting these values in R.H.S of (1), we get third approximation

$x^{(3)} = \frac{1}{83}(95 - 11y^{(2)} + 4z^{(2)}) = \frac{1}{83}(95 - 11(2) + 4(2.4)) = \frac{82.6}{83} = 0.995 \sim 1.$

$y^{(3)} = \frac{1}{52}(104 - 7x^{(2)} - 13z^{(2)}) = \frac{1}{52}(104 - 7(1.14) - 13(2.4)) = \frac{64.82}{52} = 1.25.$

$z^{(3)} = \frac{1}{29}(71- 3x^{(2)} - 8y^{(2)}) = \frac{1}{29}(71 - 3(1.14) - 8(2)) = \frac{51.58}{29} = 1.78.$

Now Substitute these values in (1), we get fourth approximation

$x^{(4)} = \frac{1}{83}(95 - 11y^{(3)} + 4z^{(3)}) = \frac{1}{83}(95 -11(1.25)+ 4(1.78)) = \frac{88.37}{83} = 1.1.$

$y^{(4)} = \frac{1}{52}(104 - 7x^{(3)} - 13z^{(3)}) = \frac{1}{52}(104 - 7(1) - 13(1.78)) = \frac{73.86}{52} = 1.42.$

$z^{(4)} = \frac{1}{29}(71- 3x^{(3)} - 8y^{(3)}) = \frac{1}{29}(71 - 3(1) - 8(1.25)) = \frac{58}{29} = 2.$

Substitute these values in (1), we get fifth approximation

$x^{(5)} = \frac{1}{83}(95 - 11y^{(4)} + 4z^{(4)}) = \frac{1}{83}(95 -11(1.42)+ 4(2)) = \frac{-87.38}{83} = 1.1.$

$y^{(5)} = \frac{1}{52}(104 - 7x^{(4)} - 13z^{(4)}) = \frac{1}{52}(104 - 7(1.1) - 13(2)) = \frac{70.3}{52} = 1.4.$

$z^{(5)} = \frac{1}{29}(71- 3x^{(4)} - 8y^{(4)}) = \frac{1}{29}(71 - 3(1.1) - 8(1.42)) = \frac{51.7}{29} = 1.78.$

$\therefore$ Hence the solution is x = 1.1, y = 1.4, z = 1.78.

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3.7 GAUSS SEIDEL METHOD:

* Find the solution of the system by Gauss Seidel Method 83x + 11y - 4z = 95; 7x + 52y + 13z = 104; 3x +8y + 29z = 71?

Solution:-

Given equations can be written as $x = \frac{1}{83}(95 - 11y + 4z)$,

$y = \frac{1}{52}(104 - 7x - 13z)$,

$z = \frac{1}{83}(71 - 3x - 8y)$------(1).

Let us take the first approximation to be $x^{(1)} = 0$, $y^{(1)} = 0$, $z^{(1)} = 0$.

Then the second approximation using (1), we get

$$x^{(2)} = \frac{1}{83}(95 - 11y^{(1)} + 4z^{(1)}) = \frac{1}{83}(95 - 0 + 0) = 1.14$$

Put $x^{(2)} = 1.14$, $y^{(1)} = 1.85$, $z^{(1)} = 0$ in (3), we get

$$y^{(2)} = \frac{1}{52}(104 - 7x^{(2)} - 13z^{(1)}) = \frac{1}{52}(104 - 7(1.14) - 0) = \frac{96.02}{52} = 1.85.$$

Put $x^{(2)} = 1.14$, $y^{(2)} = 1.85$, $z^{(1)} = 0$ in (3), we get

$$z^{(2)} = \frac{1}{83}(71 - 3x^{(2)} - 8y^{(2)}) = \frac{1}{83}(71 - 3(1.14) - 8(1.85)) = \frac{52.78}{29} = 1.82.$$

Put $x^{(2)} = 1.14$, $y^{(2)} = 1.85$, $z^{(2)} = 1.82$ in (3), we get

$$x^{(3)} = \frac{1}{83}(95 - 11y^{(2)} + 4z^{(2)}) = \frac{1}{83}(95 - 11(1.85) + 4(1.82)) = \frac{81.93}{83} = 0.99.$$

Put $x^{(3)} = 1.14$, $y^{(2)} = 1.85$, $z^{(2)} = 1.82$ in (3), we get

$$y^{(3)} = \frac{1}{52}(104 - 7x^{(3)} - 13z^{(2)}) = \frac{1}{52}(104 - 7(0.99) - 13(1.82)) = \frac{73.41}{52} = 1.41$$

Put $x^{(3)} = 0.99$, $y^{(3)} = 1.41$, in (3) we get

$$z^{(3)} = \frac{1}{83}(71 - 3x^{(3)} - 8y^{(3)}) = \frac{1}{83}(71 - 3(0.99) - 8(1.41)) = \frac{56.75}{29} = 1.95.$$

Again put $x^{(3)} = 0.99$, $y^{(3)} = 1.41$, $z^{(3)} = 1.95$ in (3) we get

$$x^{(4)} = \frac{1}{83}(95 - 11y^{(3)} + 4z^{(3)}) = \frac{1}{83}(95 - 11(0.99) + 4(1.95)) = \frac{87.29}{83} = 1.05.$$

Put $x^{(4)} = 1.05$ $z^{(3)} = 1.95$ in (2) we get $y^{(4)} = \frac{1}{52}(104 - 7x^{(3)} - 13z^{(2)}) =$

$$\frac{1}{52}(104 - 7(1.05) - 13(1.95)) = \frac{71.3}{52} = 1.37.$$

Put $x^{(4)} = 1.05$, $y^{(4)} = 1.37$ in (3), we get

$$z^{(4)} = \frac{1}{83}(71 - 3x^{(4)} - 8y^{(4)}) = \frac{1}{83}(71 - 3(1.05) - 8(1.37)) = \frac{56.89}{29} = 1.96.$$

Thus $x^{(4)}, y^{(4)}, z^{(4)}$ are sufficiently close to $x^{(3)}, y^{(3)}, z^{(3)}$ respectively

$\therefore$ Hence the solution is $x = 1.05$, $y = 1.37$, $z = 1.96$.

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* Solve by Gauss Seidel method $10x_1 + x_2 + x_3 = 12$, $2x_1 + 10x_2 + x_3 = 13$, $2x_1 + 2x_2 + 10x_3 = 14$.

Solution:-

From the given equations we have

$$X_1 = \frac{1}{10}[12 - x_1 - x_2] \text{---------(1)}$$

$$X_2 = \frac{1}{10}[13 - 2x_1 - x_3] \text{---------(2)}$$

$$X_3 = \frac{1}{10}[14 - 2x_1 - 2x_2] \text{---------(3)}$$

Putting x = 0 in right side of the equation (1), we get $x_1^{(1)} = \frac{12}{10} = 1.2$

Putting $x_1 = x_1^{(1)} = 1.2$, $x_3 = 0$ in (2), we get $x_2^{(1)} = \frac{1}{10}(13 - 2.4 - 0) = 10.6/10 = 1.06$

Putting $x_1 = x_1^{(1)} = 1.2$, and $x_2 = x_2^{(1)} = 1.06$ in (3), we get

$x_3^{(2)} = \frac{1}{10}[14 - 2.4 - 2.12] = 0.948.$

For the second iteration we have $x_1^{(2)} = \frac{1}{10}[12 - 1.06 - 0.948] = 0.9992.$

$x_2^{(2)} = \frac{1}{10}[13 - 2(0.9992) - 0.948] = 1.00536.$

$x_3^{(2)} = \frac{1}{10}[14 - 2(0.9992) - 2(1.00536)] = 0.999098.$

Thus the iteration process is continued

The results are tabulated as follows correcting to four decimal places.

i	$x_1^{(i)}$	$x_2^{(i)}$	$x_3^{(i)}$
0	1.2000	0.0000	0.0000
1	1.2000	1.0600	0.9480
2	0.9992	1.0054	0.9991
3	0.9996	1.0000	1.0000
4	1.0000	1.0000	1.0000
5	1.0000	1.0000	1.0000

$\therefore$ The exact value of the roots are $x_1 = 1$, $x_2 = 1$, $x_3 = 1$.

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Network Analysis in linear system of Equations:

Set up a system of linear equations to represent the network shown in the figure. Then solve the system.

Solution:-

Each of the network's five junctions give rise to a linear equations as follows:

$X_1 + X_2 = 20$ Junction 1

$X_3 - X_4 = -20$ Junction

$X_2 + X_3 = 20$ Junction 3

$X_1 - X_5 = -10$ Junction 4

$-X_4 + X_5 = -10$ Junction 5

The augmented matrix for this system is

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 20 \\ 0 & 0 & 1 & -1 & 0 & -20 \\ 0 & 1 & 1 & 0 & 0 & 20 \\ 1 & 0 & 0 & 0 & -1 & -10 \\ 0 & 0 & 0 & -1 & 1 & -10 \end{bmatrix}$$

Gauss Jordan elimination produces the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & -10 \\ 0 & 1 & 0 & 0 & 1 & 30 \\ 0 & 0 & 1 & 0 & -1 & -10 \\ 0 & 0 & 0 & 1 & -1 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

From the matrix above, you can see that

$X_1 - X_5 = -10$, $X_2 - X_5 = 30$,

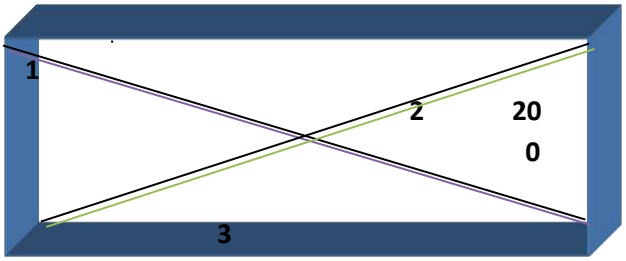
$X_3 - X_5 = -10$ and $X_4 - X_5 = -10$

Letting $t = X_5$, we have

$X_1 = t - 10$, $X_2 = -t + 30$, $X_3 = t + 30$, $X_4 = t + 10$, $X_5 = t$

Where t is the real number, so this system has infinitely many solutions.

1



2

4

5

Figure

In example 5, suppose you could control the mount of flow along the branch labeled X_5. using the solution of example 5 you could then control the flow represented by each of the other variables. For instance, letting t =10 would reduces the flow of X_1 and X_3 to zero, as shown In the figure.

4.LAPLACE TRANSFORM

4.1:Definition of Laplace Transform:It is piecewise continuous function,the Laplace Transform method of solving initial value problems

The method of Laplace transforms is a system that relies on algebra (rather than calculus-based methods) to solve linear differential equations. While it might seem to be a somewhat cumbersome method at times, it is a very powerful tool that enables us to readily deal with linear differential equations with discontinuous forcing functions.

***Definition*:** Let $f(t)$ be defined for $t \geq 0$. The Laplace transform of $f(t)$, denoted by $F(s)$ or L[f(t)] is an *integral transform* given by the *Laplace integral*:

$$L[f(t)]=F(s)=\int_0^\infty e^{-st} f(t)\, dt .$$

Provided that this (improper) integral exists, i.e. that the integral is convergent.

The Laplace transform is an operation that transforms a function of t (i.e., a function of *time domain*), defined on $[0, \infty)$, to a function of s (i.e., of *frequency domain*)*. $F(s)$ is the *Laplace transform*, or simply *transform*, of $f(t)$.

Together the two functions $f(t)$ and $F(s)$ are called a *Laplace transform pair*.

For functions of t continuous on $[0, \infty)$, the above transformation to the frequency domain is *one-to-one*. That is, different continuous functions will have different transforms.

History of Laplace Transforms

Transformation in mathematics deals with the conversion of one function to another function that may not be in the same domain. The transform method finds its application in those problems which can't be solved directly. This transform is named after the mathematician and renowned astronomer Pierre Simon Laplace who lived in France.

He used a similar transform on his additions to the probability theory. It became popular after World War Two. This transform was made popular by Oliver Heaviside, an English Electrical Engineer. Other famous scientists such as Niels Abel, Mathias Lerch, and Thomas Bromwich used it in the 19th century.The complete history of the Laplace Transforms can be tracked a little more to the past, more specifically 1744. This is when another great mathematician called Leonhard Euler was researching on other types of integrals. Euler however did not pursue it very far and left it. An admirer of Euler called Joseph Lagrange; made some modifications to Euler's work and did further work. LaGrange's work got Laplace's attention 38 years later, in 1782 where he continued to pick up where Euler left off. But it was not 3 years later; in 1785 where Laplace had a stroke of genius and changed the way we solve differential equations forever. He continued to work on it and continued to

unlock the true power of the Laplace transform until 1809, where he started to use infinity as a integral condition.

4.2 Properties of Laplace Transform

Some of the Laplace transformation properties are:

If $f_1(t) \leftrightarrow F_1(s)$ and [note: $\leftrightarrow$ implies Laplace Transform]

$f_2(t) \leftrightarrow F_2(s)$, then

Linearity Property	$A\ f_1(t) + B\ f_2(t) \leftrightarrow A\ F_1(s) + B\ F_2(s)$
Frequency Shifting Property	$\int_0^s e\ d\lambda \leftrightarrow F(s - s_0)$
Integration	$\int_0^t f(\lambda)\ d\lambda \leftrightarrow \frac{1}{s}F(s)$
Multiplication by Time	$T\ f(t) \leftrightarrow (-d\ \frac{F(s)}{ds})$
Complex Shift Property	$f(t)\ e^{-at} \leftrightarrow F(s + a)$
Time Reversal Property	$f(-t) \leftrightarrow F(-s)$
Time Scaling Property	$f(\frac{t}{a}) \leftrightarrow a\ F(as)$

4.3 Laplace Transform Table

The following Laplace transform table helps to solve the differential equations for different functions:

SI.No.	f(t)	L(f(t)) = F(p)
1	1	$\frac{1}{p}$
2	t^n at t = 1,2,3,…	$\frac{n!}{p(n+1)}$
3	√(t)	$\frac{\sqrt{\pi}}{2p^{\frac{3}{2}}}$
4	sin(at)	$\frac{a}{(p^2+a^2)^2}$
5	sinht(at)	$\frac{a}{(p^2-a^2)}$
6	$e^{(at)}$ sin(bt)	$\frac{b}{(p-a)^2+b^2}$

SI.No.	f(t)	L(f(t)) = F(p)
7	$e^{(ct)}f(t)$	F(p-c)
8	f'(t)	pF(p) – f(0)
9	$e^{(at)}$	$\frac{1}{(p-a)}$
10	t^nat p>-1	$\frac{\Gamma(p+1)}{p^{(p+1)}}$
11	$t^{(n-\frac{1}{2})}$ at n = 1,2,..	$\frac{(1.2.3..(2n-1)}{2^n\, p^{(n+\frac{1}{2})}}$
12	cos(at)	$\frac{p}{(p^2+a^2)}$
13	sin(at)	$\frac{a}{p^2+a^2}$
14	t cos(at)	$\frac{p^2-a^2}{(p^2+a^2)^2}$
15	sinht	$\frac{a}{p^2-a^2}$
16	cosh(at)	$\frac{p}{p^2-a^2}$

SI.No.	f(t)	L(f(t)) = F(p)
17	$e^{(at)}\cos(bt)$	$\frac{(p-a)}{(p-a)^2+b^2}$
18	t^nf(t) at n = 1,2,3..	$(-1)^n F^n$ s
19	f'(t)	$p^2F(p) - pf(0) - f'(0)$
20	$e^{at}t^n$	$\frac{n!}{(p-a)^{n+1}}$

Laplace transforms examples:

Example:1

Find $\mathcal{L}\{\sinh at\}$

Solution: $\mathcal{L}\{\sinh at\} = \mathcal{L}\left\{\frac{e^{at} - e^{-at}}{2}\right\}$

$= \frac{1}{2}\mathcal{L}\{e^{at}\} - \frac{1}{2}\mathcal{L}\{e^{-at}\}$

$= \frac{1}{2}\left\{\frac{1}{s-a} - \frac{1}{s+a}\right\} = \frac{a}{s^2-a^2}$

Example:2

Find $\mathcal{L}\{(t^3+1)^3\}$

Solution:Given,

$\mathcal{L}\{(t^3+1)^3\}$

$\Rightarrow \mathcal{L}\{t^6\} + L\{1\} + 2L\{t^3\}$

$$= \frac{6!}{p^{6+1}} + \frac{1}{p} + 2.\frac{3!}{p^{3+1}}$$

$$= \frac{6.5.4.3.2.1}{p^7} + \frac{1}{p} + 2.\frac{3.2}{p^4}$$

$$= \frac{720}{p^7} + \frac{1}{p} + \frac{12}{p^4}$$

$$= \frac{720}{p^7} + \frac{12}{p^4} + \frac{1}{p}$$

Example:3

Find $\mathbf{L\{\sin^2 4t\}}$

Solution: We know that,

$$\text{Cos2t} = 1 - 2\sin^2 t$$

$$\text{Cos6t} = 1 - 2\sin^2 3t$$

$$\text{Cos8t} = 1 - 2\sin^2 4t$$

$$\text{L}\left\{\frac{(1-\cos 8t)}{2}\right\}$$

$$\Rightarrow \text{L}\left\{\frac{1}{2}\right\} - \frac{1}{2}\text{L}\{cos8t\}$$

$$= \frac{1}{2}\{1\} - \frac{1}{2}\text{L}\{\text{cos8t}\}$$

$$= \frac{1}{2}.\frac{1}{p} - \frac{1}{2}\left[\frac{p}{p^2+8^2}\right]$$

$$\mathrm{L}\{\sin^2 4t\} = \frac{1}{2}\left[\frac{1}{p} - \frac{p}{p^2+64}\right]$$

Example:4

Find $\mathcal{L}\{\mathbf{sint + cost}\}^2$

Solution: Given $\mathcal{L}\{\mathrm{sint} + \mathrm{cost}\}^2$

$$\mathcal{L}\{\sin^2 t + \cos^2 t + 2sintcost\}$$

$$\Rightarrow \mathcal{L}\{1 + 2sintcost\}$$

$$\Rightarrow \mathcal{L}\{1 + sin2t\}$$

$$\Rightarrow \mathcal{L}\{1\} + \mathcal{L}\{\, sin2t\}$$

$$\mathcal{L}\{\mathrm{sint} + \mathrm{cost}\}^2 = \frac{1}{p} + \frac{2}{p^2+4}$$

Example:5

Find $\mathcal{L}\{\boldsymbol{sinhat.cosat}\}$

Solution: Given $\mathcal{L}\{sinht.cosat\}$

Sinht=($e^{ax}-e^{-ax}$ 、

$$\Rightarrow \mathrm{L}\{(\frac{e^{ax}-e^{-ax}}{2})cosat\}$$

$$=> \frac{1}{2}[\mathrm{L}\{e^{ax}cosat\} - \mathrm{L}\{e^{-ax}\}]$$

$$\mathrm{L}\{sinht.cosat\} = \frac{1}{2}\left\{\frac{p-a}{(p-a)^2+a^2} - \frac{p+a}{(p+a)^2+a^2}\right\}$$

4.4:Theorem:

FIRST SHIFTING THEOREM OF LAPLACE TRANSFORM:

STATEMENT: If $\mathcal{L}\{F(t)\}=f(p)$ then $\mathcal{L}\{e^{at}F(t)\} = f(p-a)$

Proof:- By the definition of Laplace Transform

$\mathcal{L}\{F(t)\}=f(p)$

$\Rightarrow \mathcal{L}\{F(t)\}=\int_0^\infty e^{-pt}F(t)dt$

$\Rightarrow \mathcal{L}\{e^{at}F(t)\} = \int_0^\infty e^{at}\, e^{-pt}F(t)dt$

$\Rightarrow \mathcal{L}\{e^{at}F(t)\} = \int_0^\infty e^{(a-p)t}F(t)dt$

Example:1

Find $\mathcal{L}\{e^{3t}\sin^2 t\}$

Solution: Given $\mathcal{L}\{e^{3t}\sin^2 t\}$

We know $\mathcal{L}\{\sin^2 t\} = \mathcal{L}\left\{\frac{1-cos2t}{2}\right\}$

$= \frac{1}{2}[\mathcal{L}\{1\} - \mathcal{L}\{\cos 2t\}]$

$$= \frac{1}{2}\left[\frac{1}{p} - \frac{p}{p^2+4}\right]$$

$$= \frac{2}{p(p^2+4)} = f(p), (say)$$

By First Shifting Theorem, $\mathcal{L}\{e^{3t}\sin^2 t\} = f(p-3)$

$L\{e^{-2t}\sin 2t.\sin 3t\} = \left[\frac{2}{p(p^2+4)}\right]_{p\to p-3}$

$$= \frac{2}{(p-3)[(p-3)^2+4]}$$

$\mathcal{L}\{e^{3t}\sin^2 t\} = \frac{2}{(p-3)(p^2-6p+13)}$

Example:**2**

Evaluate $\mathrm{L}\left\{e^t\left(\boldsymbol{cos2t}+\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{sinh2t}\right)\right\}$

Solution: Given $\mathrm{L}\left\{e^t\left(cos2t+\frac{1}{2}sinh2t\right)\right\}$

$\mathrm{L}\left\{cos2t+\frac{1}{2}sinh2t\right\}$

$\mathrm{L}\{cos2t\}+\mathrm{L}\{\frac{1}{2}sinh2t\}$

$$=\frac{p}{p^2+2^2}+\frac{1}{2}.\frac{2}{p^2-2^2}$$

$$=\frac{p}{p^2+4}+\frac{1}{p^2-4}=f(p),say$$

$By\ first\ shifting\ theroem,\ \mathrm{L}\ \left\{e^t\left(cos2t+\frac{1}{2}sinh2t\right)\right\}=f(p-1)$

$=\left(\frac{p}{p^2+4}+\frac{1}{p^2-4}\right)_{\mathrm{p}\to\mathrm{p\text{-}1}}$

$$=\frac{p-1}{(p-1)^2+4}+\frac{1}{(p-1)^2-4}$$

$\mathcal{L}\left\{\mathrm{e}^{\mathrm{t}}\left(\mathrm{cos2t}+\frac{1}{2}\mathrm{sinh2t}\right)\right\}=\frac{\mathrm{p-1}}{\mathrm{p^2-2p+5}}+\frac{1}{\mathrm{p^2-2p-3}}$

Example:**3**

Find $\mathbf{L\left\{e^{-2t}sin2t.\,sin3t\right\}}$.

Solution: Given $\mathrm{L}\{\mathrm{e}^{-2\mathrm{t}}\mathrm{sin2t.\,sin3t}\}$

[2sinA.sinB=cos(A-B)-cos(A+B)]

$$L\left[\frac{cost-cos5t}{2}\right]$$

$$= \frac{1}{2}[L(cost) - L(cos5t)]$$

$$= \frac{1}{2}\left[\frac{p}{p^2+1} - \frac{p}{p^2+25}\right]$$

$$= \frac{12p}{(p^2+1)(p^2+25)}$$

By first shifting theorem, $L\{e^{-2t}sin2t.sin3t\} = \left[\frac{12p}{(p^2+1)(p^2+25)}\right]_{p\to p+2}$

$$= \frac{12(p+2)}{\{(p+2)^2+1\}\{(p+2)^2+25\}}$$

$\rightarrow L\{e^{-2t}sin2t.sin3t\} = \frac{12p+24}{(p^2+4p+5)(p^2+4p+29)}$

4.5: Inverse Laplace Transform

In the inverse Laplace transform, we are provided with the transform F(s) and asked to find what function we have initially. The inverse transform of the function F(s) is given by:

$f(t) = L^{-1}\{F(s)\}$

For example, for the two Laplace transform, say F(s) and G(s), the inverse Laplace transform is defined by:

$L^{-1}\{aF(s)+bG(s)\}= a\ L^{-1}\{F(s)\}+bL^{-1}\ \{G(s)\}$

Where a and b are constants.

In this case, we can take the inverse transform for the individual transforms, and add their constant values in their respective places, and perform the operation to get the result.

Example: 1

Find $\mathbf{L^{-1}\left\{\frac{3(p^2-1)^2}{2p^5}\right\}}$

Solution: Given $L^{-1}\left\{\frac{3(p^2-1)^2}{2p^5}\right\}$

$$L^{-1}\left\{3\frac{(p^4+1-2p^2)}{2p^5}\right\}$$

$$=>\frac{3}{2}\left[L^{-1}\left\{\frac{p^4}{p^5}+\frac{1}{p^5}-2.\frac{p^2}{p^5}\right\}\right]$$

$$=>\frac{3}{2}\left[L^{-1}\left\{\frac{1}{p}\right\}+L^{-1}\left\{\frac{1}{p^5}\right\}-2L^{-1}\left\{\frac{1}{p^3}\right\}\right]$$

$$=\frac{3}{2}[1+\frac{t^4}{4!}-2.\frac{t^2}{2!}$$

$$=\frac{3}{2}\left[1+\frac{t^4}{24}-t^2\right]$$

$$=\frac{3}{2}+\frac{3}{2}.\frac{t^4}{24}-\frac{3}{2}t^2$$

$$L^{-1}\left\{\frac{3(p^2-1)^2}{2p^5}\right\}=\frac{3}{2}+\frac{t^4}{16}-\frac{3}{2}t^2$$

Example:2

Find $\boldsymbol{L^{-1}\left\{\frac{1}{(p+1)(p+2)}\right\}}$

Solution: Given $L^{-1}\left\{\frac{1}{(p+1)(p+2)}\right\}$

Take $\left\{\frac{1}{(p-1)(p+2)}\right\}=\frac{A}{p+1}+\frac{B}{p+2}\ldots\ldots\ldots(1)$

$$\left\{\frac{1}{(\mathrm{p}+1)(\mathrm{p}+2)}\right\} = \frac{\mathrm{A}(\mathrm{p}+2)+\mathrm{B}(\mathrm{p}+1)}{(\mathrm{p}+1)(\mathrm{p}+2)}$$

$$=> 1 = A(p+2)+B(p+1) \ldots\ldots\ldots (2)$$

Put p=-2 in (2),we get

1=0+B(-1)

B=-1

Put p=-1 in (2),we get

1=A+0

A=1

From (1)

$$\left\{\frac{1}{(p-1)(p+2)}\right\} = \frac{1}{p+1} - \frac{1}{p+2} \ldots\ldots\ldots (1)$$

Apply inverse Laplace on both sides we get

$$L^{-1}\left\{\frac{1}{(p-1)(p+2)}\right\} = L^{-1}\left\{\frac{1}{p+1}\right\} - L^{-1}\left\{\frac{1}{p+2}\right\}$$

$$= e^{-t} - e^{-2t}$$

Example:3

Find $\boldsymbol{L^{-1}\left\{\frac{1}{p-2}+\frac{2}{p+5}+\frac{6}{p^4}\right\}}$

Solution: Given $L^{-1}\left\{\frac{1}{p-2}+\frac{2}{p+5}+\frac{6}{p^4}\right\}$

$$=> L^{-1}\left\{\frac{1}{p-2}\right\} + L^{-1}\left\{\frac{2}{p+5}\right\} + L^{-1}\left\{\frac{3!}{p^{3+1}}\right\}$$

$$= e^{2t} + 2e^{5t} + t^n$$

4.6:THEOREM

SECOND SHIFTING THEOREM OF INVERSE LAPLCE TRANSFORM:

STATEMENT: If $L^{-1}\{f(p)\} = F(t), then\ L^{-1}\{e^{-ap}f(p)\} = G(t)$

$$G(t)= \begin{cases} F(t-a), t > a \\ 0 \qquad , t < a \end{cases}$$

Proof: $L\{G(t) = \int_0^\infty e^{-pt}G(t)dt$

$$= \int_0^a e^{-pt}G(t)dt + \int_a^\infty e^{-pt}G(t)dt$$

$$= \int_0^a e^{-pt}\,0\,dt + \int_a^\infty e^{-pt}F(t-a)dt$$

$$= 0 + \int_a^\infty e^{-pt}F(t-a)dt$$

$$= \int_a^\infty e^{-p(a+u)}F(u)du\,, t-a = u$$

$$= \int_a^\infty e^{-ap}\,e^{-pu}F(u)du$$

$$= e^{-ap}\int_a^\infty e^{-pu}F(u)du$$

$$= e^{-ap} L\{F(t)\}$$

$$= e^{-ap} f(p)$$

$$Thus, L\{G(t)\} = e^{-ap} f(p)$$

$$\rightarrow G(t) = L^{-1}\{e^{-ap} f(p)$$

Example:1

Find the inverse Lapalce transform of $\frac{e^{-2p}}{(p-2)^2}$

Solution: Given $\frac{e^{-2p}}{(p-2)^2}$

Take $L^{-1}\left[\frac{1}{(p-2)^2}\right]$

$$=> e^{2t} L^{-1}\left[\frac{1}{p^2}\right]$$

$$= e^{2t} t$$

$$By\ \ second\ shifting\ theorem, we\ have\ L^{-1}\left[\frac{e^{-2p}}{(p-2)^2}\right]$$

$$= \begin{cases} e^{2(t-2)}(t-2), t > 2 \\ 0 \qquad\qquad\quad , t < 2 \end{cases}$$

4.7: Laplace Transform of Differential Equation

The Laplace transform is a well established mathematical technique for solving a differential equation. Many mathematical problems are solved using transformations. The

idea is to transform the problem into another problem that is easier to solve. On the other side, the inverse transform is helpful to calculate the solution to the given problem.

For better understanding, let us solve a first-order differential equation with the help of Laplace transformation.

Example:1

Consider $y' - 2y = e^{3x}$ and $y(0) = -5$. Find the value of L(y).

First step of the equation can be solved with the help of the linearity equation:

Sol: $L[(y' - 2y)] = L(e^{3x})$

$L(y') - L(2y) = \frac{1}{(s-3)}$

(because $L(e^{ax}) = \frac{1}{(s-a)}$

$L(y') - 2s(y) = \frac{1}{(s-3)}$

$sL(y) - y(0) - 2L(y) = \frac{1}{(s-3)}$

(Using Linearity property of the Laplace transform)

$L(y)(s-2) + 5 = \frac{1}{(s-3)}$ (Use value of y(0) ie -5 (given))

$L(y)(s-2) = \frac{1}{(s-3)} - 5$

$L(y) = \frac{(-5s+16)}{(s-2)(s-3)}$.....(1)

Here $\frac{(-5s+16)}{(s-2)(s-3)}$ can be written as $\frac{-6}{(s-2)} + \frac{1}{(s-3)}$ using partial fraction method

(1) implies $L(y) = \frac{-6}{(s-2)} + \frac{1}{(s-3)}$

$L(y) = -6e^{2x} + e^{3x}$

Example: 2

$y'' - 4y = 0, \quad y(0) = 1, \; y'(0) = 2$

Solution: Take Laplace Transform on both sides,

$$L\{ y'' - 4y \} = L\{ 0 \}$$

$$\text{or} \quad L\{ y'' \} - 4 L\{ y \} = 0$$

$$s^2 L\{ y \} - s\, y(0) - y'(0) - 4 L\{ y \} = 0$$

$$\text{or} \quad s^2 L\{ y \} - s - 2 - 4 L\{ y \} = 0$$

$$\Rightarrow L\{ y \} = \frac{s + 2}{s^2 - 4} = \frac{1}{s - 2}$$

$$\Rightarrow y(t) = e^{2t}$$

Example:3

$y'' - 3y' + 2y = 4t - 6, \quad y(0) = 1, \; y'(0) = 3$

Solution: $(s^2 \bar{y} - s - 3) - 3 (s \bar{y} - 1) + 2 \bar{y} = \frac{4}{s^2} - \frac{6}{s}$

$$\Rightarrow \bar{y} = \frac{s^2 + 2s - 2}{s^2 (s - 1)} = \frac{1}{s - 1} + \frac{2}{s^2}$$

$$\therefore \quad y = \mathcal{L}^{-1}\left\{ \frac{s^2 + 2s - 2}{s^2 (s - 1)} \right\}$$

$$= \mathcal{L}^{-1}\left\{ \frac{1}{s - 1} + \frac{2}{s^2} \right\} = e^t + 2t$$

Solving initial value problems using the method of Laplace transforms

To solve a linear differential equation using Laplace transforms, there are only 3 basic steps:

1.Take the Laplace transforms of both sides of an equation.

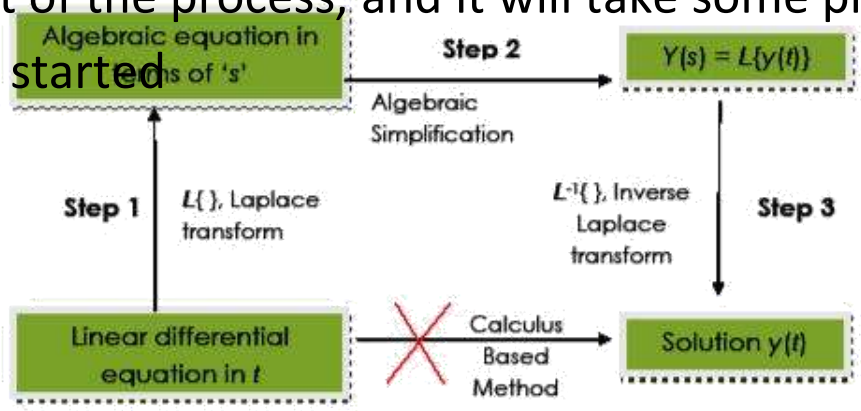
2.Simplify algebraically the result to solve for $\mathcal{L}\{y\} = Y(s)$ in terms of s.

3.Find the inverse transform of $Y(s)$. (Or, rather, find a function $y(t)$ whose Laplace transform matches the expression of $Y(s)$.) This inverse transform, $y(t)$, is the solution of the given differential equation.

The nice thing is that the same 3-step procedure works whether or not the differential equation is homogeneous or nonhomogeneous. The first two steps in the procedure are rather mechanical. The last step is the heart of the process, and it will take some practice. Let's get started

The Laplace Transform Method of Solving an Initial Value Problem

Flow Chart



Applications of Laplace transform

This section describes the applications of laplace transform in the domain of engineering. Laplace transform has several applications in almost all Engineering disciplines such as System Modelling, Analysis of Electrical and Electronic Circuits, Digital Signal Processing and Process Controls. Laplace transforms are critical for process controls. It helps analyze the variables, which when altered, produces desired manipulations in the result. For example, while study heat experiments, Laplace transform is used to find out to what extent the given input can be altered by changing temperature, hence one can alter temperature to get desired output for a while. This is an

efficient and easier way to control processes that are guided by differential equations

Electric Circuit theory

Example 1 :: The switching transient phenomenon in the RL , RC or RLC circuits can be solved by laplace

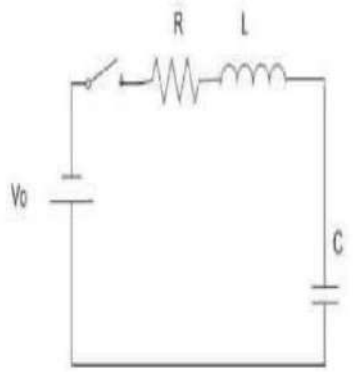


FIG. 1: Series RLC circuit

transform. Let us consider a series RLC circuit as shown figure 1 to which a dc voltage V 0 is suddenly applied.

Now applying kirchoff‘s voltage law (KVL) to the circuit , we

have Differentiating both sides,

$$L\frac{d^2i}{dt^2}+\frac{1}{C\,i}+R\frac{d\,i}{d\,t}=0\ \ or$$

Applying laplace transform to this equation, let us assume

$$\frac{d^2i}{d\,t^2}+\frac{R}{L}\frac{d\,i}{d\,t}+\frac{1}{L\,C}\,i=0\quad\rightarrow(1)$$

that the solution of this equation is

$$\mathbf{i(\,t\,)}=e^{sT}$$

$$L\,K\,s^2\,e^{s\,T}+R\,K\,e^{s\,T}+\frac{1}{C\,K}\,e^{s\,T}=0$$

5.CURVE FITTING

5.1 INTRODUCTION :

In this chapter we are concerned with the problem of fitting

an equation or a curve to data involving period values. An approximate non-mathematical relationship between the two variables , can be established by a diagram called **scatter diagram .** The exact mathematical relationship between the two variables is given by simple algebra expression called **curve fitting .** Though there are infinite variety of curve in mathematics , the curve used for the purpose of curve fitting are relatively limited in type. The straight line is the simple and one of the most important curve used.

The straight line :

The equation $y = a + bx$…. (1) Is an equation of the degree in x and y .It represents a straight line .

The difference $y_i - (a + bx_i)$….. (2)

Is zero if and only if the point (x_i, y_i) lies on the line given by (1)

Fitting a straight line:

Usually fitting a straight line means finding the value of the parameter a and b of the straight line given by (1) , as well as actually constructing the line itself .The Graphic method and the method of least square are two useful method for fitting a straight line .

The graphical method:

The method can be used whenever the given formula can be plotted as a straight line either directly or after suitable transformation. The straight line drawn after a careful visual estimate of its position has been made with the aid of a ruler. The coordinated of any two points on the measures and substitute in equation (1). The resulting equations in a and b are then solved for their *parameters.* Graphical method , whatever its theoretical attractions , suffers from the disadvantage that it is difficult apply in practice except for the straight line .This method will give fairly good results when finely divided coordinate paper is used , but in general it is recommended .

An alternative method, which id in almost universal use at present time, is known as the of **Least Squares** and we proceed to discuss it at length.

The method of lest squares:

This method of curve fitting was suggested early in the nineteenth century by the French mathematician Adrian Legendre. The method of least squares assumes that the best fitting line in the curve for which the sum of the squares of the vertical distances of the points (x_r , y_r) from the line is minimum. This method is more accurate that the graphical method.

Let $y = a + bx$ Refer (1)

Be the equation of the line. The ordinate of any point Q_r on the line vertically above or below a given point P_r , can be found by substituting the abscissa x_r , in the right-hand side of (1) . The two coordinates of Q_r will be $(x_r , a + bx_r)$ (see in the above figure). The vertical distances e_r from the line of any point p_r with coordinates (x_r , y_r) will therefore be given by the equation

$$e_r = y_r - (a + x_r) \ldots\ldots (3)$$

We may say that e_r represents the difference between the actual ordinate y_r , of a pint and its theoretical ordinate $a + bx_r$.

Let $q = \sum_{r=1}^{n} e_r^2 = \sum_{r=1}^{n} (y_r - (a + bx_r))^2$ ……. (4)

The best fitting line is that line for which the sum of the square, $q = \sum e_r^2$ is a minimum. We find the value of a and b which make q minimum as follows

:Differentiating q partially with respect to a and b equally these partially to zero , we obtain

$$\therefore \frac{\partial_q}{\partial_a} = \sum_{r=1}^{n}(-2)[y_r - (a + bx_r)] = 0 \ ,$$

And $\frac{\partial_q}{\partial_o} = \sum_{r=1}^{n}(-2)x_r[y_r - (a + bx_r)] = 0$,

Which yield the so – called **System of normal equations,**

$$\sum_{r=1}^{n} y_r = an + b\sum_{r=1}^{n} x_r \quad ,\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

And $\sum_{r=1}^{n} x_r\, y_r \ = a\ \sum_{r=1}^{n} x_r + \ b\ \sum_{r=1}^{n} x_r^2$ ……..(6)

Solving this system of equations we get

$$a = x = \frac{\sum_1^n x_r^2\ \sum_1^n y_r - \sum_1^n x_r \sum x_r y_r}{n\sum_1^n x_r^2 - [\sum_1^n x_r}$$ …….. (7)

$$b = \frac{n[\sum_{r=1}^{n} x_r y_r] - [\sum_{r=1}^{n} x_r][\sum_{r=1}^{n} x_r^2]}{n[\sum_{r=1}^{n} x_r^2] - [\sum_{r=1}^{n} x_r]^2}$$,……..(8)

Where n is the number of points $(x_r\ , y_r)$.

To simplify the for formulae, we get

$$t_r = \frac{x_r - \bar{x}}{h} \ ,$$ (9)

Where h is the interval between successive value of x assumed to be equally spaced. Shifting the origin to the mean $\bar{x}$ and compressing the intervals to unity we get transformed equation of (2) referred to the new (t, y) axes as

$$Y = \bar{a} + \bar{b}t \ ,$$ (10)

Where $\bar{a}\ and\ \bar{b}$ are the parameter in the new equation . In terms of the new coordinate formula (7) and (8) becomes

$$\bar{a} = \frac{\sum_1^n t_r^2 \sum_1^n y_r - \sum_1^n t_r \sum t_r y_r}{n \sum t_r^2 - (\sum t_r)^2} \quad , \qquad (11)$$

$$\bar{b} = \frac{n \sum_1^n t_r y_1 - \sum_1^n t_r \sum t_r y_r}{n \sum_1^n t_r^2 - [\sum_1^n t_r]^2} \quad , \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (12)$$

Transformation (9) replace the x_r $\grave{s}$ by unit deviations t_r from the mean $\bar{x}$.

Since

$$\sum_1^n t_r = 0$$

The formula (11)and (12) reduce to

$$\bar{a} = \frac{\sum_1^n t_r^2 \sum_1^n y_r - 0}{n \sum t_r^2 - 0}$$

And

$$\bar{b} = \frac{n \sum_1^n t_r y_r - 0}{n \sum t_r^2 - 0}$$

Which further simplify to the form

5.2 Fitting a parabola:

When a set of point exhibits a parabolic trend , the fitting of a quadratic functions

$$y = a + bx + cx^2 \ ,$$

To the data may be carried out by the method of least square , Which leads to the three equations

$$na + b\sum_1^n x_r + c\sum x_r^2 \;=\; \sum_1^n y_r$$

$$a\sum_1^n x_r + b\sum_1^n x_r^2 + c\sum_1^n x_r^3 = \sum_1^n x_r y_r \,.$$

$$a\sum_1^n x_r^2 + b\sum_1^n x_r^3 + c\sum_1^n x_r^4 = \sum_1^n x_r^2 y_r \,.$$

The above equations can be solve for $a\,, b, c$.

5.3 EXPONETIAL FUNCTION Y= Ae^{bx}

Transforming the exponential equation $y = ae^{bx}$ by taking logarithms on both sides we get

$$\log y = \log a + bx \log e \,.$$

If we replace $\log y$ by y and the constant $\log a$ and b $\log e$ by $\grave{a}$ and $\acute{b}$

Respectively , we obtain

$$y = \acute{a} + b\acute{x} \;,$$

Which defines a straight line .

Example:

1.Find the lest square line fir the data points (-1,10) ,(0,9),(1,7),(,4,5), (,2,5),(3,4),(4,3),(5,0) and (6,-1).

Solution: Here

x_r	y_r	x_r^2	y_r^2
-1	10	1	-10
0	9	0	0
1	7	1	7
2	5	4	10
3	4	9	12
4	3	9	12
5	0	25	0
6	-1	36	-6
20	37	92	25

From the table we have

$$n = 8\,, \sum x_r = 20\,, \sum y_r = 37, \sum x_r^2 = 92\,, and\ \sum x_r y_r = 25\,.$$

The normal equation are

$$na + b\sum x_r = \sum y_r\,,$$

$$a\sum x_r + \sum x_r^2 = \sum x_r y_r\,.$$

Putting the value in normal equations

$$8a + 20b = 37\,,$$

And

$$20a + 97b + 25\,,$$

And solving these equations we get

$$a = 8.6428571\,,$$

$$b = -1.6071429 \ .$$

Therefore the least square line is

$$y = 8.6428571 + (-1)1.6071429x \ ,$$

i.e.,

$$y = -1.6071429x + 8.6428571 \ .$$

Example 2: **Find the least square line $y = a + bx$ for the data**

X	**-2**	**-1**	**0**	**1**	**2**
Y	**1**	**2**	**3**	**3**	**4**

Solution:

x_r	y_r	x_r^2	$x_r y_r$
-2	1	4	-2
-1	2	1	-2
0	3	0	0
1	3	1	3
2	4	4	8
0	13	10	7

$$\therefore In\ this\ case\ \ n = 5 \ , \sum x_r = 0 \ , \sum y_r = 13 \ , \sum x_{r2} = 10 \ , \sum x_r y_r = 7.$$

The normal equation are

$$na + b\sum x_r = \sum y_r \ ,$$

$$a\sum x_r + b\sum x_r^2 = \sum x_r y_r \ .$$

Putting the value of $n\ , \sum x_r\ , \sum y_r\ , \sum x_r^2\ and\ \sum x_r y_r$ in the above equation we get

$$5a = 13\ , 10b = 7\ .$$

Solving

$$a = 2.6\ , b = 0.7\ .$$

∴The required line of fit is

$$y = 2.6 + b = (0.7)x\ .$$

Example 3:

Find a formula for the line of the form $y = a + bx + cx^2$ Which will fit the following data

X	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Y	3.1950	3.2299	3.2532	3.2516	3.2516	3.2282	3.1807	3.1266	3.0594	2.9759

Solution :

The normal equation are

$$na = b\sum x_r + c\sum x_r^2 = \sum y_r^2 \ ,$$

$$a\sum x_r + b\sum x_r^2 + c\sum x_r^3 = \sum x_r y_r$$

$$a\sum x_r^2 + b\sum x_r^3 + c\sum x_r^4 = \sum x_r^2 y_r \ .$$

Substituting the value of $\sum x_r\ , \sum x_r^2\ , \sum x_r^3, \sum x_r y_r, \sum x_r^2\ y_r, \sum y_r, and\ n, we\ get$

$$10a + 4.5b + 2.85c = 31.7616 \ ,$$

$$4.5a + 2.85b + 2.025c = 14.0896 \ ,$$

$$2.85a + 2.025b + 1.5333 = 8.82881 \ ,$$

And solving these equations we obtain

$$a = 3.1951 \, , b = 0.44254 \, , c = -0.76531.$$

$\therefore \ The \ requied \ equation \ is$

$$y = 3.1951 + 0.44254x - 0.76531x^2.$$

Example 4:

Fit second degree parabola to the following data

X	0	1	2	3	4
y	0	1.8	1.3	2.5	6.3

Solution : The values of x are 0,1,2,3,4

The number of value is odd. Shifting the origin to the middle value 2 of x, and making the substituting

$$u = x - 2, v = y \, ,$$

The curve of fit as

$$v = a + bu + cu^2 \ ,$$

We obtain

x	v	u	v	uv	u^2	u^2v	u^3	u^4
0	1	-1	1	-2	4	4	-8	16
1	1.8	-2	1.8	1.6	1	1.8	-1	1
2	1.2	0	1.3	0	0	0	0	0

3	2.5	1	2.5	2.5	1	2.5	1	1
4	6.3	2	6.3	12.6	4	25.2	8	16
		0	12.9	11.3	10	33.5	0	34

The normal equation are

$$\sum u = na + b\sum u + c\sum u_r^2 \quad ,$$

$$\sum uv = a\sum u + bu^2 + c\sum u^3 \quad ,$$

$$\sum u^2 v = a\sum u^2 + b\sum u^3 + c\sum u^4 \quad .$$

Putting the values of n ,$\sum u$, $\sum v$, $\sum uv$, … … … . , etc,

$$12.9=5a+40c,$$

$$11.3 = 10b \quad ,$$

$$33.5=10a+34c,$$

Solving these normal equations we get

$$a = 1.48, b = 1.13, c = 0.550.$$

∴The required equation is

$$u = 1.48 + 1.13u + 1.55u^2 \quad ,$$

Substituting $u = x - 2$, $v = y$ in the above equation we get,

$$y = 1.42 + 1.13(x - 2) + (0.55)(x - 2)^2 \quad .$$

$$\therefore The\ requird\ curve\ of\ fit\ is\ the\ parabola$$

$$y = 1.42 - 1.07x + 0.55x^2.$$

5.4 Solved problems:

1)Find the lest square line $y = a + bx$ *for the data*

X	-4	-2	0	2	4
Y	1.2	2.8	6.2	7.8	13.2

A)	X	y	x^2	xy
	-4	1.2	16	-4.8
	-2	2.8	4	-5.6
	0	6.2	0	0
	2	7.8	4	15.6
	4	13.2	16	52.8
	0	31.2	4.	58

The normal equations are

$$\sum y = na + b\sum xi$$

$$\sum xy = a\sum x_i + b\sum x_i^2$$

31.2=5a

$$a = \frac{31.2}{5}$$

= 6.24

$$58 = b(40)$$

$\therefore y = 6.24 + 1.45$.

2) **Fit a straight line to the following data regarding x as the independent variable**

X	0	1	2	3	4
Y	1	1.8	3.3	4.5	6.3

A)	x	y	x^2	xy
	0	1	0	0
	1	1.8	1	1.8
	2	3.3	4	6.6
	3	4.5	4	13.5
	4	6.3	16	25.2
	10	16.9	30	47.1

$$na + b\sum xi = \sum y$$

$$5a + b(10) = 16.9 \qquad (1)$$

$$a\sum x + b\sum x^2 = \sum xy$$

$$a(10) + b(30) = 47.1$$

$$1*2 \rightarrow 10a + 20b = 33.8$$

$$\underline{2 \rightarrow 10a + 30b = 47.1}$$

$$-10b = -11.3$$

$$b = \frac{13.3}{10}$$

$$=1.33$$

Substituting b in (1)

$$5a + (1.33)10 = 16.9$$

$5a = 16.9 - 13.3$

$a = \frac{3.6}{5}$

=0.72

$\therefore y = 0.72 + 1.33x$

3) Find the least square line $y = a + bx$

X	-2	0	2	4	6
Y	1	3	6	8	13

A)	X	y	x^2	xy
	-2	1	4	-2
	0	3	0	-2
	2	6	4	12
	4	8	16	12
	6	13	36	78
	10	31	60	78

$na + b\sum x_i = \sum xy$

$5a + 10b = 31$ (1)

$a\sum x + b\sum x^2 = \sum xy$

$10a + 60b = 102$ (2)

1*2→ $10a + 20b = 62$

10a+60b=120

$-40b = -58$

$$b = 1.45$$

Substituting b in (1)

$$5a + (1.45)10 = 31$$

$$5a = 31 - 14.5$$

$$a = \frac{16.5}{5}$$

$$=3.3$$

$$a = 3.3 \ , b = 1.45$$

$$\therefore y = 3.3 + 1.45x$$

4) Find the least squares parabolic lift $y = a + bx + cx^2$

A)	x	y	x^2	x^3	x^4	xy	x^2y
	-3	15	9	-27	81	-45	135
	-1	5	1	-1	1	-5	5
	1	1	1	1	1	1	1
	3	5	9	27	81	15	45
	0	26	20	0	164	-34	186

$$\sum y = na + b\sum x + c\sum x^2$$

$$26 = 4a + 0 + 20c$$

$$4a + 20c = 26 \qquad (1)$$

$$\sum xy = a\sum ax + b\sum x^2 + c\sum x^3$$

$$-34 = 0 + b(20) + c(0)$$

$$20b = -34$$

$$b = -\frac{34}{20}$$

$$= -1.7$$

$$\sum x^2 y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

$$186 = 20a + 164c \qquad (2)$$

$$1*5 \quad \rightarrow \quad 20a + 100c = 130$$

$$2 \quad \rightarrow \quad 20a + 164c = 186$$

$$-64c = -56$$

$$c = 0.875$$

$$4a + 20(0.875) = 26$$

$$4a = 36 - 17.5$$

$$a = \frac{8.5}{4}$$

$$= 2.125$$

$$y = a + bx + cx^2$$

$$\therefore y = 2.215 - 1.70x + 0.875x^2$$

5.5 APPLICATIONS OF CURVE FITTING

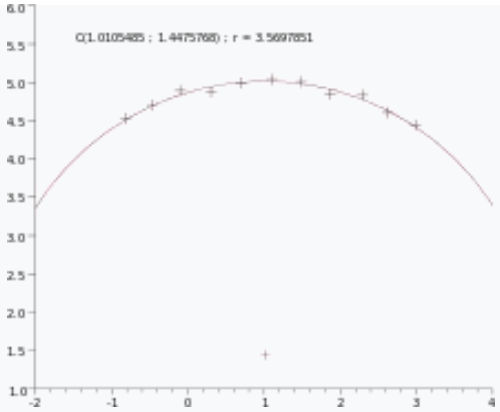
Fitting plane curves to data points :

If a function of the form cannot be postulated, one can still try to fit a plane curve.

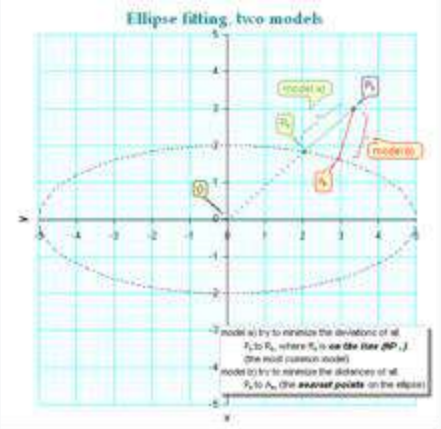
Other types of curves, such as conic sections (circular, elliptical, parabolic, and hyperbolic arcs) or trigonometric functions (such as sine and cosine), may also be used, in certain cases. For example, trajectories of objects under the influence of gravity follow a parabolic path, when air resistance is ignored. Hence, matching trajectory data points to a parabolic curve would make sense. Tides follow sinusoidal patterns, hence tidal data points should be matched to a sine wave, or the sum of two sine waves of different periods, if the effects of the Moon and Sun are both considered.

For a parametric curve, it is effective to fit each of its coordinates as a separate function of arc length; assuming that data points can be ordered, the chord distance may be used.

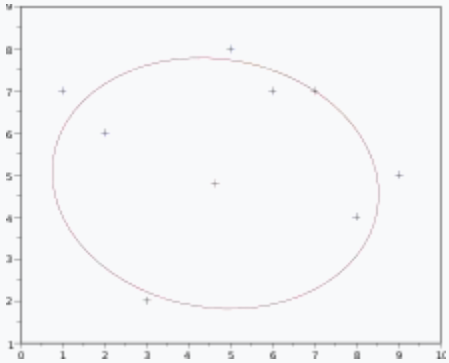
Fitting a circle by geometric fit



Circle fitting with the Coope method, the points describing a circle arc, centre (1 ; 1), radius 4.



different models of ellipse fitting



Ellipse fitting minimizing the algebraic distance (Fitzgibbon method).

Cooper[22] approaches the problem of trying to find the best visual fit of circle to a set of 2D data points. The method elegantly transforms the ordinarily non-linear problem into a linear problem that can be solved without using iterative numerical methods, and is hence an order of magnitude faster than previous techniques.

Fitting an ellipse by geometric fit

The above technique is extended to general ellipses[23] by adding a non-linear step, resulting in a method that is fast, yet finds visually pleasing ellipses of arbitrary orientation and displacement.

Application to surfaces

Further information: Computer representation of surfaces

Note that while this discussion was in terms of 2D curves, much of this logic also extends to 3D surfaces, each patch of which is defined by a net of curves in two parametric directions, typically called **u** and **v**. A surface may be composed of one or more surface patches in each direction.

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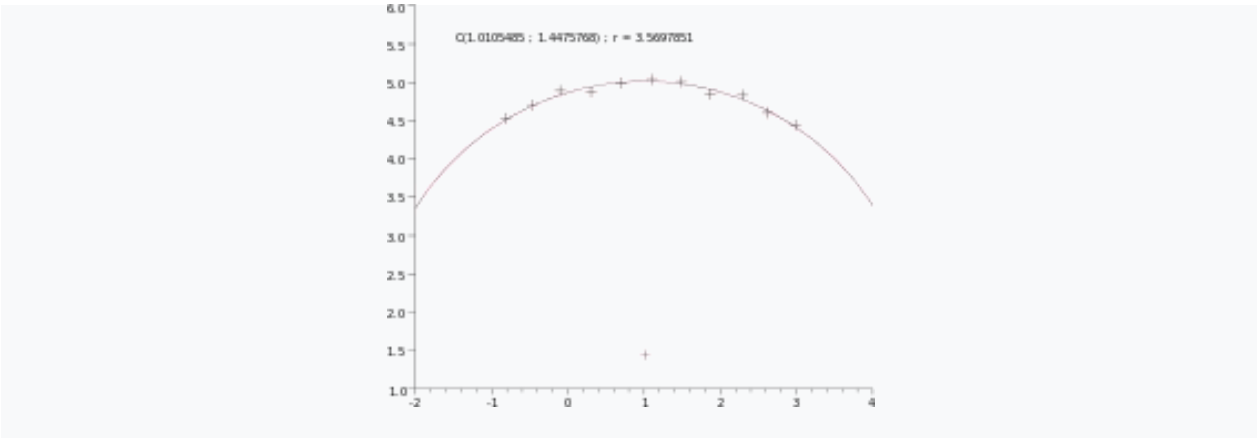

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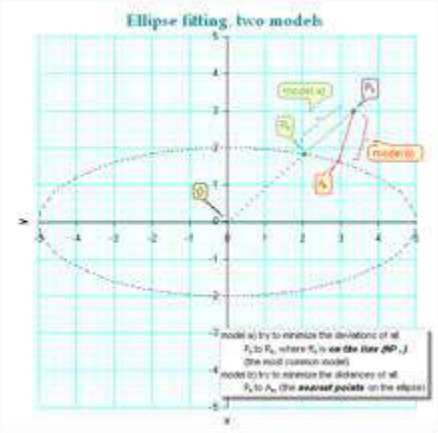

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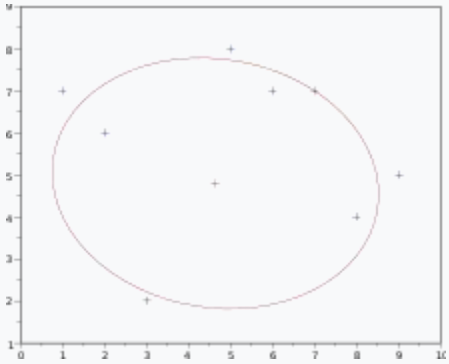

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A STUDY ON

INTERPOLATION WITH EQUAL INTERVALS

A Group Project submitted to KRISHNA UNIVERSITY

KRISHNA
UNIVERSITY

In partial fulfilment of the requirement For the award of the degree of BACHELOR OF SCIENCE

Submitted by

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UNDER THE GUIDENCE OF

SK.PARVEEN LECTURER IN MATHEMATICS

S.R.R &C.V.R G OVT.DEGREE COLLEGE MACHAVARAM,

VIJAYAWADA-520004.

S.R.R & C.V.R GOVT.DEGREE COLLEGE

CERTIFICATE

This is to certificate that the project entitled (“INTERPOLATION WITH EQUAL INTERVALS”) bonafide workdone by B.GANESH-18301202, P.BHAVANI-18301203, B.PRIYANKA- 18301207, N.ANUSHA-18301208, D.SIDDI VINAY- 18301209, P.DHANA LAKSHMI-18301234.
belongs to department of bachelor of science studies S.R.R & C.V.R GOVT. DEGREE COLLEGE,VIJAYAWADA under my guidance in partial fulfilment of the award for the degree in bachelor of science.

SK.PARVEEN

(PROJECT

GUIDE)

K.V. NAGALAKSHMI EXTERNAL EXAMINER

(H.O.D Of department of mathematics)

DECLARATION

We **B.GANESH -18301202,P.BHAVANI-18301203, B.PRIYANKA- 18301207, N.ANUSHA-18301208, D.SIDDI VINAY- 18301209, P.DHANA LAKSHMI-1830123.**
by declare that the project entitled “INTERPOLATION WITH EQUAL INTERVALS” has been prepared by in a partial fulfilment of the requirement for the award of the degree of bachelor of science submitted at S.R.R & C.V.R GOVT DEGREE COLLEGE affiliated to KRISHNA UNIVERSITY is an authentic project record of our own work .

The matter presented as not be submitted to any University or Institute for the award of bachelor of science Degree .

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We also express our heart thanks of to all my faculty for their support and encouragement in completed this project report in an elegant from last but not least, we extended our gratitude and courtesy of our family members and friends who have help us in completion of our project work .

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1. Interpolation

1.1 Introduction

Brahmagupta's interpolation formula is a second – order polynomial interpolation formula developed by the Indian mathematician and astronomer Brahmagupta [598 – 668 CE] in the early 7th century CE. The Interpolation Theory suggests that the sporophyte generation progenated from a haploid, gren algal thallus in which repeated mitotic cell divisions of a zygote produced an embryo retained on the thallus and gave rise to the diploid phase [sporophyte]. Ensuing evolution caused the sporophyte to become increasingly complex, both organographically and anatomically.

The interpolation Theory was introduced by Celakovsky [1874] as the Antithetic Theory. Bower [1889] further developed this theory and renamed it the Interpolation Theory . The theory was later supported by Overton [1893], Scott [1896] , Strasburger [1897], Williams [1904], and others.The gradual evolution of an independent, sporophyte phase was viewed by Bower as being closely related to the transition from aquatic to terrestrial plant life on Earth.

Evidence supporting this theory can be found in the life cycle of modern Bryophytes in which the sporophyte is physiologically dependent on the gametophyte.Competing theories include the Transformations theory, which was introduced as the Homologous theory by Celakovsky, and also renamed by Bower.

Interpolation literally refers to introducing something additional or extraneous between other things or parts. In numerical analysis, interpolation is a method of constructing new data points within a discrete set of known data points, using finite differences. The process of obtaining function values outside (in the vicinity) the given range is called extrapolation.

In this chapter we shall extend the applications of differencing techniques to interpolate and extrapolate data points within a given range, for equal as well as well us unequal interval lengths.

1.2 INTERPOLATION METHODS :

(i) Graphical method

(i) Method of curve fitting

(iii) Applications of the calculus of finite differences formula.

1.2.1 Graphical Method :

Let y =f(x), then plot a graph for different values of x corresponding to the value of y. From the graph, we can find the value of y for given value of x.

For example

Year(x)	1891	1901	1911	1921	1931
Population (Y)(in 1000's)	46	66	81	93	101

Interpolate population for the year 1919.

The following steps are to be maintained

1. Take a suitable scale for x and y plot the different points on the graph for given data.
2. Join the point by hand curve.
3. Find the point on the curve corresponding to x = 1919 and find corresponding value of y

Remarks : This is simply an approximate method of evaluating the value of y and is not reliable in number of cases.

1.2.2 Method of Curve Fitting :

This method is applicable in some cases in which the form of the function is known. In such cases, by the method of least squares, we can fit the curve of known form to given set of values and afterwards we can find the unknown value with the help of fitted curve.

Remarks :

(1) This is not exact method.

(2) In case of number of values are sufficiently large this method is complicated

(3) The form of the function in this method for given set of observations is assumed to be known.

1.2.3 Applications of the Calculus of Finite Differences :

(1) Newton-Gregory formula for interpolation. If n is a integer and h is the interval differencing, then $f(a+ nh) = f(a)+{}^{n}c_1 \Delta f(a)+ {}^{n}c_2\Delta^2 f(a)+....+ {}^{n}c_n \Delta^{n}{}_{f(a)}$

We prove the theorem by using the method of induction.

we know that $\Delta f(a) = f(a + h) - f(a)$

$f(a+h)= f(a) +\Delta f(a)$

The theorem is true for n = 1.

Assume that the theorem is true for n=k.

$f(a+kh)=f(a)+{}^{k}c_1\,\Delta^2 f(a)+\ldots.+{}^{k}c_n\,\Delta^k f(a)$

We have $\Delta f(a+kh) = f(a+(k+1)h)-f(a+kh)$

$f(a + (k + 1)h) = f(a+kh) + h + \Delta f(a + kh)$

$= [f(a)+{}^{k}c_1\Delta f(a)+ {}^{k}c_2\Delta^r f(a)+\ldots.+ \Delta^k f(a)]$

$[\Delta^r f(a)+{}^{k}c_1\Delta^r f(a)+ {}^{k}c_1\Delta^3 f(a)+\ldots..+ \Delta^{k+1} f(a)]$

$= f(a)+({}^{k}c_1+1)\,\Delta f(a)+({}^{k}c_1+ {}^{k}c_2)\,\Delta^2 f(a)+\ldots$

$\ldots+ \Delta^{k+1} f(a)$

$= f(a)+{}^{k+1}c_1+ \Delta f(a)+{}^{k+1}c_2\,\Delta^2 f(a)+\ldots\ldots+ \Delta^{k+1} f(a)$

The theorem is true for n=k+1

By the method of induction, the theorem is true for any integer.

$f(a+ nh) = f(a)+{}^{n}c_1\,\Delta f(a)+ {}^{n}c_2\Delta^2 f(a)+\ldots.+ {}^{n}c_n\,\Delta^{n}{}_{f(a)}$

Note :

1. $f(a+ nh) = f(a)+{}^{n}c_1\,\Delta f(a)+ {}^{n}c_2\Delta^2 f(a)+\ldots.+ {}^{n}c_n\,\Delta^{n}{}_{f(a)}$

2. Put a=0, h=1

$f(h)=f(0)+{}^{n}c_1\,\Delta f(0)+{}^{n}c_2+ {}^{n}c_2\,\Delta^2 f(0)+\ldots\ldots$

$f(x)=f(0)+\frac{x}{1!}\,\Delta f(0)+ \frac{x(x-1)}{2!}\,\Delta^2 f(0)+\ldots\ldots.$

This formula is used only when the first term a =0 and h =1.

Alter : Let y=f(x) represent a function which assumes the values f(a),f(a+h)....f(a+nh) for (n +1) equidistant values a, a+ h, a +2h, a+nh of the independent variable x and let $P_n(x)$ be a polynomial in x of degree n.

This polynomial can be expressed in the form.

$P_n(x) = Bo +B\,(x-a) + B2(x-a)(x-a-h)+\ldots\ldots\ldots+ B_n(x-a)(x-a-h)(x-a-2h)\ldots.(x-a-(n-1)h)\ldots.[1]$

where B_o. B_1, B_n are constants and are choosen such that

$P_n(a) = f(a)$, $P_n(a + h) = f(a + h)$, . . . $P_n(a + nh) = f(a + nh)$

By substituting successively the values a, a+ h, a +2h . a+nh in equation(1) and writing the values of $P_n(a)$, $P_n(a+h)$... P(a + nh), we get $f(a)= B_0$ B1=f(a + h)-f(a)

$f(a) = B_0$, $f(a + h) = B_0+ hB_1$

$B_0 =f(a)$ or $B_1 = \frac{f(a+h)-f(a)}{h} = \frac{\Delta f(a)}{h}$

$f(a + 2h) = B_0 + 2hB_1 + h^rB_2$ (or)

A2 = f(a + 2h)-2{f(a+ h)-f(a)}-f(a)/2h^2

$= \frac{1}{2!h^r}\,\Delta^2 f(a)$

Similarly, we get $B_3 = \frac{1}{3!h^3}\ \Delta^3 f(a)$…… $B_n = \frac{1}{n!h^n}\ \Delta^n f(a)$

Substituting the values of $B_0, B_1, \ldots, B_n$, we get

$P_n(x) = f(a) + \frac{\Delta f(a)}{h}(x-a) + \frac{\Delta^2 f(a)}{2!h^r}(x-a)(x-a-h) + \ldots + \frac{\Delta^n f(a)}{n!h^n}(x-a)(x-a-h)\ldots(x-a-(n-1)h)$

This is Newton-Gregory formula for forward interpolation putting $\frac{x-a}{h}$ = u or x=a+hu then(1) can be expressed as

$P_n(x)=P_n(a+nh) = f(a)+u\Delta f(a)+\frac{u(u-1)}{2!}\ \Delta^2 f(a)+\ldots+\frac{u(u-1)\ldots\ldots(u-n+1)}{n!}\ \Delta^n f(a)$

Newton-Gregory Formula for Backward Interpolation

In this case , we write

$P_n(x) = C_0 + C_1(x-a-nh) + C_2(x-a-nh) + \ldots + C_n(x-a-nh)(x-a-nh+h)\ldots(x-a-h)$ ……(1)

where Co, C_1 ,C_2… C_n are constants and are choosen such that

$f_n(a+nh) = f(a+nh), \ldots, P_n(a) = f(a)$

Substituting in (1), x=a + nh, a + nh-h, we have

$C_0 = f(a+nh)$, $f(a+nh-h) = C_o + C_1(-h)$

$C_1 = \frac{f(a+nh)-f(a+nh-h)}{h} = \frac{\Delta f(a+nh)}{h}$

Similarly $C_2 = \frac{1}{2!h^r}\ \nabla^2 f(a+nh)$

$C_3 = \frac{1}{3!h^3}\ \nabla^3 f(a+nh)$

……

$C_n = \frac{1}{n!h^n}\ \nabla^n f(a+nh)$

Substituting these values of $C_o, C_1, \ldots, C_n$ in (1), we obtain

$P_n(x) = f(a+nh) + \frac{\nabla f(a+nh)}{1!h}(x-a+nh) + \frac{\nabla^2 f(a+nh))}{2!h}(x-a+nh)\ (x-(a+nh-h)) + \ldots + \frac{(\nabla^n f(a+nh)}{n!h^n}(x-a+nh)\ (x-(a+nh-h))\ldots\ldots(x-(a+h))$ ……(2)

This is Newton-Gregory formula for backward interpolation. Substituting $n=\frac{x-(a+nh)}{h}$ or

x= a + nh + hl in eq. (2), then we obtain $P_n(x) = P_n(a + nh + h) = f(a + nh) + u\nabla f(a+nh) + \frac{u(u+1)}{2!}\nabla^2 f(a+nh) + \ldots\ldots\ldots + \frac{u(u+1)\ldots\ldots(u+n-1)}{n!}\nabla^n f(a+nh)$ Where $P_n(x) = f(x)$.

SOLVED PROBLEMS

1. Find the second degree polynomial from the data is then find f(5)

0	1	2	3	4
3	6	11	17	27

Solution .

X	F(x)	$\Delta \mathbf{f(x)}$	$\Delta^2 \mathbf{f(x)}$	$\Delta^3 \mathbf{f(x)}$
0	3			
		3		
1	6		2	
		5		0
2	11		2	
		7		0
3	18		2	
		9		
4	27			

Newton's Gregory's formula is

f(x) = f(0)+$\frac{x}{1!}\Delta f(0)+\frac{x(x-1)}{2!}\Delta^2 f(0)$

a=0 , h=1

a+uh = 5 u =5

f(x) = 3+x(3)+$\frac{x(x-1)}{2}(2)$

= 3+3x + x^2-x = x^2 + 2x + 3

f(5) = $(5)^2$ + 2(5) + 3=25+10+ 3 = 38.

2. Find the polynomial fits the data in the following table then also find f(13).

X	3	5	7	9	11
Y	6	24	58	109	174

solution. a=3, h = 2

Let a+uh = x = 13 3+2u = x

u = $\frac{x-3}{2}$

X	Y =f(x)	$\Delta \mathbf{f(x)}$	$\Delta^{\mathbf{2}}\mathbf{f(x)}$	$\Delta^{\mathbf{3}}\mathbf{f(x)}$	$\Delta^{\mathbf{4}}\mathbf{f(x)}$
3	6				
		18			
5	24		16		
		34		0	
7	58		16		0
		50		0	
9	109		16		
		66			
11	174				

Newton's Gregory's formula is

$$f(a+uh) = f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)$$

$$= 6+\frac{x-3}{1!}(18)+\frac{\left(\frac{x-3}{2}\right)\left(\frac{x-3}{2}-1\right)}{2!}(16)$$

$f(a+uh) = 6+9x-27+2(x-3)(x-5)=9x-21+2x^2-16x+30$

$f(x) = 2x^2-7x+9$

$f(13) = 2(13)^2-7(13)+9=2(169)-91+9=338+9-91$

$f(13) = 2.56$

3. Given log x for x = 40, 45, 50, 55, 60, 65 for the following data :

x	40	45	50	55	60	65
logx	1.60206	1.65321	1.69877	1.74036	1.7785	1.81291

Estimate the value of log (58.75) by using Newton backward interpolation formula.

Solution.

X	10^5xlogx=f(x)	10^5x∇f(x)	10^5x∇^2f(x)	10^5x∇^3f(x)	10^5x∇^4f(x)
40	160206				
		5115			
45	165321		-539		
		4516		102	
50	169892		-437		-25
		4319		77	
55	174036		-360		-20
		3729		57	
60	177815		-330		
		3476			
65	181291				

(a + nh) + uh = 58.75

65 +5u = 58.75

u = 1.25

Newton backward interpolation formula

$f(a+nh)+nh) = f(a+nh) + \frac{h}{1!}\Delta f(a+nh) + \frac{u(u+1)}{2!}\Delta^r f(a+nh) + \frac{u(u+1)(u+2)}{3!}\Delta^r f(a+nh)$

$10^5 f(58.75) = 181291 - 1.25x(3476) + \frac{(-1.25)(-1.25u)x(-30.5)}{2!} + \frac{(-1.25)(-1.25+1)x(-1.25+2)}{6}x(5)$

= 181291-4345-47.3438+2.2267 = 176900.89

F(58.75) = 1.7690089

4. Find the cubic polynomial for the following data hence find f(u)

X	0	1	2	3
F(x)	-2	1	4	6

Solution :

x	**F(x)**	Δf(x)	Δ^2**f(x)**	Δ^3**f(x)**
0	-2			
		3		
1	1		0	-1
		3		
2	4		-1	
		2		
3	6			

Newton's Gregory's formula is

$f(x) = f(0) + \frac{x}{1!}\Delta f(0) + \frac{x(x-1)}{2!}\Delta^2 f(0) + \frac{x(x-1)(x-2)}{3!}\Delta^3 f(0)$

$= -2 + \frac{x}{1!}(3) + \frac{x(x-1)}{2!}(0) + \frac{x(x-1)(x-2)}{3!}(-1)$

$= -2+3x-\frac{(x^3+3x^2+2x)}{6} = \frac{-12-18x-x^3+3x^2-2x}{6}$

$= \frac{x^3+3x^2+16x-12}{6} = \frac{-(4)^3+3(4)^2+16(4)-12}{6}$

$= \frac{-64+48+64-12}{6} = \frac{36}{6} = 6$

f(4) = 6

5. Find f(1.5) by using Newton's advancing difference formula given that f(o)= +2, f(1)=4, f(2)=7, f(3)=8

x	F(x)	$\Delta f(x)$	Δ^2**f(x)**	Δ^3**f(x)**
0	**2**			
		2		
1	4		1	
		3		**-3**
2	7		-2	
		1		
3	8			

Solution. Newton's advancing formula is

$$f(x) = f(0)+\frac{x}{1!}\Delta f(0)+\frac{x(x-1)}{2!}\Delta^2 f(0)+\frac{x(x-1)(x-2)}{3!}\Delta^3 f(0)$$

$$= 2+\frac{x}{1!}(2)+\frac{x(x-1)}{2!}(1)+\frac{x(x-1)(x-2)}{3!}(-3)$$

$$= 2+2x+\frac{x^2-x}{2!}+\frac{x^3-3x^2+2x}{2!}(-1)$$

$$= \frac{4+4x+x^2-x-x^3+3x^2-2x}{2}$$

$$F(x) = \frac{-(1.5)^3+4(1.5)^2+(1.5)+4}{2}$$

$$= \frac{-3.375+9+1.5+4}{2} = \frac{11-125}{2} = 5.5625$$

$$f(1.5) = 5.5625$$

6. Find the polynomial and hence find f(5) from the following data by using Newton's Gregory's formula.

x	**0**	**1**	**2**	**3**	**4**
F(x)	**0**	**7**	**26**	**63**	**124**

Solution.

X	F(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	0				
		7			
1	7		12		
		19		6	
2	26		18		0
		37		6	
3	63		24		
		61			
4	124				

$F(x) = f(0)+\frac{x}{1!}\Delta f(0) + \frac{x(x-1)}{2!}\Delta^2 f(0) + \frac{x(x-1)(x-2)}{3!}\Delta^3 f(0)$

$= 0+\frac{x}{1!}(7) + \frac{x(x-1)}{2!}(12) + \frac{x(x-1)(x-2)}{3!}(6)$

$= 7x+6x^2-6x+x^3 - 3x^2 + 2x = x^3 + 3x^2 + 7x - 4x$

$F(x) = x^3 - 3x^2 + 2x$

$F(5) = 5^3+3(5)^2+3(5) = 125+75+15 = 215$

$F(5) = 215$

7. Find the no. of men getting the wages less than is rupees from the following :

0-10	10-20	20-30	30-40
9	30	35	42

Solution .

Wages x	No. of Men	$\Delta\mathbf{f(x)}$	$\Delta^2\mathbf{f(x)}$	$\Delta^3\mathbf{f(x)}$
0-10	9			
		21		
10-20	30		-16	
		5		18
20-30	35		2	
		7		
30-40	42			

Newton's advancing formula is

$f(a+uh) = f(a)+\frac{u}{1!}\Delta f(a)+ \frac{u(u-1)}{2!}\Delta^2 f(a)+ \frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)$

$f(15) = 9+\frac{1}{2}(21)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(-16)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(18)$

$= 9+\frac{21}{2}+2+\frac{3}{8}\left(\frac{18}{16}\right) = 11+\frac{21}{2}+\frac{9}{8}$

$= 11+10.5+1.125- 22.625 = 23$

8. Find the value of f(35) from the following table :

X	Y=f(x)	$\Delta\mathbf{f(x)}$	$\Delta^2\mathbf{f(x)}$	$\Delta^3\mathbf{f(x)}$
20	512			
		-73		
30	439		-20	
		-93		10
40	346		-10	
		-103		
50	243			

Solution .

Newton's advancing formula is

F(a+uh) =

$f(a)+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)+\frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(a)$

a=20 , h=10 => a+uh=35a=30, h=10

20+10h=35 (or) 30+10u=35

h= $\frac{3}{2}$,u= $\frac{1}{2}$

$f(35)=439+\frac{1}{2}(-93)+\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(-10)+\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(-10)$

$=439-\frac{93}{2}+\frac{10}{8}+\frac{30}{48}$

=439-46.5+1,25+6.625=440.875-46.5=394.375

f(35)=394.

9. Second degree polynomial passes through the points (o, 1), (1, 3), (2, 7),(3, 13) then find the polynomial or find f(x). Also find the value of f(1.5).

Solution.

X	Y = f(x)	$\Delta\mathbf{f(x)}$	$\Delta^2\mathbf{f(x)}$	$\Delta^3\mathbf{f(x)}$
0	1			
		2		
1	3		2	
		4		0
2	7		2	
		6		
3	13			

Newton's Gregory's formula is

$f(x) = f(0)+\frac{x}{1!}\Delta f(0)+\frac{x(x-1)}{2!}\Delta^2 f(0)$

$= 1+x(2)+\frac{x(x-1)}{2}(2)$

$f(x) = 1 + 2x + x^2-x$

$= x^2 + x + 1$

f(1.5) = $(1.5)^2+(1.5)+1$

= 2.25+1.5+1 = 4.75

f(1.5) = 4.75

10 . From the following table find the no. of students who obtained less than 45 marks.

x	30-40	40-50	50-60	60-70	70-80
F(x)	31	42	51	35	31

Solution .

Marks less than x	No.of students	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
40	31				
		42			
50	73		9		
		51		-25	
60	124		-16		37
		35		12	
70	159		-4		
		31			
80	190				

a=40,h=10

let a +uh = 45

u(10) = 45-40

$$u = \frac{5}{10} = \frac{1}{2}$$

Newton's advancing formula is

F(a+uh) = f(a)

$$+\frac{u}{1!}\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)+\frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(a)$$

$$= 31+\frac{1}{2}(42)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(9)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(-25)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\left(\frac{1}{2}-3\right)}{4!}(37)$$

$$= 31+21-\frac{9}{8}+\left(-\frac{75}{48}\right)-\frac{15}{16}\left(\frac{37}{21}\right)$$

= 52-1.125-1.5625-1.4453125

= 47.8671875 = 47

: The no. of students getting less than 45 marks is 47 and the no. of students who getting more than 45 is 190-47-143

Central Difference Interpolation Formulae

1.3. INTRODUCTION :

In the previous chapter, we derived and discussed Newton forward and backward interpolation formula, which are applicable for interpolation, near the beginning and end respectively of tabulated data. We shall now discuss in the present chapter, about the central difference formula, which are suitable for interpolation near the middle of a tabulated data.

The central difference operator (δ) is defined as $\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$

The first central difference operator δf(x) defined by

$$\delta f(x) = f(x+\frac{h}{2}) - f(x-\frac{h}{2})$$

$$\delta f(x) = (E^{\frac{1}{2}} - E^{-\frac{1}{2}}) f(x)$$

We have $E = 1 + \Delta$

$$\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}} = E^{-\frac{1}{2}}(E-1) = E^{-\frac{1}{2}}\Delta \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Again by the first backward difference of f(x)

We have $\nabla f(x) = f(x)-f(x-h) = f(x)-f(x-h) = E(x)- E^{-1}f(x)$

$$\nabla f(x) = (1- E^{-1})f(x)$$

$$\nabla = (1- E^{-1})$$

$$\nabla E^{\frac{1}{2}} = E^{\frac{1}{2}} - E^{-\frac{1}{2}} = \delta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From(1) and (2) $\delta = \Delta E^{-\frac{1}{2}} = \nabla E^{\frac{1}{2}}$ $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$

The average operator u is defined as

$$U = \frac{1}{2}[E^{\frac{1}{2}} + E^{-\frac{1}{2}}]$$

Results :

(1) $U^2 = 1+\frac{1}{4}\delta^2 u^2 f(x) = \frac{1}{2}(E^{\frac{1}{2}} + E^{-\frac{1}{2}})^2$

$= \frac{1}{4}(E^{\frac{1}{2}} + E^{-\frac{1}{2}})^2 = \frac{1}{4}(E+E^{-1}+2) = \frac{1}{4}(E+E^{-1}-2+4)$

$= \frac{(u+k-1)^{(2k+1)}}{(2k+1)!} = -\frac{(k-v)^{(2k+1)}}{(2k+1)!}$

$$= \frac{v(v^2-1)(v^2-4)\ldots\ldots((v^2-k^2)}{(2k+1)!}$$

From (3), we obtain

$$y_u = vy_o + \frac{v(v^2-1)}{3!}\Delta^2 y_{-1} + \frac{v(v^2-1)(v^2-4)}{5!}\Delta^4 y_{-2} + \ldots\ldots uy_1 + \frac{u(u^2-1)}{3!}\Delta^2 y_0 + \frac{u(u^2-1)(u^2-4)}{5!}\Delta^4 y_{-2} + \ldots\ldots\ldots + \frac{u(u^2-1)(u^2-4)(u^2-k^2)}{(2k+1)!}\Delta^{2k} y_{1-k} + \ldots\ldots\ldots\ldots \quad \ldots\ldots\ldots\

Results : (1) $\delta = \Delta E^{-\frac{1}{2}} = \nabla E^{\frac{1}{2}}$

(2) $\Delta\nabla = \nabla\Delta = \Delta - \nabla = \delta^2$

(3) u= $\delta = \frac{1}{2}(\nabla + \Delta) = \frac{1}{2}\Delta E^{-1} + \frac{1}{2}\Delta$ (4) $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$)

Proof :

(1) $\delta = \Delta E^{-\frac{1}{2}} = \nabla E^{\frac{1}{2}}$

$\delta f(x) = f(x+\frac{h}{2}) - f(x-\frac{h}{2}) = \nabla f(x+\frac{h}{2}) = \nabla E^{\frac{1}{2}} f(x)$

$\delta = \nabla E^{\frac{1}{2}}$

(2) (a) $\Delta\nabla f(x) = \Delta\Delta E^{-1} f(x) = \Delta^2 E^{-1} f(x)$

(b) $\Delta\nabla f(x) = \Delta E^{-1} f(x) = \Delta^2 E^{-1} f(x)$

(c) $(\Delta - \nabla) f(x) = (\Delta - \nabla E^{-1}) f(x) = \Delta(1-\frac{1}{E}) f(x)$

$= \frac{\Delta(E-1)}{E} f(x) = \Delta.\Delta E^{-1} f(x) = \Delta^2 E^{-1} f(x)$

(d) $\delta^2 f(x) = (\Delta E^{-\frac{1}{2}})^2 f(x) = \Delta^2 E^{-1} f(x)$

$\Delta\nabla = \nabla\Delta = \Delta - \nabla = \delta^2$

(3) $u\delta f(x) = \frac{(E^{\frac{1}{2}} + E^{-\frac{1}{2}})}{2}(E^{\frac{1}{2}} - E^{-\frac{1}{2}}) f(x)$

$= \frac{1}{2}(e - e^{-1}) f(x) = \frac{1}{2}(e - \frac{1}{E}) f(x) = \frac{1}{2}(E^2 - \frac{1}{E}) f(x)$

$= \frac{(E-1)(E+1)}{2E} f(x) = \frac{\Delta}{2}(1+\frac{1}{E}) f(x)$

We know that $\nabla = \Delta E^{-1} = \frac{1}{2}(\Delta + \nabla) f(x) = \frac{1}{2}(\Delta + \nabla E^{-1}) f(x)$

$= \frac{1}{2}\nabla E^{-1} + \frac{1}{2}\Delta$

$U\delta = \frac{1}{2}(\Delta + \nabla E^{-1})$

(4) $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$

$(\frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}) f(x) = (\frac{\Delta}{\Delta E^{-1}} - \frac{\Delta E^{-1}}{\Delta}) f(x) = (E - \frac{1}{E}) f(x) = (\frac{E^2 1}{E}) f(x)$

$= (E-1)\frac{(E+1)}{E} f(x) = \Delta(1+E^{-1}) f(x) =$

$$(\Delta + \nabla E^{-1})f(x)$$

$$= (\Delta + \nabla)f(x)$$

$$(\frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}) = \Delta + \nabla$$

1.4 Interpolation within Equal Intervals

Let y = f(x) be an explicitly unknown function, with given discrete set of points (x_i,y_i), i=0,1,2,3,…,n, where x_i 's are equispaced. The process of obtaining the values $f(x_i + ph)$,-1 < p < 1, where height of the interval (h) is fixed, is known as interpolation within equal intervals. There are several methods of interpolating data points within a given range, depending upon the location where the value is to be interpolated as given below:

i. Newton's forward interpolation formula
ii. Newton's backward interpolation formula
iii. Gauss's forward difference formula
iv. Gauss's backward difference formula
v. Stirling's central difference formula
vi. Bessels's interpolation formula

We shall discuss these methodologies one by one in the coming sections.

1.4.1 Newton's Forward Interpolation Formula

Newton's forward interpolation formula is used to interpolate the values of the function y=f(x) near the beginning ($x > x_0$) and to extrapolate the values when ($x < x_0$), within the range of given data points (x_i,y_i), I = 0,1,2,3,…..,n.

Let f(x) take the values $y_0, y_1, y_2, ...y_n$; for the independent variable x taking values $x_0, x_1, x_2, ...x_n$, where height of the interval (h) is fixed, such that

$x_1 = x_0 + h$, $x_2 = x_0 + 2h$, …… , $x_n = x_0 + nh$.

Then to evaluate f(x) for $x = x_0 + ph$, $-1 < p < 1$

We have $f(x) = f(x_0 + ph) = E^p f(x_0) \equiv (1 + \Delta)^p y_0$

$\because E \equiv 1 + \Delta$ and $f (x_0) = y_0$

$\rightarrow f(x) \equiv (1 + p\,\Delta + [p(p-1)/2!]\,\Delta^2 + [p(p-1)(p-2)/3!]\,\Delta^3 + ...)y_0$, $x= x_0 + ph$

$\therefore f(x) \equiv y_0 + p\,\Delta y_0 + [p(p-1)/2!]\,\Delta^2 y_0 + [p(p-1)(p-2)/3!]\,\Delta^3 y_0 + ...$, $p = x-x_0/h$

This is Newton's forward interpolation formula and is used to interpolate or extrapolate values near the beginning of the table.

- In Newton's forward method, p is taken as 0 where $x = x_0$ and all the differences are evaluated taking p = 0 as reference point.
- Value which is to be interpolated i.e. f(x) may be denoted by y_p,| p < 1 |

Example1 Use Newton's forward interpolation formula to find the values of ¡.f(1.4) ¡¡. f(0.9) for the given set of values

x	1	2	3	4
f(x)	6	11	18	27

Solution: Function has to be evaluated near the starting of the table, thereby constructing forward difference table for the function y = f(x)

X	p	f(x)	Δ	Δ^2	Δ^3
1	0	6			
			5		
2	1	11		2	
			7		0
3	2	18		2	
			9		
4	3	27			

Newton's forward interpolation formula given by:

$f(x) \equiv y_0 + p\,\Delta y_0 + [p(p-1)/2!]\,\Delta^2 y_0 + [p(p-1)(p-2)/3!]\,\Delta^3 y_0 + \ldots\ldots\ldots,\ p = x - x_0/h \ldots\ldots(1)$

¡. To find f(1.4)

$X_0 = 1,\ y_0 = 6,\ x = 1.4,\ h = 1 \quad \therefore p = 1.4 - 1/1 = 0.4$

Also from table $\Delta y_0 = 5,\ \Delta^2 y_0 = 2,\ \Delta^3 y_0 = 0$

Substituting these values in (1), we get

$f(1.4) \equiv 6 + (0.4)(5) + [0.4(0.4-1)/2](2) + 0 = 7.76$

¡¡. To find f(0.9)

$X_0 = 1,\ y_0 = 6,\ x = 0.9,\ h = 1 \quad \therefore p = 0.9 - 1/1 = -0.1$

Also from table $\Delta y_0 = 5,\ \Delta^2 y_0 = 2,\ \Delta^3 y_0 = 0$

Substituting these values in (1), we get

$f(0.9) \equiv 6 + (-0.1)(5) + [-0.1(-0.1-1)/2](2) + 0 = 5.61$

Example 2 Find the cubic polynomial with given set of points

x	0	1	2	3
f(x)	5	6	3	14

Hence or otherwise evaluate .

Solution: Constructing forward difference table for the function

X	p	y	Δ	Δ^2	Δ^3
0	0	5			
			1		
1	1	6		-4	
			-3		18
2	2	3		14	
			11		
3	3	14			

Newton's forward interpolation formula given by:

$f(x) \equiv y_0 + p\,\Delta y_0 + [p(p-1)/2!]\,\Delta^2 y_0 + [p(p-1)(p-2)/3!]\,\Delta^3 y_0 + \ldots\ldots\ldots$, $p = x - x_0/h \ldots\ldots(1)$

$x_0 = 0$, $y_0 = 5$, $h = 1$, $\therefore p = x - 0/1 = x$

Also from table $\Delta y_0 = 1$, $\Delta^2 y_0 = -4$, $\Delta^3 y_0 = 18$

Substituting these values in (1), we get

$f(x) \equiv 5 + x(1) + [x(x-1)/2!](-4) + [x(x-1)(x-2)/3!](18)$

$f(x) \equiv 5 + x + (-2)x(x-1) + (3)x(x-1)(x-2)$

$f(x) \equiv 3x^3 - 11x^2 + 9x + 5$

Also $f(0.5) \equiv 3(0.5)^3 - 11(0.5)^2 + 9(0.5) + 5 = 7.125$

1.4.2 Newton's Backward Interpolation Formula

Newton's backward interpolation formula is used to interpolate the values of y = f(x) near the end ($x < x_n$)) and to extrapolate the values when ($x > x_n$), within the range of given data points (x_i, y_i) , I = 0,1,2,3,….,n.

Let f(x) take the values $y_0, y_1, y_2, \ldots y_n$; for the independent variable x taking values $x_0, x_1, x_2, \ldots x_n$, where height of the interval (h) is fixed, such that

$x_1 = x_0 + h$, $x_2 = x_0 + 2h$, …… , $x_n = x_0 + nh$.

Then to evaluate f(x) for $x = x_n + ph$, $-1 < p < 1$

We have $f(x) = f(x_n + ph) = E^p f(x_n) \equiv (1 - \nabla)^{-p} y_n$

$\therefore E \equiv (1 - \nabla)^{-1}$ and $f(x_n) = y_n$

$\rightarrow f(x) \equiv (1 + p\nabla + [p(p+1)/2!]\nabla^2 + [p(p+1)(p+2)/3!]\nabla^3 + \ldots\ldots)y_n$, $x = x_n + ph$

$\therefore f(x) \equiv y_n + p\nabla y_n + [p(p+1)/2!]\nabla^2 y_n + [p(p+1)(p+2)/3!]\nabla^3 y_n + \ldots\ldots$, $p = x - x_n/h$

This is Newton's backward interpolation formula and is used to interpolate or extrapolate values near the end of the table.

- P is the index which is 0 , where $x = x_n$

Example 3 Following table gives the census population of a state for the years 1971 to 2011 . Estimate the population for the years 1974 and 2005 by using appropriate interpolation technique.

Year	1971	1981	1991	2001	2011
Population (Million)	46	66	81	93	101

Solution: Function has to be evaluated near the beginning and also near the end of the table, thereby constructing difference table for the function y = f(x)

Year x	Population f(x)	1st diff	2nd diff	3rd diff	4th diff
1971	46				
		20			
1981	66		-5		
		15		2	
1991	81		-3		-3
		12		-1	
2001	93		-4		
		8			
2011	101				

To calculate the population for the year 1974 , using Newton's forward interpolation formula given by:

$f(x) \equiv y_0 + p\Delta y_0 + [p(p-1)/2!]\Delta^2 y_0 + [p(p-1)(p-2)/3!]\Delta^3 y_0 + \ldots\ldots$, $p = x - x_0/h$……(1)

$x_0 = 1971$, $y_0 = 46$, $x = 1974$, $h = 10$ $\therefore$ $p = 1974 - 1971/10 = 0.3$

Also from table $\Delta y_0 = 20$, $\Delta^2 y_0 = -5$, $\Delta^3 y_0 = 2$, $\Delta^4 y_0 = -3$

Substituting these values in (1), we get

f(1974) ≡ 46 + (0.3)(20) + [0.3(0.3-1)/2](-5) +[0.3(0.3-1)(0.3-2)/6] (2) + [0.3(0.3-1)(0.3-2)(0.3-3)/24](-3)

≡ 46 + 6 + 0.525 + 0.119 + 0.1205

= 52.7645 Million

To calculate the population for the year 2005 , using Newton's backward interpolation formula given by:

$f(x) \equiv y_n + p \nabla y_n + [p(p+1)/2!] \nabla^2 y_n + [p(p+1)(p+2)/3!] \nabla^3 y_n + \ldots\ldots , p = x-x_n/h$ ……(2)

$x_n = 2011$, $y_n = 101$, x = 2005 , h = 10 , ∴ $p = 2005-2011/10 = -0.6$

Also from table $\nabla y_n = 8$, $\nabla^2 y_n = -4$, $\nabla^3 y_n = -1$, $\nabla^4 y_n = -3$

Substituting these values in (2) , we get

F(2005) ≡ 101 + (-0.6)(8) + [-0.6(-0.6+1)/2](-4) + [-0.6(-0.6+1)(-0.6+2)/6](-1) +

[0.6(-0.6+1)(-0.6+2)(-0.6+3)/24](-3)

≡ 101 – 4.8 + 0.48 + 0.056 + 0.1008

= 96.837 Million

Example 4 Given a set of points for the function f(x) , evaluate f(2.8) and f(3.5) .

X	0	1	2	3
f(x)	1	2	1	10

Solution: Function has to be evaluated near the end of the table, thereby constructing backward difference table for the function y = f(x)

X	f(X)	∇	∇^2	∇^3
0	1			
		1		
1	2		-2	
		-1		12
2	1		10	
		9		
3	10			

Newton's backward interpolation formula given by:

$f(x) \equiv y_n + p \nabla y_n + [p(p+1)/2!] \nabla^2 y_n + [p(p+1)(p+2)/3!] \nabla^3 y_n + \ldots\ldots\ldots, p = x-x_n/h$ .....(1)

¡. To find f(2.8)

$x_n = 3, y_n = 10, x = 2.8, h = 1 \therefore p = 2.8-3/1 = -0.2$

Also from table $\nabla y_n = 9, \nabla^2 y_n = 10, \nabla^3 y_n = 12$

Substituting these values in (1) , we get

$f(2.8) \equiv 10 + (-0.2)(9) + [-0.2(-0.2+1)/2](10) + [-0.2(-0.2+1)(-0.2+2)/6](12)$

$= 6.824$

¡¡. To find f(3.5)

$x_n = 3, y_n = 10, x = 3.5, h = 1 \therefore p = 3.5-3/1 = 0.5$

Also from table $\nabla y_n = 9, \nabla^2 y_n = 10, \nabla^3 y_n = 12$

Substituting these values in (1) , we get

$f(3.5)) \equiv 10 + (0.5)(9) + [0.5(0.5+1)/2](10) + [0.25(0.5+1)(0.5+2)/6](12)$

$= 22$

1.4.3 Gauss's Forward and Backward Difference Formulae

Gauss central difference formula is used to interpolate the values of y near the middle of the table.

Newton's forward difference formula is given by:

$f(x) \equiv y_0 + p \Delta y_0 + [p(p-1)/2!] \Delta^2 y_0 + [p(p-1)(p-2)/3!] \Delta^3 y_0 + \ldots\ldots\ldots, p = x-x_0/h$……(1)

Now $\Delta^3 y_{-1} = \Delta^2 y_0 - \Delta^2 y_{-1}$

$\Delta^2 y_0 = \Delta^2 y_{-1} + \Delta^3 y_{-1}$ ……(2)

Similarly $\Delta^3 y_0 = \Delta^3 y_{-1} + \Delta^4 y_{-1}$ ……(3)

$\Delta^4 y_0 = \Delta^4 y_{-1} + \Delta^5 y_{-1}$ ……(4)

Substituting $\Delta^2 y_0, \Delta^3 y_0, \Delta^4 y_0$ from (2) , (3) , (4) in (1) , we get

$f(x) \equiv y_0 + p \Delta y_0 + [p(p-1)/2!][\Delta^2 y_{-1} + \Delta^3 y_{-1}] + [p(p-1)(p-2)/3!][\Delta^3 y_{-1} + \Delta^4 y_{-1}] +$

$[p(p-1)(p-2)(p-3)/4!][\Delta^4 y_{-1} + \Delta^5 y_{-1}] + \ldots\ldots\ldots$

Rewriting by collecting the coefficients of $\Delta^2 y_{-1}, \Delta^3 y_{-1}, \Delta^4 y_{-1}$..... , we get

$f(x) \equiv y_0 + p\,\Delta y_0 + [p(p-1)/2!]\,\Delta^2 y_{-1} + [(p+1)\,p(p-1)/3!]\,\Delta^3 y_{-1} +$

$[(p+1)p(p-1)(p-2)/4!]\,\Delta^4 y_{-1} + \ldots\ldots$ .....(5)

Again $\Delta^4 y_{-1} = \Delta^4 y_{-2} + \Delta^5 y_{-2}$ .....(6)

Using ⑥ in (5) , we get

$f(x) \equiv y_0 + p\,\Delta y_0 + [p(p-1)/2!]\,\Delta^2 y_{-1} + [(p+1)\,p(p-1)/3!]\,\Delta^3 y_{-1} +$

$[(p+1)p(p-1)(p-2)/4!]\,\Delta^4 y_{-2} + \ldots\ldots$ .....(7)

Expression given by ⑦ is known as **Gauss forward interpolation formula**

Again $\Delta^2 y_{-1} = \Delta y_0 + \Delta y_{-1}$

$\Delta y_0 = \Delta y_{-1} + \Delta^2 y_{-1}$ .....(2)'

Similarly $\Delta^2 y_0 = \Delta^2 y_{-1} + \Delta^3 y_{-1}$ .....(3)'

$\Delta^3 y_0 = \Delta^3 y_{-1} + \Delta^4 y_{-1}$ .....(4)'

Substituting Δy_0, $\Delta^2 y_0$, $\Delta^3 y_0$ from (2)' , (3)' , (4)' in (1) , we get

$f(x) \equiv y_0 + p\,(\Delta y_{-1} + \Delta^2 y_{-1}) + [p(p-1)/2!][\Delta^2 y_{-1} + \Delta^3 y_{-1}] + [p(p-1)(p-2)/3!]\,[\Delta^3 y_{-1} +$

$\Delta^4 y_{-1}] + [p(p-1)(p-2)(p-3)/4!]\,[\Delta^4 y_{-1} + \Delta^5 y_{-1}] + \ldots\ldots$

Rewriting by collecting the coefficients of Δy_{-1}, $\Delta^2 y_{-1}$, $\Delta^3 y_{-1}$, $\Delta^4 y_{-1}$, we get

$f(x) \equiv y_0 + p\,\Delta y_{-1} + [p(p+1)/2!]\,\Delta^2 y_{-1} + [(p+1)\,p(p-1)/3!]\,\Delta^3 y_{-1} +$

$[(p+1)p(p-1)(p-2)/4!]\,\Delta^4 y_{-1} + [p(p-1)(p-2)(p-3)/4!]\,\Delta^5 y_{-1} + \ldots$ .....(5)'

Again $\Delta^3 y_{-1} = \Delta^3 y_{-2} + \Delta^4 y_{-2}$ and $\Delta^4 y_{-1} = \Delta^4 y_{-2} + \Delta^5 y_{-2}$ .....(6)'

Using(6)' in (5)' , we get

$f(x) \equiv y_0 + p\,\Delta y_{-1} + [(p+1)p/2!]\,\Delta^2 y_{-1} + [(p+1)p(p-1)/3!]\,[\Delta^3 y_{-2} +$

$\Delta^4 y_{-2}] + [(p+1)p(p-1)(p-2)/4!]\,[\Delta^4 y_{-2} + \Delta^5 y_{-2}] + \ldots\ldots$

$f(x) \equiv y_0 + p\,\Delta y_{-1} + [(p+1)p/2!]\,\Delta^2 y_{-1} + [(p+1)p(p-1)/3!]\,\Delta^3 y_{-2}$

$+ [(p+2)(p+1)p(p-1)/4!]\,\Delta^4 y_{-2} + \ldots\ldots$ .....(7)'

Expression given by is known as **Gauss backward interpolation formula**

Example 5 Given a set of points for the function , evaluate using Gauss's forward Gauss's backward interpolation formulae

X	25	30	35	40
f(x)	0.25	0.3	0.33	0.37

Solution: Function has to be evaluated near centre of the table, thereby constructing difference table for the function y = f(x), taking $x_0 = 30$

X	p	f(x)	Δ	Δ^2	Δ^3
25	-1	0.25			
			0.05		
30	0	0.3		-0.02	
			0.03		0.03
35	1	0.33		0.01	
			0.04		
40	2	0.37			

¡.Gauss forward interpolation formula is given by

$f(x) \equiv y_0 + p\,\Delta y_0 + [p(p-1)/2!]\,\Delta^2 y_{-1} + [(p+1)\,p(p-1)/3!]\,\Delta^3 y_{-1} +$

$[(p+1)p(p-1)(p-2)/4!]\,\Delta^4 y_{-2} + \ldots\ldots\ldots,\ p = x-x_0/h$ ……..(1)

To find f(33),

$X_0 = 30$, $y_0 = 0.3$, $x = 33$, $h = 5$ ∴ $p = 33-30/5 = 0.6$

Also $\Delta^n y_0$, n = 1,2 lie along the dotted line as shown

∴ From table $\Delta y_0 = 0.03$, $\Delta^2 y_{-1} = -0.02$, $\Delta^3 y_{-1} = 0.03$

Substituting these values in (1) , we get

$f(33) \equiv 0.3 + (0.6)(0.03) + [0.6(0.6-1)/2]\,(-0.02) + [(0.6+1)0.6(0.6-1)/6](0.03)$

$f(33) = 0.3185$

¡¡. Gauss backward interpolation formula is given by

$f(x) \equiv y_0 + p\,\Delta y_{-1} + [(p+1)p/2!]\,\Delta^2 y_{-1} + [(p+1)p(p-1)/3!]\,\Delta^3 y_{-2}$, $p = x-x_0/h$ ……(2)

To find f(33),

$X_0 = 30$, $y_0 = 0.3$, $x = 33$, $h = 5$ ∴ $p = 33-30/5 = 0.6$

Also $\Delta^n y_0$, n = 1,2 lie along the dotted line as shown

. . From table $\Delta y_{-1} = 0.05$, $\Delta^2 y_{-2} = -0.02$, $\Delta^3 y_{-1} = 0$

Substituting these values in (2) , we get

$f(33) \equiv 0.3 + (0.6)(0.05) + [0.6(0.6-1)/2](-0.02) + [(0.6+1)0.6(0.6-1)/6](0)$

$f(33) = 0.3324$

Example 6. Find the value of cos 51^0. 42' by Gauss backward formula given that

X	50^0	51^0	52^0	53^0	54^0
Cosx	0.6428	0.6293	0.6157	0.6018	0.5878

Solution : Let us take a =50^0 as origin h =1^0

Put a+uh = $51^0 42^1, 52^0$ +u = $51^0 42^l$ => u= -18^l = $\frac{-18}{60}$ = 0.3, u = -0.3

u	10^4 f(u)	$10^4\ \delta f(u)$	$10^4\ \delta^2$ f(u)	$10^4\ \delta$f(u)	$10^4\ \delta^4$ f(u)
-2	6428	-135			
-1	6293	-136	-1	-2	4
0	6157	-139	-3	2	
1	6018	-140	-1		
2	5878				

1.4.4 Stirling's Central Difference Formula

Stirling gave the most general formula for interpolating values near the centre of the table by taking mean of Gauss forward and Gauss backward interpolation formulae.

Taking mean of expressions given by (7) and (7)' respectively, we get

$f(x) \equiv y_0 + p\,[\Delta y_0 + \Delta y_{-1}/2] + [p(p-1)/2! + [(p+1)p/2!]\,(\Delta^2 y_{-1}/2) + [\,(p+1)\,p(p-1)/3!]$

$[\Delta^3 y_{-1} + \Delta^3 y_{-2}/2]\,[\,(p+1)p(p-1)(p-2) + (p+2)(p+1)p(p-1)/4!]\,[\Delta^2 y_{-2}/2] + \ldots$

$f(x) \equiv y_0 + p\,[\Delta y_0 + \Delta y_{-1}/2] + (p^2/2!)\,\Delta^2 y_{-1} + [\,p(p^2-1)/3!][\Delta^3 y_{-1} + \Delta^3 y_{-2}/2]$

$+[\,p^2(p^2-1)/4!]\,\Delta^4 y_{-2} + \ldots$ .....(8)

Expression given in (8) is known as **Stirling's central difference formula** Putting ½ ($\Delta y_0 + \Delta y_{-1}$) = ½ ($\delta y_{1/2} + \delta y_{-1/2}$) = $\mu\,\delta y_0$

$$\tfrac{1}{2}(\Delta^3 y_{-1}+\Delta^3 y_{-2}) = \tfrac{1}{2}(\delta^3 y_{1/2} + \delta^3 y_{-1/2}) = \mu\,\delta^3 y_0$$

.

.

.

In terms of central differences, (8)takes the form

$$f(x) \equiv y_0 + p\mu\delta y_0 + (p^2/2!)\delta^2 y_0 + [\,p(p^2-1)/3!]\,\mu\delta^3 y_0 + [\,p^2(p^2-1)/4!]\delta^4 y_0 + \ldots$$

This is another form of **Stirling's central difference formula.**

The difference table used to evaluate f(x) as per Stirling's formula, is shown below. Column wise averages of differences (shown in boxes) are taken while evaluation phase.

X	p	f(x)	1st diff	2nddiff	3rd diff	4th diff
x_{-2}	-2	y_{-2}				
			Δy_{-2}			
x_{-1}	-1	y_{-1}				
			Δy_{-1}		$\Delta^3 y_{-2}$	
x_0	0	y_0		$\Delta^2 y_{-1}$		$\Delta^4 y_{-2}$
			Δy_0		$\Delta^3 y_{-1}$	
x_1	1	y_1		$\Delta^2 y_0$		
			Δy_1			
x_2	2	y_2				

Example 7 Given a set of points for the function , evaluate using Stirling's central difference formula.

x	25	30	35	40	45
f(x)	0.25	0.3	0.33	0.37	0.43

Solution: Function has to be evaluated near centre of the table, thereby constructing difference table for the function y = f(x) , taking $x_0 = 35$. Also $\Delta^n y_0$, n = 1,2 lie along the dotted line as shown.

X	p	f(x)	Δ	Δ^2	Δ^3	Δ^4
25	-2	0.25				
			0.05			
30	-1	0.3		-0.02		
			0.03		0.03	
35	0	0.33		0.01		-0.02
			0.04		0.01	
40	1	0.37		0.02		
			0.06			
45	2	0.43				

Stirling's central differences formula is given by

$f(x) \equiv y_0 + p\,[\Delta y_0 + \Delta y_{-1}/2] + (p^2/2!)\,\Delta^2 y_{-1} + [\,p(p^2-1)/3!][\Delta^3 y_{-1} + \Delta^3 y_{-2}/2]$

$+[\,p^2(p^2-1)/4!]\,\Delta^4 y_{-2} + \ldots \quad , p = x - x_0/h \qquad \ldots\ldots..(1)$

To find f(33),

$X_0 = 35 \,,\ y_0 = 0.33 \,,\ x = 33 \,,\ h = 5 \qquad \therefore\ p = 33 - 35/5 = -0.4$

Also from table $\Delta y_0 = 0.04$, $\Delta y_{-1} = 0.03$, $\Delta^2 y_{-1} = 0.01$, $\Delta^3 y_{-1} = 0.01$, $\Delta^3 y_{-2} = 0.03$, $\Delta^4 y_{-2} = -0.02$. All the positions have been shown, enclosed in boxes.

Substituting these values in (1) , we get

$f(33) \equiv 0.33 + (-0.4)(-0.04 + 0.03/2) + [(-0.4)^2/2]\,(0.01) + [-0.4((0.4)^2 - 1)/6][0.01 + 0.03/2] + [(-0.4)^2\,((0.4)^2 - 1)/24]\,(-0.02)$

$\rightarrow f(33) = 0.33 - 0.014 + 0.0008 + 0.0011 + 0.0001 = 0.318$

Example 8 Use Stirling's formula to evaluate given that

x	1	1.1	1.2	1.3	1.4
f(x)	0.841	0.891	0.932	0.963	0.985

Solution: Function has to be evaluated near centre of the table, thereby constructing difference table for the function y = f(x) , taking $x_0 = 1.2$. Also $\Delta^n y_0$, n = 1,2 lie along the dotted line as shown.

X	p	f(x)	Δ	Δ^2	Δ^3	Δ^4
1.0	-2	0.841				
			0.050			
1.1	-1	0.891		-0.009		
			0.041		-0.001	
1.2	0	0.932		-0.01		0.002
			0.031		0.001	
1.3	1	0.963		-0.009		
			0.022			
1.4	2	0.985				

Stirling's central differences formula is given by

$f(x) \equiv y_0 + p\,[\Delta y_0 + \Delta y_{-1}/2] + (p^2/2!)\,\Delta^2 y_{-1} + [\,p(p^2-1)/3!][\Delta^3 y_{-1} + \Delta^3 y_{-2}/2]$

$+[\,p^2(p^2-1)/4!]\,\Delta^4 y_{-2} + \ldots$, $p = x - x_0/h$ ……..(1)

To find f(1.22),

$X_0 = 1.2$, $y_0 = 0.932$, $x = 1.22$, $h = 0.1$ $\therefore$ $p = 1.22 - 1.2/0.1 = 0.2$

Also from table $\Delta y_0 = 0.031$, $\Delta y_{-1} = 0.041$, $\Delta^2 y_{-1} = -0.01$, $\Delta^3 y_{-1} = 0.001$, $\Delta^3 y_{-2} = -0.001$, $\Delta^4 y_{-2} = 0.002$. All the positions have been shown, enclosed in boxes.

Substituting these values in (1) , we get

$f(1.22) \equiv 0.932 + (0.2)(0.031 + 0.041/2) + [(0.2)^2/2]\,(-0.01) + [0.2((0.2)^2 - 1)/6][0.001 - 0.001/2] + [(0.2)^2\,((0.2)^2 - 1)/24]\,(0.002)$

$\rightarrow$ $f(1.22) = 0.932 + 0.0072 - 0.0002 + 0 - 0.0000032 = 0.9390$

1.4.5 Bessel's Formula

Gauss's forward interpolation formula is

$f(x) \equiv y_0 + p\,\Delta y_0 + [p(p-1)/2!]\,\Delta^2 y_{-1} + [\,(p+1)\,p(p-1)/3!]\,\Delta^3 y_{-1} +$

$[\,(p+1)p(p-1)(p+1)/4!]\,\Delta^4 y_{-2} + \ldots\ldots$

We have $\Delta^2 y_0 - \Delta^2 y_{-1} = \Delta^3 y_{-1}$ ……..(1)

i.e., $\Delta^2 y_{-1} = \Delta^2 y_0 - \Delta^3 y_{-1}$ …….(2)

Similarly $\Delta^4 y_{-2} = \Delta^4 y_{-1} - \Delta^4 y_{-2}$ etc

Now (1) can be written as

$f(x) \equiv y_0 + p\Delta y_0 + [p(p-1)/2!][1/2\ \Delta^2 y_{-1} + 1/2\ \Delta^2 y_{-1}] + [\ p(p^2-1)/3!]\ \Delta^3 y_{-1} +$

$[\ p(p^2-1)(p-2)/4!][1/2\ \Delta^4 y_{-2} + 1/2\ \Delta^4 y_{-2}] + \ldots\ldots\ldots$

$= y_0 + p\Delta y_0 + [\ ½\ p(p+1)/2!]\ \Delta^2 y_{-1} + [1/2\ \ p(p-1)/2!]\ [\Delta^2 y_0 + \Delta^3 y_{-1}\ ] +$

$[\ p(p^2-1)/3!]\ \Delta^3 y_{-1} + [1/2\ \ p(p^2-1)(p-2)/4!]\ \Delta^4 y_{-2} + [1/2\ \ p(p^2-1)(p-2)/4!]$

$[\Delta^4 y_{-2} - \Delta^5 y_{-1}] + \ldots\ldots\ldots$ [using (2) , (3) etc]

Hence

$f(x) \equiv y_0 + p\Delta y_0 + [p(p-1)/2!][\ \Delta^2 y_{-1} + \Delta^2 y_0/2\ ] + [(p-1/2)\ p(p-1)/3!]\ \Delta^3 y_{-1}$ ……(4)

Which is known as **Bessel's formula.**

Example 9. Apply Bessel's formula to obtain y_{25}, given $y_{20} = 2854$, $y_{24} = 3162$, $y_{32} = 3992$

Solution. take 24 as origin and 4 $u = \frac{25-24}{4} = \frac{1}{4} = 0.25$

Difference table is

$\mathbf{X}$	$\mathbf{u = \frac{x-24}{4}}$	$\mathbf{Y_u}$	$\boldsymbol{\delta y_u}$	$\boldsymbol{\delta^2 y_u}$	$\boldsymbol{\delta^3 y_u}$
20	-1	2854			
			308		
24	0	3162		74	
			382		-8
28	1	3544		64	
			448		
32	2	3992			

Using bessel's formula, we get

$Y_{1/4} = \frac{1}{2}(3162+3544) + (\frac{1}{4} - \frac{1}{2})382 + \frac{\frac{1}{4}(\frac{1}{4}-\frac{1}{2})}{2!}(\frac{14+66}{2}) + \frac{(\frac{1}{4}-\frac{1}{2})(\frac{1}{4})(\frac{1}{4}-1)}{3!}(-8)$

$= 3353 - 95.5 - 6.5625 - 0.0625 = 3250.875.$

Example 10.Given $y_{20} = 24$, $y_{24} = 32$, $y_{28} = 35$, $y_{32} = 40$ find y_{25} by Bessel's formula.

Solution : Taking origin at x =24 and u as the unit.

Find y_u for $u = \frac{25-24}{4} = \frac{1}{4} = 0.25$

The difference table is

X	$U=\frac{x-24}{4}$	Y_u	δy_u	$\delta^2 y_u$	$\delta^3 y_u$
20	-1	24			
			8		
24	0	32		-5	
			3		7
28	1	35		2	
			5		
32	2	40			

By using Bessel's formula we have

$$y_{\frac{1}{4}} = \frac{32+35}{2} + (\frac{1}{4} - \frac{1}{2})3 + \frac{\frac{1}{4}\left(\frac{1}{4}-1\right)\left(\frac{-5+2}{2}\right)}{2} + \frac{(\frac{1}{4}-\frac{1}{2})\frac{1}{4}\left(\frac{1}{4}-1\right)}{6}(7)$$

= 33.5-0.75+ 0.1406+0.54687 = 32.945287.

1.5 Applications in engineering :

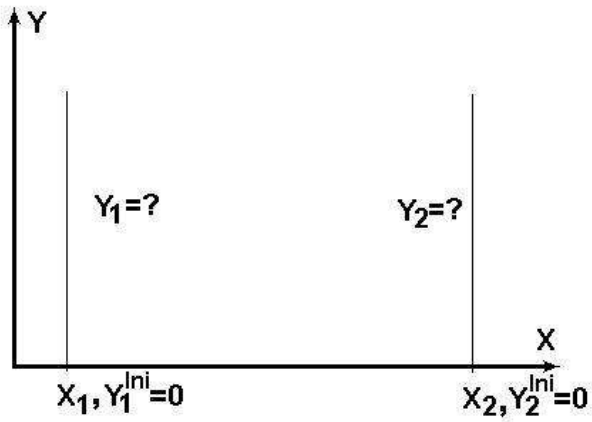
General Ideas Regarding the Use of the Interpolation in the Numerical Analysis

There are certain cases when approximation solutions are useful within the numerical methods field itself.

Some of these cases are presented in the next sections.

Approximation of the Initial Solution for Iterative Methods

Let us consider a theoretical problem where the formulation of the problem depends on some parameters and an iterative solution is possible. It is well known that the final solution depends on the initial approximation in terms of accuracy and of computer runtime.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X
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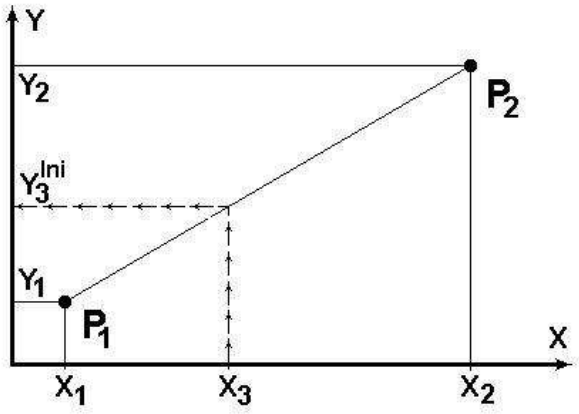
Y2=?
X1, Y1Ini=0
X2, Y2Ini=0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Y3Ini
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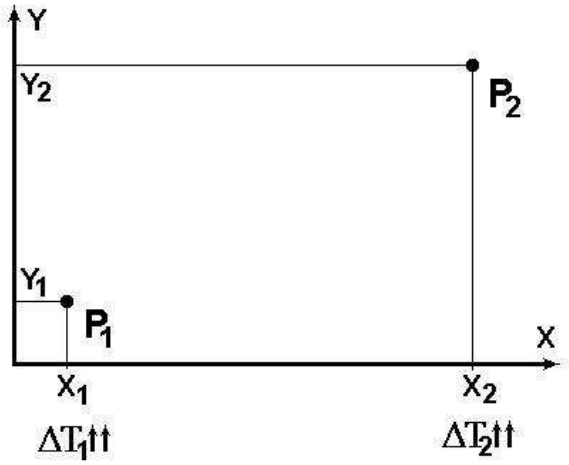
X1
X3
X2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Y1
P1
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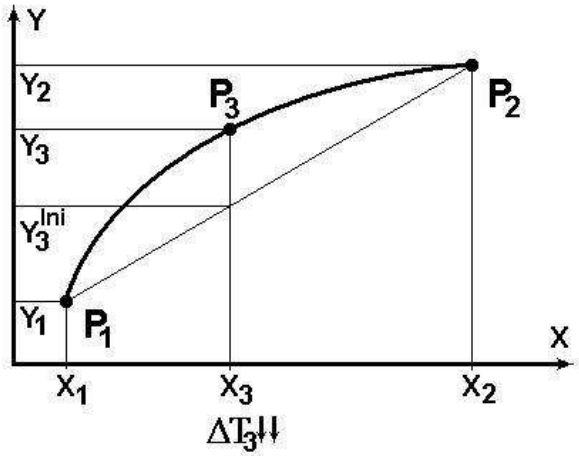
X2
ΔT1↑↑
ΔT2↑↑


Y
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Y2
P3
P2
Y3
Y3Ini
Y1
P1
X1
X3
X2
ΔT3↓↓

Calculus of the Integrals :

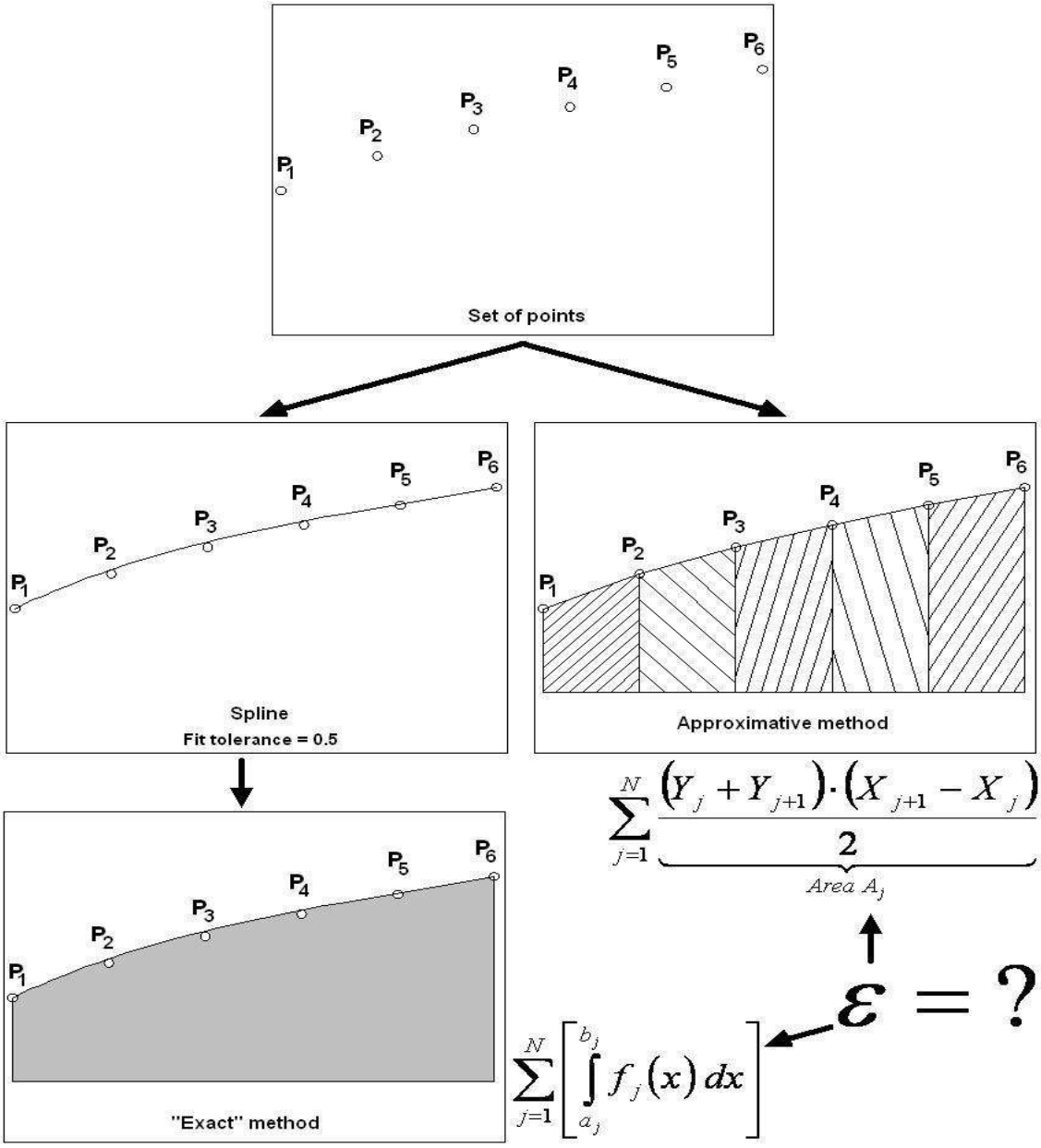
Let us consider that a set of points are known, that is $P_j\, x_j , y_j , j\ 1, N$. Usually these points result from experimental studies. There must be computed the integral of the function approximated by this set of points.

A first method to compute the integral is to use the summation of the areas of the trapezoids defined by two consecutive points. This is a coarse approximation, but if the points are close one to the other the accuracy may be considered satisfactory.

The second method has two stages. The first one is to approximate the set of points using one of the previously mentioned interpolation methods, and it results a set of equations like (6). The second stage is to analytically integrate the functions and to calculate the result. In his case the source of errors is in the interpolation stage.

A similar approach can be used to compute the derivative of the functions, either using the initial points, or using the appropriate interpolation functions.

The error for each of the methods may be evaluated for a certain dataset. Basically, it is necessary to use as many methods exist, in order to have several results which, ideally, must be close one to the other.


P1
P2
P3
P4
P5
P6
Set of points
Spline
Fit tolerance = 0.5
Approximative method
"Exact" method
$\sum_{j=1}^{N} \underbrace{\frac{(Y_j + Y_{j+1}) \cdot (X_{j+1} - X_j)}{2}}_{Area\ A_j}$
$\varepsilon = ?$
$\sum_{j=1}^{N} \left[\int_{a_j}^{b_j} f_j(x)\, dx \right]$

2. NUMERICAL INTEGRATION

2.1 INTRODUCTION :

The numerical integration is the process of evaluating the approximate Value of the integration $\int_a^b f(x)dx$ from a set of numerical values of f(x). If the problem is to be applied to the integration of function of a single variable, then the process is called as quadrature.

The problem of numerical integration is evolved by first approximating the f(x) by a polynomial with the help of interpolation formula and then integrating this expression between the limits.

$\therefore$ To find the value of $\int_a^b f(x)\,dx$, first express the function f(x) by interpolation formula, say p(x) and then integrate between the limits.

$$\therefore \int_a^b f(x)dx = \int_a^b p(x)dx$$

The error is given by $\int_a^b f(x)dx - \int_a^b p(x)dx$

2.2 A GENERAL QUADRATURE FORMULA FOR NUMERICAL INTEGRATION

Consider the integral $\int_a^b f(x)dx$. Let f(x) be given for some equally spaced values of the orguments say ,x_0 , x_0+h ,x_0+2h,…. Let the range (b-a) be divided into n equal parts, each of which is of width h.

$$b-a = nh$$

$$h = \frac{b-a}{n}$$

let $x_0 = a, x_1 = x_0+h = a+h$, $x_2 = a+2h$ ….. $x_n = a+nh = b$

$$I = \int_a^{a+nh} f(x)dx$$

Putting $x = a+uh$

$$dx = hdu$$

x -> a -> u ->0

x ->a+uh -> u->n

$I = \int_a^n f(a+uh)du = h\int_a^n f(a+uh)du$

No write as f(a+uh) on the Newton's interpolation formula as

$= h\int_0^n [f(a) + \frac{u}{1!}\Delta f(a) + \frac{u(u-1)}{2!}\Delta^2 f(a) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(a) + \cdots + \frac{u(u-1)\ldots(u-n+1)}{n!}\Delta^n f(a)\,]\,du$

$= h\,[f(a).u + \frac{u^2}{2}\Delta f(a) + (\frac{u^3}{3} - \frac{u^2}{2})\,\Delta^2 f(a) + \frac{(\frac{u^4}{4} - \frac{3u^3}{3} + \frac{2u^2}{2})}{6}\Delta^3 f(a) + \cdots +]$

$= h[nf(a) + \frac{n^2}{2}\Delta f(a) + (\frac{n^2}{6} - \frac{n^2}{4})\,\Delta^2 f(a) + (\frac{n^4}{24} - \frac{n^3}{6} + \frac{n^2}{6})\Delta^3 f(a) +]$

$\int_a^b f(x)dx = h[ny_0 + \frac{n^2}{2}(y_1-y_0) + (\frac{n^3}{6} - \frac{n^2}{4})(y_2-2y_1+y_0) + \left(\frac{n^4}{24} - \frac{n^3}{6} + \frac{n^2}{6}\right)$

$(y_3-3y_2+3y_1-y_0)+\ldots\ldots\ldots] \ldots\ldots\ldots\ldots(1)$

$\int_a^b f(x)dx = \int_a^{a+nh} f(x)dx$

This is known as general quadrature formula we deduce the following important quadratic formulas by taking n= 1,2,3….

2.3 TRAPEZOIDAL RULE :

Put n=1 eq(1) and neglect the second and higher order differences when n=1 , the argument are a, a+h and its corresponding functional values are y_0 and y_1 thus we have

$\int_a^{a+nh} f(x)dx = h[y_0 + \frac{1}{2}(y_1-y_0)] = \frac{h}{2}(y_0+y_1)$

$I_1 = \int_a^{a+nh} f(x)dx = \frac{h}{2}(y_0+y_1)$

$I_2 = \int_a^{a+nh} f(x)dx = \frac{h}{2}(y_0+y_1)$

$I_3 = \int_{a+2h}^{a+3h} f(x)dx = \frac{h}{2}(y_2+y_3)$

……..

……..

$I_n = \int_{a+(n-1)h}^{a+nh} f(x)dx = \frac{h}{2}(y_{n-1}+y_n)$

Adding these n integrals, we have

$\int_a^{a+nh=b} f(x)dx$ = $\frac{h}{2}$ [(y_0+y_n)+2(y_1+y_2+y_3+…+y_{n-1})]

This rule is called the trapezoidal rule.

2.4 SIMPSON'S $\frac{1}{3}$ RULE :

Put n=2 in the equation (1) and neglect the third and higher order difference. In this case the arguments are a, a+h, a+2h the corresponding entries are y_0,y_1 and y_2, thus we have

$\int_a^{a+2h} f(x)dx$ = h[2y_0 + 2(y_1-y_0)+($\frac{y}{3}$ -1)(y_2-2y_1+y_0)]

= $\frac{h}{3}$[y_0+4y_1+y_2]

Similarly I_2 = $\int_a^{a+2h} f(x)dx$ = $\frac{h}{3}$ (y_0+4y_1+y_2)

I_3 = $\int_{a+4h}^{a+6h} f(x)dx$ = $\frac{h}{3}$ (y_4+4y_5 +y_6)

……..

……..

I_n = $\int_{a+(n-2)h}^{a+6h} f(x)dx$ = $\frac{h}{3}$ (y_{n-2}+4y_{n-1}+ y_n)

Adding these integrals, we have I = $\int_a^b f(x)dx$

= $\frac{h}{3}$ [(y_0+y_n)+(y_1+y_3+y_5+…+y_{n-1})+2(y_2+y_4+….+y_{n-2})]

This is known as SIMPSON'S $\frac{1}{3}$ RULE

2.5 SIMPSON'S $\frac{3}{8}$ RULE :

Put n=3 in equation(1) and neglect fourth and higher order differences. In this case, the arguats are a, a+h, a+2h and a+3h and the corresponding fractional values are y_0,y_1,y_2 and y_3, thus we have

$\int_a^{a+3h} f(x)dx$ = n[3y_0+$\frac{9}{2}$(y_1-y_0)+($\frac{9}{2}$ − $\frac{9}{4}$)(y_2-2y_1+y_0) + ($\frac{27}{8}$ − $\frac{27}{6}$ − $\frac{9}{6}$)(y_3- 3y_2+3y_1-y_0)]

= h[3y_0+$\frac{9}{2}$(y_1-y_0)+($\frac{9}{2}$ − $\frac{9}{4}$)(y_2-2y_1+y_0) + ($\frac{27}{8}$ − $\frac{27}{6}$ − $\frac{9}{6}$)(y_3- 3y_2+3y_1-y_0)]

= h[3y_0+$\frac{9}{2}$(y_1-y_0)+($\frac{9}{4}$)(y_2-2y_1+y_0) + ($\frac{3}{8}$)(y_3- 3y_2+3y_1-y_0)]

$I_1 = \int_a^{a+3h} f(x)dx = \frac{3h}{8}(y_0+3y_1+3y_2+y_3)$

$I_2 = \int_{a+4h}^{a+6h} f(x)dx = \frac{3h}{8}(y_3+3y_4+3y_5+y_6)$

……..

……..

$I_n = \int_{a+(n-2)h}^{a+6h} f(x)dx = \frac{3h}{8}(y_{n-3}+3y_{n-2}+3y_{n-1}+y_n)$

Adding all these integrals we get

$\int_a^{a+nh} f(x)dx = \frac{3h}{8}[(y_0+y_n)+3(y_1+y_2+y_4+\ldots+y_{n-1})+2(y_3+y_6+\ldots.+y_{n-3})]$

This formulas is called the simpson's rule.

Note : Applying this formula, the number of sub-internals should be taken as multiple of three

2.6 WEDDLE'S RULE :

put n=6 in equation (1) and neglect the seventh and higher order differences. in this case the arguments are from a to a+6h and there will be only seven functional values, thus we get

$\int_a^{a+6h} f(x)dx = h[6y_0+18\Delta y_0+27\Delta^2 y_0+24\Delta^3 y_0+\frac{123}{10}\Delta^4 y_0+\frac{33}{10}\Delta^5 y_0+\frac{41}{140}\Delta^6 y_0]$

Replacing $\frac{41}{140}\Delta^6 y_0$ by $\frac{3}{10}\Delta^6 y_0$,

Since the error involved is negligible.

Thus above equation can be expressed as

$\int_a^{a+6h} f(x)dx = h[6y_0+18\Delta y_0+27\Delta^2 y_0+24\Delta^3 y_0+\frac{123}{10}\Delta^4 y_0+\frac{33}{10}\Delta^5 y_0+\frac{3}{10}\Delta^6 y_0]$

$= \frac{3h}{10}(y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6)$

Similarly $\int_{a+6h}^{a+12h} f(x)dx = \frac{3h}{10}(y_6+5y_7+y_8+6y_9+y_{10}+5y_{11}+y_{12})$

……..

……..

$\int_{a+(n-6)h}^{a+nh} f(x)dx = \frac{3h}{10}(y_{n-6}+5y_{n-5}+y_{n-4}+6y_{n-3}+y_{n-2}+5y_{n-1}+y_n)$

Adding these integrals, we have

$\int_a^{a+nh} f(x)dx = \frac{3h}{10}(y_0+5y_1+y_2+ 6y_3+y_4+5y_5+2y_6+ 5y_7+y_8 +\ldots\ldots\ldots)$

This formula is known as Weddle's rule.

Note:

1. This rule is more accurate than the Simpson's rule, but it requires atleast seven consecutive values of the function
2. This method is applicable when n is a multiple of 6.

2.7 ERROR IN QUADRATURE FORMULA :

The error terms in the quadrature formula is given by

$$E = \int_a^b y dx - \int_a^b p(x)dx$$

where P(x) is the polynomial representing the function y = f(x) in the interval [a, b].

(i) Error in Trapezoidal Rule : Expanding y = f(x), about $x = x_o$ by Taylor's series, we obtain

$$y = y_0 + (x-x_0)y_0 + \frac{(x-x0)^2}{2!} y_0^{11} + \ldots\ldots\ldots(1)$$

$$\int_{x_0}^{x_0+h} y\, dx = \int_{x_0}^{x_0+h} [y_0 + (x - x_0)y_0 + \frac{(x-x_0)^2}{2!} y_0 + \cdots]dx$$

$$= y_0 n + \frac{h^2}{2!} y_0 + \frac{h^2}{3!} y_0 + \ldots\ldots\ldots\ldots(2)$$

But area of the first trapezium in the interval $[x_o, x_1] = \frac{1}{2}h(y_0 + y_1)$ ....(3)

Then putting $x = x_o + h$ and $y = y_1$ in (1), we obtain

$$y_1 = y_0 + h\, y_0 + \frac{h^2}{2!} y_0 + \ldots\ldots\ldots\ldots$$

Putting this value of y, in (3), we have

$$B = \frac{1}{2} h[y_0 + y_0 + h\, y_0 + \frac{h^2}{2!} y_0 + \ldots\ldots\ldots] = h\, y_0 + \frac{h^2}{2!} y_0 + \frac{h^3}{2.2!} y_0 + \ldots\ldots \quad \ldots\ldots(4)$$

Hence the error in the interval $[x_o, x_1]$

$$= \int_{x_0}^{x_1} y\, dx - B_1 = [\frac{1}{3!} - \frac{1}{2.2!}] h^3 y_0 + \ldots\ldots\ldots\ldots = -\frac{h^3}{12} y_0 + \ldots\ldots.$$

Principal part of the error in $[x_o, x_1] = -\frac{h^3}{12} y_0$

And similarly principal part of the error in $[x_1, x_2] = -\frac{h^3}{12} y_1$ etc.,

Thus, the total error $= -\frac{h^3}{12} [y_0 + y_1 + y_{n-1}] = E$, say

Then assuming that y''(x) is the largest of the n quantities $y_o , y_1 , \ldots , y_{n-1}$, we can say that

$E < -\frac{nh^3}{12} y^{11}(x) = -\frac{(b-a)h^2}{12} y^{11}(x)$ as b-a = nh ……(5)

Thus the error in the trapezoidal rule is of the order h^2.

(ii) Error in Simpson's $\frac{1}{3}$ Rule : Expanding y = f(x) about $x = x_o$ by Taylor's series, we have

$$y = y_0 + (x-x_0)y_0 + \frac{(x-x0)^2}{2!} y_0 + \ldots\ldots$$

$$\int_{x_0}^{x^2} y\,dx = \int_{x_0}^{x_0+2h} [y_0 - (x - x_0) + \frac{(x-x_0)^2}{2!} y_{0+\ldots}]\ dr$$

$$2hy_0 + \frac{4h^2}{2!}y_0 + \frac{8h^3}{3!}y_0 + \frac{16h^4}{4!}y_0 + \ldots + \frac{32h^5}{5!}y_0 + \ldots\ldots\ldots \quad \ldots\ldots\ldots(2)$$

and area over the first double strip by Simpson's 1/3 rule

$$= \frac{1}{3} h(y_0 + 4y_1 + y_2) = A_1 \text{ say}$$

Then putting $x = x_0 + h$ and $y = y_1$ in (1), we get

$$y_2 = y_0 + 2hy_0 + \frac{4h^2}{2!}y_0 + \frac{8h^3}{3!}y_0 + \ldots\ldots\ldots$$

Further putting $x = x_o + 2h$ and y = 1/2 in (1), we get

$$y_2 = y_0 + 2hy_0 + \frac{4h^2}{2!}y_0 + \frac{8h^3}{3!}y_0 + \ldots\ldots\ldots$$

(3) → $A_1 = \frac{h}{3} [y_0 + 4(y_0 + hy_0 + \frac{h^2}{2!} y_{0+\ldots\ldots\ldots}) + (y_0 + 2h\, y_0 + \frac{4h^2}{2!} y_0 + \frac{8h^3}{3!} y_0 + \ldots\ldots\ldots)]$

$$= 2hy_0 + 2h^2y_0 + \frac{4h^3}{3} y_0 + \frac{2h^2}{8} y_0 + \frac{5h^5}{18} y_0 + \ldots\ldots\ldots \quad \ldots\ldots\ldots\ldots\ldots(4)$$

Hence the error in the interval $[x_o, x_2] = \int_{x_0}^{x} y\,dx - A_1$

$$= [\frac{4}{15} - \frac{5}{18}] h^5 y_0^5 + \ldots\ldots \quad \ldots\ldots\ldots\ldots\ldots(5)$$

and the principal part of the error in $[x_o, x_2]$

$$= [\frac{4}{15} - \frac{5}{18}] h^5 y_0^5 = - -\frac{h^5}{90} y_0^5$$

On the same lines principal part of the error in $[x_2, x_4]$

$$= \frac{h^5}{90} y_2^5, \text{ etc.,}$$

Total error = $- -\frac{x^5}{90} [y_0^4 + y_2^4 + \ldots\ldots\ldots + y_2^4 (n-1)] = E$, Say ……(6)

The assuming $y_{1v}(x)$ as the largest of $y_0^{1V}, y_2^{1V} \ldots\ldots, y_{2n-2}^{1V}$

We readily obtain $E < -\frac{n/5}{90} y_0^{1V}(x) = -\frac{(b-a)h^y y^5(x)}{180}$ ……(7) [2nh = b-a]

(iii) Error in Simpson's 3/8 rule : Proceeding as above, we have the

principal part of the error in the interval [x_o, x_3l as = $\frac{3h^5}{80}y^5$

(iv) Error in Weddle's rule: Principal part of the error in the interval[x_0,x_6] = -$(h^7/140)y_0^5$

2.8 SOLVED PROBLEMS ::

1.Evalaute the value of the integral using $\int_2^{1.4}(sinx - logx + e^x)$dx

(a) The Trapoidal rule, (b) Simpson's one-third rule,

(c) Simpson's three eight rule.

Solution. In this case divide the range of integration into twelve equal parts by taking h = -1 and then the values of the function y = sin x -$\log_e x$ +e are computed for each point of subdivision which are as under

x	Sinx	$\text{Log}_e x=\log_{10} x x 2.30258$	e^x	$Y=\sin x-\log_e x+e^x$
0.2	0.19867	-1.60943	1.22140	3.02950
0.3	0.29552	-1.20397	1.34986	2.84935
0.4	0.38942	-0.91629	1.49182	2.79753
0.5	0.47943	-0.69315	1.64872	2.82130
0.6	0.5646	-0.51083	1.82212	2.89759
0.7	0.64422	-0.35667	2.01375	3.01464
0.8	0.71736	-0.22314	2.22554	3.16604
0.9	0.78333	-0.10537	2.45960	3.34830
1.0	0.84147	+0.00000	2.71828	3.55975
1.1	0.89121	0.09530	3.00417	3.80008
1.2	0.93204	0.18232	3.32012	4.06984
1.3	0.96356	0.26236	3.66930	4.37050
1.4	0.98545	0.33647	4.05520	4.70418

(a)By Trapezoidal rule, we have

$\int_2^{1.4}$(sin x -$\log_e^x$ + e^x)dx = $\int_2^{2+12x\ 0.1} y\ dx$

Here n = 12; h = 0.1, x_o = 0.2, y = sin x -$\log_e^r$+ e^x

Again w.k.t. $\int_{x0}^{xo\ +nh} y\ dx$ = $\frac{h}{2}$[y_0+y_{n+2}(y_1+y_2+…+y_{n-1})]

Substituting these values of, h_1, y_o, y_n…, in the above formula we get

$\int_2^{1.4}(\sin x - \log ex + e^x)$ dx = $\frac{1}{20}$[3.02950+ 4.70418+2(2.8493+…+4.37050)]

= $\frac{1}{20}$[7.73368+2(36.69492)]

$$= 4.05617$$

(b) By Simpson's rule $\frac{1}{3}$ we have

$$\int_{x0}^{xo+nh} y\,dx = \frac{h}{3}[y_0+y_6+4(y_1+y_3+\ldots)+2(y_2+y_4+\ldots)]$$

Put the values of h, y_o, $y_{1..}$... from the table, we have

$$= \int_2^{1.4}(\sin x - \log ex + e^x)\,dx$$

$$= \frac{1}{3}[7.73368+ 4(20.20417)-3(16.49075)]$$

$$= \frac{1}{3}[121.53186] = 4.05106$$

(c) By Simpson's 3/8 rule we have

$$\int_{x0}^{xo+nh} y\,dx = \frac{3h}{8}[(y_0+ y_n)+3(y_1+y_2+y_4+\ldots+y_{n-1})+2(y_3+y_6+\ldots.+y_{n-3})]$$

$$= \frac{3(0.1)}{8}[7.73368+ (26.90750)+ 2(9.78742)] = 4.05116$$

2. A curve is drawn to pass through the points given by the following table.

x	1	1.5	2	2.5	3	3.5	4
y	2	2.4	2.7	2.8	3	2.6	2.1

Estimate the area bounded by the curve, the x-axis and the lines x=1,x=4.

Solution. By Simpson's rule, we have required area

$$= \frac{h}{3}[y_0+4y_1+2y_2+4y_3+\ldots\ldots.+4y_{n-1}+y_n]$$

$$= \frac{0.5}{3}[2+ 9.6+5.4+11.2+10.4+ 2.1]$$

$$= \frac{50}{30}x46.7 = \frac{23.35}{3} = 7.783 \text{ unit of area}$$

By Weddle's rule, we have required area

$$= \frac{3h}{10}[y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6]$$

$$= \frac{3x0.5}{10}[2+12.0+2.7+16.8+3.13.0+2.1] = 7.74 \text{ units of area}$$

3. Calculate by Simpson's rule an approximate value of by taking seven equidistant ordinates. Compare it with the exact value and the value obtained by using the Trapezoidal rule.

Solution. Let us divide the range of integration (-3, 3) into six equal parts each of width $\frac{3(-3)}{6}$ = 1 unit. Hence h-1, value of the function for each point of sub division are as given below.

x	$y= x^4$
-3	$(-3)^4 = 81$
-2	$(-2)^4 = 16$
-1	$(-1)^4 = 1$
0	$(0)^4 = 0$
1	$(1)^4 = 1$
2	$(2)^4 = 16$
3	$(3)^4 = 81$

Now by Simpson's rule, we obtain

$$\int_{-3}^{3} x^4 \, dx = \frac{h}{3}[y_0+y_6+4(y_1+y_3+y_5)+2(y_2+y_4)]$$

$$= \frac{1}{3}[162+4x32+ 2x2] = \frac{1}{3}x294 = 98$$

But the exact value of

$$\int_{-3}^{3} x^4 \, dx = \left[\frac{x^5}{5}\right]_{-3}^{3} = \frac{1}{5}[(3)^5 -(-3)^5] = \frac{1}{5}x486 = 97.2$$

Also by Trapezoidal rule, we have

$$\int_{-3}^{3} x^4 \, dx = \frac{h}{2}[y_0+y_6+2(y_1+ y_{2+}y_3+ y_{4+}y_5)] = \frac{1}{2}[162+2x34]$$

Thus it is obvious that the Trapezoidal rule does not give an accurate result.

4.Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rule. Hence obtain the approximate value of π in each case.

Solution. Divide the range into six equal parts each of with h = $\frac{1}{6}$ and compute

The value of y = $\frac{1}{1+x^2}$ at each point of sub division.

These values are given below.

X	$Y_x = 1/(1+x^2)$
0	1.0000
1/6	36/37 = 0.97297
2/6	36/40 = 0.90000
3/6	36/45 = -0.80000
4/6	36/52 = 0.69231
5/6	

1	36/61 = 0.59016 1 / 2 = 0.5000

By Simpson's rule, we have

$\int_{x_0}^{x_0+6h} y\,dx = \frac{h}{3}[y_0+y_6+4(y_1+y_3+y_5)+2(y_2+y_4)]$

$= \frac{1}{18}$ [1.00 + 0.500 + 4(0.97292 + 0.50000 + 0.59016) +

2(0.900+ 0.69231)] ……(1)

= 0.785395

By Simpson's 3/8 rule we have

$\int_{x_0}^{x_0+6h} y\,dx = \frac{3h}{8}[y_0+y_6+3(y_1+y_2+ y_4+y_5)+2(y_3)]$

$= \frac{1}{16}$ [1.5000 + 3 (0.97297 + 0.900 + 0.69231+0.59016 +

2 x (0.800))] = 0.785395 ……(2)

Again, $\int_0^1 \frac{dx}{1+x^2} = [\tan^{-1} x]^1_0 = \pi/4$ …….(3)

From (1) and (2), we get π = 3.14188 and from (2) and (3) we get

π = 3.141580

5. Evaluate $\int_2^{10} \frac{dx}{x}$ by dividing the range into '8' equal parts by using Trapezoidal rule.

Solution. Given integral, $\int_2^{10} \frac{dx}{x}$

Let f(x) = $\frac{1}{4}$

a = 2, b = 10;n = 8

h = $\frac{b-a}{n} = \frac{10-2}{8} = 1$

x	y = f(x) = $\frac{1}{x}$

2	1/2 = 0.5000
3	1/3 = 0.3333
4	1/4 = 0.2500
5	1/5 = 0.2000
6	1/6 = 0.1666
7	1/7 = 0.1428
8	1/8 = 0.1250
9	1/9 = 0.1000
10	1/10 = 0.1000

Trapezoidal rule is $\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

$\int_2^{10}\frac{dx}{x} = \frac{h}{2}[(y_0+y_8)+2(y_1+y_2+y_3+ y_{4+} y_{5+} y_6+y_7)]$

$= \frac{1}{2}[(0.5+ 0.1)+ 2(0.333+ 0.25)+ 0.2+ 0.125+ 0.166 + 0.1428 +0.1111]$

$= \frac{1}{2}[(0.6)+2(1.3288)]$

$= 0.3+1.3288$

$= 1.62880$

6. Evaluate $\int_2^{10}\frac{dx}{1+x}$ by dividing the range into '8' equal parts by using Trapezoidal rule.

Solution. Let $f(x) = \frac{1}{1+x}$

a = 2, b = 10, n = number of equal parts = 8

$h = \frac{b-a}{n} = \frac{10-2}{8} = 1$

h = 1

x	$f(x) = \frac{1}{1+x}$

2	1/3 = 0.333
3	1/4 = 0.2500
4	1/5 = 0.2000
5	1/6 = 0.1666
6	1/7 = 0.1428
7	1/8 = 0.1250
8	1/9 = 0.1111
9	1/10 = 0.1000
10	1/11 = 0.0909

Trapezoidal rule is $\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

$\int_2^{10} \frac{dx}{1+x} = \frac{h}{2}[(y_0+y_8)+2(y_1+y_2+y_3+ y_{4+} y_{5+} y_6+y_7)]$

$= \frac{1}{2}[(0.333+ 0.090)+ 2(0.25+ 0.2+ 0.1661 + 0.1425+ 0.125+$

$0.111+ 0.1)]$

$= \frac{1}{2}[0.4242+2(1.0955)] = 0.2121 + 1.0955 = 1.3076$

8. Evaluate $\int_4^{5.2} f(x)dx$ by using Trapezoidal rule from the following.

X	4	4.2	4.4	4.6	4.8	5.0	5.2
Y	1.4	1.4	1.5	1.5	1.6	1.6	1.6

Where h = 0.2

Solution. Trapezoidal rule is, $\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

$\int_4^{5.2} f(x)dx = \frac{0.2}{2}[(1.4+ 1.6)+ 2(1.4+1.5+ 1.5+1.6+ 1.6)]$

$= 0.1[3+2(7.6)]$

$= 0.1[18.2] = 1.82$

9. Evaluate $\log_e 7$ from the integral $\int_1^7 \frac{dx}{x}$

Solution. Given f(x) = $\frac{1}{x}$ a = 1, b = 7, n = 6, h = $\frac{b-a}{n} = \frac{7-1}{6} = \frac{6}{6} = 1$

x	y= f(x)
1	1.0000
2	0.5000
3	0.333
4	0.2500
5	0.2000
6	0.1666
7	0.1428

Trapezoidal rule is, $\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})]$

$= \frac{h}{2}[(1+ 0.1428)+ 20.5+ 0.333+0.25+ 0.2+ 0.1666)]$

$= \frac{1}{2}[1.1428+ 2(1.4499)]$

$= \frac{1}{2}[4.0426]$ and $\int_1^7 \frac{dx}{x} = [\log_e x]_1^7$

$= \log_e 7 = \log_e 1 = \log_e 7$

$\log_e 7 = 2.0113$

10. A curve is drawn to passes through the point given by the following table.

X	1	1.5	2	2.5	3	3.5	4
Y	2	2.4	2.7	2.8	3	2.6	2

Solution. Estimate the area bounded by the curve, the axis-x and the lines x=1,x=4by using Trapezoidal rule.

Trapezoidal rule is h= $\frac{4-1}{6}$=0.5

$\int_a^b f(x)dx = \frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+y_4+y_5)]$

$= \frac{0.5}{2}[2+2.1+2+ (2.4 +2.7+2.8+2.6+3)]$

$\frac{0.5}{2}(4.1+27.0) = \frac{15.35}{2} = 7.775$ units of area

11. calculate e $\int_2^{10} \frac{dx}{1+x}$ by dividing the arrange into 8 equal parts by using Simpson's 1/3 rule.

Solution.: Simpson's 1/3 rule is

$$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_5+.....)+2(y_2+ y_4+.....)]$$

$$\int_2^{10} \frac{dx}{1+x} = \frac{h}{3}[(O.333+ 0.0909)+ 4(0.25+ 0.1666+ 0.125+0.1) + 200.2+ 0.142+ 0.111)]$$

$$= \frac{1}{3}[0.4242+ 4(0.6416)+ 2(0.4539)) = \frac{1}{3}[3.8984] = 1.2994666$$

12. calculate $\int_a^b f(x)dx$ by using Simpson's $\frac{1}{3}$ rule

X	4	4.2	4.4	4.6	4.8	5.0	5.2
Y	1.4	1.4	1.5	1.5	1.6	1.6	1.6

where h = 0.2

Solution: Simpson's $\frac{1}{3}$ rule is

$$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_5+.....)+2(y_2+ y_4+.....)]$$

$$\int_4^{5.2} f(x)dx = \frac{0.2}{3}[(1.4+1.6)+ 4(1.4+1.5+ 1.6)+ 2(1.5+ 1.6)]$$

$$= \frac{0.2}{3}[3+4(4.5)+2(3.1)] = \frac{0.2}{3}[27.2]$$

$$= 1.813333.....$$

13. Evaluate $\int_2^{10} \frac{dx}{1+x}$ by dividing the range iato 8 equal parts by using Simpson's $\frac{1}{3}$ rule.

Solution:. Let f(x)= $\frac{1}{x}$

a = 2, b = 10, n =number of equal parts = 8

h = $\frac{b-a}{n} = \frac{8}{8} = 1$

X	Y = f(x) = $\frac{1}{x}$
2	0.5000
3	0.3333
4	0.2500
5	0.2000

6	0.1666
7	0.1428
8	0.1250
9	0.1111
10	0.1000

Simpson's $\frac{1}{3}$ rule is

$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_5+.....+y_{n-1})+2(y_2+ y_4+y_6.....+y_{n-2})]$

$= \frac{1}{3}$ [I0.6)+ 4(0.3333+ 0.2+ 0.1428+ 0.1111)] +2(0.25+0.1666+0.125)]

$=\frac{1}{3}[0.6+4(0.7872+ 1.0832)]$

$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_6+.....)+2(y_2+ y_4+.....)]$

$\int_0^1 \frac{dx}{1+x^2} = \frac{1}{6*3}$ [(1 +0.5)+ 4(0. 9729+0.8+0.3+ 2(09+0.6923)]

$= \frac{1}{18}[1.5+9.452+3.1846] = \frac{1}{18}[14.1366]$

= 0.7853666

On the other hand $\int_0^1 \frac{1}{1+x^2}dx = (\tan^{-1})^1_0$

$= \tan^{-1}(1)-\tan^{-1}(0)$

$= \frac{\pi}{4}-0 = \frac{\pi}{4}$

$= \frac{\pi}{4} = 0.7853$

$= \pi = 3142$

14. Evaluate $\int_0^4 e^x\,dx$ by using Simpson's rule using the data

e = 2°.72, e2= 739, e3=20.09, e4=54.60 and compare it will the actual value.

Solution.: Divide the whole range (0,4) into four equal parts by taking h=1. By Simpson's rule 1/3 we have

$\int_0^4 e^x$ dx = $\int_{x_0}^{x_0+4h} y\,dx$ = $\frac{h}{3}$(y_0+y_4+4(y_1+y_3)+242)

= $\frac{1}{3}$[1+54.60+4(2.72+20.09)+2(7.39)]

= $\frac{1}{3}$[55.60+91.24+14.78] = $\frac{1}{3}$[161.62]

= 553.373

The actual value of the integral is

$\int_0^4 e^x$ dx = e^4 - e^0 = 54.60 – 1.0 = 53.60

15. Evaluate $\int_4^{5.2} \log, x$ dx by (a) Simpson's $\frac{1}{3}$ (b) $\frac{3}{8}$ rule . After finding the true value of the integral.

Solution. Divide the whole range of integration ie., (4, 5, 2) into six equal parts taking h 0.2 and then compute the values of the function f(x)= $\log^x_e$ for each of sub division. These computed values are as follows.

X	Y = $\log^x_e$ x	X	Y = $\log^x_e$ x
Xo = 4.0	1.38629436	Xo+ 4h= 4.8	1.56861592
Xo+ h =4.2	1.43508453		
		Xo+5h= 5.0	1.60943791
X_0 + 2h= 4.4	1.48160454		
		Xo+ 6h= 5.2	1.64865863
Xo +3h= 4.6	1.52605630		

(a) By Simpson's$\frac{1}{3}$ rule, we have

$\int_4^{5.2}$ log, x dx = $\frac{h}{3}$ [y_0+y_6+(y_1+y_3++y_5)+2(y_2+y_4)]

= $\frac{2}{8}$ [3.03495299+ 4(4.57056874)+ 2(3.05022046)]

=1.82784726.

(b) By Simpson's 3/8 rule, we have

$\int_4^{5.2}$ loge, x dx = $\frac{3h}{8}$ [y_0+y_2+3(y_1+y_2++y_4+ y_5)+2 y_3]

$= \frac{3(0.2)}{8}$[03495299+ 3(6.09424290)+ 2 (1.52605630)]

$= \frac{0.3}{4}$[24.371294291]

= = 1.82784707

Again exact value of the integral is

$\int_4^{5.2} \log_e x \, dx$ = x($\log_e$ x-1)$\int_{4.0}^{5.4} 1.82784741$

16. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ rule.

Solution : let f(x) = $\frac{1}{1+x^2}$ let n =6 , a= -0 , b = 1 ,h = $\frac{b-a}{n}$ = $\frac{1-0}{6}$ = $\frac{1-0}{6}$= $\frac{1}{6}$ = 0.1666

X	f(x) = $\frac{1}{1+x^2}$
0	1
1/6	0.9729
2/6	0.9000
3/6	0.8000
4/6	0.6923
5/6	0.5901
$\frac{6}{6}$=1	0.5000

Simpson's $\frac{1}{3}$ rule is

a = 0, b = 6, n= 6

h = $\frac{b-a}{n}$ = $\frac{6-0}{6}$ = 1

y_0 = f(0) -= $\frac{1}{1+0^2}$ = $\frac{1}{1}$ = 1

y_1 = f(1) = $\frac{1}{1+1^2}$ = $\frac{1}{2}$ = 0.5000

$y_2 = f(2) = \frac{1}{1+2^2} = \frac{1}{5} = 0.2000$

$y_3 = f(3) = \frac{1}{1+3^2} = \frac{1}{10} = 0.1000$

$y_4 = f(4) = \frac{1}{1+4^2} = \frac{1}{17} = 0.0588$

$y_5 = f(5) = \frac{1}{1+5^2} = \frac{1}{26} = 0.0385$

$y_6 = f(6) = \frac{1}{1+6^2} = \frac{1}{37} = 0.0270$

Simpson's $\frac{1}{3}$ rule is

$$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_n)+4(y_1+ y_3+ y_5+.....)+2(y_2+ y_4+.....)]$$

$$= \frac{1}{3}[(1 + 0.027)+ 4(0.5+ 0.1 + 0.0385)+ 2(0.2+ 0.0588)]$$

$$= \frac{1}{3} [3.7104] = 1.2368$$

17. Find the value of $\int_0^1 \frac{x^2}{1+x^2}$ dx by using Simpson's $\frac{1}{3}$ rule and also find the error

Solution. : Dividing the whole range (o, 1) into 4 equal parts nd making use of Simpson's $\frac{1}{3}$ rule we have

$$\int_0^1 \frac{x^2}{1+x^2} dx = \frac{h}{3}[(y_0+y_4)+4(y_1+ y_3+)+2y_2)]$$

$h = \frac{1}{4}$, a =0 , b= 1 ,n = $\frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4}$

X	F(x)
0	0
0.25	0.0615
0.5	0.2222
0.75	0.3956
1.0	0.5000

$\int_0^1 \frac{x^2}{1+x^2}$ dx = $\frac{1}{12}$ [(0+ 0.5)+ 4C0.0615+ 0.3956)+ 2(0.222)]

= $\frac{1}{12}$ [0.5+ 1.8284+ 0.444]

=$\frac{2.7728}{12}$ = 0.2310

18. Find $\int_4^{5.2} \log x_e$ dx by using Simpson's $\frac{1}{3}$ rule.

solution.: Given integral is $\int_4^{5.2} \log x_e$ dx

A = 4, b = 52 let n= 6 = $\frac{b-a}{n}$ = $\frac{5.2-2}{6}$ = 0.2

X	F(x)
4	1.3862
4.2	1.4350
4.4	1.4816
4.6	1.5260
4.8	1.5686
5.0	1.6094
5.2	1.6486

$\int_4^{5.2} \log x_e$ = $\frac{h}{3}$[(y_0+y_6)+4(y_1+ y_3+ y_5)+2(y_2+ y_4)]]

= $\frac{0.2}{3}$[(1.3862+1.6481+4(1.435071 +1.5260+1.6094)

+2(1.4816+1.5686)]

= $\frac{0.2}{3}$ [(3.0348)+ 4 (4.5704)+ 2(3.05027)]

= $\frac{0.2}{3}$[3.0348+18.2816+ 6.1004] = $\frac{5.48336}{3}$ = 1.8277866.

19. Evaluate $\int_{-3}^{3} x^4$dx by using Simpson's $\frac{1}{3}$ rule.

Solution. We know the integral is $\int_{-3}^{3} x^4$dx

Where $h = \frac{b-a}{h} = \frac{3-(-3)}{6} = \frac{6}{6} = 1$

x	F(x)
-3	81
-2	16
-1	1
0	0
1	1
2	16
3	81

Simpson's $\frac{1}{3}$ rule is

$$\int_b^a f(x)dx = \frac{h}{3}[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)]$$

$$= \frac{1}{3}[(81+81)+4(16+0+16)+2(1+1)]$$

a = 3.b = 3 , n = 6

$$h = \frac{b-a}{h} = \frac{3-(-3)}{6} = 1$$

let $f(x) = x^2$

$y_0 = f(-3) = (-3)^2 = 9$

$y_1 = f(-2) = (-2)^2 = 4$

$y_2 = f(-1) = (-1)^2 = 1$

$y_3 = f(0) = (0)^2 = 0$

$y_4 = f(1) = (1)^2 = 1$

$y_5 = f(2) = (2)^2 = 4$

$y_6 = f(3) = (3)^2 = 9$

Simpson's $\frac{1}{3}$ rule

$$\int_a^b f(x) = \frac{h}{3}[(y_0+y_n)+4(y_1+y_{3+\ldots})+2(y_2+y_{4+\ldots})]$$

$= \frac{1}{3}[(9+9)+4(4+0+4)+2(1+1)]$

$= \frac{54}{3} = 18 = \frac{1}{3}[162 + 128 + 4] = \frac{1}{3}[294] = 98$

20 .calculate an approximate value of $\int_a^{\pi/2} sinx\ dx$ by (a) Trapezoidal rule (b) Simpson's rule using 11 ordinates

Solution : Divide the range of integration into a ten equal parts taking h = $\pi/20$ and then compute the values of the function f(x) = sin x for each point of sub-division. The computed values are as follows :

X	Y=sin x
0	0.00000
$\pi/20$	0.15663
$2\pi/20$	0.30902
$3\,\pi/20$	0.45399
$4\pi/20$	0.58778
$5\pi/20$	0.707111
$6\,\pi/20$	0.80900
$7\,\pi/20$	0.89101
$8\,\pi/20$	0.95106
$9\pi/20$	0.98769
$10\,\pi/20$	1.00000

(a) By the trapezoidal rule, we have

$$\int_0^{\pi/2} sinx\ \mathrm{dx} = \frac{h}{3}[(y_0+y_{10})+2(y_1+y_{2+\ldots})]$$
$$= \frac{\pi/20}{2}[1.0000+2(5.85311)]$$
$$= \frac{\pi}{40}(12.70622) = 0.9981$$

(b) By Simpson's $\frac{1}{3}$ rule , we have

$$\int_0^{\pi/2} sinx\ \mathrm{dx} = \frac{h}{3}[(y_0+y_n)+4(y_1+y_{3+\ldots})+2(y_2+y_{4+\ldots})]$$
$$= \frac{\pi}{60}[1.00000+4(3.19623)+2(2.65688)]$$

$= \frac{\pi}{60}[1.00000+12.78492+5.31376]$

$= \frac{\pi}{60}[19.09868] = 1.0006$

Again the exact value of the integral

$\int_0^{\pi/2} sinx \ \mathrm{dx} = [-\cos x]_0^{\pi/2} = 1.0000$

The error (i) Due to Simpson's $\frac{1}{3}$ rule , we have = -0.0006

(ii) Due to trapezoidal rule = 0.0019

21.Evaluate $\int_a^b f(x)dx$,by Simpson's $\frac{3}{8}$ rule from the following

X	4	4.2	4.4	4.6	4.8	5.0	5.2
Y	1.4	1.4	1.5	1.5	1.6	1.6	1.6

Where

Solution : Simpson's $\frac{3}{8}$ rule is

$\int_a^b f(x)dx = \frac{3h}{8}[y_0+3y_1+2y_2+3y_2+2y_3+3y_4+3y_5+y_6]$

$= \frac{3(0.2)}{8}[1.4+3(1.4)+3(1.5)+2(1.5)+3(1.6)+3(1.6)+1.6]$

$= \frac{0.6}{8}[1.4+4.2+4.5+3.0+4.8+4.8+1.6]$

$= \frac{0.6}{8}(24.3) = 1.8225$

22. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's $\frac{3}{8}$ rule.

Solution : By the above problem we have y_0 =1 ,y_1 = 0.5000,y_2 = 0.1000,

y_4 = 0.0588 , y_5 = 0.0385, y_6 = 0.0270 and h = 1

Simpson's $3/8\ rule\ is$

$\int_a^b f(x)dx = \frac{3h}{8}[(y_0+ y_6)+3y_1+2y_3+3y_4+3y_5]$

$\int_0^6 \frac{\mathrm{dx}}{1+x^2} = \frac{3}{8}[(1+0.027)+3(0.5+0.2+0.0588+0.0385+2(0.1))]$

$= \frac{3}{8}[1.027+2.3919+0.2] = 13.09437/8 = 1.6770$

23.Evaluate $\int_{-3}^3 x^2dx$, using by Simpson's $\frac{3}{8}$ rule we know by the above problem

Solution. $y_0=9$,$y_1= 4$,$y_2= 1$, $y_3 = 0$,$y_4 = 1$, $y_5 = 4$,$y_6 = 9$

Simpson's $3/8\ rule\ is$

$$\int_a^b f(x)dx = \frac{3h}{8}[(y_0+ y_6)+3y_1+2y_3+3y_4+3y_5]$$

$$= \frac{3h}{8}[(9+9)+3(4+1+4+1)+0]$$

$$= \frac{3}{8}[48] = 18$$

24. Evaluate $\int_{-3}^{3} x^4 dx$,by using Simpson's $\frac{3}{8}$ rule

Solution. we know by the above problem.

Let f(x) = x^4

X	F(X)
-3	81
-2	16
-1	1
0	0
1	1
2	16
3	81

Where h=1

Simpson's 3/8 rule is

$$\int_a^b f(x)dx = \frac{3h}{8}[(y_0+ y_6)+3y_1+2y_3+3y_4+3y_5]$$

$$= \frac{3}{8}[81+81+3(16+1+1+16)]$$

$$= \frac{3}{8}[162+102] = \frac{3}{8}(264) = 99$$

25. A curve is drawn to passes through the part given by the following .

X	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
Y	**2**	**2.4**	**2.7**	**2.8**	**3**	**2.6**	**2.1**

Estimate the area bounded by the curve the x- axis and the lines x =1, x=4 by using Simpson's $\frac{3}{8}$rule.

Solution. h = $\frac{4-1}{6}$ = 0.5

Simpson's 3/8 rule is

$$\int_a^b f(x)dx = \frac{3h}{8}[(y_0+ y_6)+3y_1+2y_3+3y_4+3y_5]$$

$$= \frac{3(0.5)}{8}[2+2.1+3(2.4+2.7+3+2.6)+2(2.8)]$$

$$= \frac{6.35}{8} = 8.04375 \text{ units of area}$$

26. Evaluate $\int_a^b f(x)$ dx by using weddles formula from the following.

x	4	4.2	4.4	4.6	4.8	5.0	5.2
F(x)	1.4	1.4	1.5	1.5	1.6	1.6	1.6

Where h=0.4

Weddle's formula is

$$\int_a^b f(x)dx = \frac{3h}{10}[y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6]$$

$$= \frac{3(0.2)}{10}[(1.4+1.6)+5(1.4+1.6)+(1.5+1.6)+6(1.5)]$$

$$= \frac{0.6}{10}[3+15+3.1+9]$$

$$= \frac{0.6}{10}(30.1)$$

$$= 1.806$$

27. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using weddle's formula.

Solution. a =0 , b= 1 ,h = $\frac{1-0}{6} = \frac{1}{6} = 0.1666$

X	f(x)
0	1
1/6	0.9729
2/6	0.9000
3/6	0.8000
4/6	0.6923
5/6	0.5901
	0.5000

$\frac{6}{6}=1$	

Weddle's formula is

$$\int_a^b f(x)dx = \frac{3h}{10}[y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6]$$

$$= \frac{3}{6x10}[(1+0.5)+5(0.972+0.590)+(0.9+0.6923)+6(0.8)]$$

$$= \frac{1}{20}[1.5+7.815+1.5923+4.8]$$

$$= \frac{1}{20}[15.703]$$

$$= 0.785365$$

2.9 Applications of Numerical Methods

Considering the following model equation:

$$(\mu u_y)_y + y^n u_y = C, \qquad (1)$$

defined in a domain $\Omega = [0\ 1]$.

Where $\mu = (1 - exp(-y/\varepsilon) + \delta)/Re$, $\varepsilon << 1$, $\delta << 1$, $Re >> 1$, $n > 0$.

The first term simulates dissipative terms in the Navier-Stokes equations, whereas the second term models the contribution of convective terms and the right hand side represents the pressure gradient term or source in the transport equations. The "viscosity" coefficient μ corresponds to the effective viscosity coefficient. The coefficient is rapidly changed from a relatively small value $\mu_l = \delta /Re$ (laminar viscosity) to a "turbulent" viscosity $\mu_t \approx 1/Re$.

The equation simulates the low-Reynolds-number effects and can be considered as a model equation for the LR model. The left hand side point in the domain Ω will be

treated as a “wall”. The low-Reynolds-number effects occur nearby this point. If we set $\mu = (1+ \delta)/Re$ in (1), we have the HR approach.

Furthermore, we will consider the following values for the constants: $Re = 10^2$, $\varepsilon = 3*10^{-2}$, $\delta = 10^{-2}$. For simplicity the right-hand side will be considered as the constant $C = -1$

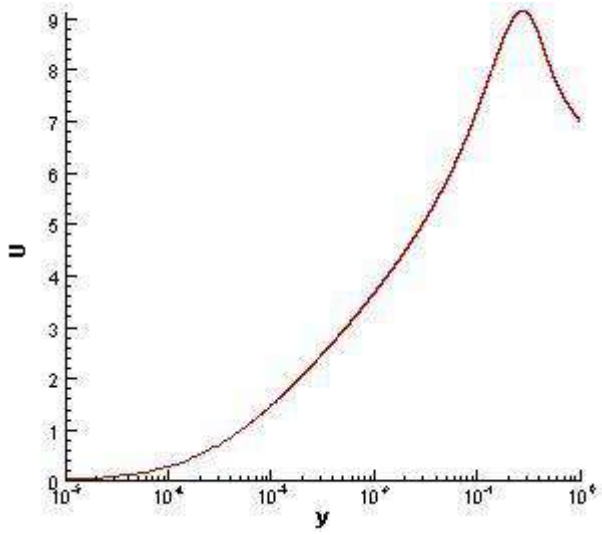
although this assumption is not important for our consideration.

Consider the following boundary-value Dirichlet problem:

$$(\mu u_{y})_y + y^n u_y = C \qquad (2)$$

$$u(0) = u_0 \quad u(1) = u_1$$

In the case of $n = 2$, $u_0 = 0$ *and* $u_1 = 7$, the solution is shown in Figure 1. The profile includes both the linear near-wall and logarithmic parts. Near the wall, $u = \boldsymbol{\varepsilon}\, ln(1+y/(\boldsymbol{\varepsilon\delta}))$. The thickness of the viscous “sub-layer”, where u is a linear function, can be approximated as $y_v \approx \boldsymbol{\varepsilon\delta}$.



In the calculations, the compact scheme [4] is used. The scheme allows us to calculate both the function and its derivative with a fourth-order of approximation. In Table 1, the results of calculations of the "friction" $\tau_w = \mu_w du/dy(0)$ (here $\mu_w = \mu(0)$) are given on different meshes. To exclude questions concerned with a mesh adaptation, a uniform mesh is used

Grid	10^2	$5*10^2$	10^3	$5*10^3$	10^4
τ_w	$3.1*10^{-1}$	$3.2*10^{-1}$	$3.3*10^{-1}$	$3.36*10^{-1}$	$3.36*10^{-1}$

Table 1. Calculation of τ_w on different meshes.

If we use the HR model with the same boundary conditions, then $\tau_w = 4.9*10^{-1}$, which shows the importance of the sub-layer.

3. Linear System of Equations

Solutions of simultaneous linear equations:

Rate, Distance and Time:

You can calculate the best routes for your running or cycling schedule by creating a mathematical expression that takes into account the distance and your average speed for various parts of the route. You can use the

equations to set different goals, such as to maximize time for build endurance, or to maximize speed for performance.

Planes, Trains and Automobiles:

The same formula used to calculate running times can be used to determine speed, distances and time duration when traveling by car, plane or train and you want to know the values for the unknown variables in your travel situations.

The Best Deal:

You want to find out the better deal when renting a car, and you're comparing two rental companies. By putting the variable and fixed costs, such as the per - mile and daily rate, into an algebraic expression, then solving for the total cost, you ca see which company saves you money for different amounts of driving.

The Best Plan:

You can use this same process with a system of equations when trying to decide on the best call phone plan, determining at how many minutes both companies charge the same amount and deciding from there which is the best plan for you and your intended usage.

Deciding on a Loan:

Simultaneous equations can be used to determine the best loan

choice to make when buying a car or a house when you consider the duration of the loan, the interest rate and the monthly payment of the loan. Other variables may be involved as well, with the information at hand, you can calculate which loan is the best choice for you.

Cost and Demand:

Simultaneous equations can be used when considering the relationship between the price of a commodity and the quantities of the commodity people want to buy at a certain price. An equation can be written that describes the relationship between quantity, price and other variables, such as income. These relationship equations can be solved

simultaneously to determine the best way to price the commodity and sell it.

In the Air:

An air traffic controller can use Simultaneous equations to ensure two airplanes don't intersect at the same time.

The Best job for the Money:

System of equations can be used when trying to determine if you'll make more money at one job or another, taking multiple variables into account, such as salary, benefits and commissions.

Investing Wisely:

You can use Simultaneous equations to decide on your best investment option, taking into account the duration of the investment, the interest it will accrue, as well as other variables that will affect the end result. If you know the amount you'd like to accrue, you can set the options equal to each other and figure out which option is best for your future.

Mixing it up:

With respect to mixtures Simultaneous equations can be used for achieving a certain consistency in a resultant product, which is dependent on the consistency of the compounds mixed together to produce it.

Rates :

Linear equations can be useful tool for comparing rates of pay. For example, if one company offers to pay you $450 per week and other offers $10 per hour, and both ask you to work 40 hours per week, which company is offering the better rate of pay? A linear equation can help you figure it out! The first company's offer expressed as 450=40x. the second company's offer is expressed as y=10(40). After computing the two offers, the equations tell you that the first company is offering the better rate of pay at $11.25 per hour.

Variable costs:

Imagine that you taking a taxi while on vacation. You know that the taxi service charges $9 to pick your family up form your hotel and another $0.15 per mile for the trip. Without knowing how many miles it will be to each destination, you can set up a linear equation that can be used to find the cost of any taxi trip you take on your trip. By using "x" to represent the number of miles to your destination and "y" to represent the cost of that taxi ride, the linear equation would be: y= 0.15x+9.

3.1 MATRIX INVERSION METHOD:

* Solve x + 2y = 10, 2x - y = 15 by matrix inverse method?

Solution:-

Let $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 10 \\ 15 \end{bmatrix}$

Given equations are equivalent to AX = B

=> $X = A^{-1} B$.

$$A^{-1} = \frac{1}{-1-4}\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix} \frac{1}{-1-4} = -\frac{-1}{5}\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix}$$

$$\therefore X = A^{-1} B = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} \frac{1}{5} \begin{bmatrix} 10 \\ 15 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 10 + 30 \\ 20 - 15 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 40 \\ 5 \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

$\therefore$ x = 18, y = 1 is the solution.

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* Solve the equations 3x + 2y + 4z = 7; 2x + y + z = 7; x + 3y + 5z = 2. by matrix inversion method ?

Solution:-

Let $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 7 \\ 7 \\ 2 \end{bmatrix}$

Given equations are equivalent to AX = B

det A = 3(5 - 3) - 2(10 - 1) +4(6 - 1) = 6 - 18 + 20 = 8 ≠ 0.

$\therefore$ $X = A^{-1} B$ is the solution.

Now adj $A = \begin{bmatrix} 2 & 2 & -2 \\ -9 & 11 & 5 \\ 5 & -7 & -1 \end{bmatrix}$

$\Rightarrow A^{-1} = \frac{AdjA}{\det A} = \frac{1}{8} \begin{bmatrix} 2 & 2 & -2 \\ -9 & 11 & 5 \\ 5 & -7 & -1 \end{bmatrix}$

$\therefore X = A^{-1} B = \frac{1}{8} \begin{bmatrix} 2 & 2 & -2 \\ -9 & 11 & 5 \\ 5 & -7 & -1 \end{bmatrix} \begin{bmatrix} 7 \\ 7 \\ 2 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 24 \\ 24 \\ -16 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ -2 \end{bmatrix}$

$\therefore$ x = 3, y = 3, z = -2 is the solution.

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3.2 GAUSS ELIMINATION METHOD:

* Solve the equations 2x - y + 4z = 12; 8x - 3y =2z = 23; 4x + 11y - z = 33 by Gauss Elimination Method ?

Solution:-

Given equations are 2x - y + 4z = 12, 8x - 3y + 2z = 23, 4x + 11y - z = 33.

Dividing first equation by 2, we get x - $\frac{1}{2}$y+ 2z = 6 ----------(1)

Now subtract (1) after multiplying by 8 and 4 respectively from second and third equations we get y - 14z = -25----->(2), 13y - 9z = 9 ------(3)

Subtract (2) after multiplied by 13 from (3) we get 173z = 334

=> z = $\frac{334}{173}$ = 1.931

From (3); 13y - 9$\left(\frac{334}{173}\right)$ = 9 => 13y = 9 + $\frac{3006}{173}$ => y = $\frac{4563}{2249}$ = 2.029

From (1); x = 6+$\frac{1}{2}\left(\frac{4563}{2249}\right)$ - 2$\left(\frac{334}{173}\right)$ = 6+$\frac{1}{2}$(2.029) - 2(1.931)

= 6 + 1.0145 - 3.862 = 3.153.

$\therefore$ The solution is x = 3.153, y = 2.029, z = 1.931.

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* Solve the system of equations 2x + y + z = 10, 3x + 2y + 3z = 18, x + 4y + 9z = 16 by Gauss Elimination Method.

Solution:-

We have 2x + y + z = 10------(1), 3x + 2y + 3z = 18------(2), x + 4y + 9z = 16-----(3)

Divide (1) by (2) and subtract multiplied by 3 from (2) then subtract from (3), we get

$x + \frac{1}{2}y + \frac{1}{2}z = 5$ -----(4), $\frac{1}{2}y + \frac{3}{2}z = 3$ -----(5), $\frac{7}{2}y + \frac{17}{2}z = 11$ ----(6)

Now divide (5) by $\frac{1}{2}$ and then subtract after multiplied by $\frac{7}{2}$ from (6) we get

$x + \frac{1}{2}y + \frac{1}{2}z = 5$ ------(7), y + 3z = 6 ----(8), -2z = -10 -----(9)

From back substitution form (9), (8) & (7) we get z = 5 and y + 3z = 6 => y + 3(5) = 6

=> y = 6 - 15 => y = -9 and

$x + \frac{1}{2}(-9) + \frac{1}{2}(5) = 5 \Rightarrow x = 5 + \frac{9}{2} - \frac{5}{2} = 7$

$\therefore$ Hence the solution is x = 7, y = -9, z = 5.

3.3GAUSS JORDAN METHOD:

* Solve the system of equations 2x + y + z = 1, x -2y - 3z = 1, 3x + 2y + 4z = 5 by Gauss Jordan Method.

Solution:-

The augmented matrix is $\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & -2 & -3 & 1 \\ 3 & 2 & 4 & 5 \end{bmatrix}$

$2R_2 - R_1, 2R_3 - 3R_1 \sim \begin{bmatrix} 2 & 1 & 1 & 1 \\ 0 & -5 & -7 & 1 \\ 0 & 1 & 5 & 7 \end{bmatrix}$

$5R_1 + R_2, 5R_3 + R_2 \sim \begin{bmatrix} 10 & 0 & -2 & 6 \\ 0 & -5 & -7 & 1 \\ 0 & 0 & 18 & 36 \end{bmatrix}$

$9R_1 + R_3, 18R_2 + 7R_3 \sim \begin{bmatrix} 90 & 0 & 0 & 90 \\ 0 & -90 & 0 & 270 \\ 0 & 0 & 18 & 36 \end{bmatrix}$ $\frac{1}{90}$

$R_1, \frac{1}{90}R_2, \frac{1}{18}R_3 \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

$\therefore$ **By** Gauss Jordan method, x = 1, y = -3, z = 2.

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* Solve the equations by Gauss Jordan Method . 5x + 2y + z = 12, x + 4y + 2z = 15, x + 2y + 5z = 20.

Solution:-

The augmented matrix is $\begin{bmatrix} 5 & 2 & 1 & 12 \\ 1 & 4 & 2 & 11 \\ 1 & 2 & 5 & 20 \end{bmatrix}$

$5R_2 - R_1, 5R_3 - R_1 \sim \begin{bmatrix} 5 & 2 & 1 & 12 \\ 0 & 18 & 9 & 63 \\ 0 & 8 & 24 & 88 \end{bmatrix}$

$9R_1 - R_2, 9R_3 - 4R_1 \sim \begin{bmatrix} 45 & 0 & 0 & 45 \\ 0 & 18 & 9 & 63 \\ 0 & 0 & 180 & 540 \end{bmatrix}$

$180R_3 \sim \begin{bmatrix} 45 & 0 & 0 & 45 \\ 0 & 18 & 9 & 63 \\ 0 & 0 & 1 & 3 \end{bmatrix}$ $R_2 - 9R_3 \sim \begin{bmatrix} 45 & 0 & 0 & 45 \\ 0 & 18 & 0 & 36 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

$\frac{1}{45}R_1, \frac{1}{18}R_2 \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

$\therefore$ **By** Gauss Jordan method, x = 1, y = 2, z = 3.

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3.4 LU - DECOMPOSITION METHOD:

* Solve the system of equations $2x_1 + x_2 + x_3 = 2$, $x_1 + 3x_2 + 2x_3 = 2$, $3x_1 + x_2 + 2x_3 = 2$ by LU Decomposition Method?

Solution:-

Given equations are $2x_1 + x_2 + x_3 = 2$----------(1)

$x_1 + 3x_2 + 2x_3 = 2$---------(2)

$3x_1 + x_2 + 2x_3 = 2$---------(3)

Here the coefficient matrix A is given by $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 3 & 1 & 2 \end{bmatrix}$ and

$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$

$\therefore AX = B$

Now $A = LU \Rightarrow \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$

$$= \begin{bmatrix} u_{11} & u_{12} & u_{12} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{bmatrix}$$

On comparing two matrices, we get $U_{11} = 2$, $U_{12} = 1$, $U_{13} = 1$;

$L_{21}U_{11} = 1 \Rightarrow L_{21} = \frac{1}{U_{11}} = \frac{1}{2}$; $L_{31}U_{11} = 3 \Rightarrow L_{31} = \frac{3}{U_{11}} = \frac{3}{2}$;

$L_{21}U_{12}+U_{22} = 3 \Rightarrow 3 - L_{21}U_{12} = 3 - \frac{1}{2}(1) = \frac{5}{2}$

$L_{21}U_{13}+U_{2} = 3 \Rightarrow 2 - L_{21}U_{12} = 2 - \frac{1}{2}(1) = \frac{3}{2}$

$L_{31}U_{12}+L_{32}U_{22} = 1 \Rightarrow L_{32} = \frac{1-L_{31}U_{12}}{U_{22}} = \frac{1-\frac{3}{2}}{\frac{5}{2}}(1) = \frac{-1}{5}$

$L_{31}U_{13}+L_{32}U_{23}+U_{33} = 2$

$\Rightarrow U_{33} = 2 - L_{31}U_{13} - L_{32}U_{23} = 2 - \frac{3}{2} + \frac{1}{5}\left(\frac{3}{2}\right) = 2 - \frac{3}{2} + \frac{3}{10} = \frac{8}{10} = \frac{4}{5}$

Thus $L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{3}{2} & \frac{-1}{5} & 1 \end{bmatrix}$ and

$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 0 & \frac{5}{2} & \frac{3}{2} \\ 0 & 0 & \frac{4}{5} \end{bmatrix}$

LY = B => $\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{3}{2} & \frac{-1}{5} & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$

From this we get $y_1 = 1$, $\frac{1}{2}y_1 + y_2 = 2$, $\frac{3}{2}y_1 - \frac{1}{5}y_2 + y_3 = 2$.

On solving, we get $y_1 = 2$, $y_2 = 1$, $y_3 = \frac{-4}{5}$.

And since, we have UX =Y => $\begin{bmatrix} 2 & 1 & 1 \\ 0 & \frac{5}{2} & \frac{3}{2} \\ 0 & 0 & \frac{4}{5} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ \frac{-4}{5} \end{bmatrix}$

$2x_1 + x_2 + x_3 = 2$, $\frac{5}{2}x_2 + \frac{3}{2}x_3 = 1$, $\frac{4}{5}x_3 = \frac{-4}{5}$

By back substitution, we get $x_1 = 1$, $x_2 = 1$, $x_3 = -1$.

$\therefore$ Hence, solution is $x_1 = 1$, $x_2 = 1$, $x_3 = -1$

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* Solve the equations 2x + 3y + z = 9, x + 2y + 3z = 6, 3x + y 2z = 8 by LU Decomposition method.

Solution:-

Given equations can be written as AX= B where A = $\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$, X= $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, B = $\begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$

Let A = LU

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{12} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{bmatrix}$$

Comparing, we get $U_{11}= 2$, $U_{12}= 3$, $U_{13}= 1$ and $L_{21}U_{11}= 1 \Rightarrow L_{21}= \frac{1}{2}$;

$L_{21}U_{12}+U_{22}= 2 \Rightarrow U_{22}= 2 - L_{21}U_{12} = 2 - \frac{3}{2} = \frac{1}{2}$;

$L_{21}U_{13}+U_{23}= 3 \Rightarrow U_{23}= 3 - L_{21}U_{13} = 3 - \frac{1}{2} = \frac{5}{2}$;

$L_{31}U_{11} = 3 \Rightarrow L_{31}= \frac{3}{U_{11}} = \frac{3}{2}$ and

$L_{31}U_{12} +L_{32}U_{22} = 1 \Rightarrow L_{32}= \frac{1-L_{31}U_{12}}{U_{22}} = \frac{1-\frac{9}{2}}{\frac{1}{2}} = -7$

$L_{31}U_{13} +L_{32}U_{23} = 2 \Rightarrow U_{33}= 1 - L_{31}U_{13} - L_{32}U_{23} = 2 - \frac{3}{2} - (-7)\left(\frac{5}{2}\right) = 2 - \frac{3}{2} + \frac{35}{2} = \frac{36}{2} = 18.$

Thus L and U are given by L = $\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -7 & 1 \end{bmatrix}$, U = $\begin{bmatrix} 2 & 3 & 1 \\ 0 & 1/2 & 5/2 \\ 0 & 0 & 18 \end{bmatrix}$

LY = B => $\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -7 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$

From this we get $y_1 = 9$, $\frac{1}{2}y_1 + y_2 = 6$, $\frac{3}{2}y_1 - 7y_2 + y_3 = 8$.

On solving, we get $y_1 = 9$, $y_2 = \frac{3}{2}$, $y_3 = 5$.

Further since UX = Y => $\begin{bmatrix} 2 & 3 & 1 \\ 0 & 1/2 & 5/2 \\ 0 & 0 & 18 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 3/2 \\ 8 \end{bmatrix}$

2x + 3y + z = 9, 1/2y + 5/2z = 3/2, 18z = 5.

By back substitution, we get z = 5/18 and 1/2y + 5/2(5/18) = 3/2

=> y = 3 - 25/18 = 29/18

And 2x + 3(29/18) + (5/18) = 9 => 2x = 9 - 29/6 - 5/18 = 9 - 92/18

=> 2x = 70/18 => x = 35/18

Hence, the solution is x = 35/18, y = 29/18, z = 5/18.

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3.5 ILL CONDITIONED OR WELL CONDITIONED:

* Establish whether the following matrix is singular, ill conditioned or well conditioned: $\begin{bmatrix} 3 & -1 & 3 \\ 1 & 0 & 4 \\ 2 & 2 & 1 \end{bmatrix}$?

Solution:-

Let A = $\begin{bmatrix} 3 & -1 & 3 \\ 1 & 0 & 4 \\ 2 & 2 & 1 \end{bmatrix}$

det A = 3(0 - 8) + 1(1 - 8) + 3(2 - 0) = -24 - 7 + 6 = -25

$\therefore$ A is not singular.

$\begin{vmatrix} 3.01 & -1 & 3 \\ 1 & 0 & 4 \\ 2 & 2 & 1 \end{vmatrix}$ = (3.01)(0 - 8) + 1(1 - 8) + 3(2 - 0)= -24.08-7+6 =-25.08.

$\therefore$ The matrix is well conditioned.

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* Show that A = $\begin{bmatrix} -73 & 78 & 24 \\ 92 & 66 & 25 \\ -80 & 37 & 10 \end{bmatrix}$ is ill- conditioned?

Solution :-

The ,matrix A = $\begin{bmatrix} -73 & 78 & 24 \\ 92 & 66 & 25 \\ -80 & 37 & 10 \end{bmatrix}$ is well conditioned, since $|A|$ = 1 whereas

$\begin{vmatrix} -73 & 78 & 24 \\ 92.01 & 66 & 25 \\ -80 & 37 & 10 \end{vmatrix}$ = 2.08, $\begin{vmatrix} -73 & 78.01 & 24 \\ 92 & 66 & 25 \\ -80 & 37 & 10 \end{vmatrix}$ = -28.20

and $\begin{vmatrix} -73 & 78 & 24 \\ 92 & 66 & 25 \\ -80 & 37 & 10.01 \end{vmatrix}$ = -118.94

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3.6 GAUSS - JACOBI METHOD:

* Solve the following equations by Jacobi method $20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25$?

Solution:-

Given equations can be written as $x = \frac{1}{20}(17 - y + 2z)$---------(1)

$$y = \frac{1}{20}(-18 - 3x + z)$$--------(2)

$$z = \frac{1}{20}(25 - 2x + 3y)$$--------(3)

Let us take first approximation $x^{(1)} = 0$, $y^{(1)} = 0$, $z^{(1)} = 0$.

Then the second approximation using (1), we get

$$x^{(2)} = \frac{1}{20}(17 - y^{(1)} + 2z^{(1)}) = \frac{17}{20} = 0.85$$

$$y^{(2)} = \frac{1}{20}(-18 - 3x^{(1)} + z^{(1)}) = \frac{1}{20}(-18 - 0 + 0) = -\frac{18}{20} = -0.9.$$

$$z^{(2)} = \frac{1}{20}(25 - 2x^{(1)} + 3y^{(1)}) = \frac{1}{20}(25 - 0 + 0) = \frac{25}{20} = 1.25.$$

Substituting these values in R.H.S of (1), we get third approximation

$$x^{(3)} = \frac{1}{20}(17 - y^{(2)} + 2z^{(2)}) = \frac{1}{20}(17 + 0.9 + 2(1.25)) = \frac{20.4}{20} = 1.02.$$

$$y^{(3)} = \frac{1}{20}(-18 - 3x^{(2)} + z^{(2)}) = \frac{1}{20}(-18 - 3(0.85) + 1.25) = \frac{-19.3}{20} = -0.965.$$

$$z^{(3)} = \frac{1}{20}(25 - 2x^{(2)} + 3y^{(2)}) = \frac{1}{20}(25 - 2(0.85) + 3(-0.9)) = \frac{20.6}{20} = 1.03.$$

Now Substitute these values in (1), we get fourth approximation

$$x^{(4)} = \frac{1}{20}(17 - y^{(3)} + 2z^{(3)}) = \frac{1}{20}(17 + 0.965 + 2(1.03)) = \frac{20.025}{20} = 1.0013.$$

$$y^{(4)} = \frac{1}{20}(-18 - 3x^{(3)} + z^{(3)}) = \frac{1}{20}(-18 - 3(1.02) + 1.03) = -\frac{20.03}{20} = -1.0015.$$

$z^{(4)} = \frac{1}{20}(25- 2x^{(3)} + 3y^{(3)}) = \frac{1}{20}(25 - 2(1.02)+3(-0.965))= \frac{20.065}{20} =$ 1.0032.

Substitute these values in (1), we get fifth approximation

$x^{(5)} = \frac{1}{20}(17 - y^{(4)} + 2z^{(4)}) = \frac{1}{20}(17 +1.0015+ 2(1.0032))= \frac{20.0079}{20} =$

1.0004.

$y^{(5)} = \frac{1}{20}(-18 - 3x^{(4)} + z^{(4)}) = \frac{1}{20}(-18 - 3(1.0013) + 1.0032) =$

$- \quad = \quad -\frac{20.0007}{20} = -1.00003.$

$z^{(5)} = \frac{1}{20}(25- 2x^{(4)} + 3y^{(4)}) = \frac{1}{20}(25 - 2(1.0013)+3(1.0015))= \frac{19.9929}{20} =$

1.9996.

Substitute these values in (1), we get sixth approximation

$x^{(6)} = \frac{1}{20}(17 - y^{(5)} + 2z^{(5)}) = \frac{1}{20}(17 + 1.00003 + 2(0.9996))= \frac{19.99923}{20} =$

0.99996.

$y^{(6)} = \frac{1}{20}(-18 - 3x^{(5)} + z^{(3)}) = \frac{1}{20}(-18 - 3(1.0004) + 0.9996) = \frac{-20.0016}{20} =$

-1.00008.

$z^{(6)} = \frac{1}{20}(25- 2x^{(5)} + 3y^{(5)}) = \frac{1}{20}(25 - 2(1.0004)+3(-1.00003))= \frac{19.99911}{20} =$

0.99995.

The values in the fifth and sixth approximation are nearly same.

$\therefore$ Hence the solution is x = 1, y = -1, z = 1.

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* Solve the equations by Jacobi Method 83x + 11y - 4z = 95; 7x + 52y + 13z = 104; 3x +8y + 29z = 71?

Solution:- Given equation can be written as

$x = \frac{1}{83}(95 - 11y + 4z)$, $y = \frac{1}{52}(104 - 7x - 13z)$, $z = \frac{1}{83}(71 - 3x - 8y)$------(1).

Let us take the first approximation to be $x^{(1)} = 0$, $y^{(1)} = 0$, $z^{(1)} = 0$.

Then the second approximation using (1), we get

$x^{(2)} = \frac{1}{83}(95 - 11y^{(1)} + 4z^{(1)}) = \frac{1}{83}(95 - 0 + 0) = 1.14$

$y^{(2)} = \frac{1}{52}(104 - 7x^{(1)} - 13z^{(1)}) = \frac{1}{52}(104 - 0 - 0) = \frac{104}{52} = 2.$

$z^{(2)} = \frac{1}{29}(71 - 3x^{(1)} - 8y^{(1)}) = \frac{1}{29}(71 - 0 - 0) = \frac{71}{23} = 2.4.$

Substituting these values in R.H.S of (1), we get third approximation

$x^{(3)} = \frac{1}{83}(95 - 11y^{(2)} + 4z^{(2)}) = \frac{1}{83}(95 - 11(2) + 4(2.4)) = \frac{82.6}{83} = 0.995 \sim 1.$

$y^{(3)} = \frac{1}{52}(104 - 7x^{(2)} - 13z^{(2)}) = \frac{1}{52}(104 - 7(1.14) - 13(2.4)) = \frac{64.82}{52} = 1.25.$

$z^{(3)} = \frac{1}{29}(71 - 3x^{(2)} - 8y^{(2)}) = \frac{1}{29}(71 - 3(1.14) - 8(2)) = \frac{51.58}{29} = 1.78.$

Now Substitute these values in (1), we get fourth approximation

$x^{(4)} = \frac{1}{83}(95 - 11y^{(3)} + 4z^{(3)}) = \frac{1}{83}(95 - 11(1.25) + 4(1.78)) = \frac{88.37}{83} = 1.1.$

$y^{(4)} = \frac{1}{52}(104 - 7x^{(3)} - 13z^{(3)}) = \frac{1}{52}(104 - 7(1) - 13(1.78)) = \frac{73.86}{52} = 1.42.$

$z^{(4)} = \frac{1}{29}(71 - 3x^{(3)} - 8y^{(3)}) = \frac{1}{29}(71 - 3(1) - 8(1.25)) = \frac{58}{29} = 2.$

Substitute these values in (1), we get fifth approximation

$x^{(5)} = \frac{1}{83}(95 - 11y^{(4)} + 4z^{(4)}) = \frac{1}{83}(95 - 11(1.42) + 4(2)) = \frac{-87.38}{83} = 1.1.$

$y^{(5)} = \frac{1}{52}(104 - 7x^{(4)} - 13z^{(4)}) = \frac{1}{52}(104 - 7(1.1) - 13(2)) = \frac{70.3}{52} = 1.4.$

$z^{(5)} = \frac{1}{29}(71 - 3x^{(4)} - 8y^{(4)}) = \frac{1}{29}(71 - 3(1.1) - 8(1.42)) = \frac{51.7}{29} = 1.78.$

$\therefore$ Hence the solution is x = 1.1, y = 1.4, z = 1.78.

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3.7 GAUSS SEIDEL METHOD:

* Find the solution of the system by Gauss Seidel Method 83x + 11y - 4z = 95; 7x + 52y + 13z = 104; 3x +8y + 29z = 71?

Solution:-

Given equations can be written as $x = \frac{1}{83}(95 - 11y + 4z)$,

$y = \frac{1}{52}(104 - 7x - 13z)$,

$z = \frac{1}{83}(71 - 3x - 8y)$------(1).

Let us take the first approximation to be $x^{(1)} = 0$, $y^{(1)} = 0$, $z^{(1)} = 0$.

Then the second approximation using (1), we get

$x^{(2)} = \frac{1}{83}(95 - 11y^{(1)} + 4z^{(1)}) = \frac{1}{83}(95 - 0 + 0) = 1.14$

Put $x^{(2)} = 1.14$, $y^{(1)} = 1.85$, $z^{(1)} = 0$ in (3), we get

$y^{(2)} = \frac{1}{52}(104 - 7x^{(2)} - 13z^{(1)}) = \frac{1}{52}(104 - 7(1.14) - 0) = \frac{96.02}{52} = 1.85.$

Put $x^{(2)} = 1.14$, $y^{(2)} = 1.85$, $z^{(1)} = 0$ in (3), we get

$z^{(2)} = \frac{1}{83}(71 - 3x^{(2)} - 8y^{(2)}) = \frac{1}{83}(71 - 3(1.14) - 8(1.85)) = \frac{52.78}{29} = 1.82.$

Put $x^{(2)} = 1.14$, $y^{(2)} = 1.85$, $z^{(2)} = 1.82$ in (3), we get

$x^{(3)} = \frac{1}{83}(95 - 11y^{(2)} + 4z^{(2)}) = \frac{1}{83}(95 - 11(1.85) + 4(1.82)) = \frac{81.93}{83} = 0.99.$

Put $x^{(3)} = 1.14$, $y^{(2)} = 1.85$, $z^{(2)} = 1.82$ in (3), we get

$y^{(3)} = \frac{1}{52}(104 - 7x^{(3)} - 13z^{(2)}) = \frac{1}{52}(104 - 7(0.99) - 13(1.82)) = \frac{73.41}{52} = 1.41$

Put $x^{(3)} = 0.99$, $y^{(3)} = 1.41$, in (3) we get

$z^{(3)} = \frac{1}{83}(71 - 3x^{(3)} - 8y^{(3)}) = \frac{1}{83}(71 - 3(0.99) - 8(1.41)) = \frac{56.75}{29} = 1.95.$

Again put $x^{(3)} = 0.99$, $y^{(3)} = 1.41$, $z^{(3)} = 1.95$ in (3) we get

$x^{(4)} = \frac{1}{83}(95 - 11y^{(3)} + 4z^{(3)}) = \frac{1}{83}(95 - 11(0.99) + 4(1.95)) = \frac{87.29}{83} = 1.05.$

Put $x^{(4)} = 1.05$ $z^{(3)} = 1.95$ in (2) we get $y^{(4)} = \frac{1}{52}(104 - 7x^{(3)} - 13z^{(2)}) =$

$\frac{1}{52}(104 - 7(1.05) - 13(1.95)) = \frac{71.3}{52} = 1.37.$

Put $x^{(4)} = 1.05$, $y^{(4)} = 1.37$ in (3), we get

$z^{(4)} = \frac{1}{83}(71 - 3x^{(4)} - 8y^{(4)}) = \frac{1}{83}(71 - 3(1.05) - 8(1.37)) = \frac{56.89}{29} = 1.96.$

Thus $x^{(4)}, y^{(4)}, z^{(4)}$ are sufficiently close to $x^{(3)}, y^{(3)}, z^{(3)}$ respectively

$\therefore$ Hence the solution is $x = 1.05$, $y = 1.37$, $z = 1.96$.

-----0-----

* Solve by Gauss Seidel method $10x_1 + x_2 + x_3 = 12$, $2x_1 + 10x_2 + x_3 = 13$, $2x_1 + 2x_2 + 10x_3 = 14$.

Solution:-

From the given equations we have

$$X_1 = \frac{1}{10}[12 - x_1 - x_2] \text{---------(1)}$$

$$X_2 = \frac{1}{10}[13 - 2x_1 - x_3]\text{---------(2)}$$

$$X_3 = \frac{1}{10}[14 - 2x_1 - 2x_2]\text{---------(3)}$$

Putting x = 0 in right side of the equation (1), we get $x_1^{(1)} = \frac{12}{10} = 1.2$

Putting $x_1 = x_1^{(1)} = 1.2$, $x_3 = 0$ in (2), we get $x_2^{(1)} = \frac{1}{10}(13 - 2.4 - 0) = 10.6/10 = 1.06$

Putting $x_1 = x_1^{(1)} = 1.2$, and $x_2 = x_2^{(1)} = 1.06$ in (3), we get

$x_3^{(2)} = \frac{1}{10}[14 - 2.4 - 2.12] = 0.948.$

For the second iteration we have $x_1^{(2)} = \frac{1}{10}[12 - 1.06 - 0.948] = 0.9992.$

$x_2^{(2)} = \frac{1}{10}[13 - 2(0.9992) - 0.948] = 1.00536.$

$x_3^{(2)} = \frac{1}{10}[14 - 2(0.9992) - 2(1.00536)] = 0.999098.$

Thus the iteration process is continued

The results are tabulated as follows correcting to four decimal places.

i	$x_1^{(i)}$	$x_2^{(i)}$	$x_3^{(i)}$
0	1.2000	0.0000	0.0000
1	1.2000	1.0600	0.9480
2	0.9992	1.0054	0.9991
3	0.9996	1.0000	1.0000
4	1.0000	1.0000	1.0000
5	1.0000	1.0000	1.0000

$\therefore$ The exact value of the roots are $x_1 = 1$, $x_2 = 1$, $x_3 = 1$.

-----0-----

Network Analysis in linear system of Equations:

Set up a system of linear equations to represent the network shown in the figure. Then solve the system.

Solution:-

Each of the network's five junctions give rise to a linear equations as follows:

$X_1 + X_2 = 20$ Junction 1

$X_3 - X_4 = -20$ Junction

$X_2 + X_3 = 20$ Junction 3

$X_1 - X_5 = -10$ Junction 4

$-X_4 + X_5 = -10$ Junction 5

The augmented matrix for this system is

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 20 \\ 0 & 0 & 1 & -1 & 0 & -20 \\ 0 & 1 & 1 & 0 & 0 & 20 \\ 1 & 0 & 0 & 0 & -1 & -10 \\ 0 & 0 & 0 & -1 & 1 & -10 \end{bmatrix}$$

Gauss Jordan elimination produces the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & -10 \\ 0 & 1 & 0 & 0 & 1 & 30 \\ 0 & 0 & 1 & 0 & -1 & -10 \\ 0 & 0 & 0 & 1 & -1 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

From the matrix above, you can see that

$X_1 - X_5 = -10$, $X_2 - X_5 = 30$,

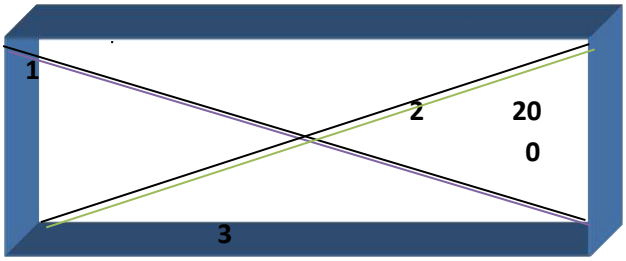
$X_3 - X_5 = -10$ and $X_4 - X_5 = -10$

Letting $t = X_5$, we have

$X_1 = t - 10$, $X_2 = -t + 30$, $X_3 = t + 30$, $X_4 = t + 10$, $X_5 = t$

Where t is the real number, so this system has infinitely many solutions.

1



2

4

5

Figure

In example 5, suppose you could control the mount of flow along the branch labeled X_5. using the solution of example 5 you could then control the flow represented by each of the other variables. For instance, letting t =10 would reduces the flow of X_1 and X_3 to zero, as shown In the figure.

4. LAPLACE TRANSFORM

4.1 Definition and properties of Laplace Transform, piecewise continuous functions, the Laplace Transform method of solving initial value problems

The method of Laplace transforms is a system that relies on algebra (rather than calculus-based methods) to solve linear differential equations. While it might seem to be a somewhat cumbersome method at times, it is a very powerful tool that enables us to readily deal with linear differential equations with discontinuous forcing functions.

***Definition*:** Let $f(t)$ be defined for $t \geq 0$. The Laplace transform of $f(t)$, denoted by $F(s)$ or $\mathcal{L}\{f(t)\}$, is an *integral transform* given by the *Laplace integral*:

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\, dt .$$

Provided that this (improper) integral exists, i.e. that the integral is convergent.

The Laplace transform is an operation that transforms a function of t (i.e., a function of *time domain*), defined on $[0, \infty)$, to a function of s (i.e., of *frequency domain*)*. $F(s)$ is the *Laplace transform*, or simply *transform*, of $f(t)$. Together the two functions $f(t)$ and $F(s)$ are called a *Laplace transform pair*.

For functions of t continuous on $[0, \infty)$, the above transformation to the frequency domain is *one-to-one*. That is, different continuous functions will have different transforms.

The integral is divergent whenever $s \leq 0$. However, when $s > 0$, it converges to

$$\frac{1}{s}(0 - e^0) = \frac{-1}{s}(-1) = \frac{1}{s} = F(s)$$

***Example*:** Let $f(t) = t$, then $F(s) = \dfrac{1}{s^2}$, $s > 0$.

[This is left to you as an exercise.]

The integral is divergent whenever s ≤ a. However, when s > a, it converges to

$$\frac{1}{a-s}(0-e^{0})=\frac{1}{a-s}(-1)=\frac{1}{s-a}=F(s).$$

***Definition*:** A function $f(t)$ is called *piecewise continuous* if it only has finitely many (or none whatsoever – a continuous function is considered to be "piecewise continuous"!) discontinuities on any interval $[a, b]$, and that both one-sided limits exist as t approaches each of those discontinuity from <u>within</u> the interval. The last part of the definition means that f could have removable and/or jump discontinuities only; it cannot have any infinity discontinuity.

Theorem: Suppose that

1.f is piecewise continuous on the interval $0 \le t \le A$ for any $A > 0$.

2.$|f(t)| \le K e^{at}$ when $t \ge M$, for any real constant a, and some positive constants K and M. (This means that f is "of *exponential order*", i.e. its rate of growth is no faster than that of exponential functions.)

3.Then the Laplace transform, $F(s) = \mathcal{L}\{f(t)\}$, exists for $s > a$.

> ***Note*:** The above theorem gives a <u>sufficient</u> condition for the existence of Laplace transforms. It is <u>not</u> a <u>necessary</u> condition. A function does not need to satisfy the two conditions in order to have a Laplace transform.
>
> Examples of such functions that nevertheless have Laplace transforms are logarithmic functions and the unit impulse function.
>
> **<u>Some properties of the Laplace Transform</u>**:
>
> 1. $\mathcal{L}\{0\} = 0$
> 2. $\mathcal{L}\{f(t) \pm g(t)\} = \mathcal{L}\{f(t)\} \pm \mathcal{L}\{g(t)\}$
> 3. $\mathcal{L}\{c\, f(t)\} = c\, \mathcal{L}\{f(t)\}$, for any constant c.

Properties 2 and 3 together means that the Laplace transform is *linear*.

[The derivative of Laplace transforms]

$\mathcal{L}\{(-t)f(t)\} = F'(s)$ or, equivalently $\mathcal{L}\{t\,f(t)\} = -F'(s)$

Example: $\mathcal{L}\{t^2\} = -(\mathcal{L}\{t\})' = -\frac{d}{ds}\left(\frac{1}{s^2}\right) = -\left(\frac{-2}{s^3}\right) = \frac{2}{s^3}$

In general, the derivatives of Laplace transforms satisfy

$\mathcal{L}\{(-t)^n f(t)\} = F^{(n)}(s)$ or, equivalently $\mathcal{L}\{t^n f(t)\} = (-1)^n F^{(n)}(s)$

Warning: The Laplace transform, while a linear operation, is **not** *multiplicative*. That is, in general $\mathcal{L}\{f(t)\,g(t)\} \neq \mathcal{L}\{f(t)\}\,\mathcal{L}\{g(t)\}$.

Exercise: (a) Use property 4 above, and the fact that $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$,

to deduce that $\mathcal{L}\{t\,e^{at}\} = \frac{1}{(s-a)^2}$. (b) What will $\mathcal{L}\{t^2 e^{at}\}$ be?

Exercises C-1.1:

1 – 5 Use the (integral transformation) definition of the Laplace transform to find the Laplace transform of each function below.

1. t^2
2. $t\,e^{6t}$
3. $\cos 3t$
4. $e^{-t}\sin 2t$

5.* $e^{i\alpha t}$, where i and α are constants, $i = \sqrt{-1}$.

6 – 8 Each function $F(s)$ below is defined by a definite integral. Without integrating, find an explicit expression for each $F(s)$.

[Hint: each expression is the Laplace transform of a certain function. Use your knowledge of Laplace Transformation, or with the help of a table of common Laplace transforms to find the answer.]

6. $\int_{+0}^{\infty} e^{-(s+7)t}\,dt$

7. $\int_{+0}^{\infty} t^2 e^{-(s-3)t}\,dt$

8. $\int_{+0}^{\infty} 4e^{-st} \sin 6t\,dt$

9. Let $f(t)$ be a continuous function and $F(s)$ be its Laplace transform. Suppose $f(t) \geq 0$ for $t \geq 0$. (a) Show that $F(0)$ is equal to the area of the region under the curve $y = f(t)$, $0 \leq t \leq \infty$. (b) Verify this fact by comparing the value of $F(0)$ with the area under the curve obtained the usual way (i.e. by integration) for each of the functions e^{-t}, te^{-2t}, and t^2e^{-4t}.

Answers C-1.1:

1. $\dfrac{2}{s^3}$

2. $\dfrac{1}{(s-6)^2}$

3. $\dfrac{s}{s^2+9}$

4. $\dfrac{2}{s^2+2s+5}$

5. $\dfrac{s}{s^2+\alpha^2} + i\dfrac{\alpha}{s^2+\alpha^2}$

Note: Since the Euler's formula says that $e^{i\alpha t} = \cos \alpha t + i \sin \alpha t$, therefore, $\mathcal{L}\{e^{i\alpha t}\} = \mathcal{L}\{\cos \alpha t + i \sin \alpha t\}$. That is, the real part of its Laplace transform corresponds to that of $\cos \alpha t$, the imaginary part corresponds to that of $\sin \alpha t$. (Check it for yourself!)

6. $\dfrac{1}{s+7}$

7. $\dfrac{2}{(s-3)^3}$

8. $\dfrac{24}{s^2+36}$

9. Set $s = 0$ in the definition $F(s) = \int_{+0}^{\infty} e^{-st} f(t)\,dt$ to show that

$F(0) = +_0^{\infty} f(t)dt = A$.

Solution of Initial Value Problems :

We now shall meet "the new System": how the Laplace transforms can be used to solve linear differential equations algebraically.

Theorem: [**Laplace transform of derivatives**] Suppose *f* is of exponential order, and that *f* is continuous and *f* ′ is piecewise continuous on any interval $0 \le t \le A$. Then $\mathcal{L}\{f'(t)\} = s\,\mathcal{L}\{f(t)\} - f(0)$

Applying the theorem multiple times yields:

$$\mathcal{L}\{f''(t)\} = s^2\,\mathcal{L}\{f(t)\} - s f(0) - f'(0),$$

$$\mathcal{L}\{f'''(t)\} = s^3\,\mathcal{L}\{f(t)\} - s^2 f(0) - s f'(0) - f''(0),$$

:

:

$$\mathcal{L}\{f^{(n)}(t)\} = s^n \mathcal{L}\{f(t)\} - s^{n-1} - s^2 f^{(n-3)}(0) - s f^{(n-2)}(0) - f$$

This is an extremely useful aspect of the Laplace transform: that it changes differentiation with respect to t into multiplication by s (and, as seen a little earlier, differentiation with respect to s into multiplication by $-t$, on the other hand). Equally importantly, it says that the Laplace transform, when applied to a differential equation, would change derivatives into algebraic expressions in terms of s and (the transform of) the dependent variable itself. Thus, it can transform a differential equation into an algebraic equation.

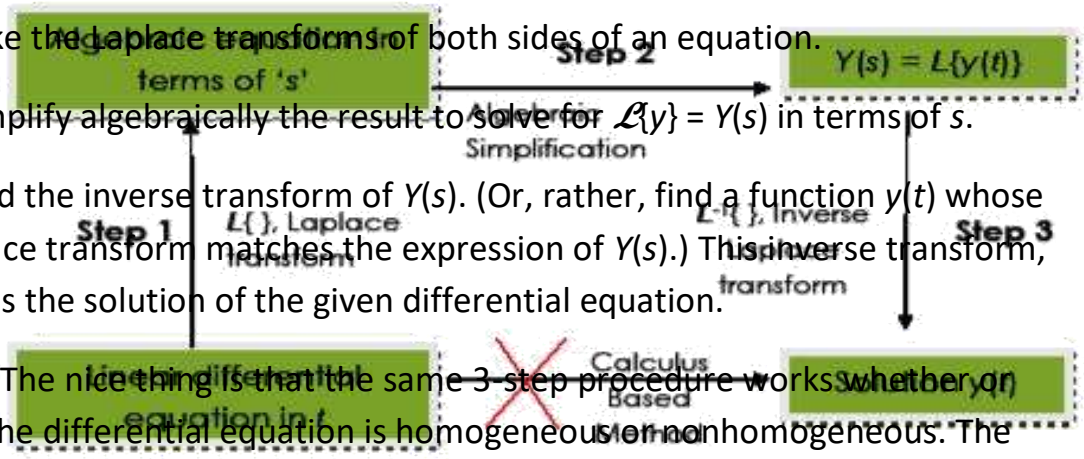
We are now ready to see how the Laplace transform can be used to solve differentiation equations.

The Laplace Transform Method of Solving an Initial Value Problem

Solving initial value problems using the method of Laplace transforms

To solve a linear differential equation using Laplace transforms, there are only 3 basic steps:

Flow Chart



1.Take the Laplace transforms of both sides of an equation.

2.Simplify algebraically the result to solve for $\mathcal{L}\{y\} = Y(s)$ in terms of s.

3.Find the inverse transform of $Y(s)$. (Or, rather, find a function $y(t)$ whose Laplace transform matches the expression of $Y(s)$.) This inverse transform, $y(t)$, is the solution of the given differential equation.

The nice thing is that the same 3-step procedure works whether or not the differential equation is homogeneous or nonhomogeneous. The first two steps in the procedure are rather mechanical. The last step is the heart of the process, and it will take some practice. Let's get started

***Example*:** $y'' - 6y' + 5y = 0, \qquad y(0) = 1, y'(0) = -3$

[Step 1] Transform both sides

$\mathcal{L}\{y'' - 6y' + 5y\} = \mathcal{L}\{0\}$

$(s^2\,\mathcal{L}\{y\} - s\,y(0) - y'(0)) - 6(s\mathcal{L}\{y\} - y(0)) + 5\mathcal{L}\{y\} = 0$

[Step 2] Simplify to find $Y(s) = \mathcal{L}\{y\}$

$(s^2\,\mathcal{L}\{y\} - s - (-3)) - 6(s\,\mathcal{L}\{y\} - 1) + 5\mathcal{L}\{y\} = 0\ (s^2 - 6\,s + 5)$

$\mathcal{L}\{y\} - s + 9 = 0(s^2 - 6\,s + 5)\ \mathcal{L}\{y\} = s - 9$

[Step 3] Find the inverse transform $y(t)$

Use partial fractions to simplify,

$$\mathcal{L}\{y\} = \frac{s-9}{s^2-6s+5} = \frac{a}{s-1} + \frac{b}{s-5}$$

$$\frac{s-9}{s^2-6s+5} = \frac{a(s-5)}{(s-1)(s-5)} + \frac{b(s-1)}{(s-5)(s-1)}$$

$s - 9 = a(s - 5) + b(s - 1) = (a + b)s + (-5a - b)$

Equating the corresponding coefficients:

$1 = a + b \qquad a = 2$

$-9 = -5a - b \qquad b = -1$

Hence,

$$\mathcal{L}\{y\} = \frac{s-9}{s^2-6s+5} = \frac{2}{s-1} - \frac{1}{s-5}\ .$$

The last expression corresponds to the Laplace transform of $2e^{t} - e^{5t}$. Therefore, it must be that, $y(t) = 2e^{t} - e^{5t}$.

Many of the observant students no doubt have noticed an interesting aspect (out of many) of the method of Laplace transform: that it finds the particular solution of an initial value problem directly, without solving for the general solution first. Indeed, it usually takes more effort to find the general solution of an equation than it takes to find a particular solution!

The Laplace Transform method can be used to solve linear differential equations of any order, rather than just second order equations as in the previous example. The method will also solve a nonhomogeneous linear differential equation directly, using the exact same three basic steps, without having to separately solve for the complementary and particular solutions. These points are illustrated in the next two examples.

Example: $y' + 2y = 4t\,e^{-2t}$, $y(0) = -3$.

[Step 1] Transform both sides

$$\mathcal{L}\{y' + 2y\} = \mathcal{L}\{4t\,e^{-2t}\}$$

$$(s\mathcal{L}\{y\} - y(0)) + 2\mathcal{L}\{y\} = \mathcal{L}\{4t\,e^{-2t}\} = \frac{4}{(s+2)^2}$$

$$(s\mathcal{L}\{y\} - (-3)) + 2\mathcal{L}\{y\} = \frac{4}{(s+2)^2}$$

$$(s+2)\,\mathcal{L}\{y\} + 3 = \frac{4}{(s+2)^2}$$

$$(s+2)\,\mathcal{L}\{y\} = \frac{4}{(s+2)^2} - 3$$

$$(s+2)^2$$

$$\mathcal{L}\{y\} = \frac{4}{3} - \frac{3}{s+2} = \frac{4-3(s+2)}{3} \quad 2 = \frac{-3s^2-12s-8}{3}$$
$$(s+2) \qquad (s+2) \qquad (s+2) \qquad (s+2)$$

[Step 3] Find the inverse transform $y(t)$

By partial fractions,

$$\mathcal{L}\{y\} = \frac{-3s^2 \quad -12s-8}{3} = \frac{a}{3} + \frac{b}{2} + \frac{c}{s+2}.$$
$$(s+2) \qquad (s+2) \qquad (s+2) \qquad s+2$$

$$\frac{-3s^2-12s-8}{(s+2)^3} = \frac{a}{(s+2)^3} + \frac{b(s+2)}{(s+2)^3} + \frac{c(s+2)^2}{(s+2)^3}$$

$$= \frac{a+bs+2b+cs^2 \quad +4cs+4c}{(s+2)^3} = \frac{cs^2+(b+4c)s+(a+2b+4c)}{(s+2)^3}$$

$-3 = c$ $\qquad$ $a = 4$

$-12 = b + 4c$ $\qquad$ $b = 0$

$-8 = a + 2b + 4c$ $\qquad$ $c = -3$

$$\mathcal{L}\{y\} = \frac{-3s^2 - 12s - 8}{(s+2)^3} = \frac{4}{(s+2)^3} - \frac{3}{s+2}.$$

This expression corresponds to the Laplace transform of $2t^2 e^{-2t} - 3e^{-2t}$. Therefore,

$$y(t) = 2t^2 e^{-2t} - 3e^{-2t}.$$

Note: $\mathcal{L}\{t^n e^{at}\} = \frac{}{(s-a)^{n+1}}$

Example: $y'' - 3y' + 2y = e^{3t}$, $y(0) = 1, y'(0) = 0$

[Step 1] Transform both sides

$(s^2 \mathcal{L}\{y\} - s\,y(0) - y'(0)) - 3(s\mathcal{L}\{y\} - y(0)) + 2\mathcal{L}\{y\} = \mathcal{L}\{e^{3t}\}$

[Step 2] Simplify to find $Y(s) = \mathcal{L}\{y\}$

$(s^2 \mathcal{L}\{y\} - s - 0) - 3(s\mathcal{L}\{y\} - 1) + 2\mathcal{L}\{y\} = 1 / (s - 3)$

$(s^2 - 3s + 2)\,\mathcal{L}\{y\} - s + 3 = 1 / (s - 3)$

$(s^2 - 3$

s

$$+ 2)\,\mathcal{L}\{y\} = s - 3 + \frac{1}{s-3} = \frac{(s-3)^2 + 1}{s-3}$$

$$\mathcal{L}\{y\} = \frac{s^2 - 6s + 10}{(s^2 - 3s + 2)(s-3)} = \frac{s^2 - 6s + 10}{(s-1)(s-2)(s-3)}$$

[Step3] Find the inverse transform $y(t)$

By partial fractions,

$$\mathcal{L}\{y\} = \frac{s^2 - 6s + 10}{(s-1)(s-2)(s-3)} = \frac{5}{2}\cdot\frac{1}{s-1} - 2\cdot\frac{1}{s-2} + \frac{1}{2}\cdot\frac{1}{2s-3}.$$

Therefore, $y(t) = \frac{5}{2}e^{t} - 2e^{2t} + \frac{1}{2}e^{3t}$.

For the next example, we will need the following Laplace transforms:

$$\mathcal{L}\{\cos bt\} = \frac{s}{s^2 + b^2}, \quad s > 0$$

$$\mathcal{L}\{\sin bt\} = \frac{b}{s^2 + b^2}, \quad s > 0$$

$$\mathcal{L}\{e^{at}\cos bt\} = \frac{s-a}{(s-a)^2 + b^2}, \quad s > a$$

$$\mathcal{L}\{e^{at}\sin bt\} = \frac{b}{(s-a)^2 + b^2}, \quad s > a$$

$$\frac{}{(s-a)^2+b^2}$$

***Note*:** The values of a and b in the last two expressions' denominators can be determined without using the method of completing the squares. Any irreducible quadratic polynomial $s^2 + Bs + C$ can always be written in the

required from of $(s-a)^2 + b^2$ by using the quadratic formula to find (necessarily complex-valued roots) s . The value a is the real part of s, and the value b is just the absolute value of the imaginary part of s. That is, if $s = \lambda \pm \mu\, i$, then $a = \lambda$ and $b = \mu$.

Example: $y'' - 2y' + 2y = \cos(t)$, $y(0) = 1, y'(0) = 0$

[Step 1] Transform both sides

$(s^2\, \mathcal{L}\{y\} - s\, y(0) - y'(0)) - 2(s\mathcal{L}\{y\} - y(0)) + 2\mathcal{L}\{y\} = \mathcal{L}\{\cos(t)\}$

[Step 2] Simplify to find $Y(s) = \mathcal{L}\{y\}$

$(s^2\, \mathcal{L}\{y\} - s - 0) - 2(s\mathcal{L}\{y\} - 1) + 2\mathcal{L}\{y\} = s\,/\,(s^2 + 1)$

$(s^2 - 2\,s + 2)\, \mathcal{L}\{y\} - s + 2 = s\,/\,(s^2 + 1)$

$$(s^2 - 2\,s + 2)\, \mathcal{L}\{y\} = s - 2 + \frac{s}{s^2+1} = \frac{(s-2)(s^2+1) + s}{s^2+1}$$

$$\mathcal{L}\{y\} = \frac{s^3 - 2s^2 + s - 2 + s}{(s^2+1)(s^2 - 2s + 2)} = \frac{s^3 - 2s^2 + 2s - 2}{(s^2+1)(s^2 - 2s + 2)}$$

[Step 3] Find the inverse transform $y(t)$

By partial fractions,

$$\mathcal{L}\{y\} = \frac{s^3 - 2s^2 + 2s - 2}{} = \frac{1\,s - 2}{} + \frac{4s - 6}{}$$

$$\frac{}{(s^2+1)(s^2-2s+2)} - \frac{}{5s^2+1} \quad \frac{}{s^2-2s+2}$$

$$\frac{1}{5}\left[\frac{s}{s^2+1} - \frac{2}{s^2+1} + \frac{4(s-1)}{(s-1)^2+1^2} - \frac{2}{(s-1)^2+1^2}\right]$$

$$= -\frac{1}{5}\left[\frac{s}{s^2+1} - \frac{2}{s^2+1} + \frac{s-1}{(s-1)^2+1^2} - \frac{2}{(s-1)^2+1^2}\right]$$

which corresponds to

$$y(t) = -\frac{1}{5}[\cos(t) - 2\sin(t) + 4e^t\cos(t) - 2e^t\sin(t)]$$

Examples: Find the inverse Laplace transform of each

(i) $F(s) = \dfrac{2s-5}{s^2+4s+8}$

Rewrite $F(s)$ as:

$$F(s) = \frac{2s-5}{(s+2)^2+2^2} = \frac{2(s+2)}{(s+2)^2+2^2} - \frac{9}{2}\cdot\frac{2}{(s+2)^2+2^2}$$

Answer: $f(t) = 2e^{-2t}\cos(2t) - \frac{9}{2}e^{-2t}\sin(2t)$

(ii) $F(s) = \frac{s+4}{(s-2)^3}$

Use partial fractions to rewrite $F(s)$ as:

$$F(s) = \frac{1}{(s-2)^2} + \frac{6}{(s-2)^3}$$

Answer: $f(t) = te^{2t} + 3t^2e^{2t}$

Appendix A

4.2 Some Additional Properties of Laplace Transforms

I. Suppose $f(t)$ is discontinuous at $t = 0$, then the Laplace transform of its derivative becomes

$$\mathcal{L}\{f'(t)\} = s\,\mathcal{L}\{f(t)\} - \lim_{t \to 0-} f(t).$$

Suppose $f(t)$ is a periodic function of period T, that is, $f(t + T) = f(t)$, for all t in its domain, then

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\, dt .$$

***Comment*:** This property is convenient when finding the Laplace transform of a discontinuous periodic function, whose discontinuities (necessarily there are infinitely many, due to the periodic nature of the function) would make the usual approach of integrating from 0 to ∞ unwieldy.

Let c > 0 be a constant, the *time-scaling* property of Laplace transform states that

$$\mathcal{L}\{f(ct)\} = \frac{1}{c} F\left(\frac{s}{c}\right).$$

IV. We have known that $\mathcal{L}\{-t f(t)\} = F'(s)$. Taking the inverse transforms on both sides yields $-t f(t) = \mathcal{L}^{-1}\{F'(s)\}$. Therefore, $f(t) = -\frac{1}{t}\mathcal{L}^{-1}\{F'(s)\}$. That is, if we know how to invert the function $F'(s)$, then we also know how to find the inverse of its anti-derivative $F(s)$. Formally,

$$\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(u)\, du .$$

V. Similarly, dividing $F(s)$ by s corresponding an integral with respect to t.

$$\mathcal{L}\left\{\int_0^t f(u)\, du\right\} = \frac{F(s)}{s}$$

Comment: Therefore, dividing a function by its independent variable has the effect of anti-differentiation with respect to the other independent variable for Laplace transforms. These two properties (IV and V) are the

counter parts of the multiplication-corresponds-to-differentiation properties seen earlier.

VI. After learning the fact that $\mathcal{L}\{f(t)\, g(t)\} \neq \mathcal{L}\{f(t)\}\, \mathcal{L}\{g(t)\}$, one might have wondered whether there is an operation of two functions f and g whose result has a Laplace transform equal to the product of the individual transforms of f and g. Such operation does exist:

$$\mathcal{L}\{ \int_0^t f(\tau)g(t-\tau)d\tau \} = \mathcal{L}\{f(t)\}\mathcal{L}\{g(t)\}.$$

In addition, observe that, by taking the inverse Laplace transform of both sides the property VI, we have

$$\int_0^\tau f(\tau)g(t-\tau)d\tau = \mathcal{L}^{-1}\{\mathcal{L}\{f(t)\}\mathcal{L}\{g(t)\}\}.$$

In other words, we can obtain the inverse Laplace transform of a (simple) function of s that is itself a product of Laplace transforms of two known functions of t by taking the convolution of the two functions of t.

Example: Find the inverse Laplace transform of

$$F(s) = \frac{1}{s(s^2 + 4)}.$$

First, note that

$$F(s) = \frac{1}{s} \times \frac{1}{s^2 + 4} = \frac{1}{s}\,\frac{1}{2} \qquad \mathcal{L}\ \mathcal{L}$$

$$\{1\}\ \{\sin 2t\}$$

Therefore, its inverse can be found by the convolution integral (letting $f(t) = 1$ and $g(t) = \sin 2t$):

4.3 Applications of Laplace transform

This section describes the applications of laplace transform in the domain of engineering. Laplace transform has several applications in almost all Engineering disciplines such as System Modelling, Analysis of Electrical and Electronic Circuits, Digital Signal Processing and Process Controls. Laplace transforms are critical for process controls. It helps analyze the variables, which when altered, produces desired manipulations in the result. For example, while study heat experiments, Laplace transform is used to find out to what extent the given input can be altered by changing temperature, hence one can alter temperature to get desired output for a while. This is an efficient and easier way to control processes that are guided by differential equations

Electric Circuit theory

Example 1 :: The switching transient phenomenon in the RL , RC or RLC circuits can be solved by laplace transform. Let us consider a series RLC circuit as shown figure 1 to which a dc voltage V_0 is suddenly applied.

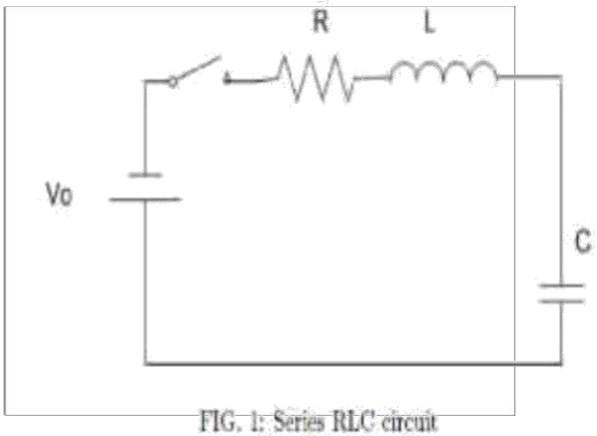


FIG. 1: Series RLC circuit

Now applying kirchoff‟s voltage law (KVL) to the circuit , we have

Differentiating bothsides, $$R_i + L\frac{d\,i}{d\,t} + \frac{1}{C}\int i\,dt = V_0$$

$$L\frac{d^2i}{dt^2} + \frac{1}{C\,i} + R\frac{d\,i}{d\,t} = 0 \;\; or$$

$$\frac{d^2i}{d\,t^2} + \frac{R}{L}\frac{d\,i}{d\,t} + \frac{1}{L\,C}\,i = 0 \quad \rightarrow (1)$$

Applying laplace transform to this equation, let us assume that the solution of this equation is

$$i\,(t) = K\,e^{s\,T}$$

$$L\,K\,s^2\,e^{s\,T} + R\,K\,e^{s\,T} + \frac{1}{C\,K}\,e^{s\,T} = 0$$

5.CURVE FITTING

5.1 INTRODUCTION :

In this chapter we are concerned with the problem of fitting

an equation or a curve to data involving period values. An approximate non-mathematical relationship between the two variables , can be established by a diagram called **scatter diagram .** The exact mathematical relationship between the two variables is given by simple algebra expression called **curve fitting .** Though there are infinite variety of curve in mathematics , the curve used for the purpose of curve fitting are relatively limited in type. The straight line is the simple and one of the most important curve used.

The straight line :

The equation $y = a + bx$…. (1) Is an equation of the degree in x and y .It represents a straight line .

The difference $y_i - (a + bx_i)$….. (2)

Is zero if and only if the point (x_i, y_i) lies on the line given by (1)

Fitting a straight line:

Usually fitting a straight line means finding the value of the parameter a and b of the straight line given by (1) , as well as actually constructing the line itself .The Graphic method and the method of least square are two useful method for fitting a straight line .

The graphical method:

The method can be used whenever the given formula can be plotted as a straight line either directly or after suitable transformation. The straight line drawn after a careful visual estimate of its position has been made with the aid of a ruler. The

coordinated of any two points on the measures and substitute in equation (1). The resulting equations in a and b are then solved for their *parameters.* Graphical method , whatever its theoretical attractions , suffers from the disadvantage that it is difficult apply in practice except for the straight line .This method will give fairly good results when finely divided coordinate paper is used , but in general it is recommended .

An alternative method, which id in almost universal use at present time, is known as the of **Least Squares** and we proceed to discuss it at length.

The method of lest squares:

This method of curve fitting was suggested early in the nineteenth century by the French mathematician Adrian Legendre. The method of least squares assumes that the best fitting line in the curve for which the sum of the squares of the vertical distances of the points (x_r , y_r) from the line is minimum. This method is more accurate that the graphical method.

Let $y = a + bx$ Refer (1)

Be the equation of the line. The ordinate of any point Q_r on the line vertically above or below a given point P_r , can be found by substituting the abscissa x_r , in the right-hand side of (1) . The two coordinates of Q_r will be $(x_r , a + bx_r)$ (see in the above figure). The vertical distances e_r from the line of any point p_r with coordinates (x_r , y_r) will therefore be given by the equation

$$e_r = y_r - (a + x_r) \ldots\ldots \quad (3)$$

We may say that e_r represents the difference between the actual ordinate y_r , of a pint and its theoretical ordinate $a + bx_r$.

Let $q = \sum_{r=1}^{n} e_r^2 = \sum_{r=1}^{n} (y_r - (a + bx_r))^2$ ……. (4)

The best fitting line is that line for which the sum of the square, $q = \sum e_r^2$ is a minimum. We find the value of a and b which make q minimum as follows :Differentiating q partially with respect to a and b equally these partially to zero , we obtain

$$\therefore \frac{\partial_q}{\partial_a} = \sum_{r=1}^{n}(-2)[y_r - (a + bx_r)] = 0 \ ,$$

And $\frac{\partial_q}{\partial_o} = \sum_{r=1}^{n}(-2)x_r[y_r - (a + bx_r)] = 0$,

Which yield the so – called **System of normal equations,**

$$\sum_{r=1}^{n} y_r = an + b\sum_{r=1}^{n} x_r \quad ,\text{.....(5)}$$

And $\sum_{r=1}^{n} x_r\, y_r \ = a\ \sum_{r=1}^{n} x_r + \ b\ \sum_{r=1}^{n} x_r^2$ .....(6)

Solving this system of equations we get

$$a = x = \frac{\sum_1^n x_r^2\ \sum_1^n y_r - \sum_1^n x_r \sum x_r y_r}{n\sum_1^n x_r^2 - [\sum_1^n x_r}$$ ..... (7)

$$b = \frac{n[\sum_{r=1}^{n} x_r y_r] - [\sum_{r=1}^{n} x_r][\sum_{r=1}^{n} x_r^2]}{n[\sum_{r=1}^{n} x_r^2] - [\sum_{r=1}^{n} x_r]^2} \ ,\text{.....(8)}$$

Where n is the number of points $(x_r\ , y_r)$.

To simplify the for formulae, we get

$$t_r = \frac{x_r - \bar{x}}{h} \ , \qquad (9)$$

Where h is the interval between successive value of x assumed to be equally spaced. Shifting the origin to the mean $\bar{x}$ and compressing the intervals to unity we get transformed equation of (2) referred to the new (t, y) axes as

$$Y=\bar{a} + \bar{b}t \ , \qquad (10)$$

Where $\bar{a}\ and\ \bar{b}$ are the parameter in the new equation . In terms of the new coordinate formula (7) and (8) becomes

$$\bar{a} = \frac{\sum_1^n t_r^2 \sum_1^n y_r - \sum_1^n t_r \sum t_r y_r}{n\sum t_r^2 - (\sum t_r)^2} \ , \qquad (11)$$

$$\bar{b} = \frac{n\sum_1^n t_r y_1 - \sum_1^n t_r \sum t_r y_r}{n\sum_1^n t_r^2 - [\sum_1^n t_r]^2} \quad , \qquad \ldots\ldots\ldots\ldots\ldots \qquad (12)$$

Transformation (9) replace the x_r $\grave{s}$ by unit deviations t_r from the mean $\bar{x}$.

Since

$$\sum_1^n t_r = 0$$

The formula (11)and (12) reduce to

$$\bar{a} = \frac{\sum_1^n t_r^2 \sum_1^n y_r - 0}{n\sum t_r^2 - 0}$$

And

$$\bar{b} = \frac{n\sum_1^n t_r y_r - 0}{n\sum t_r^2 - 0}$$

Which further simplify to the form

5.2 Fitting a parabola:

When a set of point exhibits a parabolic trend , the fitting of a quadratic functions

$$y = a + bx + cx^2 ,$$

To the data may be carried out by the method of least square , Which leads to the three equations

$$na + b\sum_1^n x_r + c\sum x_r^2 \;=\; \sum_1^n y_r$$

$$a\sum_1^n x_r + b\sum_1^n x_r^2 + c\sum_1^n x_r^3 = \sum_1^n x_r y_r \,.$$

$$a\sum_1^n x_r^2 + b\sum_1^n x_r^3 + c\sum_1^n x_r^4 = \sum_1^n x_r^2 y_r \,.$$

The above equations can be solve for a , b, c.

5.3 EXPONETIAL FUNCTION Y= Ae^{bx}

Transforming the exponential equation $y = ae^{bx}$ by taking logarithms on both sides we get

$$\log y = \log a + bx \log e \,.$$

If we replace $\log y$ by y and the constant $\log a$ and b $\log e$ by $\grave{a}$ and $\acute{b}$

Respectively , we obtain

$$y = \acute{a} + b\acute{x} \;,$$

Which defines a straight line .

Example:

1.Find the lest square line fir the data points (-1,10) ,(0,9),(1,7),(,4,5), (,2,5),(3,4),(4,3),(5,0) and (6,-1).

Solution: Here

x_r	y_r	x_r^2	y_r^2
-1	10	1	-10
0	9	0	0
1	7	1	7
2	5	4	10
3	4	9	12
4	3	9	12
5	0	25	0
6	-1	36	-6
20	37	92	25

From the table we have

$$n = 8\,, \sum x_r = 20\,, \sum y_r = 37, \sum x_r^2 = 92\,, and\ \sum x_r y_r = 25\,.$$

The normal equation are

$$na + b\sum x_r = \sum y_r\,,$$

$$a\sum x_r + \sum x_r^2 = \sum x_r y_r\ .$$

Putting the value in normal equations

$$8a + 20b = 37\ ,$$

And

$$20a + 97b + 25\quad ,$$

And solving these equations we get

$$a = 8.6428571\ ,$$

$$b = -1.6071429\ \ .$$

Therefore the least square line is

$$y = 8.6428571 + (-1)1.6071429x \ ,$$

i.e.,

$$y = -1.6071429x + 8.6428571 \ .$$

Example 2: **Find the least square line $y = a + bx$ for the data**

X	**-2**	**-1**	**0**	**1**	**2**
Y	**1**	**2**	**3**	**3**	**4**

Solution:

x_r	y_r	x_r^2	$x_r y_r$
-2	1	4	-2
-1	2	1	-2
0	3	0	0
1	3	1	3
2	4	4	8
0	13	10	7

$\therefore$ *In this case* $n = 5 \ , \sum x_r = 0 \ , \sum y_r = 13 \ , \sum x_{r2} = 10 \ , \sum x_r y_r = 7.$

The normal equation are

$$na + b\sum x_r = \sum y_r \ ,$$

$$a\sum x_r + b\sum x_r^2 = \sum x_r y_r \ .$$

Putting the value of n , $\sum x_r$, $\sum y_r$, $\sum x_r^2$ and $\sum x_r y_r$ in the above equation we get

$$5a = 13 \ , 10b = 7 \ .$$

Solving

$$a = 2.6 \ , b = 0.7 \ .$$

∴The required line of fit is

$$y = 2.6 + b = (0.7)x \ .$$

Example 3:

Find a formula for the line of the form $y = a + bx + cx^2$ Which will fit the following data

X	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Y	3.1950	3.2299	3.2532	3.2516	3.2516	3.2282	3.1807	3.1266	3.0594	2.9759

Solution :

The normal equation are

$$na = b\sum x_r + c\sum x_r^2 = \sum y_r^2 \ ,$$

$$a\sum x_r + b\sum x_r^2 + c\sum x_r^3 = \sum x_r y_r$$

$$a\sum x_r^2 + b\sum x_r^3 + c\sum x_r^4 = \sum x_r^2 y_r \ .$$

$Substituting\ the\ value\ of\ \sum x_r \ , \sum x_r^2 \ , \sum x_r^3 , \sum x_r y_r , \sum x_r^2 \ y_r , \sum y_r , and\ n , we\ get$

$$10a + 4.5b + 2.85c = 31.7616 \ ,$$

$$4.5a + 2.85b + 2.025c = 14.0896 \ ,$$

$$2.85a + 2.025b + 1.5333 = 8.82881 \ ,$$

And solving these equations we obtain

$$a = 3.1951 \, , b = 0.44254 \, , c = -0.76531.$$

$\therefore \ The \ requied \ equation \ is$

$$y = 3.1951 + 0.44254x - 0.76531x^2.$$

Example 4:

Fit second degree parabola to the following data

X	0	1	2	3	4
y	0	1.8	1.3	2.5	6.3

Solution : The values of x are 0,1,2,3,4

The number of value is odd. Shifting the origin to the middle value 2 of x, and making the substituting

$$u = x - 2, v = y \ ,$$

The curve of fit as

$$v = a + bu + cu^2 \ ,$$

We obtain

x	v	u	v	uv	u^2	u^2v	u^3	u^4
0	1	-1	1	-2	4	4	-8	16
1	1.8	-2	1.8	1.6	1	1.8	-1	1
2	1.2	0	1.3	0	0	0	0	0

3	2.5	1	2.5	2.5	1	2.5	1	1
4	6.3	2	6.3	12.6	4	25.2	8	16
		0	12.9	11.3	10	33.5	0	34

The normal equation are

$$\sum u = na + b\sum u + c\sum u_r^2 \quad ,$$

$$\sum uv = a\sum u + bu^2 + c\sum u^3 \quad ,$$

$$\sum u^2 v = a\sum u^2 + b\sum u^3 + c\sum u^4 \quad .$$

Putting the values of n ,$\sum u$, $\sum v$, $\sum uv$, … … … . , etc,

12.9=5a+40c,

$11.3 = 10b$,

33.5=10a+34c,

Solving these normal equations we get

$$a = 1.48, b = 1.13, c = 0.550.$$

∴The required equation is

$$u = 1.48 + 1.13u + 1.55u^2 \quad ,$$

Substituting $u = x - 2$, $v = y$ in the above equation we get,

$$y = 1.42 + 1.13(x - 2) + (0.55)(x - 2)^2 \quad .$$

$$\therefore The\ requird\ curve\ of\ fit\ is\ the\ parabola$$

$$y = 1.42 - 1.07x + 0.55x^2.$$

5.4 Solved problems:

1)Find the lest square line $y = a + bx$ for the data

X	-4	-2	0	2	4
Y	1.2	2.8	6.2	7.8	13.2

A)	X	y	x^2	xy
	-4	1.2	16	-4.8
	-2	2.8	4	-5.6
	0	6.2	0	0
	2	7.8	4	15.6
	4	13.2	16	52.8
	0	31.2	4.	58

The normal equations are

$$\sum y = na + b\sum xi$$

$$\sum xy = a\sum x_i + b\sum x_i^2$$

31.2=5a

$$a = \frac{31.2}{5}$$

= 6.24

$$58 = b(40)$$

$$\therefore y = 6.24 + 1.45 \ .$$

2) **Fit a straight line to the following data regarding x as the independent variable**

X	0	1	2	3	4

Y	1	1.8	3.3	4.5	6.3

A)	x	y	x^2	xy
	0	1	0	0
	1	1.8	1	1.8
	2	3.3	4	6.6
	3	4.5	4	13.5
	4	6.3	16	25.2
	10	16.9	30	47.1

$$na + b\sum xi = \sum y$$

$$5a + b(10) = 16.9 \qquad (1)$$

$$a\sum x + b\sum x^2 = \sum xy$$

$$a(10) + b(30) = 47.1$$

$$1*2\rightarrow\ 10a + 20b = 33.8$$

$$\underline{2\rightarrow\ 10a + 30b = 47.1}$$

$$-10b = -11.3$$

$$b = \frac{13.3}{10}$$

$$=1.33$$

Substituting b in (1)

$$5a + (1.33)10 = 16.9$$

$5a = 16.9 - 13.3$

$a = \frac{3.6}{5}$

=0.72

$$\therefore y = 0.72 + 1.33x$$

3) Find the least square line $y = a + bx$

	X	-2	0	2	4	6
	Y	1	3	6	8	13

A)	X	y	x^2	xy
	-2	1	4	-2
	0	3	0	-2
	2	6	4	12
	4	8	16	12
	6	13	36	78
	10	31	60	78

$$na + b\sum x_i = \sum xy$$

$5a + 10b = 31$ (1)

$$a\sum x + b\sum x^2 = \sum xy$$

$10a + 60b = 102$ (2)

1*2→ $10a + 20b = 62$

10a+60b=120

$$-40b = -58$$

$$b = 1.45$$

Substituting b in (1)

$$5a + (1.45)10 = 31$$

$$5a = 31 - 14.5$$

$$a = \frac{16.5}{5}$$

$$=3.3$$

$$a = 3.3 \ , b = 1.45$$

$$\therefore y = 3.3 + 1.45x$$

4) Find the least squares parabolic lift $y = a + bx + cx^2$

A)	x	y	x^2	x^3	x^4	xy	x^2y
	-3	15	9	-27	81	-45	135
	-1	5	1	-1	1	-5	5
	1	1	1	1	1	1	1
	3	5	9	27	81	15	45
	0	26	20	0	164	-34	186

$$\sum y = na + b\sum x + c\sum x^2$$

$$26 = 4a + 0 + 20c$$

$$4a + 20c = 26 \qquad (1)$$

$$\sum xy = a\sum ax + b\sum x^2 + c\sum x^3$$

$$-34 = 0 + b(20) + c(0)$$

$$20b = -34$$

$$b = -\frac{34}{20}$$

$$= -1.7$$

$$\sum x^2y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

$$186 = 20a + 164c \qquad (2)$$

1*5 $\rightarrow$ $20a + 100c = 130$

2 $\rightarrow$ $20a + 164c = 186$

$$-64c = -56$$

$$c = 0.875$$

$$4a + 20(0.875) = 26$$

$$4a = 36 - 17.5$$

$$a = \frac{8.5}{4}$$

$$= 2.125$$

$$y = a + bx + cx^2$$

$$\therefore y = 2.215 - 1.70x + 0.875x^2$$

5.5 APPLICATIONS OF CURVE FITTING

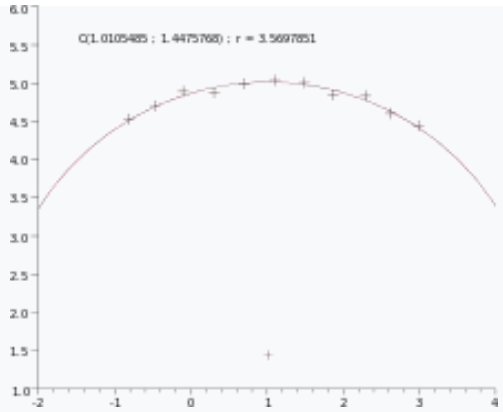
Fitting plane curves to data points :

If a function of the form cannot be postulated, one can still try to fit a plane curve.

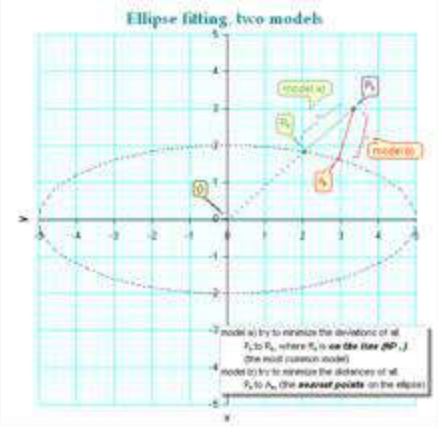
Other types of curves, such as conic sections (circular, elliptical, parabolic, and hyperbolic arcs) or trigonometric functions (such as sine and cosine), may also be used, in certain cases. For example, trajectories of objects under the influence of gravity follow a parabolic path, when air resistance is ignored. Hence, matching trajectory data points to a parabolic curve would make sense. Tides follow sinusoidal patterns, hence tidal data points should be matched to a sine wave, or the sum of two sine waves of different periods, if the effects of the Moon and Sun are both considered.

For a parametric curve, it is effective to fit each of its coordinates as a separate function of arc length; assuming that data points can be ordered, the chord distance may be used.

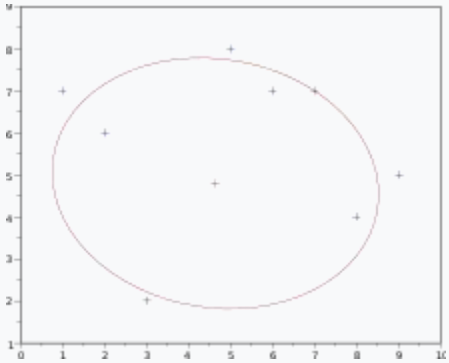
Fitting a circle by geometric fit



Circle fitting with the Coope method, the points describing a circle arc, centre (1 ; 1), radius 4.



different models of ellipse fitting



Ellipse fitting minimizing the algebraic distance (Fitzgibbon method).

Cooper[22] approaches the problem of trying to find the best visual fit of circle to a set of 2D data points. The method elegantly transforms the ordinarily non-linear problem into a linear problem that can be solved without using iterative numerical methods, and is hence an order of magnitude faster than previous techniques.

Fitting an ellipse by geometric fit

The above technique is extended to general ellipses[23] by adding a non-linear step, resulting in a method that is fast, yet finds visually pleasing ellipses of arbitrary orientation and displacement.

Application to surfaces

Further information: Computer representation of surfaces

Note that while this discussion was in terms of 2D curves, much of this logic also extends to 3D surfaces, each patch of which is defined by a net of curves in two parametric directions, typically called **u** and **v**. A surface may be composed of one or more surface patches in each direction.

APPLICATIONS OF CURVE FITTING

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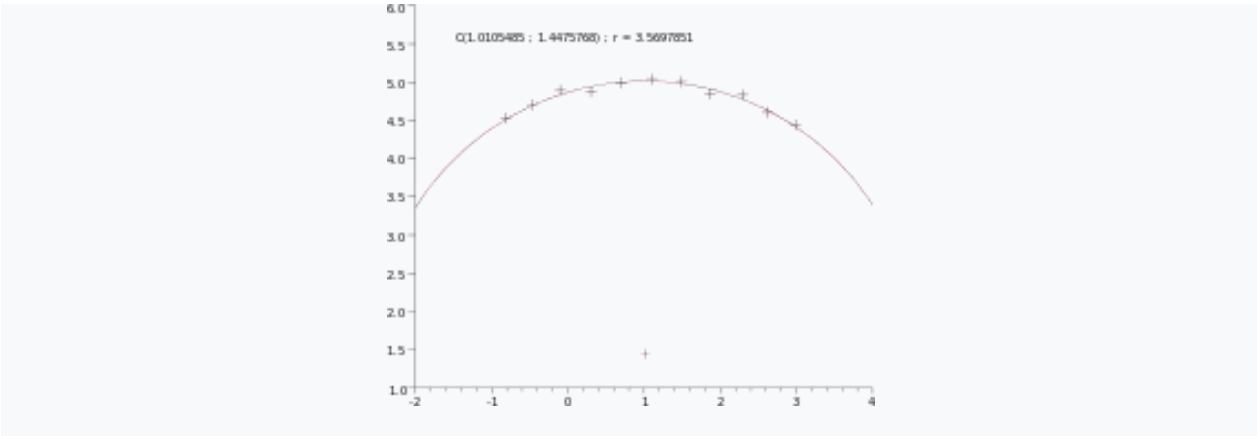
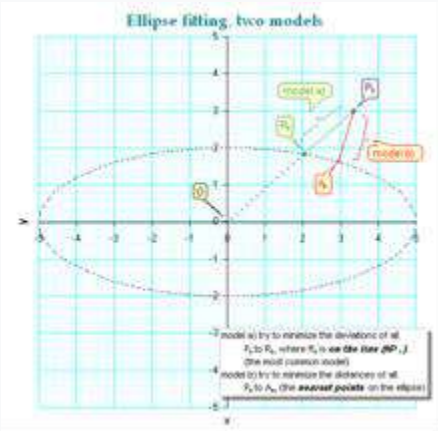
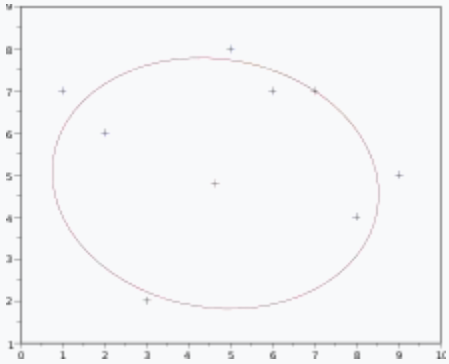
Fitting a circle by geometric fit



Circle fitting with the Coope method, the points describing a circle arc, centre (1 ; 1), radius 4.



different models of ellipse fitting



Ellipse fitting minimizing the algebraic distance (Fitzgibbon method).

Cooper[22] approaches the problem of trying to find the best visual fit of circle to a set of 2D data points. The method elegantly transforms the ordinarily non-linear problem into a linear problem that can be solved without using iterative numerical methods, and is hence an order of magnitude faster than previous techniques.

Fitting an ellipse by geometric fit

The above technique is extended to general ellipses[23] by adding a non-linear step, resulting in a method that is fast, yet finds visually pleasing ellipses of arbitrary orientation and displacement.

Application to surfaces

Further information: Computer representation of surfaces

Note that while this discussion was in terms of 2D curves, much of this logic also extends to 3D surfaces, each patch of which is defined by a net of curves in two parametric directions, typically called **u** and **v**. A surface may be composed of one or more surface patches in each direction.

A STUDY ON

APPLICATIONS OF NUMERICAL ANALYSIS

A Group Project submitted to KRISHNA UNIVERSITY

KRISHNA UNIVERSITY

In partial fulfilment of the requirement For the award of the degree of BACHELOR OF SCIENCE

Submitted by

V.MOHAN	18303228
B.VAMSI	18303230
G.ANILKUMAR	18303234
R.ESWARA RAO	18303238
K.GOWRI SANKAR	18303242

UNDER THE GUIDENCE OF SK.PARVEEN LECTURER IN MATHEMATICS.

S.R.R &C.V.R GOVT.DEGREE COLLEGE

MACHAVARAM, VIJAYAWADA-520004

S.R.R & C.V.R GOVT.DEGREE COLLEGE

CERTIFICATE

This is to certificate that the project entitled ("APPLICATIONS OF NUMERICAL ANALYSIS") bonafide workdone by V.MOHAN(18303228), B.VAMSI(18303230), G.ANIL KUMAR(18303234), R.ESWARA RAO(18303238), K.GOWRI SANKAR(18303242) belongs to department of bachelor science studies, S.R.R & C.V.R GOVT. DEGREE COLLEGE,VIJAYAWADA under my guidance in partial fulfilment of the award for the degree in bachelor of science.

EXTERNAL EXAMINAR K.V. NAGALAKSHMI SK.PARVEEN

(HOD of dept of mathematics) (PROJECT GUIDE)

DECLARATION

We V.MOHAN(18303228), B.VAMSI(18303230), G.ANIL KUMAR(18303234), R.ESWARA RAO(18303238), K.GOWRI SANKAR(18303242) here by declare that the project entitled “APPLICATIONS OF NUMERICAL ANALYSIS”has been prepared by in a partial fulfilment of the requirement for the award of the degree of bachelor of science submitted at S.R.R & C.V.R GOVT DEGREE COLLEGE affiliated to KRISHNA UNIVERSITY is an authentic project record of our own work .

The matter presented as not be submitted to any University or Institute for the award of bachelor of science Degree .

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ACKNOWLEDGEMENT

We would like to convey my deep felt regards to all of my well wishers, Wh have contributed their share of cooperation and help me to complete this project work .

Firstly we would like to thank. Dr.ILLA RAVI principal and SK.PARVEEN and K.V.NAGALAKSHMI HOD OF MATHEMATICS S.R.R &C.V.R GOVT.DEGREE COLLEGE VIJAYAWADA.for their Encouragement and guidance throughout my project work

We also express our heart thanks of to all my faculty for their support and encouragement in completed this project report in an elegant from last but not least, we extended our gratitude and courtesy of our family members and friends who have help us in completion of our project work .

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A PROJECT REPORT

On

A STUDY ON ADVANCED NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of **BACHELOR OF SCIENCE**

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

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CERTIFICATE

This is to certify that the project titled "A **STUDY ON ADVANCED NUMERICAL ANALYSIS & LAPLACE TRANSFORMS**" is a bonafide work done by **JALASUTRAM RAVITEJA (19304210)** under my guidance and supervision in partial fulfillment of the requirement for the award of degree of **Bachelor of Science (MSCS)**in **SRR&CVRGOVT.DEGREE COLLEGE(A)**, Vijayawada, affiliated to **Krishna University, Machilipatnam**, during the academic year **2021-2022.**

Project Guide

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EXTERNAL EXAMINER

DECLARATION

I hereby declare that this project titled **"A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"** is a bonafide work done by me under the guidance of **Dr. G. LALITHA.** This project work is submitted to **SRR&CVR GOVT. DEGREE COLLEGE(A),** Vijayawada, in partial fulfillment of the requirements for the award of **BACHELOR OF SCIENCE** during the academic year **2021-2022.**

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Date: 30-07-2022

Place: VIJAYAWADA

ACKNOWLEDGEMENT

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T. Pavleja

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Ch. Mukundapriya.

ACKNOWLEDGEMENT

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Date: 30/07/2022

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In partial fulfilment of the requirement for the award of degree of

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Date: 30-07-2020

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B. Jagadish.

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This is to certify that the project titled "A **STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS**" is a bonafide work done by **VENNA. NAGA PRASANNA ABHILASH (19304232)** under my guidance and supervision in partial fulfillment of the requirement for the award of degree of **Bachelor of Science (MSCS)**in **SRR&CVRGOVT.DEGREE COLLEGE(A)**, Vijayawada, affiliated to **Krishna University, Machilipatnam**, during the academic year **2021-2022.**

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Date: 30/07/2022.

Place: VIJAYAWADA

Venna. Naga prasanna Abhilash

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Project Guide

Dr. G. LALITHA

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Date: 30/07/2022

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G. Avinash

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J. Avinash

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EXTERNAL EXAMINER

DECLARATION

I hereby declare that this project titled **"A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"** is a bonafide work done by me under the guidance of **Dr. G. LALITHA.** This project work is submitted to **SRR&CVR GOVT. DEGREE COLLEGE(A),** Vijayawada, in partial fulfillment of the requirements for the award of **BACHELOR OF SCIENCE** during the academic year **2021-2022.**

I also declare that this project is a result of my own effort and that it has not been submitted to any other University for the award of any degree.

Date: 30/7/2022

Place: VIJAYAWADA

P Prabha kumari

ACKNOWLEDGEMENT

It is a great pleasure to take opportunity and express my gratitude to all those who helped me throughout my project work. First and foremost, I am thankful to **Dr.K.BHAGYALAKSHMI, Principal, SRR&CVR GOVT.DEGREE COLLEGE (A),** for giving me opportunity to take up the project work.

I also thank,**Dr. G. LALITHA,project guide** and**Head of the Department,K.V.N.LAKSHMI** for giving me opportunity to take up the project work and helping me through out and finally completing the project.

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J. Prabhakar

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10 NUMERICAL SOLUTIONS OF ORDINARY DIFFERENTIAL EQUATIONS 82-86

A PROJECT REPORT

On

A STUDY ON ADVANCED NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of **BACHELOR OF SCIENCE**

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

BEVARA LOKESH(19304239)

Under the Guidance of

Dr. G. LALITHA GARU

DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

This is to certify that the project titled "A **STUDY ON ADVANCED NUMERICAL ANALYSIS & LAPLACE TRANSFORMS**" is a bonafide work done by **BEVARA LOKESH (19304239)** under my guidance and supervision in partial fulfillment of the requirement for the award of degree of **Bachelor of Science** (MSCS)in **SRR&CVRGOVT.DEGREE COLLEGE(A)**, Vijayawada, affiliated to **Krishna University, Machilipatnam**, during the academic year **2021-2022.**

Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

DECLARATION

I hereby declare that this project titled **"A STUDY ON ADVANCED NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"** is a bonafide work done by me under the guidance of **Dr. G. LALITHA.** This project work is submitted to **SRR&CVR GOVT. DEGREE COLLEGE(A)**, Vijayawada, in partial fulfillment of the requirements for the award of **BACHELOR OF SCIENCE** during the academic year **2021-2022.**

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Date: 30-07-2022

Place: VIJAYAWADA

R. Lokesh

ACKNOWLEDGEMENT

It is a great pleasure to take opportunity and express my gratitude to all those who helped me throughout my project work. First and foremost, I am thankful to **Dr.K.BHAGYALAKSHMI, Principal, SRR&CVR GOVT.DEGREE COLLEGE (A),** for giving me opportunity to take up the project work.

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A PROJECT REPORT

On

A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of **BACHELOR OF SCIENCE**

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

ATMAKURI. BALARAM KRISHNA(19304246)

Under the Guidance of

Dr. G. LALITHA GARU

DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

This is to certify that the project titled "A **STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS**" is a bonafide work done by**Atmakuri. Balaram Krishna (19304246)** under my guidance and supervision in partial fulfillment of the requirement for the award of degree of **Bachelor of Science** (MSCS)in **SRR&CVRGOVT.DEGREE COLLEGE(A),** Vijayawada, affiliated to **Krishna University, Machilipatnam**, during the academic year **2021-2022.**

Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

DECLARATION

I hereby declare that this project titled **"A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"** is a bonafide work done by me under the guidance of **Dr. G. LALITHA.** This project work is submitted to **SRR&CVR GOVT. DEGREE COLLEGE(A),** Vijayawada, in partial fulfillment of the requirements for the award of **BACHELOR OF SCIENCE** during the academic year **2021-2022.**

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Date: 30-07-2022

Place: VIJAYAWADA

A. Balaram Krishna

ACKNOWLEDGEMENT

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A PROJECT REPORT

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A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of BACHELOR OF SCIENCE

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted R MAHESH BABU (19304248)

Under the Guidance of

Dr. G. LALITHA GARU

DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

This is to certify that the project titled "**A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS**" is a bonafide work done by R MAHESH BABU (19304248) under my guidance and supervision in partial fulfillment of the requirement for the award of degree of **Bachelor of Science (Mscs)**in **SRR&CVRGOVT.DEGREE COLLEGE(A)**, Vijayawada, affiliated to **Krishna University, Machilipatnam**, during the academic year **2021-2022.**

Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

DECLARATION

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Date: 30-07-2022

Place: VIJAYAWADA

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P. Mahesh Babu

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UNIT-I LAPLACE TRANSFORM

1).Find Laplace Transform of $[t^2+1]^2$

Sol: $[t^2+1]^2=t^4+2t^2+1$

A PROJECT REPORT

On

A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of **BACHELOR OF SCIENCE**

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

KANAKAPUDI BHAGAVAN(19304251)

Under the Guidance of

Dr. G. LALITHA GARU

DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

This is to certify that the project titled "A **STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"** is a bonafide work done by KANAKAPUDI BHAGAVAN **(19304251)** under my guidance and supervision in partial fulfillment of the requirement for the award of degree of **Bachelor of Science (MSCS)**in **SRR&CVRGOVT.DEGREE COLLEGE(A)**, Vijayawada, affiliated to **Krishna University, Machilipatnam**, during the academic year **2021-2022.**

Project Guide

Dr. G. LALITHA

EXTERNAL EXAMINER

DECLARATION

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Date: _____

Place: VIJAYAWADA

ACKNOWLEDGEMENT

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K. Bhagavan

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A PROJECT REPORT

On

A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS

Submitted to

SRR&CVR GOVT DEGREE COLLEGE (A), VIJAYAWADA, in partial fulfillment of the requirement for the award of Degree of **BACHELOR OF SCIENCE**

In partial fulfilment of the requirement for the award of degree of

BACHELOR OF SCIENCE

Submitted by

shaik ahammad vali (19304254)

Under the Guidance of

Dr. G. LALITHA GARU

DEPARTMENT OF MATHEMATICS

SRR&CVR GOVT. DEGREE COLLEGE(A)

MACHAVARAM, VIJAYAWADA

2021-2022

CERTIFICATE

This is to certify that the project titled **"A STUDY ON NUMERICAL ANALYSIS & LAPLACE TRANSFORMS"** is a bonafide work done by shaik ahammad vali (19304254) DSSunder my guidance and supervision in partial fulfillment of the requirement for the award of degree of **Bachelor of Science (Mscs)**in **SRR&CVRGOVT.DEGREE COLLEGE(A)**, Vijayawada, affiliated to **Krishna University, Machilipatnam**, during the academic year **2021-2022.**

Project Guide

Dr. G. LALITHA

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Date: 30/07/2022

Place: VIJAYAWADA

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Sk. Ahammad vali

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SRR AND CVR GOVERNMENT DEGREE COLLEGE(A), VIJAYAWADA.

COMMUNITY SERVICE PROJECT

MENTOR: Smt.P.Sailaja M.Sc.M.Phil

DEPARTMENT OF Electronics 2021-2022

Regd no:- 21 30 3205

ENERGY AUDIT

In

KAVULURU. G. Konduru

CSP on SHALABHA

I Year B.Sc.MECs

01-06-2022 to 25-06-2022

II SEMESTER / SUMMER

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CERTIFICATION

This is to certify that it is a bonafide Community Service Project Work completed by. SK. ARiff[redacted] in partial fulfillment of his UG programme in the period of study from 2022 to 2023.

Signature of the Mentor

i/c PRINCIPAL
SRR & CVR Govt. Degree College (A)
VIJAYAWADA-520004

28/9/22

Signature of the Principal 29/9/22

i/c PRINCIPAL
SRR & CVR Govt. Degree College (A)
VIJAYAWADA-520004

27/9/22

DECLARATION

I declare that this project log represents my ideas in my own words and wherever others' ideas or words have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my project work. I promise I maintained minimum 75% attendance. I understand that any violation of the above invites disciplinary action by the Institute.

SK. Arifa

Signature of the Student

CSP Mentor Smt. Dr. R. Kameshwari
Lecture in Electronics.

S.R.R. & C.V.R. GOVT. DEGREE COLLEGE
(Autonomous)
NAAC accredited with 'B' Grade
Dr. K. Bhagya Lakshmi, M.Sc., M.Ed., Ph.D.
Principal

Dt.01.06.2022

Subject: Community Service Project -
Letter of Recommendation for Data Collection and Research Report as part of academic work.

To Whomsoever It May Concer

This is to certify that Mr./Ms. SHAIF ARIFF is a bonafide student of SRR & CVR Government Degree College (A), Vijayawada. His /Her Hall Ticket No is 21302205. He/She is conducting a research on the topic " ENERGY AUDIT " under the supervision of Project Mentor Smt. P.Sailaja , Lecturer in Electronics, SRR & CVR Degree College (A), Vijayawada.

The aim of the Community Service Project is to provide practical learning experience to students and expose them to the real living conditions around them and encourage them to take an active role in finding solutions to issues or concerns in the society.

As part of this project, the student needs to collect data from your City/Town /Village/ Ward/ Institute/ Workplace/ Organization through surveys, interviews and interactions.

We request you to extend your cooperation to our students and assure you that the data - collection is purely for academic purpose.

Best Regards,

PRINCIPAL
SRR & CVR GOVT. DEGREE COLLEGE
(Autonomous)
Vijayawada - 520 010

ACKNOWLEDGEMENT

We are thankful to our Principal, Dr.K. Bhagyalakshmi M.Sc,P.hd,for her permission to select this project and for her continuous encouragement and valuable suggestions.

We are also thankful to our Head of the Department Smt. P.Sailaja madam M.Sc, M.phil, for continuous and constant motivation towards completion of this project.

We wish to express our gratitude to our faculty and project guide Smt. P.Sailaja madam M.Sc, M.phil,for her timely suggestions and encouragement.

We are also thankful to our village people for their support and cooperation while doing the surveys and my special thanks to our volunteer k. Keerthi and sachivalayam incharge for giving our village information

We are thankful to Friends and Dear ones who either directly or indirectly helped us in carrying out this project successfully.

Last but not the least; we thank our parents for their moral support towards completion of this Project.

Introduction

In India the energy conservation is very much essential as the demand in the society is increasing day by day, looking over the scenario, electrical energy audit is done. It is a process of checking how energy is used and identifying the areas where wastage can be minimized if not totally eradicated. Energy audit consists of several tasks which can be carried out depending on the type of the audit and the function of the audited facility. It starts with a review of the historical data on energy consumption, which can be compiled from the electricity bills. The energy audit work was carried out by various researchers for the consumption of energy is presented here.

Principale

An energy audit is an inspection survey and an analysis of energy flows for energy conservation in a building. It may include a process or system to reduce the amount of energy input into the system without negatively affecting the output.

When the object of study is an occupied building then reducing energy consumption while maintaining or improving human comfort, health and safety are of primary concern. Beyond simply identifying the sources of energy use, an energy audit seeks to prioritize the energy uses according to the greatest to least cost-effective opportunities for energy savings.

SRR & CVR GOVERNMENT DEGREE COLLEGE(A), VIJAYAWADA

COMMUNITY SERVICE PROJECT

MENTOR: Smt. *P. Sailaja M.Sc., M.Phil.*

DEPARTMENT OF ELECTRONICS 2021-2022

WOMEN EDUCATION

In

Christurajupuramu, Vijayawada

CSP by Sk. Hashmiun

Register no:- 21303201

I Year B.Sc.(MECs)

01-06-2022 to 25-06-2022

II SEMESTER's SUMMER

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SRR & CVR GOVT. DEGREE COLLEGE(A)

CERTIFICATE

This is to certify that this Community Service Project entitled with WOMEN'S EDUCATION in Christurajupuramu emitted to SRR&CVR GOVT. DEGREE COLLEGE (A), VIJAYAWADA in partial fulfilment to the requirements for the award of the degree of BACHELOR OF COMPUTER SCIENCE is done by MS. SK. HASHMIUN B.SC (Mecs) Regd.No:21303201 for the academic year 2020-2024.

Place: Christurajupuram

Date: 27/9/2022

Signature of the mentor Coordinator

(SMT P. SAILAJA)

Signature of the Principal

(Dr. K. BHAGYA LAKSHMI)
PRINCIPAL
SRR & CVR Govt. Degree College
VIJAYAWADA.

27/9/22

27/9/22

S.R.R. & C.V.R. GOVT. DEGREE COLLEGE
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NAAC accredited with 'B++' Grade
Machavaram, VIJAYAWADA - 520 004. Krishna District.
Cell : 9848251236 Ph : 0866-2430060, Fax : 0866-2441092, www.srrcvr.ac.in srrandcvr@gmail.com

Dr. K. Bhagya Lakshmi, M.Sc., M.Phil. Ph.D.
Principal

Date .....

Dt. 01.06.2022

Subject: Community Service Project - Letter of Recommendation for Data Collection and Research Report as part of academic work.

To Whomsoever It May Concern

This is to certify that Mr./Ms.SHAIK HASHMIUN is a bonafide student of SRR & CVR Government Degree College (A), Vijayawada. His /Her Hall Ticket No is 21303201 He/She is conducting a research on the topic " WOMEN EDUCATION " under the supervision of Project Mentor Smt. P.Sailaja , Lecturer in Electronics, SRR & CVR Degree College (A), Vijayawada.

The aim of the Community Service Project is to provide practical learning experience to students and expose them to the real living conditions around them and encourage them to take an active role in finding solutions to issues or concerns in the society.

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PRINCIPAL
SRR & CVR GOVT. DEGREE COLLEGE
(Autonomous)
Machavaram, VIJAYAWADA - 520 004.

CSP Mentor: Smt.P.Sailaja,
Lecture in Electronics.
9491753148

DECLARATION

I declare that this project log represents my ideas in my own words and wherever others' ideas or words have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my project work. I promise I maintained minimum 75% attendance. I understand that any violation of the above invites disciplinary action by the Institute.

Sk Haghmun

Sk. Hashmiun

(Signature ofthestudent)

ACKNOWLEDGEMENT

We are thankful to our Principal, Dr.K. Bhagyalakshmi M.Sc,P.hd,for her permission to select this project and for her continuous encouragement and valuable suggestions.We are also thankful to our Head of the Department Smt. P.Sailaja madam M.Sc, M.phil, for continuous and constant motivation towards completion of this project.We wish to express our gratitude to our faculty and project guide Smt. P.Sailaja madam M.Sc, M.phil,for her timely suggestions and encouragement.We are also thankful to our village people for their support and cooperation while doing the surveys and my special thanks to our volunteer Koteswaramma and sachivalayam incharge for giving our village information We are thankful to Friends and Dear ones who either directly or indirectly helped us in carrying out this project successfully. Last but not the least; we thank our parents for their moral support towards completion of this Project.

Sk. Hashmiun

I B.SC (M.E.Cs)

INTRODUCTION

Nowadays, the importance of women's education is growing day by day. It is not only important to educate girls and women, but also it is necessary to provide them with basic facilities. In many countries, especially in developing countries, the literacy rate of women is low as compared to men. The main reason behind this illiteracy rate among women is the lack of proper resources. Women's Education In India, the situation of women's education is not very good. According to the 2011 census, the literacy rate of Indian women was 64.6%. This number is quite low as compared to the literacy rate of men, which is 80.9%.

There are many reasons behind this illiteracy rate among women. The most important reason is the lack of proper resources. In India, most of the women are illiterate because they are not allowed to go for education. Society thinks that men must educate their children, especially girls, because they think that women's role is only to take care of the house and family. If she starts going to school or university, then who will look after her house? Moreover, sometimes when women send their children to schools, they are not allowed to sit in the same class as their male counterparts.

This is because of the social customs and traditions which are still prevailing in our society. The Government of India has made it

mandatory for all the schools to provide education to girls till middle school. However, this is not being implemented properly because of the lack of resources. The lack of resources is not the only reason behind the illiteracy rate of women. The mind set of people is also one of the main reasons. In our society, the role of women is still considered to be limited to the house and taking care of the family. This mind set is changing slowly, but it will take some time to change completely.

India is considered to be one of the fastest-growing economies in the world. Its democracy is also quite overwhelming for the entire world as the country comprises many cultures, religions, and diversities. Since independence from the Colonial Rule, India has done miracles in different phases and made significant development. This development is also due to women's education and empowerment steps taken by the Government.

The social stigma of gender bias and inequality is changing rapidly. India is becoming a superpower due to the similar contribution of both genders. By promoting education for women, India is also achieving a higher literacy rate very fast. This, on the other hand, will impressively help our country to progress in all aspects. The literacy rate of women was 8.6% after independence and has increased to 64% within 7 decades. Despite the glorifying facts, India is still wreaked with different malpractices and social stigmas in the deeper corners

Child labour, child marriage, dowry, etc., are a few of the prime reasons for gender inequality. Women were meant to be inside the house when men gathered food. This happened thousands of years ago when humans understood the biological differences between the genders. The time has changed. We have a haven for all of us to live and prosper. Women should enjoy every right men are enjoying. One of the prime rights of women that is entitled to be is education. Education is the prime weapon to fight all such social stigmas and illogical practices.

This is the major step towards the brighter future of India. There are many reasons why women are still not given similar rights to enjoy and get educated. They are thought to be the burden of a family. Even today, female feticide is practised. It means that this gender has less value in society. We need to educate the entire society regarding the social rights of all genders. Women should get educated to understand their rights as well. If you look a little deeper, we will find that many crimes circle women, such as trafficking, rape, feticide, murder, dowry, etc. Gender-based discrimination is the prime reason behind such crimes. Until and unless both genders are considered to be equal, these crimes will carry on as usual. Women should be educated first. This is the stepping stone to such a beautiful future.

Even in the 21st century, many families feel reluctant to send their girls children to pursue schooling and higher studies. One

of the prime reasons behind such malicious thought is economic disparity. Many families are unable to send their children to schools. When it comes to gender, they choose their sons and could not afford education for girl children.

CONTENT OF WOMEN'S EDUCATION

The subject matter of the women's education should be selected at the national level and adapted to the needs of each society. The themes include, occupational enrichment, consumer economics, civics, health and family living.

The content of women's education should be the reflection of the society's idea of the role assigned to women, namely, cooking, sewing, laundry for house-keeping and home management for rearing children. Other subjects are; science, technology, commerce, and arts. These are to make them able to compete with men in getting equal opportunity and income in the labour market. The content has to be the one that girls or women bring home; from school such as useful information about health and nutrition as well as vocational skill, that can be put to good service by the family. Ideally, women are to educate themselves, to acquire information and knowledge which will help them challenge norms, values and behaviour pattern. Women need an education which will help them not only to read and understand the world, and master the 3RS (reading, writing and arithmetic), but be masters of their lives and makers of their own destinies. They need education which will help them acquire the necessary analytical skills to understand the fast changing realities of life, which will give them the confidence and strength to refuse to submit to conditions of indignity and inhumanity. It should be the type of education that will help women form strong groups so that they can gain more control over Their lives and break their silence and make them visible. The classes environment should create an atmosphere which allows women more freedom, which gives them more opportunities to realise full human potentials.

Women need an education which will not only help them in the search and acquisition of new skills and knowledge, but also help them to acquire and strengthen values like justice, equality, honesty, truthfulness and solidarity. It should also create or release energies in women to act with conviction and courage in their various struggles at different levels. Education should help women to develop an analytical and questioning mind and a scientific approach in understanding the realities around them.

SRR AND CVR GOVERNMENT DEGREE COLLEGE(A), VIJAYAWADA.

COMMUNITY SERVICE PROJECT

MENTOR: Smt.*P.SailajaM.Sc,M.Phil*

DEPARTMENT OF Electronics **2021-2022**

ENERGY AUDIT

In

NAGARAJUKUNT Prakasham District

CSP by.Y. ANIL KUMAR REDDY

Reg No: 21303202

I Year B.Sc.MECs

01-06-2022 to 25-06-2022

II SEMESTER's SUMMER

CERTIFICATION

This is to certify that it is a bonafide Community Service Project Work completed by **Mr.y. Anil Kumar Reddy** in partial fulfillment of his UG programme in the period of study from
2022 to 2023.

Signature of the Mentor

Signature of the Principal

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VAYAWADA

27/9/2022.

27/9/2022

DECLARATION

I declare that this project log represents my ideas in my own words and wherever others' ideas or words have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my project work. I promise I maintained minimum 75% attendance.I **understand that any violation of the above invites disciplinary action by the Institute.**

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ACKNOWLEDGEMENT

We are thankful to our Principal, Dr.K. BhagyalakshmiM.Sc,P.hd,for her permission to select this project and for her continuous encouragement and valuable suggestions.

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MENTOR: Smt. P.SAILAJA (M.Sc,M.phil)

DEPARTMENT OF ELECTRONICS 2021-2022

ENERGY AUDIT

IN

NAGARAJUKUNTA, PRAKASAM DISTRICT

CSP BY

M. NARENDRA

REG NO:21303203

1st YEAR B.Sc(MECs)

01-06-2022 to 25-06-2022

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DECLARATION

I declare that this project log represents my ideas in my own words and wherever others' ideas or words have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea / data / fact / source in my project work. I promise I maintained minimum 75% attendance. I understand that any violation of the above invites disciplinary action by the Institute.

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INTRODUCTION

An energy audit, sometimes referred to as an energy survey or an energy inventory, is an examination of the total energy used in a particular property. The analysis is designed to provide a relatively quick and simple method of determining not only how much energy is being consumed but where and when. The energy audit will identify deficiencies in operating procedures and in physical facilities. Once these deficiencies have been identified, it will be apparent where to concentrate efforts in order to save energy. The energy audit is the beginning of and the basis for an effective energy-management programme. Human settlements encompass a variety of buildings. Regardless of the building involved, the audit procedure is basically the same. If more than one facility is involved (as in the case of apartment buildings), an audit of each will be necessary.

No two buildings are identical regarding energy usage. This is due to the possible variables affecting the buildings, e.g., occupancy rates, the building's size and orientation, its geographic location, the type of heating and cooling systems, the amount and types of equipment in use, the type of construction, the level of insulation and so on. Because each building is unique, it is difficult to generalize about energy-consumption patterns, and so it is necessary to conduct an energy audit for each building. Most buildings were probably designed, built and equipped when cheap energy was readily available. Little attention was paid to energy efficiency. Consequently, there is a great potential for improving operating costs of existing buildings. The chapters that follow explain the procedures and computations necessary to understand all phases of energy consumption. The data compiled will be suitable for use in both manual and computer-assisted energy audits. The emphasis is on the do-it-yourself approach, and the worksheets have been designed accordingly.

THE ENERGYAUDIT

In order to audit energy consumption, several steps are necessary. The basic procedures to be followed are:

(a) The historical audit summarizes all types and amounts of energy used in the past. Data should be compiled and analysed on the totals of both energy consumption and costs. This analysis, then, becomes the base with which future energy use will be compared.

(b) The diagnostic audit is carried out to identify the users of energy and to discover any deficiencies in operating and maintenance procedures as well as in physical facilities. This part of the audit is usually done in two parts: an equipment survey and a building survey.

(c) The financial evaluation determines the most cost-effective options. This will lead to the establishment of high-priority actions to be undertaken in the energy-management programme.

Each of these steps is explained in detail in the following pages, and sample worksheets are provided, where appropriate, to assist in recording the necessary data. Once the energy audit has been completed, there will be sufficient information upon which to establish an energy-management programme. However, the existing situation should be fully understood before any attempt is made at improvements.

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ENERGY AUDIT

In

Gunadala, Vijayawada

CSP by P.OmeshDuragaRam

I Year B.Sc.MECs

01-06-2022 to 30-06-2022

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CERTIFICATION

This is to certify that it is a bonafide Community Service Project Work completed by**Mr. *Omesh Durga Ram.Pandalaneni***in partial fulfillment of hisUG programme in the period of study from 2021 to 2022.

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DECLARATION

I declare that this project log represents my ideas in my own words and wherever others' ideas or words have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my project work. I promise I maintained minimum 75% attendance. I understand that any violation of the above invites disciplinary action by the Institute.

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Introduction

In India the energy conservation is very much essential as the demand in the society is increasing day by day, looking over the scenario, electrical energy audit is done. It is a process of checking how energy is used and identifying the areas where wastage can be minimized if not totally eradicated. Energy audit consists of several tasks which can be carried out depending on the type of the audit and the function of the audited facility. It starts with a review of the historical data on energy consumption, which can be compiled from the electricity bills. The energy audit work was carried out by various researchers for the consumption of energy is presented here.

Principals

An energy audit is an inspection survey and an analysis of energy flows for energy conservation in a building. It may include a process or system to reduce the amount of energy input into the system without negatively affecting the output.

When the object of study is an occupied building then reducing energy consumption while maintaining or improving human comfort, health and safety are of primary concern. Beyond simply identifying the sources of energy use, an energy audit seeks to prioritize the energy uses according to the greatest to least cost-effective opportunities for energy savings.

CONTENT of Energy Audit (Explanation)

A **home energy audit** is a service where the energy efficiency of a house is evaluated by a person using professional equipment (such as blower doors and infrared cameras), with the aim to suggest the best ways to improve energy efficiency in heating and cooling the house.

An energy audit of a home may involve recording various characteristics of the building envelope including the walls, ceilings, floors, doors, windows, and skylights. For each of these components the area and resistance to heat flow (R-value) is measured or estimated. The leakage rate or infiltration of air through the building envelope is of concern, both of which are strongly affected by window construction and quality of door seals such as weatherstripping. The goal of this exercise is to quantify the building's overall thermal performance. The audit may also assess the efficiency, physical condition, and programming of mechanical systems such as the heating, ventilation, air conditioning equipment, and thermostat.

A home energy audit may include a written report estimating energy use given local climate criteria, thermostat settings, roof overhang, and solar orientation. This could show energy use for a given time period, say a year, and the impact of any suggested improvements per year. The accuracy of energy estimates are greatly improved when the homeowner's billing history is available

showing the quantities of electricity, natural gas, fuel oil, or other energy sources consumed over a one or two-year period.

Some of the greatest effects on energy use are user behavior, climate, and age of the home. An energy audit may therefore include an interview of the homeowners to understand their patterns of use over time. The energy billing history from the local utility company can be calibrated using heating degree day and cooling degree day data obtained from recent, local weather data in combination with the thermal energy model of the building. Advances in computer-based thermal modeling can take into account many variables affecting energy use.

A home energy audit is often used to identify cost effective ways to improve the comfort and efficiency of buildings. In addition, homes may qualify for energy efficiency grants from central government.

Recently, the improvement of smartphone technology has enabled homeowners to perform relatively sophisticated energy audits of their own homes. This technique has been identified as a method to accelerate energy efficiency improvements.

SRR AND CVR GOVERNMENT DEGREE COLLEGE(A), VIJAYAWADA

COMMUNITY SERVICE PROJECT

MENTOR : Smt. P. SAILAJA

DEPARTMENT OF Electronics 2021-2022

HEALTH AND HYGIENE

In

D.M.R.NAGAR, VIJAYAWADA

CSP BY P. RECHAL JYOTHI
Reg. No. 21303206
I Year B.Sc.MECs

01-06-2022 to 25-06-2022

II SEMESTER's SUMMER

CERTIFICATE

This is to certify that this community service project Entitled with HEALTH & HYGIENE of different people in Vijayawada, D.M.Nagar submitted to SRR & CVR Govt. Degree College (A), Vijayawda in Partial to the requirements for the award of Degree of Bachelor of Science is done by P. Rechal Jyothi, B.Sc (MECS) Reg.No:21303206 for the academic year 2021 - 2022.

P. Rechal

Signature of the Mentor

Valued

27/9/2022.

Signature of the Principal

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DECLARATION

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ACKNOWLEDGEMENT

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Best Regards,

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HEALTH & HYGIENE

INTRODUCTION

Health is a positive state of well-being in which all parts of the body and mind remain in harmony and proper balance with all other parts.

So all the organs of the body function normally. It is well said that we can call only those persons healthy who have a healthy mind in a healthy body.

Health and hygiene is a vital phenomenon for a healthy and happy life. Health education plays a vital role in community hygiene. To prevent disease and have a **positive attitude** towards health, you need a correct and complete knowledge about health.
We relate health to cleanliness, and **cleanliness is one** of the principal ways to defend infectious diseases and self-preservation. In this lesson, we will discuss the real importance of health and hygiene, so we can achieve the goal of good health through healthy habits and a healthy lifestyle.

We need to understand a few things about Health and Hygiene

- *Understand well the concept of health and hygiene.*
- *Recognition of the importance of physical, mental, and spiritual health*
- *Importance of hygiene.*
- *Specify the need for personal, environmental, and hygienic hygiene.*

What is health?

Health is a characteristic of life that allows you to live longer. According to the World Health Organization (WHO): Health is a state of complete physical, mental, spiritual, and communal well-being, not just a lack of disease.

SRR AND CVR GOVERNMENT DEGREE COLLEGE(A),

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(ENERGY AUDIT)

MENTOR: Smt.*P.Sailaja M.Sc,M.Phil*

DEPARTMENT OF Electronics **2021-2022**

ENERGY AUDIT

In

SINGHNAGAR, Vijayawada

CSP by M. Chandra sekhar

I year BSc MECs

01-06-2022to25-06-2022

II SEMESTER's SUMMER

Reg.NO :- 2130207

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CERTIFICATION

This is to certify that it is a bona fide Community Service Project Work completed by Mr. *Medasani Chandrasekhar* in partial fulfillment of his UG programme in the period of study from 2022 to 2023.

Signature of the Mentor

Signature of the Principal

PRINCIPAL
SRR & CVR Govt. Degree College
VIJAYAWADA.

27/9/2022.

27/9/22

S.R.R. & C.V.R. GOVT. DEGREE COLLEGE
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Machavaram, VIJAYAWADA - 520 004, Krishna District.
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Dr. K. Bhagya Lakshmi, M.Sc., M.Phil., Ph.D
Principal

Date .....

Dt. 01.06.2022

Subject: Community Service Project - Letter of Recommendation for Data Collection and Research Report as part of academic work.

To Whomsoever It May Concern

This is to certify that Mr./Ms.MEDASANI CHANDRASEKHAR is a bonafide student of SRR & CVR Government Degree College (A), Vijayawada. His /Her Hall Ticket No is 21303207. He/She is conducting a research on the topic " ENERGY AUDIT " under the supervision of Project Mentor Smt. P.Sailaja , Lecturer in Electronics, SRR & CVR Degree College (A), Vijayawada.

The aim of the Community Service Project is to provide practical learning experience to students and expose them to the real living conditions around them and encourage them to take an active role in finding solutions to issues or concerns in the society.

As part of this project, the student needs to collect data from your City /Town /Village/ Ward/ Institute/ Workplace/ Organization through surveys, interviews and interactions.

We request you to extend your cooperation to our students and assure you that the data - collection is purely for academic purpose.

Best Regards,

PRINCIPAL
SRR & CVR GOVT. DEGREE COLLEGE
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Machavaram, VIJAYAWADA - 520 004.

P. Sailaja

CSP Mentor: Smt.P.Sailaja.
Lecture in Electronics.
9491753148

ACKNOWLEDGEMENT

We are thankful to our Principal, Dr.K. Bhagyalakshmi M.Sc,P.hd,for her permission to select this project and for her continuous encouragement and valuable suggestions.

We are also thankful to our Head of the Department *Smt. P.Sailaja madam M.Sc, M.phil,* ***for continuous and constant motivation towards completion of this project.***

We wish to express our gratitude to our faculty and project *guide Smt. P.Sailaja madam M.Sc, M.phil,* ***for her timely suggestions and encouragement.***

We are also thankful to our village people for their support and cooperation while doing the surveys and my special thanks to our volunteer Sudha garu and sachivalayam incharge for giving our village information

We are thankful to Friends and Dear ones who either directly or indirectly helped us in carrying out this project successfully.

Last but not the least; we thank our parents for their moral support towards completion of this Project.

M. Chandra Sekhar

DECLARATION

I declare that this project log represents my ideas in my own words and wherever others' ideas or words have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my project work. I promise I maintained minimum 75% attendance. I understand that any violation of the above invites disciplinary action by the Institute.

M. Chandra Sekhar

Signature of the Student

SRR AND CVR GOVERNMENT DEGREE COLLEGE(A), VIJAYAWADA

COMMUNITY SERVICE PROJECT

MENTOR : Smt. P. SAILAJA

DEPARTMENT OF Electronics 2021-2022

HEALTH AND HYGIENE

In

VELPURU, VIJAYAWADA

CSP BY V. LAVANYA
Reg. No. 21303208

I Year B.Sc.MECs

01-06-2022 to 25-06-2022

II SEMESTER's SUMMER

ACKNOWLEDGEMENT

We are thankful to our Principal, Dr.K. Bhagyalakshmi M.Sc,P.hd,for her permission to select this project and for her continuous encouragement and valuable suggestions.

We are also thankful to our Head of the Department Smt. P.Sailaja madam M.Sc, M.phil, for continuous and constant motivation towards completion of this project.

We wish to express our gratitude to our faculty and project guide Smt. P.Sailaja madam M.Sc, M.phil,for her timely suggestions and encouragement.

We are also thankful to our village people for their support and cooperation while doing the surveys and my special thanks to our volunteer k. Keerthi and sachivalayam incharge for giving our village information

We are thankful to Friends and Dear ones who either directly or indirectly helped us in carrying out this project successfully.

Last but not the least; we thank our parents for their moral support towards completion of this Project.

V.Lavanya.

DECLARATION

`I declare that this project log represents my ideas in my own words and wherever others' ideas or words have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my project work. I promise I maintained minimum 75% attendance. I understand that any violation of the above invites disciplinary action by the Institute.

J. Lavanya.

CERTIFICATE

This is to certify that this community service project Entitled with HEALTH & HYGIENE of different people in Vijayawada, VELPURU submitted to SRR & CVR Govt. Degree College (A), Vijayawda in Partial to the requirements for the award of Degree of Bachelor of Science is done by V. Lavanya, B.Sc (MECS) Reg.No:21303208 for the academic year 2021 - 2022.

Signature of the Mentor

Signature of the Principal

PRINCIPAL
SRR & CVR Govt. Degree College
VIJAYAWADA

S.R.R. & C.V.R. GOVT. DEGREE COLLEGE
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Cell : 9848251236 Ph : 0866-2430060, Fax : 0866-2441092, www.srcvr.ac.in srrandcvr@gmail.com

Dr. K. Bhagya Lakshmi, M.Sc., M.Phil., Ph.D.
Principal

Date .....

Dt. 01.06.2022

Subject: Community Service Project - Letter of Recommendation for Data Collection and Research Report as part of academic work.

To Whomsoever It May Concern

This is to certify that Mr./Ms.VADUGU LAVANYA is a bonafide student of SRR & CVR Government Degree College (A), Vijayawada. His /Her Hall Ticket No is 21303208. He/She is conducting a research on the topic " HEALTH AND HYGIENE" under the supervision of Project Mentor Smt. P.Sailaja , Lecturer in Electronics, SRR & CVR Degree College (A), Vijayawada.

The aim of the Community Service Project is to provide practical learning experience to students and expose them to the real living conditions around them and encourage them to take an active role in finding solutions to issues or concerns in the society.

As part of this project, the student needs to collect data from your City /Town /Village/ Ward/ Institute/ Workplace/ Organization through surveys, interviews and interactions.

We request you to extend your cooperation to our students and assure you that the data - collection is purely for academic purpose.

Best Regards,

PRINCIPAL
SRR & CVR GOVT. DEGREE COLLEGE
(Autonomus)
Machavaram, VIJAYAWADA - 520 004.

CSP Mentor: Smt.P.Sailaja,
Lecture in Electronics.
9491753148

S.R.R AND C.V.R GOVERNMENT DEGREE COLLEGE (A),

VIJAYAWADA

COMMUNITY SERVICE PROJECT

MENTOR: Smt. P.SAILAJA {M.Sc,M.phil}

DEPARTMENTOF ELECTRONICS 2021-2022

WATER FACILITY AND DRINKING WATER AVAILABILITY

IN

MACHAVARAM, VIJAYAWADA

CSP BY

YELISALA CHARMI

REG NO:21303209

Ist YEAR B.Sc{MECs}

01-06-2022 to 25-06-2022

CERTIFICATION

This is to certify that it is a bonafide Community Service Project Work Ms.YELESALA CHARMI completed by in partial fulfillment of his UG programme in the period of study from 2022 to 2023.

Signature of the Mentor

Signature of the Principal

PRINCIPAL
SRR & CVR Govt. Degree College
VIJAYAWADA.

Valued

27/9/2022.

27/9/22

DECLARATION

I declare that this project log represents my ideas in my own words and wherever others' ideas or words have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my project work. I promise I maintained minimum 75% attendance.I understand that any violation of the above invites disciplinary action by the Institute.

Y. Charmi

Signature of the Student

S.R.R. & C.V.R. GOVT. DEGREE COLLEGE
(Autonomous)
NAAC accredited with 'B' Grade
Dr. K. Bhagya Lakshmi, M.Sc., M.Ed., Ph.D.
Principal

01.01.06.2022

Subject: Community Service Project - Letter of Recommendation for Data Collection and Research Report as part of academic work.

To Whomsoever It May Concern

This is to certify that Mr./Ms.YELISELA CHARMI is a bonafide student of SRR & CVR Government Degree College (A), Vijayawada. His /Her Hall Ticket No is 21303209 .He/She is conducting a research on the topic " DRINKING WATER FACILITY " under the supervision of Project Mentor Smt. P.Sailaja , Lecturer in Electronics, SRR & CVR Degree College (A), Vijayawada.

The aim of the Community Service Project is to provide practical learning experience to students and expose them to the real living conditions around them and encourage them to take an active role in finding solutions to issues or concerns in the society.

As part of this project, the student needs to collect data from your City /Town /Village/ Ward/ Institute/ Workplace/ Organization through surveys, interviews and interactions.

We request you to extend your cooperation to our students and assure you that the data - collection is purely for academic purpose.

Best Regards,

PRINCIPAL
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(AUTONOMOUS)
VIJAYAWADA

CSP Mentor: Smt.P.Sailaja,
Lecture in Electronics,
9491753148

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INTRODUCTION

Drinking water is water that is used in drink or food preparation; potable water is water that is safe to be used as drinking water. The amount of drinking water required to maintain good health varies, and depends on physical activity level, age, health-related issues, and environmental conditions. For those who work in a hot climate, up to 16 litres (4.2 US gal) a day may be required "" On average, American households use 300 gallons of water a 13) day Typically in developed countries, tap water meets drinking water quality standards, even though only a small proportion is actually consumed or used in food preparation. All public water suppliers in the US must uphold a certain standard of water quality. If the requirements are met, Americans can drink their local tap water Other typical uses for tap water include washing, toilets, and irrigation Greywater may also be used for toilets or irrigation. Its use for irrigation however may be associated with risks. Water may also be unacceptable due to levels of toxins or suspended solids.

Globally, by 2015, 89% of people had access to water from a source that is suitable for drinking - called improved water source In sub-Saharan Africa, access to potable water ranged from 40% to 80% of the population. Nearly 4.2 billion people worldwide had access to tap water, while another 2.4 billion had access to wells or public taps. The World Health Organization considers access to safe drinking-water a basic human right About 1 to 2 billion people lack safe drinking water Water can carry vectors of disease. More people die from unsafe water than from war, then-U.N. Secretary-General Ban Ki-moon said in 2010. Third world countries are most affected by lack of water, flooding, and water quality. Up to 80 percent of illnesses in developing countries are the direct result of inadequate water and sanitation.

ACKNOWLEDGEMENTS

We are thankful to our Principal, Dr.K. Bhagyalakshmi M.Sc,P.hd,for her permission to select this project and for her continuous encouragement and valuable suggestions.

We are also thankful to our Head of the Department Smt. P.Sailaja madam M.Sc, M.phil, for continuous and constant motivation towards completion of this project.

We wish to express our gratitude to our faculty and project guide Smt. P.Sailaja madam M.Sc, M.phil,for her timely suggestions and encouragement.

We are also thankful to our village people for their support and cooperation while doing the surveys and my special thanks to our volunteer VIJAY and sachivalayam incharge for giving our village information

We are thankful to Friends and Dear ones who either directly or indirectly helped us in carrying out this project successfully.

Last but not the least; we thank our parents for their moral support towards completion of this Project.

SRR & CVR GOVERNMENT DEGREE COLLEGE(A), VIJAYAWADA

COMMUNITY SERVICE PROJECT

MENTOR: Smt. *P. Sailaja M.Sc., M.Phil.*

DEPARTMENT OF ELECTRONICS 2021-2022

WOMEN EDUCATION

In

Khuddus nagar, Vijayawada

CSP by Ch. Honey

Register no:- 21303210

I Year B.Sc.(MECs)

01-06-2022 to 25-06-2022

II SEMESTER's SUMMER

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SRR & CVR GOVT. DEGREE COLLEGE(A)

CERTIFICATE

This is to certify that this Community Service Project entitled with WOMEN'S EDUCATION in Christurajupuramu emitted to SRR&CVR GOVT. DEGREE COLLEGE (A), VIJAYAWADA in partial fulfilment to the requirements for the award of the degree of BACHELOR OF COMPUTER SCIENCE is done by MS. CH. HONEY B.SC (Mecs) Regd.No:21303210 for the academic year 2020-2024.

Place: Khuddus Nagar

Date: 27/9/2022

Signature of the mentor
Coordinator

(SMT P. SAILAJA)

Signature of the Principal

(Dr. K. BHAGYA LAKSHMI)

PRINCIPAL
SRR & CVR Govt. Degree College
VIJAYAWADA.

Values

27/9/2022.

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Dr. K. Bhagya Lakshmi, M.Sc., M.Phil, Ph.D.
Principal

Date .....

Dt. 01.06.2022

Subject: Community Service Project - Letter of Recommendation for Data Collection and Research Report as part of academic work.

To Whomsoever It May Concern

This is to certify that Mr./Ms.CHIGURUPATI HONEY is a bonafide student of SRR & CVR Government Degree College (A), Vijayawada. His /Her Hall Ticket No is 21303210.He/She is conducting a research on the topic " WOMEN EDUCATION " under the supervision of Project Mentor Smt. P.Sailaja , Lecturer in Electronics, SRR & CVR Degree College (A), Vijayawada.

The aim of the Community Service Project is to provide practical learning experience to students and expose them to the real living conditions around them and encourage them to take an active role in finding solutions to issues or concerns in the society.

As part of this project, the student needs to collect data from your City /Town /Village/ Ward/ Institute/ Workplace/ Organization through surveys, interviews and interactions.

We request you to extend your cooperation to our students and assure you that the data - collection is purely for academic purpose.

Best Regards,

PRINCIPAL
SRR & CVR GOVT. DEGREE COLLEGE
(Autonomous)
Machavaram, VIJAYAWADA - 520 004.

CSP Mentor: Smt.P.Sailaja,
Lecture in Electronics.
9491753148

DECLARATION

I declare that this project log represents my ideas in my own words and wherever others' ideas or words have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my project work. I promise I maintained minimum 75% attendance. I understand that any violation of the above invites disciplinary action by the Institute.

Ch. Honey

Ch. Honey

(Signature of the student)

ACKNOWLEDGEMENT

We are thankful to our Principal, Dr.K. Bhagyalakshmi M.Sc,P.hd,for her permission to select this project and for her continuous encouragement and valuable suggestions.We are also thankful to our Head of the Department Smt. P.Sailaja madam M.Sc, M.phil, for continuous and constant motivation towards completion of this project. We wish to express our gratitude to our faculty and project guide Smt. P.Sailaja madam M.Sc, M.phil,for her timely suggestions and encouragement. We are also thankful to our village people for their support and cooperation while doing the surveys and my special thanks to our volunteer S.Triveni and sachivalayam incharge for giving our village information We are thankful to Friends and Dear ones who either directly or indirectly helped us in carrying out this project successfully. Last but not the least; we thank our parents for their moral support towards completion of this Project.

CH. HONEY

I B.SC (M.E.Cs)

SRR & CVR GOVERNMENT DEGREE COLLEGE(A), VIJAYAWADA

COMMUNITY SERVICE PROJECT

MENTOR: Smt. *P. Sailaja M.Sc., M.Phil.*

DEPARTMENT OF ELECTRONICS 2021-2022

USAGE OF ELECTRONICS GADGETS

In

MARUTHI NAGAR, Vijayawada

CSP by B. G. M. KALIDAS

I Year B.Sc. MECs

01-06-2022 to 25-06-2022

II SEMESTER's SUMMER

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SRR & CVR GOVT. DEGREE COLLEGE(A)

CERTIFICATE

This is to certify that this Community Service Project entitled with USAGE OF ELECTRONIC GADGETS in MARUTHI NAGAR submitted to SRR&CVR GOVT. DEGREE COLLEGE (A), VIJAYAWADA in partial fulfilment to the requirements for the award of the degree of BACHELOR OF COMPUTER SCIENCE is done by MR. B.G.M. KALIDAS B.SC (M.E. Cs) Regd.No:21303201 for the academic year 2020-2024.

Place: Vijayawada, Maruthi Nagar.

Date: 27-9-2022

Signature of the mentor

Coordinator

(SMT P. SAILAJA)

Signature of the Principal

(DR. K. BHAGYA LAKSHMI)

PRINCIPAL
SRR & CVR Govt. Degree College
VIJAYAWADA.

27/9/2022.

27/9/22

S.R.R. & C.V.R. GOVT. DEGREE COLLEGE
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Cell : 9848251236 Ph : 0866-2430966, Fax 0866-2441092, www.srcvr.ac.in srrandcvr@gmail.com

Dr. K. Bhagya Lakshmi, M.Sc. M.Phil Ph.D
Principal

Date .....

Dt. 01.06.2022

Subject: Community Service Project - Letter of Recommendation for Data Collection and Research Report as part of academic work.

To Whomsoever It May Concern

This is to certify that Mr./Ms.BHOGIPATHI GAYATRI MRUTHYUN JAYA KALIDAS is a bonafide student of SRR & CVR Government Degree College (A), Vijayawada. His /Her Hall Ticket No is 21303211. He/She is conducting a research on the topic " USAGE OF ELECTRONIC GADGETS " under the supervision of Project Mentor Smt. P.Sailaja , Lecturer in Electronics, SRR & CVR Degree College (A), Vijayawada.

The aim of the Community Service Project is to provide practical learning experience to students and expose them to the real living conditions around them and encourage them to take an active role in finding solutions to issues or concerns in the society.

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We request you to extend your cooperation to our students and assure you that the data - collection is purely for academic purpose.

Best Regards,

PRINCIPAL
SRR & CVR GOVT. DEGREE COLLEGE
(Autonomous)
Machavaram, VIJAYAWADA - 520 004.

CSP Mentor: Smt.P.Sailaja,
Lecture in Electronics.
9491753148

DECLARATION

I declare that this project log represents my ideas in my own words and wherever others' ideas or words have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my project work. I promise I maintained minimum 75% attendance. I understand that any violation of the above invites disciplinary action by the Institute.

B.G.M. Kalidas

Place: MARUTHI NAGAR

Date: 27-9-2022

(B.G.M.KALIDAS)

Reg. No. 21303211.

ACKNOWLEDGEMENT

We are thankful to our Principal, Dr. K. Bhagya Lakshmi M.Sc., Ph.D., for her permission to select this project and for her continuous encouragement and valuable suggestions.

We are also thankful to our Head of the Department Smt. P. Sailaja M.Sc., M.Phil., for continuous and constant motivation towards completion of this project.

We wish to express our gratitude to our faculty and project guide Smt. P. Sailaja M.Sc., M.Phil., for her timely suggestions and encouragement.

We are also thankful to our village people for their support and cooperation while doing the surveys and my special thanks to our volunteer Madhavi for giving our area information.

We are thankful to Friends and Dear ones who either directly or indirectly helped us in carrying out this project successfully.

Last but not the least, we thank our parents for their moral support towards completion of this Project.

A STUDY ON

COMPARISON OF ORGANIC AND CHEMICAL FARMING OF PADDY WITH SPECIAL REFERENCE TO PENAMALURU MANDAL AREA

A Group Project Submitted to S.R.R & C.V.R GDC(A) in partial fulfilment of the requirements for the award of degree of

BACHELOR OF SCIENCE

Submitted by

CH. Sai Babu	19304227
S. Jyothi	19304222
Sk. Parveen Sultana	19304221
T. Veena Sahithi	19304247
G.Revathi	19304204

UNDER THE GUIDANCE OF

SRI KOLLU ASHOK

LECTURER IN-CHARGE, DEPARTMENT OF STATISTICS

S.R.R & C.V.R GOVT. DEGREE COLLEGE [A]
MACHAVARAM, VIJAYAWADA-520004

2019-2022

SRR & CVR GOVT DEGREE COLLEAGE [A]
MACHAVARAM, VIJAYAWADA-520004

CERTIFICATE

This is to certify that it is a bonafide record of the project work entitled "COMPARISON OF ORGANIC AND CHEMICAL FORMING OF PADDY WITH SPECIAL REFERENCE TO PENAMALURU MANDAL AREA "done by Ch.Sai babu – 19304227, S.Jyothi – 19304222, Sk.Parveen Sultana – 19304221, T.Veenasahithi – 19304247, G.Revathi – 19304204 belong to the Department of STATISTICS in partial fulfilment of the requirements for the award of degree of Bachelor of Science[MSCs] at S.R.R&C.V.R GOVT COLLEGE(A), VIJAYAWADA during the period 2019-2022.

Project Guide(HOD)
(K.Ashok)

Head of the Dept. of Statistics
S.R.R. & C.V.P. GOVT. COLLEGE
VIJAYAWADA - 4.

Principal

PRINCIPAL
SRR & CVR Govt. Degree College
VIJAYAWADA.

External Examiner 21/8/2022

DECLERATION

We CH.Saibabu, S.Jyothi, Sk.ParveenSulthana, T.Veenasahithi, G.Revathi here by declare that the project work entitled "Comparison of Organic and Chemical farming of Paddy" is a bonafide work done by us under the guidance of Sri K.Ashok.

This project work is submitted to S.R.R & C.V.R GOVT DEGREE COLLEGE (A), Vijayawada, in partial fulfilment of the requirements for the award of BACHELOR OF SCIENCE during the academic year 2019-2022.

We also declare that this project is a result of our own effort and
that it has not been submitted to any other University for the award of any degree.

CH. Sai Babu	19304227	Ch. Saibabu
S. Jyothi	19304222	S. Jyothi
Sk. Parveen Sulthana	19304221	Sk. Parveen
T. Veena Sahithi	19304247	T. Veenasahithi
G.Revathi	19304204	G. Revathi

Date: 29/07/22
Place: VIJAYAWADA

ACKNOWLEDGEMENT

It is great pleasure to take opportunity and express our gratitude to all those who helped us throughout our project work.

First and foremost, we thankful to Dr K. BHAGYA LAKSHMI, Principal, S.R.R & C.V.R GOVT DEGREE COLLEGE (A), for giving us opportunity to take up the project work.

Our sincere thanks Sri K. ASHOK, project guide and Head of the Department of STATISTICS for giving us opportunity to take up the project work and helping us through out and finally completing the project.

We would like to express our sincere and heartful thanks to all the teaching and non-teaching staff of the department for their continuous co-operation, which has given us the guidance to build up adamant aspiration over the completion of my project.

We extend my heartful thanks to our parents and friends for their constant support and cooperation in completing this project successfully.

Finally, We thank one and all who directly and indirectly helped to complete our project successfully.

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1. INTRODUCTION

Agriculture: -

Agriculture is the art and science of cultivating the soil, production of crops, and raising livestock. Plants and animal products are being produced for people to utilize and distribute them to the markets. Agriculture was the key advancement within the rise of sedentary human civilization, whereby farming of tamed species made nourishment surpluses that sustained the advancement of civilization. The study of agriculture is known as agricultural science. The history of agriculture dates back thousands of a long time, and its improvement has been driven and characterized by incredibly distinctive climates, cultures, and advances, and modern technologies. Industrial agribusiness based on large-scale monoculture cultivating has ended up the prevailing agrarian method.

https://www.nicheagriculture.com/agriculture-definition-of-agriculture-by-niche-agriculture/#:~:text=up%20the%20prevailing%20agrarian%20methodology.

Agricultural efficiency in India depends on a few components. These include the accessibility and quality of rural inputs such as arrive, water, seeds, and fertilizers, access to agrarian credit and crop insurance, assurance of profitable costs for rural delivery, and storage and marketing infrastructure.

History of Agriculture:

The history of agriculture in India dates back to the Indus Valley civilization period and indeed even before that in a few parts of southern India. The agricultural sector is one of the foremost imperative businesses within the Indian economy, which implies it0020is additionally a colossal boss. Roughly 60 percent of the Indian populace works within the industry, contributing almost 18 percent to India's Gross Domestic Production (GDP). This share diminishes continuously with each year, with the advancement in other sectors and divisions of the country's economy**.**

Agriculture Department

Agricultural Department refers to the federal department that administers the programs and provides services to the farmers including research and conservation of soils and efforts to stabilize the economy created in 1862.

Agricultural Products

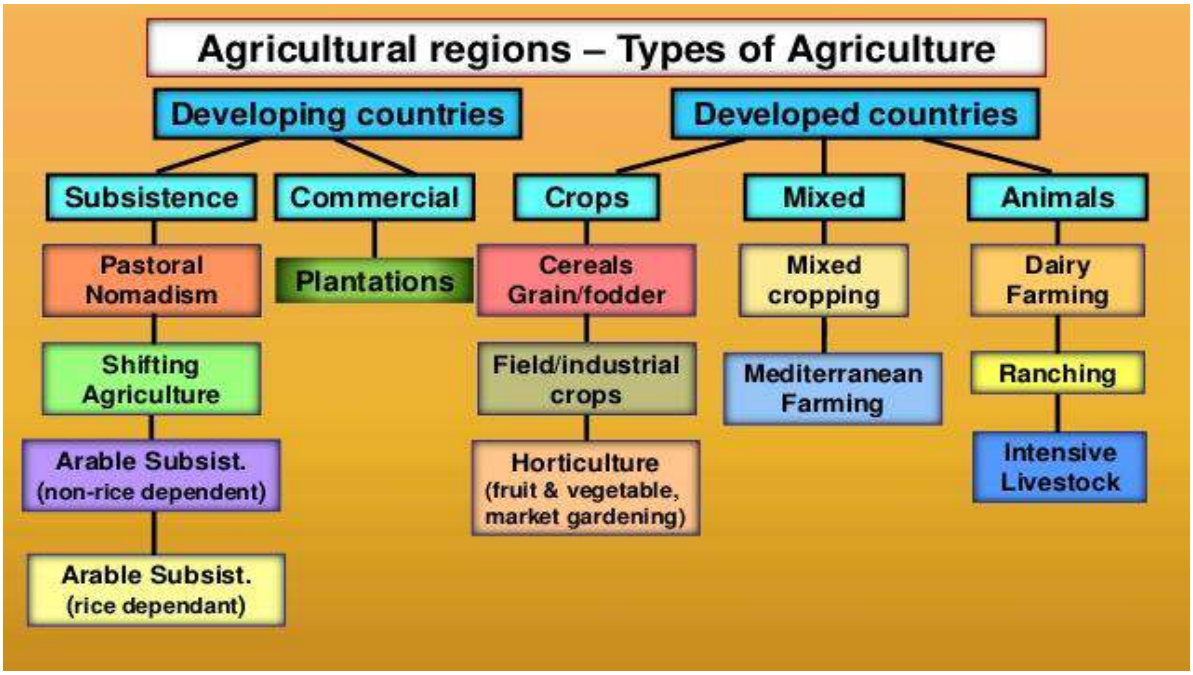
Agrarian items imply crops, animals, and livestock items, including but not constrained to field crops, natural products, vegetables, agricultural specialties, cattle, sheep, hogs, goats, steeds,poultry, fur-bearing animals, drain eggs, and hides.

Agricultural Tools

In older times, tools used in agriculture are tractor, combine, plough fertilizer, and many others. But, with these modern techniques tools like transportation, cultivation, new irrigation facilities like drip irrigation, and other pest control systems for better sustainability and cultivation.

Sustainable Agriculture

Sustainable agriculture refers to the means of meeting societies present food and with the needs of textiles without compromising the ability for other current needs and sustains them for future generations.



BENEFITS OF AGRICULTURE

- Source of.
- Livelihood. Most people's main source of livelihood is farming. ...
- Significance to the International Trade. ...
- Marketable Surplus. ...
- Source of Raw Material. ...
- Significance in Transport. ...
- Foreign Exchange Resources. ...
- Great Employment Opportunities.

PADDY

India has the largest Paddy output in the world and is also the fourth largest exporter of rice in the world. In India, West Bengal is the largest rice producing state. Paddy fields are a common sight throughout India, both in the northern genetic plains and the southern peninsular plateaus. Paddy is cultivated at least twice a year in most parts of India, the two seasons being known as Rabi and Kharif respectively. The former cultivation is dependent on irrigation, while the latter depends on Monsoon.

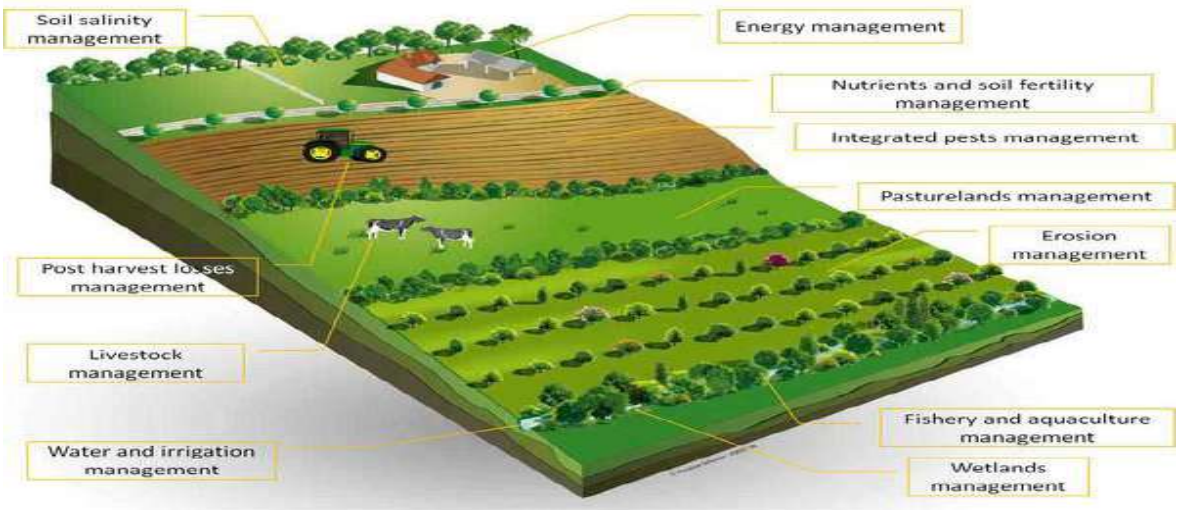
The Paddy cultivation plays a major role in Socio-culture life of rural India. Many festival such as Onam in Kerala, Bihu in Assam, Sankranti in Andhra Pradesh, Thai Pongal In Tamil Nadu, Makar Sankranti in Karnataka, Nabanna in West Bengal celebrates harvest of Paddy.Kaveri delta region of Thanavur is historically known as the rice bowl of Tamil Nadu and Kuttanadu is called the rice bowl of Kerala.

ORGANIC FARMING

Organic farming is a method of crop and livestock product that involves much more than choosing not to use pesticides, fertilizers, genetically modified organisms, antibiotics and growth hormones. Organic production is a holistic system designed to optimize the productivity and fitness of diverse communities within the agro-ecosystem, including soil organisms, plants, livestock and people. The principal goal of organic production is to develop enterprises that are sustainable and harmonious with the environment.

Organic farming promotes the use of crop rotations and cover crops, and encourages balanced host/predator relationships. Organic residues and nutrients produced on the farm are recycled back to the soil. Cover crops and composted manure are used to maintain soil organic matter and fertility.

Organic farming presents many challenges. Some crops are more challenging than others to grow organically; however, nearly every commodity can be produced organically.



https://www.indiafilings.com/learn/integrated-organic-farming-system/

Organic farming Fertilizers:-

- Neemastram
- Agrastram
- Brahmas tram
- Sour butter Milk
- Panchagavya
- Sapta dharyankura dravanam

CHEMICAL FARMING

Chemical farming in India Agriculture remains the principal source of livelihood for majority of the population in India. The increased use of synthetic fertilizers and chemical pesticides in agriculture started in India since 1960s as part of the Green Revolution. Over the positive decades synthetic fertilizers' consumption has drastically increased several folds and India is now one of the leading producers of agrochemicals in the world. Many of the chemical pesticides can have harmful effects on human beings either as acute or chronic toxicity. Acute exposure to pesticides can lead to death or serious illnesses. About 355,000 people die globally each year due to unintentional acute poisonings. Two-thirds of these deaths occur in developing countries where such poisonings are associated with excessive exposure and or inappropriate use of toxic chemicals and pesticides present in occupational and domestic environments. The cumulative health impacts of human exposures to various agrochemicals can be a factor in a range of chronic health conditions and diseases like cancer, reproductive, endocrine, immunological, congenital and developmental disorders.(Rakesh.P.Seatl,Sep.-Oct.2013,IOSR Journal Of Environmental Science, Toxicology And Food Technology (IOSR-JESTFT), Volume 5, Issue 5,Vellore 632004, Tamil Nadu, India.

Chemical farming fertilizer

- Zinc
- D.A.P
- Refit
- Namini gold
- 14-35-14
- Urea
- Synjenta
- Victoria
- Aartin
- Navacraan
- Corobaan
- 20-00-15
- Potash
- Keldon Powder
- Saahf
- Bheem

https://www.goodstuffnw.com/tag/green-revolution/

Various Types of Seeds in Kharif & Rabi (Organic & Chemical)

S.No	Seeds	Kharif	Rabi	Days	Tonne
1	Siddi (WGL - 44)	Yes	No	150	6.0 - 7.0
2	Samba Mashuri (B.PT. - 5204)	Yes	No	150	5.5 - 6.0
3	MTU 1010	No	Yes	110	6.0 - 6.5
4	Swarna (MTU 7029)	Yes	No	150	6.5 - 7.0
5	Krishna (RNR 2458)	Yes	No	135	6.5 - 7.0
6	SriDruthi (MTU 1121)	No	Yes	130	6.0 - 7.0
7	Varangal Sannalu (WGL 32100)	Yes	No	135	5.5 - 6.0
8	Praanahita (JGL 11727)	Yes	No	135	6.5 - 7.0
9	Tharamgani (MTU 1156)	No	Yes	120	6.5 - 7.0
10	Bhadrakaali (WGL 3962)	Yes	No	135	6.5 - 7.0
11	Vijetha (MTU 1001)	Yes	No	135	7.0 - 7.5
12	Chandra (MTU 1153)	No	Yes	120	6.5 - 7.5
13	Sumathi (RNR 18833)	Yes	No	140	6.5 - 7.0
14	Sugandha Samba (RNR 2465)	Yes	No	135	5.5 - 6.5
15	IR 64	No	Yes	125	6.5 - 7.0
16	Shobini (RNR 2354)	Yes	No	125	6.5 - 7.5
17	NLR 34449	No	Yes	120	6.0 - 6.5
18	Swarna Mukhi (NLR 145)	Yes	No	140	7.0 - 7.5
19	NLR 3354	No	Yes	125	6.0 - 6.5
20	Seethal (WGL 283)	No	Yes	130	6.5 - 7.0
21	Kaatan Dhora (MTU 1010)	Yes	Yes	125	7.5 - 8.5
22	Raamappa (WGL 23985)	Yes	Yes	125	6.0 - 6.5
23	RNR 15048	Yes	Yes	125	6.5 - 7.0
24	JGL 18047	Yes	Yes	120	7.0 - 7.5
25	KNM 118	Yes	Yes	125	7.0 - 7.5

2.OBJECTIVES

- To Study Organic & Chemical Farming in Paddy
- To Study Last five years data of Organic & Chemical Farming in Paddy
- To observe various Types of Seeds in Organic & Chemical Farming
- To understand Organic Fertilizers & Chemical Fertilizers
- To estimate cost of cultivation in Paddy based on the data during 2016 - 2021 (Organic & Chemical)
- To compare Organic & Chemical Farming
- To compare the yield in kharif and Rabi season

3.METHODOLOGY

Secondary Data:

The data can be collected asSecondary data. It means that the information is already available.

The data is collected from Agriculture centre& Rhythu Bharosa kendram at Penamaluru Mandal. We collect the last five years of Organic & Chemical Farming data in Paddy.

EXCEL: Microsoft Excel is a spreadsheet developed by Microsoft for Windows, macOS, Android and iOS. It features calculation or computation capabilities, graphing tools, pivot tables, and a macro programming language called Visual Basic for Applications. Excel forms part of the Microsoft Office suite of software.The data was tabulated and visualised using appropriate graphical tools.

R Programming: R is a programming language for statistical computing and graphics supported by the R Core Team and the R Foundation for Statistical Computing. Created by statisticians Ross Ihaka and Robert Gentleman, among dataminers, bioinformaticians and statisticians for data analysis and developing statistical software. R is one of the most commonly used programming languages used in data mining.

The data was analysed as per the objective of the study using R code of variance - test and t – test

4.DATA

DATA OF LAST FIVE YEARS OF ORGANIC & CHEMICAL FARMING IN PADDY (KHARIF AND RABI)

ORGANIC FARMING IN KHARIF SEASON:

S.NO	YEAR	CROP	TOTAL FARMERS (ORGANIC)	TOTAL ACRES (ORGANIC)	SEEDS	ORGANIC/CHEMICAL FERTILIZERS	CROP DAYS	COST OF CULTIVATION PER ACRE (ORGANIC)
1	2016 - 2017	Paddy	150	160	BPT-5204	Organic	140	16,500
2	2017 - 2018	Paddy	170	200	BPT-5204	Organic	140	16,500
3	2018 - 2019	Paddy	403	431.9	BPT-5204	Organic	140	16,500
4	2019 - 2020	Paddy	487	334.7	BPT-5204	Organic	140	16,500
5	2020 - 2021	Paddy	766	627.6	BPT-5204	Organic	140	16,500

BAGS PER ACRE(ORGANIC)	BAG PRICE	TOTAL AMOUNT PER ACRE(ORGANIC)	AMOUNT OF PROFIT/LOSS	PROFIT/LOSS	TOTAL BAGS	TOTAL AMOUNT
28	1,250	35,000	18,500	Profit	4,480	5,600,000
29	1,250	36,250	19,750	Profit	5,800	7,250,000
30	1,480	44,400	27,900	Profit	12,957	19,176,360
34	1,850	62,900	46,400	Profit	11,380	21,053,000
34	1,850	62,900	46,400	Profit	21,338	39,475,300

CHEMICAL FARMING IN KHARIF SEASON:

S.NO	YEAR	CROP	TOTAL FARMERS(CHEMICAL)	TOTAL ACRES (CHEMICAL)	SEEDS	ORGANIC/ CHEMICAL FERTILIZERS	CROP DAYS	COST OF CULTIVATION PER ACRE (CHEMICAL)
1	2016 - 2017	Paddy	300	1550	MTU - 1153	Chemical	120	26,850
2	2017 - 2018	Paddy	270	1305	MTU - 1153	Chemical	120	26,850
3	2018 - 2019	Paddy	270	1175	MTU - 1156	Chemical	150	26,850
4	2019 - 2020	Paddy	200	1050	MTU - 1121	Chemical	150	26,850
5	2020 - 2021	Paddy	132	1000	MTU-1061	Chemical	125	26,850

BAGS PER ACRE (CHEMICAL)	BAG PRICE	TOTAL AMOUNT PER ACRE(CHEMICAL)	AMOUNT OF PROFIT/LOSS	PROFIT/LOSS	TOTAL BAGS	TOTAL AMOUNT
32	1,250	40,000	13,150	Profit	49,600	62,000,000
33	1,250	41,250	14,400	Profit	43,065	5,38,31,250
35	1,480	51,800	24,950	Profit	41,125	6,08,65,000
35	1,850	64,750	37,900	Profit	36,750	6,65,17,500
35	1,850	64,750	37,900	Profit	35,000	6,47,50,000

ORGANIC FARMING IN RABI SEASON:

S.NO	YEAR	CROP	TOTAL FARMERS (ORGANIC)	TOTAL ACRES (ORGANIC)	SEEDS	ORGANIC/CHEMICAL FERTILIZERS	CROP DAYS	COST OF CULTIVATION PER ACRE (ORGANIC)
1	2016 - 2017	Paddy	150	160	MTU - 1010	Organic	110	16,500
2	2017 - 2018	Paddy	170	200	MTU - 1010	Organic	110	16,500
3	2018 - 2019	Paddy	503	431.9	MTU - 1010	Organic	110	16,500
4	2019 - 2020	Paddy	487	334.7	MTU - 1010	Organic	110	16,500
5	2020 - 2021	Paddy	766	627.6	MTU - 1010	Organic	110	16,500

BAGS PER ACRE(ORGANIC)	BAG PRICE	TOTAL AMOUNT PER ACRE(ORGANIC)	AMOUNT OF PROFIT/LOSS	PROFIT/LOSS	TOTAL BAGS	TOTAL AMOUNT
28	1,250	35,000	18,500	Profit	4,480	5,600,000
33	1,250	41,250	24,750	Profit	6,600	82,50,000
30	1,480	44,400	27,900	Profit	12,957	1,91,76,360
42	1,850	77,700	61,200	Profit	14,057	2,60,05,450
42	1,850	77,700	61,200	Profit	26,359	4,87,64,150

CHEMICAL FARMING IN RABI SEASON:

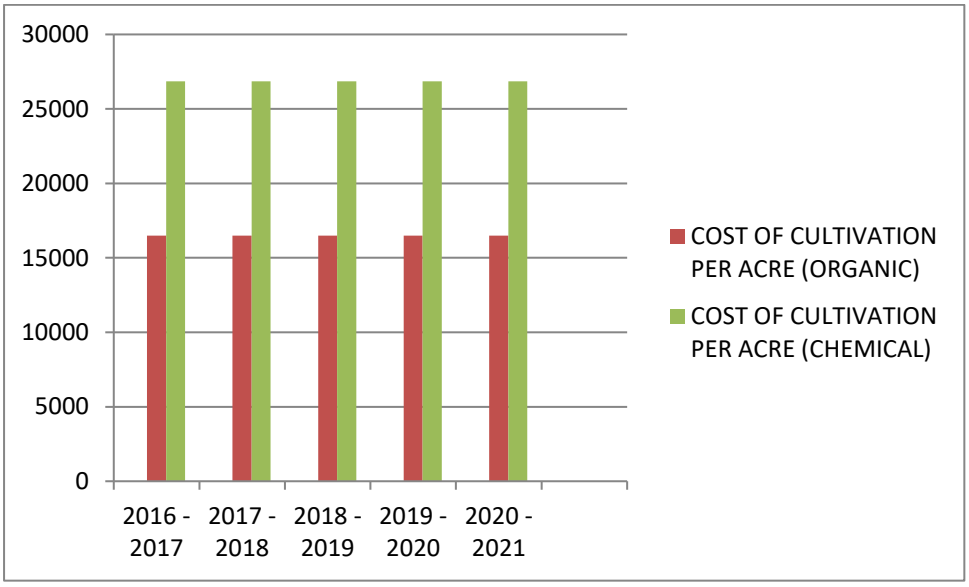
S.NO	YEAR	CROP	TOTAL FARMERS(CHEMICAL)	TOTAL ACRES (CHEMICAL)	SEEDS	ORGANIC/ CHEMICAL FERTILIZERS	CROP DAYS	COST OF CULTIVATION PER ACRE (CHEMICAL)
1	2016 - 2017	Paddy	300	1550	MTU - 1010	Chemical	110	26,850
2	2017 - 2018	Paddy	270	1305	MTU - 1010	Chemical	110	26,850
3	2018 - 2019	Paddy	270	1175	MTU - 1010	Chemical	110	26,850
4	2019 - 2020	Paddy	200	1050	MTU - 1010	Chemical	110	26,850
5	2020 - 2021	Paddy	132	1000	MTU-1010	Chemical	110	26,850

BAGS PER ACRE (CHEMICAL)	BAG PRICE	TOTAL AMOUNT PER ACRE(CHEMICAL)	AMOUNT OF PROFIT/LOSS	PROFIT/LOSS	TOTAL BAGS	TOTAL AMOUNT
32	1,250	40,000	23,500	Profit	49,600	6,20,000000
33	1,250	41,250	24,750	Profit	43,065	5,38,31,250
35	1,480	51,800	35,300	Profit	41,125	14,39,375
42	1,850	77,700	61,200	Profit	44,100	18,52,200
42	1,850	77,700	61,200	Profit	42,000	17,64,000

5. DATA ANALYSIS

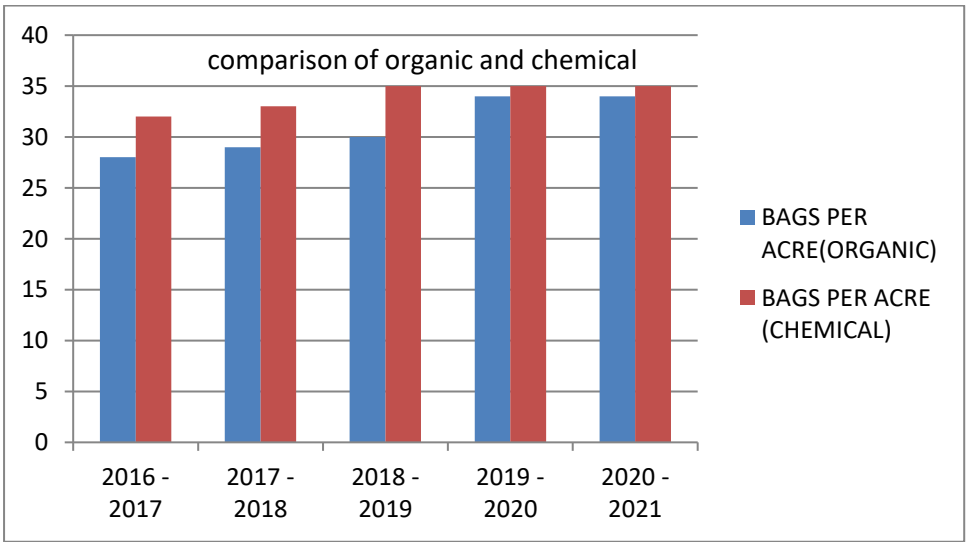
KHARIF SEASON

5.1 Comparison of Organic & Chemical Farming in Cost of Cultivation (Kharif Season):



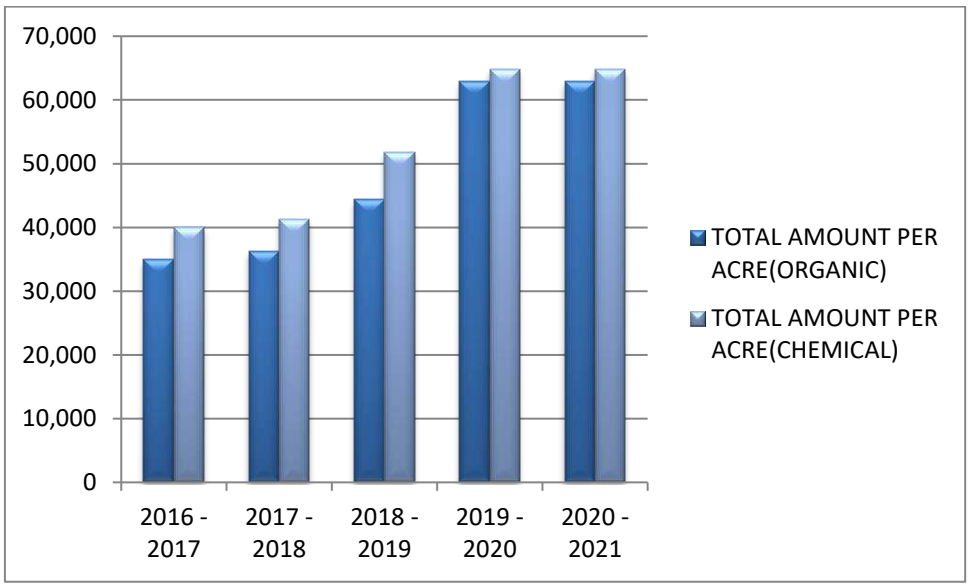
Cultivation cost of chemical farming is more than Organic farming

5.2 Comparison Of Organic & Chemical Farming in Bags Per Acre (Kharif Season)



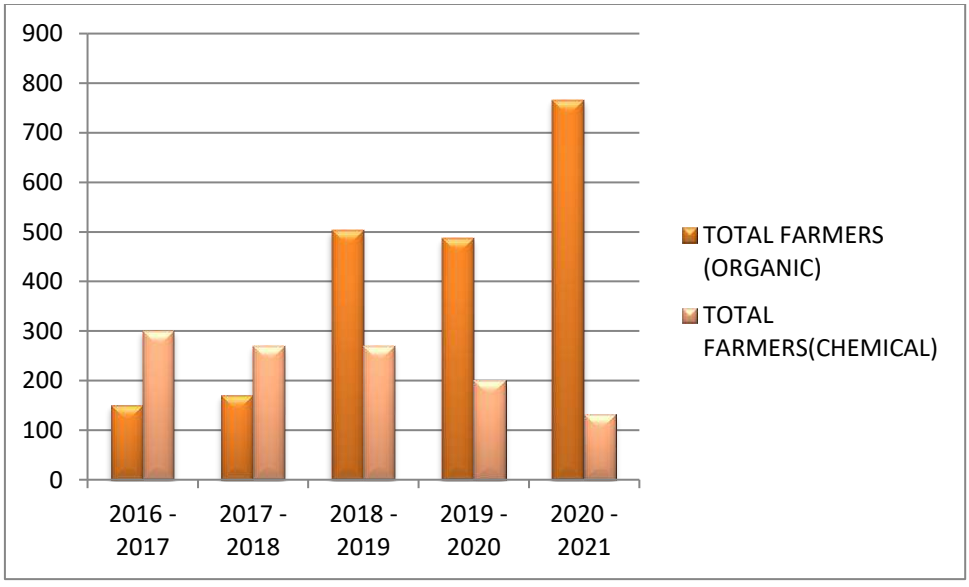
Bags per acre of chemical farming is more than organic farming

5.3 Comparison of Organic & Chemical Farming in Total Amount per Acre (Kharif Season)



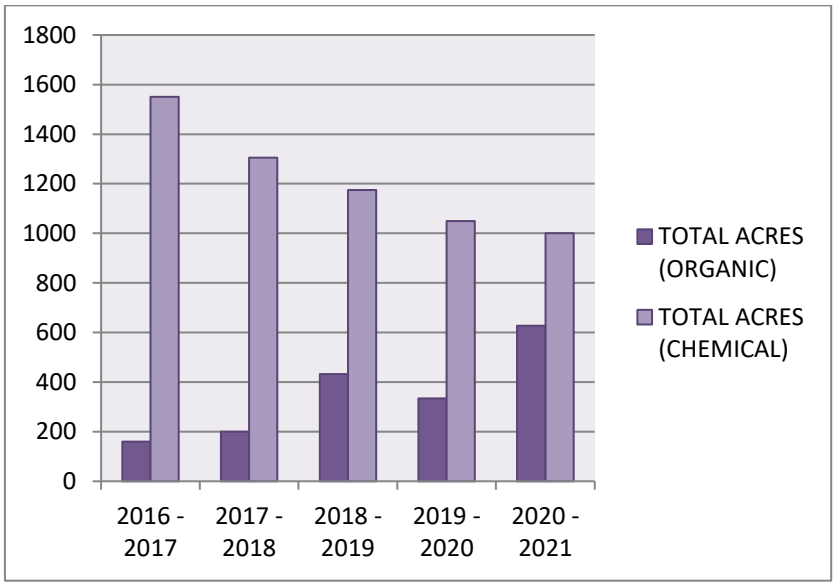
Chemical farming output is more than organic farming per acre

5.4 Comparison of Organic & Chemical Farming in Total Number of Farmers (Kharif Season) :



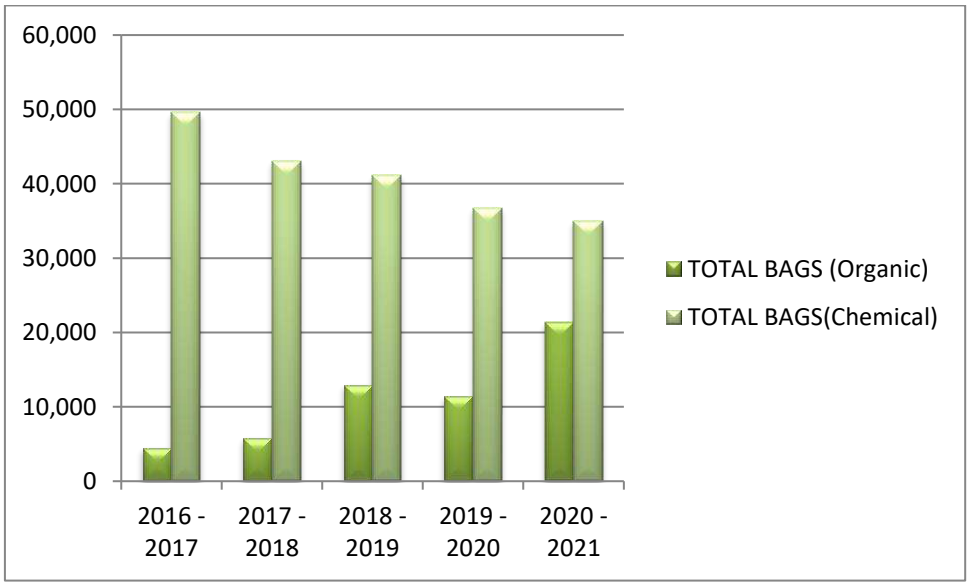
Total farmers of organic farming is more than chemical farming

5.5 Comparison of Organic & Chemical Farming in Total Number of Acres (Kharif Season) :



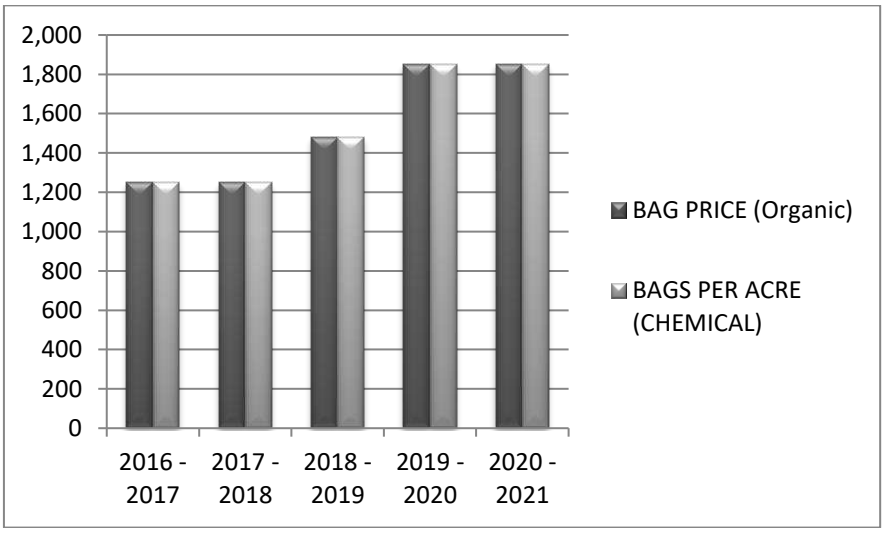
The total number of acres in chemical farming is more than organic farming

5.6 Comparison of Organic & Chemical Farming in Total Bags (Kharif Season) :



Total bags of chemical farming is more than organic farming

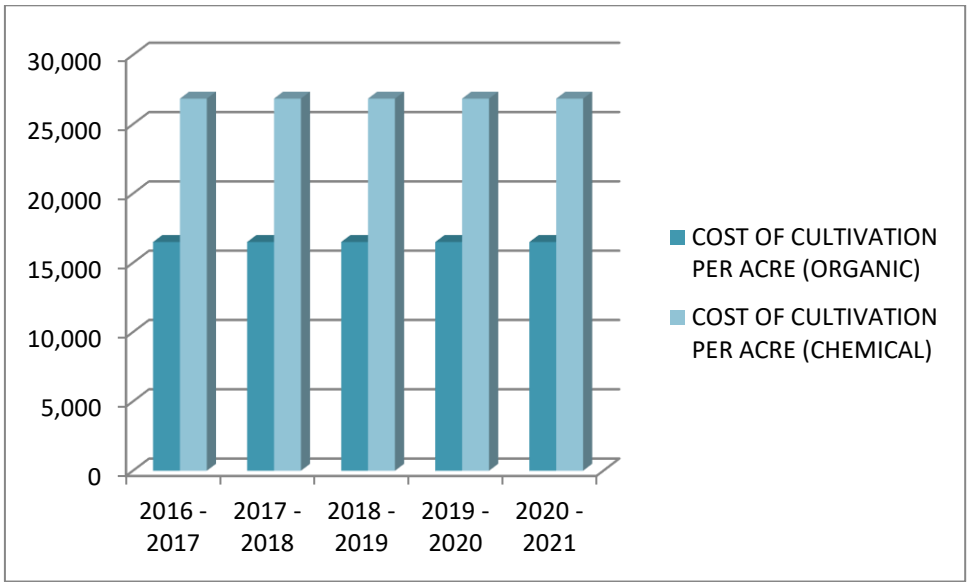
5.7 Comparison of Organic & Chemical Farming in Bag Price (Kharif Season) :



Bag price is equal in both organic and chemical farming

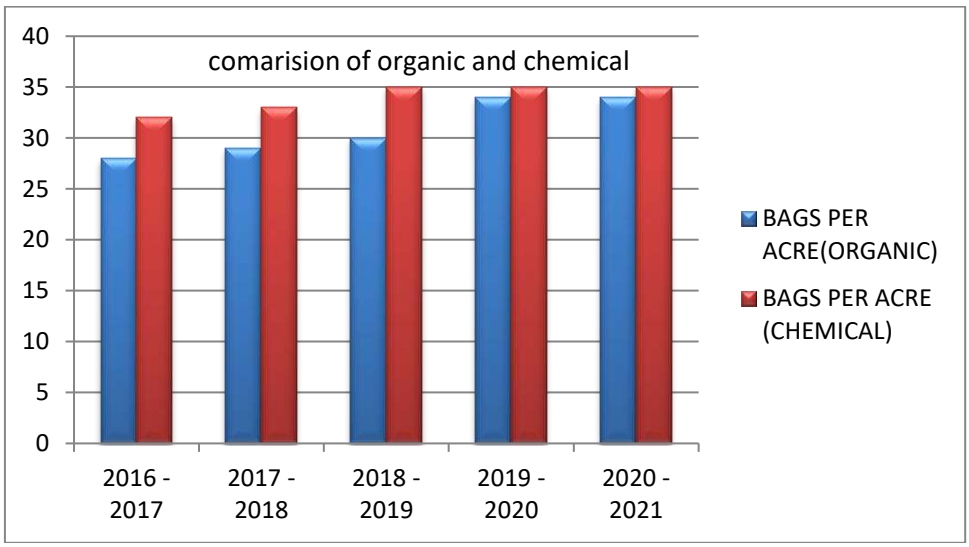
RABI SEASON

5.8 Comparison of Organic & Chemical Farming in Cost of Cultivation (Rabi Season):



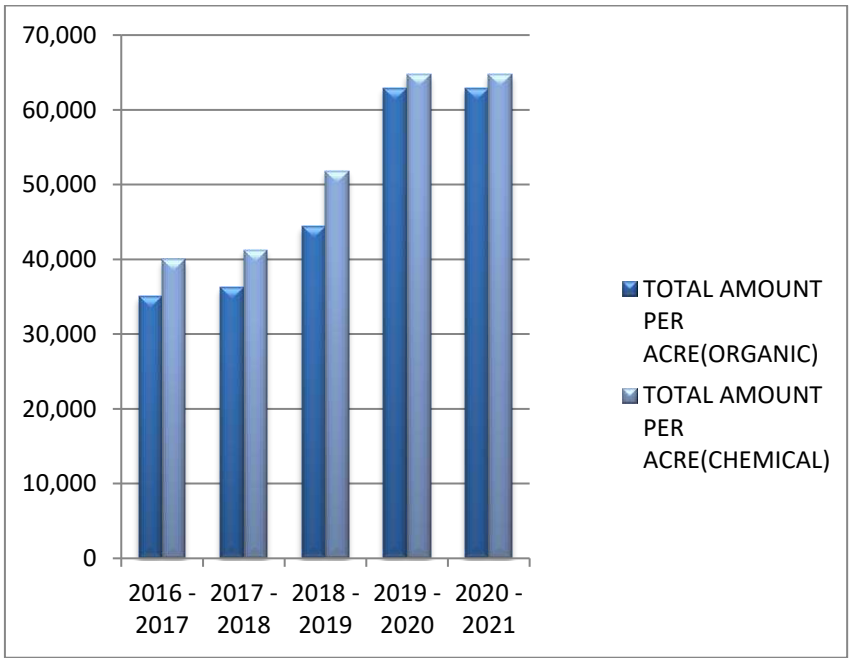
Cultivation cost of chemical farming is more than Organic farming

5.9 Comparison of Organic & Chemical Farming in Bags per Acre (Rabi Season)



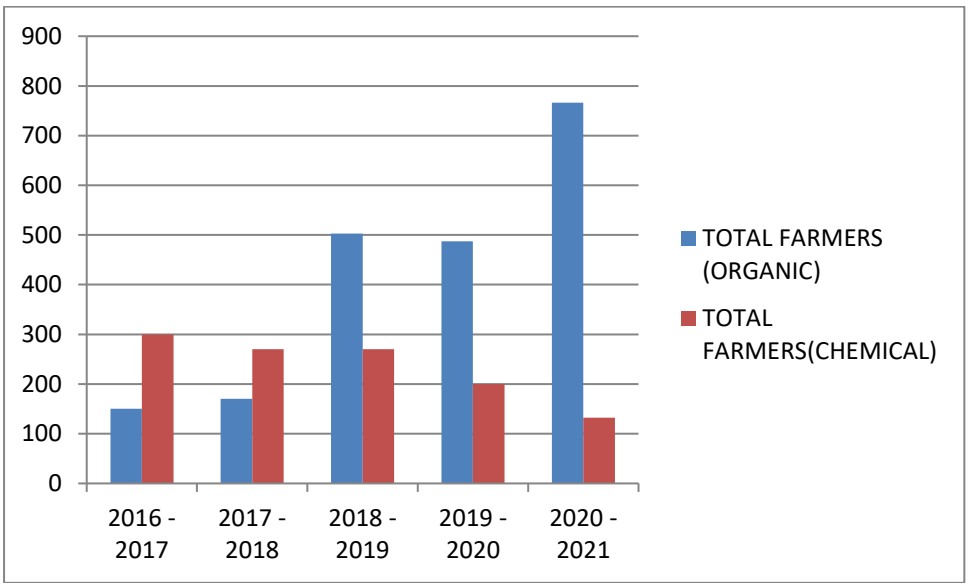
Bags per acre of chemical farming is more than organic farming

5.10 Comparison of Organic & Chemical Farming in Total Amount Per Acre (Rabi Season):



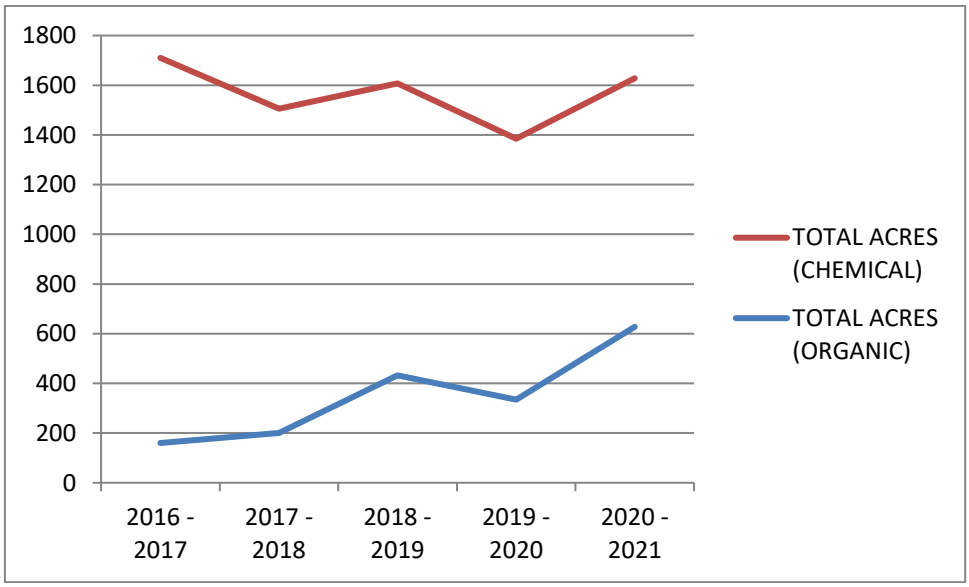
Chemical farming output is more than organic farming per acre

5.11Comparison of Organic & Chemical Farming in Total Number of Farmers (Rabi Season) :



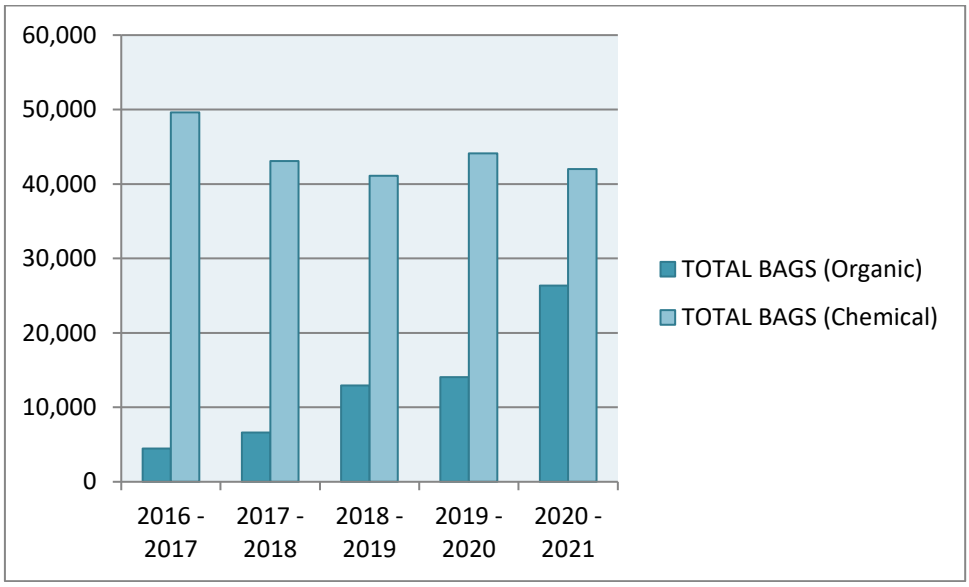
Total farmers of organic farming is more than chemical farming

5.12 Comparison of Organic & Chemical Farming in Total Number of Acres (Rabi Season) :



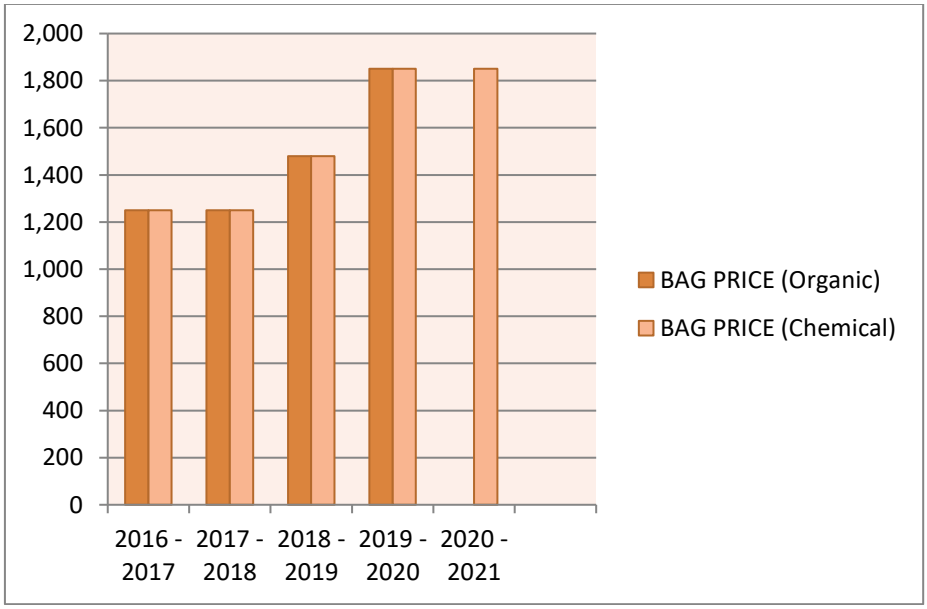
The total number of acres in chemical farming is more than organic farming

5.13 Comparison of Organic & Chemical Farming in Total Bags (Rabi Season) :

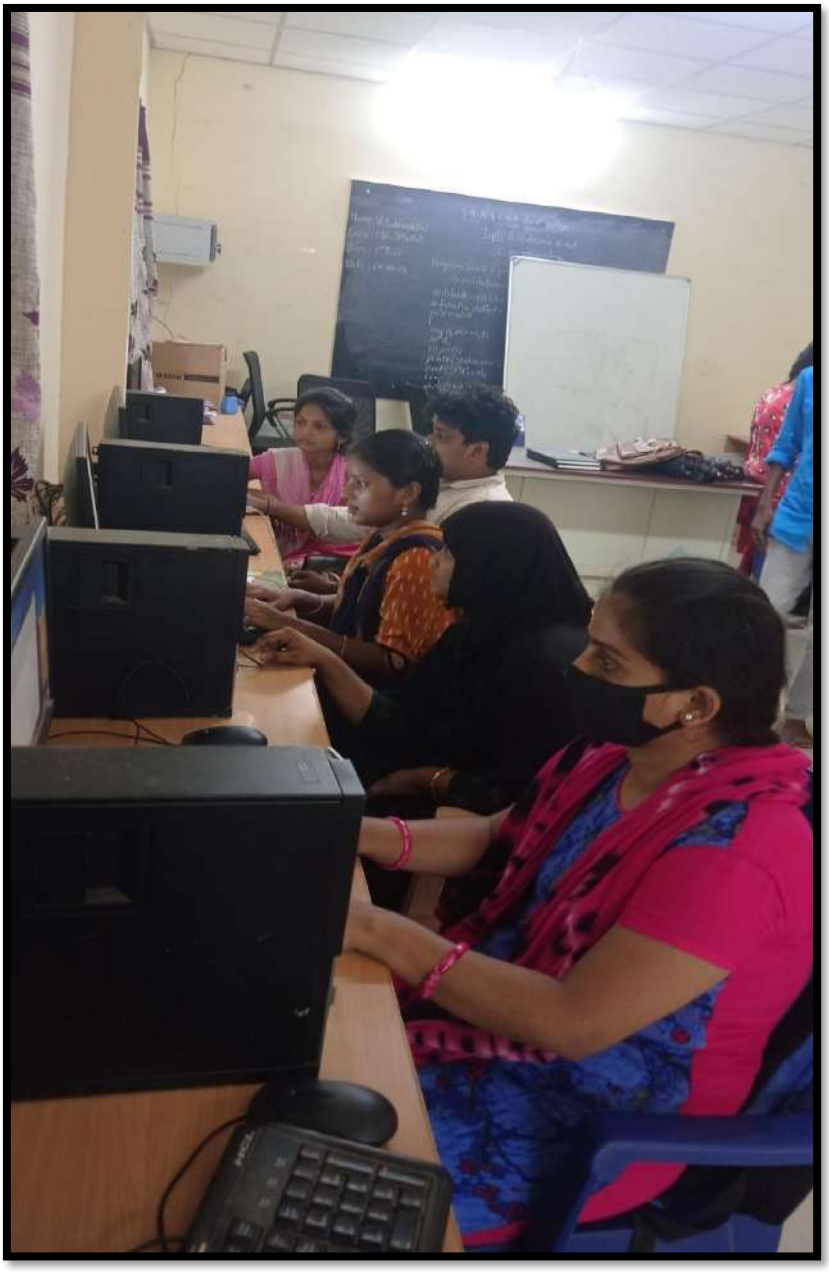


Total bags of chemical farming is more than organic farming

5.14 Comparison of Organic & Chemical Farming in Bag Price (Rabi Season) :



Bag price is equal in both organic and chemical farming



Kharif Season:

R-Code:

```
Organic_Forming<-c(28,29,30,34,34)
Chemical_Forming<-c(32,33,35,35,35)
Organic_Forming
Chemical_Forming
var.test(Organic_Forming,Chemical_Forming)
t.test(Organic_Forming,Chemical_Forming,var.equal=T)
```

Output:

```
>Organic_Forming<-c(28,29,30,34,34)
> Chemical_Forming<-c(32,33,35,35,35)
> Organic_Forming
[1] 28 29 30 34 34
> Chemical_Forming
[1] 32 33 35 35 35
> var.test(Organic_Forming,Chemical_Forming)

        F test to compare two variances

data:  Organic_Forming and Chemical_Forming
F = 4, num df = 4, denom df = 4, p-value = 0.208
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
  0.4164701 38.4181195
sample estimates:
ratio of variances
                 4
```

Since p-vale >0.05, there is no significance difference between the variances in bags per acre of organic forming and chemical forming in kharif season

Now we can test the significance difference b/w the average no.of bags per acre of organic and chemical forming with code,

```
> t.test(Organic_Forming,Chemical_Forming,var.equal=T)

        Two Sample t-test
```

data: Organic_Forming and Chemical_Forming
t = -2.1213, df = 8, p-value = 0.06669
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -6.2611823 0.2611823
sample estimates:
mean of x mean of y
      31   34

Inference: The difference b/w avg no. of bags per acre of Organic and Chemical form is not significant kharif season

Rabi Season:

R-Code:

```
Organic_Forming<-c(28,33,30,42,42)
Chemical_Forming<-c(32,33,35,35,35)
Organic_Forming
Chemical_Forming
var.test(Organic_Forming,Chemical_Forming)
t.test(Organic_Forming,Chemical_Forming,var.equal=T)
```

Result:

Organic_Forming<-c(28,33,30,42,42)
> Chemical_Forming<-c(32,33,35,35,35)
> Organic_Forming
[1] 28 33 30 42 42
> Chemical_Forming
[1] 32 33 35 35 35
> var.test(Organic_Forming,Chemical_Forming)

        F test to compare two variances

data: Organic_Forming and Chemical_Forming
F = 22, num df = 4, denom df = 4, p-value = 0.01101
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
   2.290586 211.299657
Sample estimates:
ratio of variances

22

Since p-vale <0.05, there is a significance difference between the variances in bags per acre of organic forming and chemical forming in rabi season

Now we can test the significance difference b/w the average no.of bags per acre of organic and chemical forming with code,

```
> t.test(Organic_Forming,Chemical_Forming,var.equal=F)

        Welch Two Sample t-test

data:  Organic_Forming and Chemical_Forming
t = 0.32969, df = 4.3629, p-value = 0.7569
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -7.152194  9.152194
sample estimates:
mean of x mean of y
       35        34
```

Inference: The difference b/w avg no. of bags per acre of Organic and Chemical form is not significant in rabi season also.

6. Conclusions

- Estimate of Cost of Cultivation In Paddy based on the data during 2016 - 2021
  Organic Forming (Kharif/Rabi) – 16,500 per acre
  Chemical forming (Kharif/Rabi) – 28,500 per acre
  Cultivation cost of Chemical farming is more than that of Organic farming.
- Bag Price in organic farming and chemical farming is equal

- Comparison of yield per acre b/w Organic and Chemical forming in kharif and Rabi season

Kharif season

The difference b/w avg no. of bags per acre of Organic and Chemical form is not significant in kharif season

Rabi Season

The difference b/w avg no. of bags per acre of Organic and Chemical form is not significant in rabi season also.

7. References

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